

Quantum algorithms for hypergraph bi-partitioning

Julien Rodriguez

Université Paris-Saclay, CEA, List, F-91120, Palaiseau
 julien.rodriquez@cea.fr

Université de Bordeaux, INRIA
 julien.rodriquez@inria.fr

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1 Introduction

The arrival of quantum machines may revolutionize many fields, in particular that of combinatorial algorithms. Indeed, these machines propose several optimization algorithms, such as QAOA for "Quantum Approximate Optimization Algorithm", VQE for "Variational Quantum Eigensolver" and the oracle-based search algorithm published by Lov Grover [2]. These algorithms each bring a theoretical gain on the computation time for some combinatorial problems.

However, to date there are several different technologies allowing the design of a quantum chip, none of which is dominant at the moment. In this work we are interested in machines called "gate machines" and "quantum annealing" machines. The executions for the gate machines are realized on IBM simulators and machines using the Qiskit library and the Ocean library of D-Wave for the quantum annealing machines.

The objective is to propose different models with their associated quantum circuits for the hypergraph partitioning problem and to measure the complexity in space and in number of gates of the different optimization algorithms chosen.

2 Hypergraph partitioning

A hypergraph is an extension of the notion of graph in which hyperedges can contain more than two vertices. Hypergraph partitioning consists in separating the vertices in different subsets with a similar size to $\pm \epsilon$, while minimizing the number of hyperedges between them. There are several ways to measure the value of a partition, such as the number of cut hyper-edges or the "minus one" connectivity : $\lambda_e - 1 = \sum_k \pi_k(e) - 1$, with $\pi_k(e) = 1$ if and only if at least one of the vertices connected to e is in the k part. Note that in the case of bi-partitioning, the objective $\lambda - 1$ counts exactly the number of cut hyperedges.

2.1 Problem model

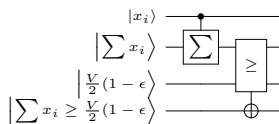
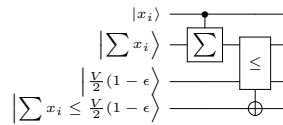
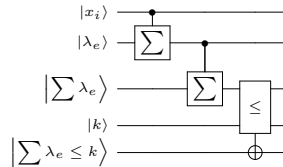
Let $H = (V, E)$ a hypergraph, then :

$$\min \sum_{e \in E} \lambda_e - 1$$

$$st. \sum_{v \in V} x_v \leq \frac{|V|}{2}(1 + \epsilon)$$

$$\sum_{v \in V} x_v \geq \frac{|V|}{2}(1 - \epsilon)$$

$$x_v \in \{0, 1\}, \forall v \in V$$



Grover’s algorithm allows a search for elements in an unstructured set. In our example, the set is the search space defined by all possible assignments of the variables x_v and the elements searched are assignments that validate the constraints of the problem.

From this modeling it is possible to build an oracle for Grover’s search algorithm. Indeed, the operations necessary to encode this circuit are : $+/-$, \leq , \geq and \neq . The construction of the final circuit is partly based on the sub-circuit codes presented in [4].

2.2 QUBO modelling

Variational algorithms are quantum approximation algorithms minimizing an Ising model that can be generated from an unconstrained quadratic function. Quantum annealing computers can also solve these models.

In [3] the authors have defined a QUBO model for the problem of partitioning in graphs. The model presented below is a possible extension for hypergraphs which is not always quadratic :

$$\min A \times \sum_e \left(1 - \prod_{v \in e} x_v - \prod_{v \in e} (1 - x_v) \right) + B \times \left(\sum_v x_v - \frac{|V|}{2}(1 + \epsilon) \right)^2$$

A hypergraph $H = (V, E)$ can also be modeled as a graph $G = (V, E')$ in which $E' = \{(x_u, x_v) | (x_u, x_v) \in e, \forall e \in E\}$ and to use the formulation of [3] for graphs, at the risk of counting a number of cut edges higher than the number of hyperedges actually cut for the same x_v assignment.

The construction of the circuits for QAOA and VQE follows in part the methodology presented in [1].

3 Conclusions and perspectives

The number of qubits evolves linearly with the size of the instances but the circuit for Grover’s algorithm requires more qubits than the circuit for QAOA. Also for the number of gates used, QAOA requires fewer gates than the Grover approach. From the point of view of complexity in space and number of gates, QAOA appears to be more interesting. However, the absence of strong constraints in QAOA disturbs the convergence towards an optimal solution contrary to Grover which accepts a constrained model.

The presented partitioning models can be extended for k -partitioning at a space cost of $O(|V| \log_2(k))$ additional qubits with the Grover approach and $O((k-1)|V|)$ qubits with QAOA.

Références

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