Computational study for OWA Traveling Salesman Problem

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1 Introduction

Given a set of cities and distances to travel between each pair of them, the Traveling Salesman Problem (TSP) is to find a shortest tour which visits every city exactly once and returns to the starting city. In some practical cases, the balance between edges in the tour is as important as the total distance. It motivates us to study an equitable version of TSP where the optimal tour satisfies : i) Pareto optimality (i.e. not improvable on all distances simultaneously), ii) fairness (or the balance between edges). In this work, we study a variant of TSP where the ordered weighted averaging (OWA) is used to control both Pareto efficiency and fairness, we call this problem OWATSP. OWA imposes implicitly the balance between edges by Pigou-Dalton transfer principle claiming that a transfer from a richer resource to a poorer one results in a fairer distribution.

OWATSP belongs to the class of fair combinatorial optimization considered in [2]. The main challenge of this problem class is the non-linearity of OWA objective. Fortunately, it can be cast to Mixed-Integer Programs (MIPs) by several existing linearization methods [3, 1]. However, the time spent to solve exact formulations increases considerably with the size of instances and might reach to hours for small-size ones. To tackle the issue, an iterative algorithm based on Lagrangian relaxation [2] is proposed and verified the efficiency on several fair optimization problems related to matching. In this paper, we focus on algorithms to solve efficiently large-size instances of OWATSP - an NP-hard problem.

2 Solving methods for OWATSP

2.1 Problem formulation

Consider a complete graph $G = (V, E)$ where $V = [n] := \{1, 2, \ldots, n\}$ and a cost vector $c \in \mathbb{R}_+^{|E|}$. OWA of $v \in \mathbb{R}^n$ is given by $OWA_{\mathbf{w}}(v) = w_1\theta_1(v) + \cdots + w_n\theta_n(v)$ where $w_i \in [0, 1]$, $w_1 > \cdots > w_n > 0$ and $\theta_i(v)$ is the ith largest component of vector $v = (v_1, v_2, \ldots, v_n)$. OWATSP is define as follows :

$$\min_{\mathcal{H} \in \Pi(G)} OWA_w((c)_{\mathcal{H}})$$

where $\Pi(G)$ is the set of all Hamiltonian cycles in G, $(c)_{\mathcal{H}}$ is a cost vector of selected edges in $\mathcal{H}$. OWATSP is a non-linear optimization problem as OWA operator. Moreover, OWATSP is NP-hard since it contains the problem of finding a Hamiltonian cycle in G.

2.2 MIP formulations for OWATSP

To get MIPs for OWATSP, we first consider a directed version of G where every edge ij is replaced by two arcs (i, j) and (j, i). The cost vector of selected edges in a tour can be rewritten as $v = (\sum_{j \in [n]} c_{i,j}x_{i,j})_{i \in [n]}$ where $\mathbf{x} \in \{0, 1\}^{n \times n}$ represents a tour in the directed graph.

Table 1.1	MIP [3]	MIP [1]	AlgoOpt [2]	
Instance	**Time**	**Time**	**Time**	**Gap**
burma14	0.83	0.45	1.62	0.06%
fri26	21.01	49.97	5.74	0.00%
hk48	1943.39	719.89	32.21	0.03%

Table 1.2	AlgoOpt [2]
Instance	**Time**
berlin52	36.02
eil76	217.552
rat99	833.335

TAB. 1 – Experiment results of all methods for small instances (Table 1.1) and the heuristic algorithm for larger ones (Table 1.2).

Two linearization methods for OWA are proposed by Ogryczak et al. [3] and Chassein et al. [1]. The key idea of Ogryczak's approach is to use Lorenz components to reformulate OWA, i.e. $\min OWA_{\mathbf{w}}(v) = \min \sum_{k\in[n]} w'_k L_k(v)$ where $L_k(v) = \sum_{i\in[k]} \theta_i(v)$ and $w'_k = w_k - w_{k+1},\ \forall k \in [n-1],\ w'_n = w_n$. Then, each Lorenz component is computed by a linear programming (LP) with $n+1$ extra variables and n new constraints. Hence, OWATSP can be represented as a MIP using n^2+n additional variables and n^2 new constraints. On the other hand, Chassein starts from the observation that $\min OWA_w(v) = \min \max_{\tau\in P} \sum_{i\in[n]} w_{\tau(i)} v_i$ where P is the set of all permutations of $\{1,\dots,n\}$. The inner optimization problem is rewritten as a LP by using the permutahedron and dualized to get a minimization problem which utilizes $2n$ additional variables and n^2 new constraints. We get an alternative formulation for OWATSP, which uses the same number of new constraints but only $2n$ extra variables instead of n^2+n one as in [3].

2.3 Heuristic algorithm for OWATSP

Although MIPs solve efficiently OWATSP in small-size instances, the CPU time increases very quickly with instances' size. To tackle this issue, a fast heuristic algorithm based on Lagrangian relaxation is employed as done in [2]. In particular, we consider Lagrangian relaxations $\mathcal{L}(\boldsymbol{\lambda})$ of MIPs where $\boldsymbol{\lambda}$ is Lagrangian multipliers. The algorithm starts with a feasible $\boldsymbol{\lambda}$, computes the associated $\mathbf{x}$ by solving $\mathcal{L}(\boldsymbol{\lambda})$ and uses the latter to iteratively improve $\boldsymbol{\lambda}$. The algorithm terminates when the max iteration has been reached or change on $\boldsymbol{\lambda}$ is small.

3 Experiment results

We utilize instances of TSPLIB to evaluate these methods. Table 1.1 shows that the CPU time spent for solving exact formulations increases exponentially with the size of instance. In contrast, the heuristic algorithm gives high-quality quasi-solutions quickly and can obtain optimal solutions for several instances. Furthermore, the running time rises acceptably when applying the heuristic algorithm for larger instances as shown in Table 1.2.

Références

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