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Ayeley Tchangani, Daniel Noyes. Modeling dynamic reliability using dynamic Bayesian networks. Journal Européen des Systèmes Automatisés (JESA), 2006, 40 (8), pp.911-935. 10.3166/jesa.40.915-935 . hal-03594445

**HAL Id: hal-03594445**

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Submitted on 2 Mar 2022

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<http://dx.doi.org/10.3166/jesa.40.915-935>

**To cite this version:**

Tchangani, Ayeley and Noyes, Daniel *Modeling dynamic reliability using dynamic Bayesian networks*. (2006) *Journal Européen des Systèmes Automatisés*, vol. 40 (n°8). pp. 911-935. ISSN 1269-6935

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# Modeling dynamic reliability using dynamic Bayesian networks

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*Abstract: This paper considers the problem of modeling and analyzing the reliability of a system or a component (system) where the state of the system and the state of process variables influences each other in addition to an exogenous perturbation influence: this is the dynamic reliability. We consider discrete time case, that is the state of the system as well as the state of process variables are observed or measured at discrete time instants. A mathematical tool that shows interesting properties for modeling and analyzing this problem is the so called Dynamic Bayesian Networks (DBN) that permit graphical representation of stochastic processes. Furthermore their learning and inference capabilities can be exploited to take into account experimental data or expert's knowledge. We will show that a complex interaction between system and process on one hand and between system, process and exogenous perturbation on the other hand can simply be represented graphically by a dynamic Bayesian network. With their extended tool, known as influence diagrams (ID) that integrate actions or decisions possibilities, one can analyze and optimize a maintenance policy and/or make reactive decision during an accident by simulating different scenarios of its evolution for instance.*

*Keywords: Dynamic Reliability, Dynamic Bayesian Networks, Influence Diagrams, Maintenance.*

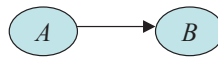
*Résumé: Nous considérons dans cet article le problème de modélisation et d'analyse de la fiabilité d'un système ou d'un composant dont l'état et celui du processus qui s'y déroule s'influencent mutuellement en plus d'une éventuelle perturbation exogène : c'est la fiabilité dynamique. Nous considérons le cas où le temps est discret : l'état du système, celui des variables du processus ainsi que la perturbation sont observés ou mesurés à des instants précis. Pour modéliser et analyser ce problème, les Réseaux Bayésiens Dynamiques (RBD) constituent un outil mathématique aux propriétés intéressantes permettant une représentation graphique des processus stochastiques. Le pouvoir d'apprentissage et d'inférence des RBD peut être exploité pour prendre en compte les données de retour d'expérience ou la connaissance des experts. Nous allons montrer qu'une interaction complexe entre l'état du système et le processus, d'une part, le processus et la perturbation externe, d'autre part, peut être représentée simplement par un réseau bayésien dynamique. L'extension des RBD, connue sous le nom de Diagrammes d'Influence qui intègrent la possibilité de prise de décision, va permettre l'analyse et l'optimisation des politiques de maintenance et/ou de prise de décision réactive en cas d'accident en simulant des scénarios possibles de l'évolution de cet accident par exemple.*

*Mots clefs: Fiabilité Dynamique, Réseaux Bayésiens Dynamiques, Diagrammes d'Influence, Maintenance.*

## 1. Introduction

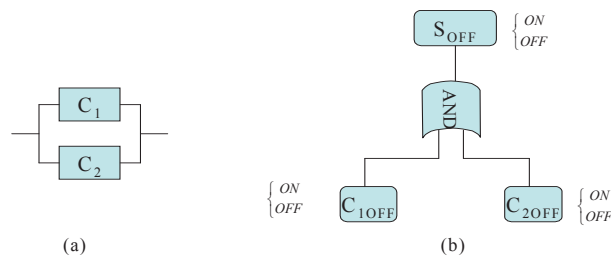
The necessity of maintaining systems performance at high level and avoiding catastrophic accidents for systems such as nuclear power plants, airplanes, chemical plants, etc. raises new and challenging aspects of research in dependability (reliability, availability, maintainability, safety, etc.). One of such challenging problem is modeling and analyzing dynamic reliability: reliability that takes into account the environment of the system in terms of mutual influence between the state (for instance the system may be functioning, state OK or failed, state OFF) of the system and the state of process variables (pressure, temperature in a tank for instance) and/or exogenous perturbation. The dynamic reliability concept is recognized as a more realistic modeling of the systems for the purposes of reliability, risk and safety analysis (Labeau *et al.*, 2000). Classically, the reliability of a system is defined for the duration of its mission in given conditions. As results, qualitative tools such as fault tree analysis (FTA, a deductive top-down method for analyzing system design and performance) and possibly quantitative tools mainly probabilities calculus and stochastic processes are sufficient for analyzing and assessing important (steady state) dependability measures of the system. Fault tree analysis involves specifying a top event to analyze (such as the failure of the system), followed by identifying all of the associated events that could lead to the top event. But this representation is mainly qualitative (or logical) because the state of the system is generally supposed to be binary stating the fact that the system is operating or not and the fault tree represents just a logical function (Pagès *et al.*, 1979). This approach has some drawbacks such as not taking into account approximate performance of the system whereas in practice it can happen that a component performs approximately and the overall performance of the system be acceptable. To overcome this, one can use many states than two to represent the functioning modes of a component; the approximate functioning will then be stated in terms of probability. A good candidate mathematical tool for this purpose (Tchangani, 2001; Bobbio *et al.*, 2001) is Bayesian Networks (BN) that are graphical representation of probabilistic relationships between variables of a knowledge domain. The terminology “Bayesian Networks” comes from the work by Thomas Bayes (Bayes, 1763, 1958) in eighteenth century. Its actually development is due to (Pearl, 1988); see (Jensen, 1999; Pearl, 1988; Becker *et al.*, 1999; Naïm *et al.*, 2004) for a good introduction to Bayesian networks. A Bayesian network consists of two components: its *structure* that is a directed acyclic graph defining some relevant relationships between nodes that represent variables of a knowledge domain (for instance components or subsystems) and its *parameters* that give conditional probability density function (or table) of each node given the evidence on its parents (nodes that have a direct link to the former one), see for instance (Jensen, 1999) for more. A typical Bayesian network is given by Figure 1 where  $A$  is the parent node that is relevant, in some sense (causality, correlation, etc.), for the knowledge of the node  $B$ ; to be complete and for a quantitative evaluation purpose, this relevancy

(structure) must be completed by a conditional probability table or density  $\Pr\{B / A\}$  that is the probability of  $B$  knowing the state of  $A$ .

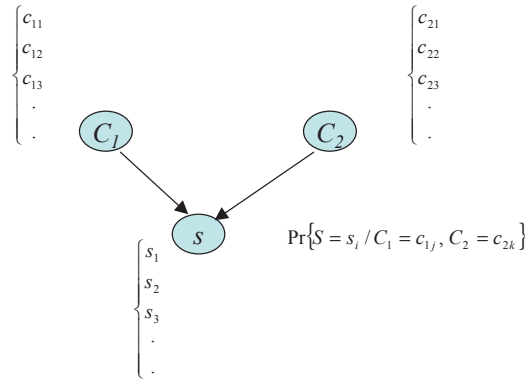


**Figure 1.** *A typical example of a Bayesian network*

Modeling a system in terms of reliability integrating an approximate functioning states can benefit of a combination of reliability diagrams or fault tree analysis approach and Bayesian networks theory, see for instance (Tchangani, 2001; Bobbio *et al.*, 2001). The fault tree analysis can be used as a top level tool to represent interactions in terms of reliability between components or functions of a system; then in a second stage, Bayesian network model can be derived by transforming the AND/OR gates of the fault tree models in probability tables and considering that components can have more than two states of functioning. To illustrate this idea, let us consider a two components redundancy system and its fault tree model depicted on Figure 2. The gate AND means that the system (S) is in the state OFF if and only if the two components ( $C_1$  and  $C_2$ ) are in their OFF states respectively; the system will be in the state OK for any other combination of components' states.



**Figure 2.** *A two components redundancy system (a) and its fault tree model (b)*



**Figure 3.** A Bayesian network model of the two components redundancy system of the Figure 2 (a)

A Bayesian network model of such system where the components as well as the system may have as many states as possible is given by the Figure 3 where the equivalent of the AND gate is determined by the conditional probability table given by the equation [1].

$$\Pr\{S = s_i / C_1 = c_{1j}, C_2 = c_{2k}\} \quad [1]$$

Besides the fact that a Bayesian network model integrates many functioning states of components, it has another advantage over the fault tree model because it has learning capabilities (see later) that can be used to derive conditional probability density functions or tables using expert knowledge and/or experimental data. Using Bayesian network approach for RAMS (Reliability – Availability – Maintainability – Safety) modeling and analysis as well as for maintenance management policy set up has gained a great interests in recent years in the literature, see for instance (Proccacia *et al.*, 2003; Tchangani, 2001; Bobbio *et al.*, 2001) and references therein.

But, fault tree analysis model and related methods (see (Labeau *et al.*, 2000) and references therein) as well as Bayesian networks model presented so far do not take into account the *time* effect (non stationary components failure rate for instance) or *exogenous uncontrollable perturbations* effect (the effect of the ambient temperature on the failure rate of an electronic component for instance) or the effect of the *state of the process* that is taking place in the system or components (for instance the pressure, the temperature and the quantity of matter in a chemical reactor or in a boiler will have an effect on the failure rate of its closing elements (valves) for instance) and so these tools are not adapted for dynamic reliability analysis. Different mathematical tools more or less complex among which are diffusion equation and Monte Carlo analysis (see (Labeau *et al.*, 2000; Marseguerra *et al.*, 1998) and references therein for an introduction to some of them) or Petri nets

(Chabot *et al.*, 2003) are used in the literature to analyze dynamic reliability of a system. In this paper, we will use dynamic Bayesian networks (DBN) as a mathematical tool to derive a generic approach for modeling and analyzing dynamic reliability; this choice is guided by the following facts:

- dynamic Bayesian networks as Bayesian networks briefly recalled in upper lines are graphical representation (and therefore an easy language to understand even for non specialists) of stochastic processes that can represent very complex relationships between variables in a given knowledge domain;
- there exists efficient algorithms, see (Murphy, 2002), for learning and inference that make it easy to integrate them into decision support systems; but one must be careful when choosing the appropriate algorithms mainly in the case of continuous dynamic variables, see (Murphy, 2002; Naïm *et al.*, 2004);
- they are widely used and have shown good results in practice in many domains such as knowledge discovery, data mining, fault diagnosis, medical diagnosis, etc;
- there exists efficient software (Hugin Explorer, Netica, BayesiaLab to name few) or Toolbox for use with Matlab (see BNT) and other scientific software that render their usage easy in practice even for non specialists;
- when extended to influence diagrams by introducing decision nodes and possibly value nodes, they can be used for performance evaluation of maintenance policies or to support planning appropriate actions in the case of accident or catastrophic event because a built model can be used to simulate and obtain the most probable outcome with regard to different scenarios;
- etc.

Nowadays, dynamic Bayesian networks as a mathematical tool for modeling and analyzing dynamic reliability is gaining a great interest in the RAMS community, see (Weber *et al.*, 2003; Weber *et al.*, 2004) and (Tchangani *et al.*, 2005), a preliminary version of this paper. As stated previously, a dynamic reliability analysis problem is very large and general and so it is important to give the border of the problem under consideration; the context of dynamic reliability problem considered in this paper is defined by the following variables and assumptions.

- $s(t) \in S = \{1, 2, \dots, s\}$  is the state (discrete) of the system, component (functioning modes) at time instant  $t$ ; it belongs to a finite set  $S$  (the system is *normally* functioning, *approximately* functioning, is *OFF*, is under *reparation*, etc. for instance).
- $x(t) \in \mathfrak{R}^n$  is the state (continuous) of the process variables at time instant  $t$ ;  $\mathfrak{R}^n$  denotes a real vector space of dimension  $n$ ; it could be for instance  $x(t) = [P(t) \ T(t) \ V(t)]^T$  in a chemical reactor or a boiler where  $P(t)$  is the pressure,  $T(t)$  is the temperature and  $V(t)$  is the quantity of the matter and the superscript  $T$  stands for the transpose of the corresponding vector or matrix.
- $y(t) \in \mathfrak{R}^l$  is the observation or measurements of the process variables available at time instant  $t$ ; it is a function of the state variables  $x(t)$  and  $l$  will be less than  $n$  in general; it could be the temperature of a chemical reactor obtained by a thermocouple for instance.

- $a(t) \in A = \{a_1, a_2, \dots, a_m\}$  is the action of the decision maker at time  $t$  that influences the state of the system; there is a finite number of stationary actions defined by the set  $A$  (have a cooling effect on a component, to lubricate, to heat up, etc.) available to the decision maker at each instant  $t$ ; notice that we do not consider the lower controller (PID controller for instance) effect that could have an influence (stabilization for example) on the process variables state.
- $w(t) \in \mathfrak{R}^p$  is an exogenous perturbation that influences the system and/or the process variables behavior; it could be for instance the effect of ambient conditions (temperature, pressure, humidity, ..) on the failure rate of a component.
- $\pi(t) = [\pi_1(t) \dots \pi_s(t)]$  where  $\pi_i(t) = \Pr\{s(t) = i\}$  is the probability that the system is in the state  $i$  at the time instant  $t$ ; these probabilities verify the condition 
$$\sum_{i=1}^s \pi_i(t) = 1.$$

The purpose of this paper is then to establish a model that describes how all these variables dynamically influence each other. The remainder of this paper is organized as follow: in the second section we will present dynamic Bayesian networks (only the concepts that are relevant to our purpose will be presented; for more formal presentation, the reader is invited to consult specialized literature such as (Murphy, 2002)) and their *learning* and *inference* capabilities that make them suitable for modeling stochastic processes; the third section will consider the usage of dynamic Bayesian networks for modeling and analyzing dynamic reliability as defined in the introduction section; finally a conclusion is presented in the fourth section. We will illustrate each modeling stage by using a small example to show how dynamic Bayesian networks may be used.

## 2. Dynamic Bayesian Networks

### 2.1. Presentation

Dynamic Bayesian networks (DBNs) are directed graphical models of stochastic processes, see (Murphy, 2002), and they generalize Hidden Markov Models (HMMs) and Linear Dynamical Systems (LDSs) by representing the hidden and observed state in terms of state variables, which can have complex interdependencies. The graphical structure provides an easy way to specify these conditional interdependencies, and hence to provide a compact parameterization of the model. A dynamic Bayesian networks is completely defined by two components: its *structure* that is a directed acyclic graph (DAG, nodes represented by ovals) representing relationships between variables and its *parameters* that represent conditional probability density (CPD) in the case of a continuous variable (the allowed values of the variable belong to a continuous set) or conditional probability table (CPT) in the case of a discrete variable (the allowed values of the variable belong to a discrete set that will be in general a finite set). A dynamic Bayesian network structure consists of an *intra slice* directed acyclic graph and an *inter slice* directed graph; slices represent time instants to describe dynamic behavior of the



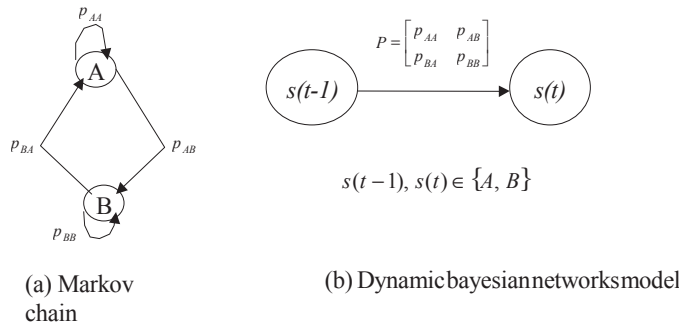
system. Intra slice graph models the instantaneous relationships of nodes (a Bayesian Network) and the inter slice graph represents the dynamics of the nodes. Intra slice parameters are CPD and/or CPT of the corresponding Bayesian network and inter slice parameters represent the dynamics of variables on one hand and their relationships with the variables that influence their behavior on the other hand. For instance a dynamic Bayesian network representing a Markov chain (Hêche *et al.*, 2003) will have a two time slices graph with *inter slice* directed graph and no intra slice graph. For instance, Figure 4 shows an example of a Markov chain (a) with two states  $A$  and  $B$  and its dynamic Bayesian network representation (b) where the generic state  $s$  can be  $A$  or  $B$  and the dynamics are captured by the transition matrix  $P = \begin{bmatrix} P_{AA} & P_{AB} \\ P_{BA} & P_{BB} \end{bmatrix}$ ; this matrix must be a stochastic matrix that is it must verify the conditions of equation [2]

$$P_{AA} + P_{AB} = 1, \quad P_{BA} + P_{BB} = 1. \quad [2]$$

The actual probability of belonging to one or other state  $A$  or  $B$  is given by the row vector  $\pi(t) = [\pi_A(t) \quad \pi_B(t)]$  and the behavior of the system is described by the following equation [3].

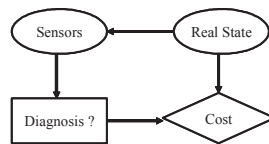
$$\pi(t) = \pi(t-1) \cdot P, \quad \pi(0) = \pi_0 \quad [3]$$

The advantage of the Bayesian network model over the Markov chain representation, besides the fact that the model is more compact is that the transition matrix  $P$  can be learnt (estimated) from the expert knowledge and/or experimental data. But, as stated in the previous lines, dynamic Bayesian networks represent more complex stochastic processes than Markov chains and so algorithms to learn parameters for a dynamic Bayesian networks for a real world problem may be very complex or necessitate an approximation scheme that must be chosen carefully (see for instance Murphy, 2002).



**Figure 4.** An example of a Markov chain (a) and its dynamic Bayesian network representation (b)

Most of the time, a system is not autonomous and there is a decision maker that can influence the behavior of the system, this can be taken into account by adding a decision node in a Bayesian network that leads to an *influence diagram*. Hence, an influence diagram is a simple visual representation of a decision problem. Influence diagrams offer an intuitive way to identify and display the essential elements, including decisions, uncertainties, and objectives, and how they influence each other. An influence diagram or decision graph (Howard *et al.*, 1984; Jensen, 1999) is a directed acyclic graph (DAG) that depicts relationships among variables in a decision problem. A typical influence diagram is shown by Figure 5 that describes the following decision problem: to monitor a machine, some *sensors* are put on it in order to give information about its *actual state*. According to this information one decides whether to stop the machine for *diagnosis* or not. Stopping the machine for diagnosis or letting it operates in bad state has a *cost*.



**Figure 5.** Example of an influence diagram

All the nodes necessary to define an influence diagram are shown on the former Figure 5, namely: *chance nodes* (ovals) that represent uncertain variables impacting the decision problem; *decision nodes* (rectangles) that represent choices open to a decision maker and *value nodes* (diamonds) that represent attributes (most of the time numeric attributes) the decision maker cares about. They are an extension of Bayesian networks or dynamic Bayesian networks by adding decision and value nodes. In an influence diagram, an arc or edge relating two chance nodes is called a *relevance arc* because it indicates that the state of the source node is relevant to the probability distribution of the destination node, arcs from decision nodes to chance nodes are known as *influence arcs* meaning that the decision influences the outcome of the chance node and arcs into decision nodes (from chance nodes) are called *information arcs* meaning that the outcome of the chance node will be known at the time the decision is taken. Decision nodes are ordered in time (sequential decisions): there is a direct link between all decision nodes. Finally, arcs from chance or decision nodes into value nodes represent *functional links*. Relevance arcs may mean many things depending on the problem at hand such as: implication, correlation, causality, etc.

The consideration of influence diagrams together with dynamic Bayesian networks in this paper is motivated by the fact that, in general, the main purpose of carrying a (dynamic) reliability study or analysis is to set up a preventive maintenance policy and so integrating decision nodes in the model to represent maintenance actions for instance is justified.

In the next paragraph we will consider the properties of dynamic Bayesian networks that make them suitable for modeling stochastic processes in general and dynamic reliability in particular.

## 2.2. Inference and learning capabilities of dynamic Bayesian networks

### 2.2.1. Learning capabilities

Though learning a dynamic Bayesian network consists in two components: learning its structure and/or learning its parameters; structure learning is more difficult than parameters learning. On the other hand, in many domains such as that we are concerned with in this paper, experts are able to establish the relationships existing between variables; that is why we consider only parameters learning. For parameters learning purpose, there exists efficient algorithms and software, see for instance (BNT) and (Murphy, 2002), that can compute conditional probability table for discrete nodes when experimental data (evidence) exist. Learning conditional probability density functions for continuous nodes necessitates in general discretization and approximation schemes that are not obvious (Murphy, 2002). For our problem of dynamic reliability analysis, discretization of continuous variables (temperature, pressure, etc.) may be straightforward because in many cases the experts reason about these variables in terms of thresholds that leads to a natural discretization see for instance (Labeau *et al.*, 2000; Marseguerra *et al.*, 1998). A possible direct application of parameters learning is the estimation of some important dependability performance measures of the system. Indeed, by simulating the obtained model, some parameters such as the steady state probability of being in a particular state or the mean transition time between two given states or the same state can be computed. For instance in the case of Markov and constant transition matrix assumption, learning parameters returns to determining the transition matrix  $P$  of the system from experimental data; from this matrix  $P$ , one can deduce some steady state performance measures such as: mean up time (MUT), mean time to repair (MTTR), mean time between failures (MTBF), mean life duration, availability, safety, etc. by applying the theory of Markov processes; the following facts are well known from this theory.

For an *irreducible* non periodic Markov chain, see (Hêche *et al.*, 2003) for definition, that can represent the behavior of a *reparable system* (there is no catastrophic states for this system), with transition matrix  $P$ , the steady state probability distribution  $\pi$  (a row vector of dimension  $s$ ) exists and is the unique solution of the equations [4]

$$\pi \cdot P = \pi, \quad \sum_{i=1}^s \pi_i \quad [4]$$

and it is known that:

- $\pi_i$  is the probability that the system is in the state  $i$  or equivalently the proportion of the time the system spent in the state  $i$  (a possible estimation of the mean up time (MUT) or mean time to repair (MTTR)) ;

- $\frac{1}{\pi_i}$  is the mean number of transitions (mean time) between two visit of state  $i$  (a possible estimation of mean time between failures (MTBF), etc.).

For an *absorbing* Markov chain, see (Hêche *et al.*, 2003), that can represent the behavior model of a system with non *reparable states* and the transition matrix  $P$  in

the canonical form  $P = \begin{pmatrix} I & 0 \\ R & Q \end{pmatrix}$  where  $I$  is an identity matrix with dimension equals

to the number of absorbent states,  $R$  and  $Q$  are constant matrices with appropriate dimensions, it is known that the steady state behavior of  $P^t$  is given by

$\lim_{t \rightarrow \infty} P^t = \begin{pmatrix} I & 0 \\ (I-Q)^{-1} R & 0 \end{pmatrix}$  and  $N = [n_{ij}] = (I-Q)^{-1}$  is called the fundamental

matrix. From this observation, the following results are known:

- the mean time that the system sojourns in the transient state  $j$  when beginning its behavior from the transient state  $i$  (a possible estimation of the mean time the system will be functioning approximately for instance) is the element  $n_{ij}$  of the fundamental matrix  $N$ ;
- the mean time before attaining an absorbent state (a possible estimation of mean life duration) when beginning its behavior from the transient state  $i$  is the sum of the elements of the  $i^{th}$  row of the fundamental matrix  $N$ ;
- the probability of being absorbed (a parameter related to the safety) by the absorbent state  $j$  when beginning the behavior from the transient state  $i$  is the element  $b_{ij}$  of the matrix  $B = NR = (I-Q)^{-1} R$ .

Notice that for any absorbent Markov chain, the canonical form of its transition matrix  $P$  can be simply obtained by rearranging the order of its states.

This simple and short recall shows how learning capabilities of dynamic Bayesian networks can be exploited for analyzing the behavior of stochastic systems in many practical domains. In the following paragraph we will consider their inference capabilities.

### 2.2.2. Inference capabilities

Another possibility offered by dynamic Bayesian networks is the inference; that is propagating a change in the system to estimate the possible outcome and identifying the most probable state of a system or the value of a variable given the observation or measurement. For instance if the exogenous perturbation behavior changes from an estimated nominal behavior, an interesting question could be: what will be the behavior of the system state (prognostics) ? There are algorithms and software (see (BNT) and (Murphy, 2002)) that handle such issues. Let us define  $O(t)$  to be a vector containing the states and/or the values of all observed nodes at time  $t$ . The general inference problem for dynamic Bayesian networks is to compute the following parameters, equation [5]

$$\Pr\{X(t) / O(\tau), t_1 \leq \tau \leq t_2\} \quad [5]$$

where  $X(t)$  represents generically the state or the value of any hidden node. The interesting and usually considered cases in practice are *filtering* ( $t = t_2$ ), *prediction* ( $t > t_2$ ) and *smoothing* ( $t_1 < t < t_2$ ). Once these probabilities are computed, one can use the so called Viterbi decoding scheme (the abduction or most probable explanation) to determine the most probable state  $s^*(t)$  of the component or node under consideration at the time instant  $t$  as given by equation [6]

$$s^*(t) = \arg \max_{i \in S} \{\Pr\{s(t) = i / O(\tau), t_1 \leq \tau \leq t_2\}\}. \quad [6]$$

The inference offers other possibilities that could permit in practice to react quickly to given evidences; a main parameter against which one fight in practice is the time (or duration). For instance knowing the duration before a likely catastrophic event given current evidence (for instance the failure of a low level controller that causes the states of process variables to grow out of limit causing damage to the system) is important for assistance purpose and this duration can be derived from inference. Let us call  $C \subset S$  the set of all catastrophic states (states to be avoided) of a system, then, given an evidence  $E(t)$  (the behavior of the perturbation  $w(t)$ , failure of a component, etc.) for a period  $[t_0, T]$  we can define the duration  $\tau(\delta)$  to go before catastrophe at the risk  $0 < \delta < 1$  as the first instant from  $t_0$  such that the probability that the state of the system at this instant belongs to the set  $C$  exceeds  $1 - \delta$ ; it is given by equation [7]

$$\tau(\delta) = \inf_{\tau} \{\Pr\{s(t_0 + \tau) \in C / E(t), t_0 + \tau, t \in [t_0, T]\} \geq 1 - \delta\} \quad [7]$$

Of course as in the learning case, inference algorithms are more or less complex depending on the nature of nodes (continuous or discrete) and the interdependency (the number of slices) among nodes. As the purpose of this paper is to show how dynamic Bayesian networks could be used for dynamic reliability modeling and analysis purpose, we consider the subtleties of choosing an appropriate inference algorithms to be out of the scope of this paper; but we would like to insist to the readers intention that the choice of an appropriate inference algorithm may be a matter of experts and encourage them to refer to appropriate literature such as (Murphy, 2002; Naïm *et al.*, 2004).

In the following section, we will show how dynamic Bayesian networks and their capabilities presented so far can be used as a modeling tool for dynamic reliability modeling, assessment and analysis purpose as defined in the introduction section.

### 3. Modeling dynamic reliability using dynamic Bayesian networks

In this section we will show how the dynamic reliability problem defined in the introduction section can be tackled using dynamic Bayesian networks and influence

diagrams as the underlying mathematical tool in different configurations (wearing away process of components, influence of exogenous perturbation, relationships between state of the system and process variables as well as the effect of decision maker's action). For sake of simplicity and without loss of generality we assume that all stochastic processes considered here are Markov Processes (MP), two slices dynamic Bayesian networks (in the case of non Markovian processes only the number of slices will change, more than two slices to take into account the history of the system for a more or less large horizon and the main difficulty will be the complexity of parameters specification and the complexity of learning and inference algorithms). We would like to precise that the Markovian hypothesis is guided by the sake of simplicity for the presentation and the fact that we conceive this paper as a tutorial or introduction of how to use dynamic Bayesian networks for dynamic reliability modeling. A stochastic process  $X(t)$  is said to be a Markov process if and only if the following equation [8] is valid.

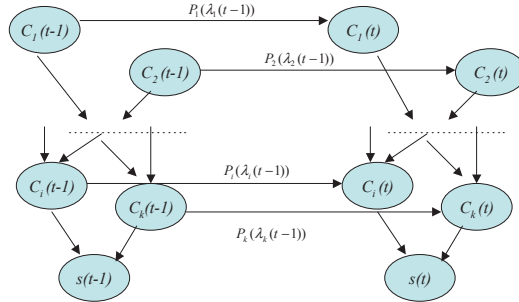
$$\Pr\{X(t) / X(t-1), \dots, X(0)\} = \Pr\{X(t) / X(t-1)\} \quad [8]$$

An influence diagram will correspond to Markov Decision Processes (MDP) that consider the possibility for a decision maker to intervene on the behavior of the system: the transition probabilities at each instant  $t$  depend on the action taken by the decision maker or agent; a cost (or benefit) may be associated with the actual state and/or the decision. The goal is to find a function, called a policy, which decides what to do (which action to take) in each state, so as to optimize some performance index (e.g. the mean or expected discounted sum of reward). The influence diagrams offer then the possibility with regard to reliability analysis to set up and evaluate maintenance policies.

In the following paragraph we consider gradually the modeling of different effects on the reliability of a system from simple consideration to more complex ones.

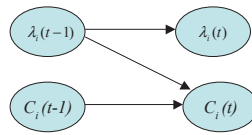
### ***3.1. Modeling the wearing away process of a system***

When aging any system will have more and more chance to fail because of a wearing away phenomenon. The wearing away process modeling (with Markov assumption) using dynamic Bayesian networks is straightforward and the corresponding model (structure) is typically given by Figure 6 where we consider that the state of the system  $s(t)$  at an instant  $t$  depends on the states of different components  $C_i(t)$  at that instant.



**Figure 6.** A typical dynamic Bayesian networks model of a wearing away process

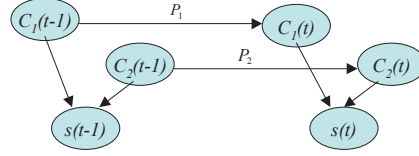
The Bayesian network of Figure 6 shows that the system is hierarchically organized with many components that can cause its failure; that is why each slice constitutes a Bayesian network. The inter slice structure shows a purely wearing away phenomenon of components because each component of the slice  $t-1$  is its unique parent in the slice  $t$ . The transition matrix  $P_i$  depends on actual value of the corresponding component failure rate  $\lambda_i(t-1)$  that has its own dynamics. Notice that the behavior of  $P_i$  could be integrated in the model by adding nodes to represent the failure rates processes as shown by Figure 7 for the component  $C_i$ .



**Figure 7.** Dynamic Bayesian network representation of the behavior of the transition matrix with regard to the failure rate process

Once the model is established, it can be used in different manner: to estimate the failure rates from experimental data by learning parameters  $P_i(t)$  or to use the model as a decision support to set up a (predictive) maintenance policy if failure rates behaviors are known.

To illustrate this idea, let us consider the system of Figure 2 (a) whose Bayesian network model is given by Figure 3. We consider, for the sake of simplicity, that the components as well as the system have only two states namely *OK* meaning that the component or the system is normally functioning and *OFF* meaning that the system or the component is out of service; furthermore we consider that the components are not repairable. The dynamic Bayesian network model (structure) of this system is given by Figure 8 ((a) represents the structure of the model, (b) is the intra slice parameters and (c) represents the inter slice parameters or transition matrices; the matrices  $A_1$  and  $A_2$  are the generating matrices of the corresponding continuous time Markov chain, see (Hêche *et al.*, 2003).



(a) structure

$C_1$	$C_2$	$\Pr\{S = OK / C_1, C_2\}$
OK	OK	1 (p)
OK	OFF	1 (p)
OFF	OK	1 (p)
OFF	OFF	0 (q)

(b) intra slice parameters

$$P_1 = e^{A_1}, P_2 = e^{A_2}$$

$$A_1 = \begin{bmatrix} -\lambda_1 & \lambda_1 \\ 0 & 0 \end{bmatrix}, A_2 = \begin{bmatrix} -\lambda_2 & \lambda_2 \\ 0 & 0 \end{bmatrix}$$

(c) inter slice parameters

**Figure 8.** Dynamic Bayesian network model of the system of Figure 2 (a)

For the intra slice parameters, notice that we did not give  $\Pr\{S = OFF / C_1, C_2\}$  because we have the following relation (equation [9])

$$\Pr\{S = OFF / C_1, C_2\} = 1 - \Pr\{S = OK / C_1, C_2\}. \quad [9]$$

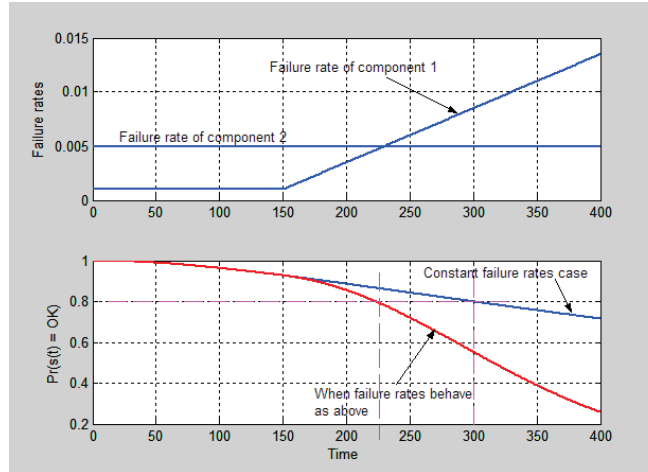
Though we could consider an approximate functioning, that is there is no certainty (probability  $p$  rather than  $1$ ) that the system functions when at least one component functions, we consider here the perfect case so that the probability of the system to be in the state  $OK$  is given by the following equation [10].

$$\pi_{OK}^S(t) = 1 - \left(1 - \pi_{OK}^{C_1}(t)\right) \left(1 - \pi_{OK}^{C_2}(t)\right) \quad [10]$$

If we consider that the predictive maintenance policy is to intervene on the system (change components for instance) if the probability of the system to be in the state  $OK$  is less than 80%, that is  $\pi_{OK}^S(t) \leq 0.8$ , then by simulating the former model, one can derive the schedule of the predictive maintenance. The Figure 9 shows simulation results with an initial conditions  $\pi_{C_1}(0) = \pi_{C_2}(0) = [1 \ 0]$ ; the first graphic of this figure shows a constant failure rate for  $C_2$  that is  $\lambda_2 = 0.5 \times 10^{-2}$  and a behavior that varies from a constant value of  $\lambda_1 = 10^{-3}$  to a linear form for the failure rate of component  $C_1$ ; the second graphic shows the behavior of the probability of the system to be in the state  $OK$  when the component  $C_1$  failure rate is considered equal to its constant part and in the general case respectively. The predictive maintenance schedule will be then to intervene after 300 time units if



failure rates are constant and after 225 time units if  $C_1$  failure rate behaves as shown on the first graphic of Figure 9.

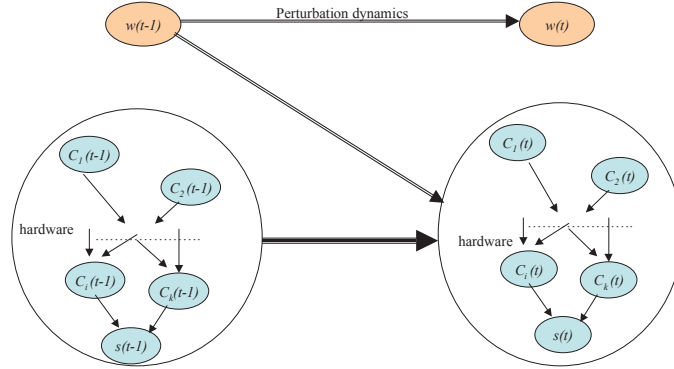


**Figure 9.** Simulation results of the model of Figure 8

In the next paragraph the effect of a possible exogenous perturbation will be introduced in the model.

### 3.2. Influence of an exogenous perturbation on the state of a system

Let us consider the problem of monitoring the state  $s(t)$  of a system that is influenced by a continuous exogenous perturbation  $w(t)$ ; observations are made at discrete instant and the perturbation is supposed to be a Markov process (its value (continuous) at time  $t$  is influenced only by its value at time  $t-1$ ). A typical model of such problem using dynamic Bayesian networks is given by Figure 10 where the state of the system at time  $t$  is influenced by the perturbation value at time  $t-1$  through the influence on its components as compared to the autonomous model of Figure 6. This assumption, once again, is made for sake of presentation simplicity and does not restrict the application of the model because in a real problem case, one can remove this assumption without altering the modeling process and result.



**Figure 10.** Dynamic Bayesian networks structure of the state of a system influenced by an exogenous perturbation

The parameters of this model are two fold. The conditional probability density (CPD) of  $w(t)$  is a function of  $w(t-1)$ . For instance if this conditional probability follows a normal distribution law with mean  $Aw(t-1)+b$  where  $A$  is a matrix and  $b$  is a vector of dimension  $p$  and covariance  $\Sigma$  then we have the following conditional probability density function  $f_w(w(t))$  for  $w(t)$  (see equation [11]).

$$f_w(w(t)) = \frac{1}{(2\pi)^{p/2} |\Sigma|^{1/2}} \exp\left(-\frac{1}{2}(w(t) - Aw(t-1) - b)^T \Sigma^{-1} (w(t) - Aw(t-1) - b)\right) \quad [11]$$

where  $|\Sigma|$  denotes the determinant of  $\Sigma$ ; the probability for  $w(t)$  to belong to a subset  $\Omega$  of  $\mathfrak{R}^p$  when  $w(t-1)$  is known is then given by equation [12]

$$\Pr\{w(t) \in \Omega / w(t-1)\} = \int_{\Omega} f_w(w) dw. \quad [12]$$

The inter slice parameters of the component consist in a conditional probability tables (CPT) where each element depends on the perturbation as given by the equation [13] whereas the intra slice parameters remain unchanged.

$$\Pr\{C_i(t) = j / C_i(t-1) = k, w(t-1) = w\} = p_{kj}^{C_i}(w) \quad [13]$$

As an illustration, let us consider the behavior of the system of Figure 2 (a) and suppose that the failure rates  $\lambda_1$  and  $\lambda_2$  are functions of a perturbation  $w$  defined by the equations [14].

$$\lambda_1(w) = \lambda_{10} + \alpha w, \quad \lambda_2(w) = \lambda_{20} + \alpha w \quad [14]$$

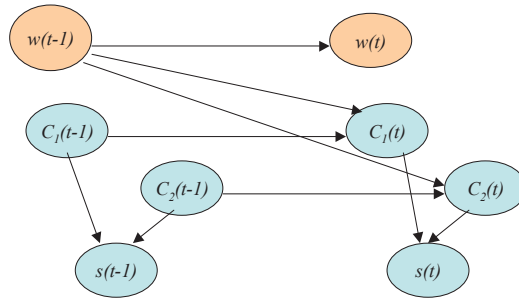
The purpose is to establish a predictive maintenance policy according to the intensity of the perturbation  $w$ . The model of this problem in terms of dynamic Bayesian network is given by Figure 11. The states of components as well as the intra slice

parameters are the same as that of former Figure 8. The transition matrices are given by equation [15]

$$P_1(w) = e^{A_1(w)}, \quad P_2(w) = e^{A_2(w)} \quad [15]$$

with

$$A_1(w) = \begin{bmatrix} -\lambda_1(w) & \lambda_1(w) \\ 0 & 0 \end{bmatrix}, \quad A_2(w) = \begin{bmatrix} -\lambda_2(w) & \lambda_2(w) \\ 0 & 0 \end{bmatrix}. \quad [16]$$

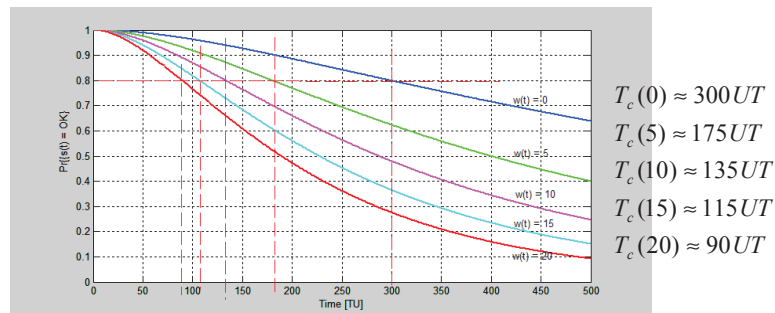


**Figure 11.** Dynamic Bayesian network model of the example of Figure 2 (a) where components are influenced by an exogenous perturbation

Simulating this model with parameters:

$$\lambda_{10} = 10^{-3}, \quad \lambda_{20} = 0.5 \times 10^{-2}, \quad \alpha = 0.2 \times 10^{-3}$$

we obtain results of Figure 12 for different value of the perturbation. Let  $T_c(w)$  denotes the time it lasts, given the perturbation intensity  $w$ , before preventive intervention on the system considering former predictive maintenance policy (intervene whenever  $\pi_{OK}^S(t) \leq 0.8$ ) then we obtain results shown on Figure 12 and one can notice the intuitive coherency for the behavior of  $T_c(w)$  with regard to the perturbation intensity.

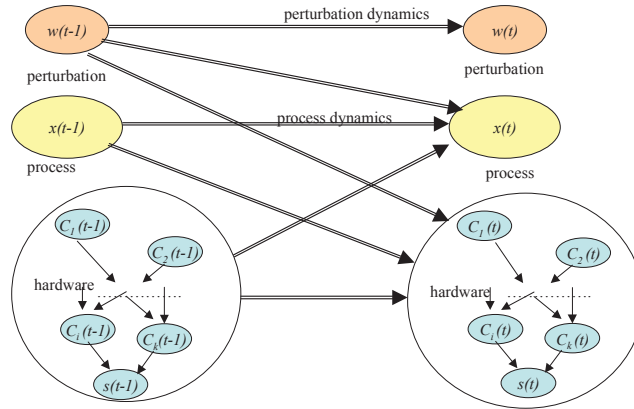


**Figure 12.** Simulation results of the model of Figure 11

In the next paragraph, we will consider a more general dynamic reliability model that integrates interactions between perturbation, process and system.

### 3.3. Interaction between process, perturbation and the system

Here we consider the case where in addition to an exogenous perturbation  $w(t)$ , there exists a relationship between the process variables  $x(t)$  and the state of the system  $s(t)$ . The exogenous perturbation is either a deterministic or a stochastic process (Markov) and it can influence either the state of the system  $s(t)$ , the process variables  $x(t)$  or both with possible complex relationships. The Figure 13 shows an example of the structure of such model.



**Figure 13.** Dynamic Bayesian networks model of interactions between the state of the system and process variables influenced by an exogenous perturbation

The link from system nodes at time  $t-1$  to the process variables at time  $t$  means that actual state of the system may influence the dynamics of the internal process; indeed, in a boiler the pressure will depend on weather the boiler is closed or open. The intra slice parameters as well as perturbation dynamics are similar to what was stated in the previous paragraph whereas the conditional probability density (CPD) of the process variables state  $x(t)$  depends on  $x(t-1)$ ,  $w(t-1)$  and  $s(t-1)$ . For instance in the case of Gaussian distribution (see equation [17])

$$\Pr(x(t) / x(t-1), w(t-1) = w, s(t-1) = i) = f_x(x(t)) \quad [17]$$

with mean  $A_i(w)x(t-1) + b_i(w)$  and covariance  $\Sigma_i(w)$  where the parameters  $A_i(w)$ ,  $b_i(w)$  and  $\Sigma_i(w)$  depend on the actual state  $i$  of the system and the actual value of the perturbation, this conditional probability density function is given by equation [18].

$$f_x(x(t)) = \frac{1}{(2\pi)^{n/2} |\Sigma_i(w)|^{1/2}} \exp\left(-\frac{1}{2}(x(t) - A_i(w)x(t-1) - b_i(w))^T \Sigma_i^{-1}(w)(x(t) - A_i(w)x(t-1) - b_i(w))\right) \quad [18]$$

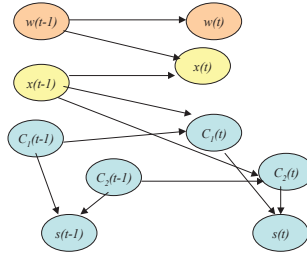
The probability that the state of the process variables belongs to a given set  $\Gamma$  at the next instant given actual conditions can then be calculated using equation [19].

$$\Pr\{x(t) \in \Gamma / x(t-1), w(t-1) = w, s(t-1) = i\} = \int_{\Gamma} f_x(x(t)) dx \quad [19]$$

The transition matrix of the state of the system is given by equation [20]

$$\Pr\{s(t) = j / s(t-1) = i, x(t-1) = x, w(t-1) = w\} = p_{ij}(x, w). \quad [20]$$

As an illustration let us consider the system depicted on Figure 2 (a) with the assumptions that failure rates of the components are functions of an internal process state (temperature, pressure)  $x(t)$  that is a positive scalar which dynamics in return are influenced by a perturbation  $w(t)$ . The structure of a dynamic Bayesian network that describes such problem is given by Figure 14.



**Figure 14.** Dynamic Bayesian network model of example of Figure 2 (a) where components are influenced by an internal process which in return is influenced by an exogenous perturbation

For simulation, let us suppose that failure rates behave as equation [21]

$$\lambda_1(x) = \lambda_{10} + \alpha x, \quad \lambda_2(x) = \lambda_{20} + \alpha x \quad [21]$$

and the dynamics of the process state is given by equation [22].

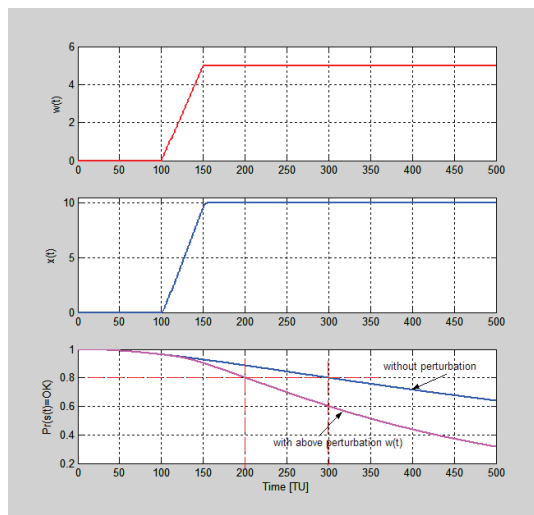
$$x(t) = \beta x(t-1) + w(t-1), \quad x(0) = 0 \quad [22]$$

Simulating this model with the parameters of equation [23]

$$\lambda_{10} = 10^{-3}, \quad \lambda_2 = 0.5 \times 10^{-2}, \quad \alpha = 0.2 \times 10^{-3}, \quad \beta = 0.5 \quad [23]$$

we obtain the results depicted on Figure 15 where the first graphic shows the perturbation  $w(t)$  and the second graphic shows the induced behavior of the process state  $x(t)$ . The third graphic represents the probability that the system is in its OK

state when there is non perturbation and when we consider a perturbation given by the first graphic. With this simulation, one can set up predictive maintenance as in the previous section without observing the actual state of the process variables just by estimating the perturbation. Let us consider as in the previous paragraph that the maintenance policy is to intervene on the system whenever  $\pi_{OK}^S(t) \leq 0.8$  and let  $T_c$  be the time it lasts before intervention, then the simulation shows that if there is no perturbation then  $T_c = 300 TU$  and if the perturbation behaves like shown by the first graphic then  $T_c = 200 TU$ ; this information can be used to set up intervention plans according to environmental changes. This model shows an hierarchic relevancy: the perturbation is relevant to the process variable which in return is relevant to the state of the system.



**Figure 15.** Simulation results of the model of Figure 14

Finally, in the next paragraph we will consider the possibility that a decision maker (in a very broad acceptance including human, computer program, robot, ..) has an effect on the behavior of the system; this leads to an influence diagram as model.

### 3.4. Introducing the effect of the decision maker action

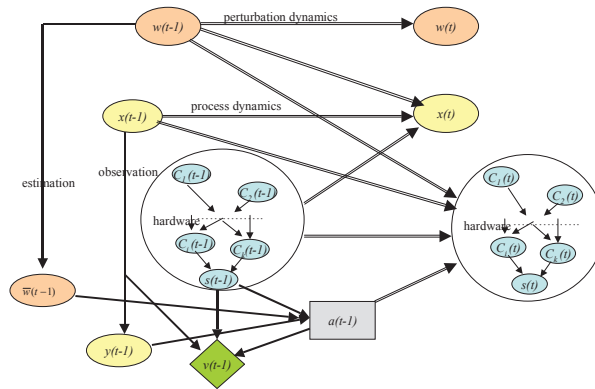
Let us suppose now that an agent or decision maker can have an action on the state of the system. From the model established in the previous paragraph, we must just add a decision node (and possibly value node) to obtain the influence diagram depicted on the Figure 16 when a Markov process assumption is considered. Notice here that we consider that the agent do not have the entire state of the process variables (this is common in practice) at the moment its decision is made but a partial observation  $y(t)$ ; we consider also the possibility to estimate the intensity of

the perturbation  $\bar{w}(t)$  that will be available to decision maker at the moment decision is made; a value node is introduced to take into account possible benefit or cost induced by the action, the state of the system and the state of process variables (this later one could measure the quality of a product for instance). In terms of parameters there is no change with regard to previous paragraph for the exogenous perturbation  $w(t)$  and for the state of the process variables  $x(t)$ . But the transition probability  $p_{ij}$

of the state of the system will depend on the perturbation value  $w(t-1)$ , the state of process variables value  $x(t-1)$  and the action  $a(t-1)$ , and so it is given by equation [24].

$$\Pr\{s(t) = j / s(t-1) = i, x(t-1) = x, w(t-1) = w, a(t-1) = a_k\} = p_{ij}^k(x, w) \quad [24]$$

The observation of  $y(t)$  from  $x(t)$  and the estimation  $\bar{w}(t)$  from  $w(t)$  may be modeled by Bayesian networks too.

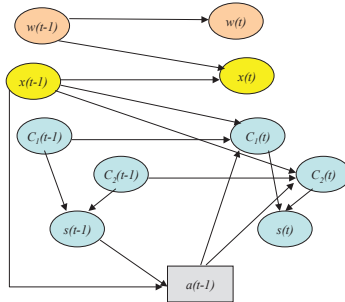


**Figure 16.** Influence diagram model of interaction between process, system, perturbation and decision maker

To illustrate this approach, let us consider the model of Figure 14 and consider that an agent has an action  $a$  that influences the components failure rates according to the following law (equation [25])

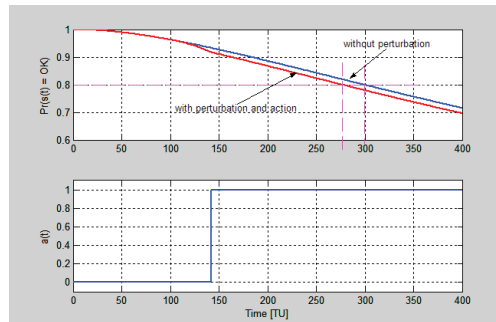
$$\lambda_1(x(t)) = \lambda_{10} + \alpha(1 - a(t))x(t), \quad \lambda_2(x(t)) = \lambda_{20} + \alpha(1 - a(t))x(t) \quad [25]$$

where either  $a(t) = 0$  (do nothing) or  $a(t) = 1$  (do something that will bring the failure rates to their initial values). This model (structure) is given by the influence diagram of Figure 17.



**Figure 17.** Influence diagram model of the example of Figure 2 corresponding to model of Figure 14 with a decision maker's action on components failure rates

Let us suppose that the decision maker observes the process variables state  $x(t)$  and takes the action if this value is beyond a threshold  $x_c$ . Simulating this model with the same parameters as in the former paragraph (Figure 14) and for the threshold value  $x_c = 8$  (80% of the final value), we obtain the results of Figure 18; the behaviors of the perturbation and the process variable states are the same as the two first graphics of Figure 15 respectively. The first graphic shows the probability for the system to be in the state OK when there is no perturbation and with perturbation and decision maker correction and the second graphic shows the behavior of the decision maker's action. Notice that here the time it lasts before intervention is 275 TU compared to 200 TU for the case where there is no action.



**Figure 18.** Simulation results of the model of Figure 18

The following paragraph gives an idea of possible approximation when learning and inference become intractable because of continuous variables.

### 3.5. Possible approximation scheme

Learning and inference with continuous variables is in general a hard task (see (Murphy, 2002)). Furthermore, in practice one may be interested only by when a continuous variable come across the border of a compact subset (when the pressure

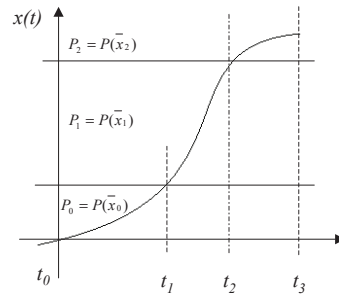


or the temperature goes beyond or below a threshold) so that it can be acceptable to approximate the problem using the piecewise constant scheme that is the transition matrix  $P$  is considered constant whenever the continuous variable belong to a given compact subset. The failure rate of a component will brutally vary if the perturbation and/or the process state goes beyond/below a threshold for instant. In this case, when we consider the transition matrix  $P$  to depend on a continuous variable  $x$ , that is  $P = P(x)$ , it means that on each previously defined compact subset  $\Omega$ , the transition matrix is a function of  $x_\Omega$  ( $P = P(x_\Omega)$ ) that represents  $x$  on  $\Omega$ ; in general it will be the mean value of  $x$  over  $\Omega$ , that is  $\bar{x}_\Omega$  is given by equation [26]

$$\bar{x}_\Omega = \frac{1}{m(\Omega)} \int_{\Omega} x d\Omega \quad [26]$$

where  $m(\Omega)$  is a measure of  $\Omega$ . For instance if the transition matrix  $P$  is a function of a time varying function  $x(t)$  that behave as shown by Figure 19, then we could divide the time interval  $[t_0, t_3]$  into three compact intervals and consider that  $P = P_i$  on each sub interval where  $P_i$  is a function of  $\bar{x}_i$  that is defined by equation [27]

$$\bar{x}_i = \frac{1}{t_{i+1} - t_i} \int_{t_i}^{t_{i+1}} x(t) dt . \quad [27]$$



**Figure 19.** An example of the transition matrix approximation

Monitoring such a system consists then essentially to monitoring when the continuous variable come across a compact domain border and simulating the corresponding model to react in consequence.

#### 4. Conclusion

The problem of modeling and analyzing dynamic reliability has been considered in this paper through dynamic Bayesian networks as the underlying mathematical tool. It is shown that their learning and inference capabilities can be exploited in order to take into account experts knowledge and experimental data to estimate the

dependability measures and to update beliefs given evidence. The existence of efficient algorithms for learning and inference make it possible to integrate them into decision support system for maintenance purpose or to construct standalone package for analysis and optimization of maintenance policy or as an aid for proactive and reactive decision making, by simulating the possible outcome of different scenarios before and during an abnormal behavior due to the growth of the perturbation or process variables out of limit for instance. The small academic example considered along the paper shows the potentiality of the approach presented so far but this potentiality must be proved by applying the approach to a real world complex example; this is the task for future works and the generality of the approach make it possible to use it in other domains. The need of expertise suggests that the use of this approach for modeling a real world problem will necessitate a multi disciplinary team. Though the exogenous perturbation and the process variables are considered to be continuous, in practice, with most of the existing software, it will be required to sample them on a given domain and this process may lead to some errors in the estimation of dependability measures; this possibility must be taken into account by the modeling team. As stated in the previous sections, the choice of appropriate algorithms and approximation schemes for a practical application may be a matter of expertise and so the modeling process must be carried up by a team comprising experts of dynamic Bayesian networks experts.

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