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On Higgs inflation in non-minimally coupled models of gravity

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Abstract

Models of inflation in which the Higgs field is non-minimally coupled to gravity lead, after a rescaling of the metric, to a complicated expression for the potential of the inflaton field. Nevertheless, this potential produces the desired features, both theoretical and experimental, after some approximations are made. In this note, we also allow for a modification of the Higgs kinetic term in such a way that the resulting potential for the inflaton field is, without any approximation, of the simplest form after the rescaling of the metric.

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1 Introduction

The Standard Model of particles physics has proved to be a robust theory in describing the observed interactions between particles at the level of particle accelerators. However, this successful theory does not seem to answer questions related to the cosmological evolution of the Universe. In particular, the nature of dark matter or dark energy as well as the early inflationary regime of the Universe remain a mystery. In this note we will be mostly concerned with the problem of inflation. Introductory reviews of cosmic inflation can be found in [1, 2, 3, 4, 5, 6].

It is widely accepted now that inflation might be driven by a scalar field moving slowly in a certain potential. The discovery of the Higgs field has completed the Standard Model and opened a new window in cosmology. It is therefore tempting to identify the Higgs field with the scalar field needed for inflation.

The issue now is how to couple the Standard Model to gravity in order to account for inflation through the Higgs field. Various models can be constructed depending on what kind of coupling one chooses. Accounts of the Higgs field in cosmology can be found in [7, 8, 9, 10]. Here we will mention only what is called the non-minimal Higgs inflation model as it is relevant to our study. In this theory, the coupling of the Higgs field to gravity is given by the action [11, 12, 13]

$$S = \int d^4x \sqrt{-g} \left[-\frac{1}{2} (M^2 + 2\xi H^\dagger H) R + \mathcal{L}_{\text{SM}} \right] . \quad (1.1)$$

The quantity \mathcal{L}_{SM} is the full Standard Model Lagrangian written in curved space-time and H is the Higgs doublet. We will return to this model of inflation in some details in section 3. It suffices to say for the moment that under a certain approximation (see later), this theory gives a viable explanation of inflation.

In order to get the standard term $\int d^4x \sqrt{-g} R$ for the gravitational sector, the authors of refs.[11, 12, 13] carried out a conformal rescaling of the metric field. This operation induced a complicated expression for the potential of the Higgs field (or equivalently, gave a non-canonical kinetic term for the Higgs field). Nevertheless, this complication disappears under the above cited approximation [12, 13] .

The aim of this note is to provide a model of Higgs inflation where no approximation is needed. We have simply modified the Higgs kinetic term in the Standard Model non-minimally coupled to gravity. More specifically, we propose the model described by the action

$$S = \int d^4x \sqrt{-g} \left[-\frac{1}{2} (M^2 + 2\xi H^\dagger H) R + F(H^\dagger H) (D_\mu H)^\dagger (D^\mu H) + \mathcal{L}'_{\text{SM}} \right] \quad (1.2)$$

to account for inflation of the early Universe. Here \mathcal{L}'_{SM} is the Standard Model Lagrangian without the gauge covariant kinetic term and the function $F(H^\dagger H)$ is determined in such a way that the resulting kinetic term of the inflaton field, after a rescaling of the metric, is in the usual canonical form. This theory could be seen as an improvement of the proposition of ref.[12]. It is also crucial to mention that the two models coincide for a large Higgs field (which is precisely the approximation made in ref.[12]).

The article is organised as follows: In the next section we review the Starobinsky model of inflation [14] as there are similarities between this model and non-minimal Higgs inflation [15]. Some details of the non-minimal coupling of the Higgs field to gravity are given in section 3. Our modification of the non-minimally coupled Higgs field is presented in section 4. We have also included, in section 5, a short study of Higgs-dilaton gravity with modified kinetic terms. The last section is dedicated to some conclusions.

2 The Starobinsky model of inflation : An inspiration

Various models dealing with the issue of the rapid expansion of the early Univers have been inspired by the Starobinsky model of inflation [15]. This is described by the action [14]

$$S_0 = \int d^4x \sqrt{-g} \left[-\frac{M_{\text{p}}^2}{2} R + \frac{1}{12M_{\text{S}}^2} R^2 \right] . \quad (2.1)$$

As it is well-known, the quadratic term in the scalar curvature can be traded for a scalar field [16]. One starts by writing the equivalent action

$$S_1 = \int d^4x \sqrt{-g} \left[-\frac{M_{\text{p}}^2}{2} \left(1 + \frac{X}{3M_{\text{p}}^2 M_{\text{S}}^2} \right) R - \frac{X^2}{12M_{\text{S}}^2} \right] , \quad (2.2)$$

where X is an auxiliary field whose equation of motion is $X = -R$. Upon replacing this in the last action we recover the Starobinsky model (2.1).

The action (2.2) can be cast in a more familiar form by going to the Einstein frame through the field redefinition

$$g_{\mu\nu} \longrightarrow e^{\omega\varphi(x)} g_{\mu\nu} \quad \text{with} \quad \left(1 + \frac{X}{3M_{\text{p}}^2 M_{\text{S}}^2} \right) e^{\omega\varphi(x)} = 1 \quad , \quad \omega = -\sqrt{\frac{2}{3}} \frac{1}{M_{\text{p}}} . \quad (2.3)$$

With this conformal rescaling of the metric, the action (2.2) yields (up to a total derivative)

$$S_2 = \int d^4x \sqrt{-g} \left[-\frac{M_{\text{p}}^2}{2} R + \frac{1}{2} g^{\mu\nu} \partial_{\mu}\varphi \partial_{\nu}\varphi - \frac{3}{4} M_{\text{p}}^4 M_{\text{S}}^2 (1 - e^{\omega\varphi})^2 \right] . \quad (2.4)$$

The two actions (2.1) and (2.4) could be thought of as dual to each other.

We see that the field $\varphi(x)$ is moving in a potential which is almost flat for very large values of $\varphi(x)$. This is the most sought feature which realises the slow-roll condition needed for inflationary models. In this case the kinetic term of the field $\varphi(x)$ is negligible and the cosmological evolution of the Universe is dominated by the constant term $\frac{3}{4} M_{\text{p}}^4 M_{\text{S}}^2$ (playing the role of a cosmological constant). The Starobinsky model is compatible with cosmological measurements for $M_{\text{S}} \approx 10^{-5}$ (see, for instance, refs.[8, 15] for the estimation of M_{S}).

It is essentially the theory in (2.4) which was reached by the non-minimally coupled Higgs field of ref.[12]. It was also shown in [15] that other models of inflation could be thought of, under certain assumptions, as descendants of the Starobinsky model.

3 The Higgs field non-minimally coupled to gravity

Here and in the rest of this note, we will assume that the Higgs doublet is in the unitary gauge

$$H(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ h(x) \end{pmatrix} . \quad (3.1)$$

The Higgs field non-minimally coupled to gravity is described, in the Jordan frame, by the action [11, 12, 13]

$$S_J = \int d^4x \sqrt{-g} \left[-\frac{1}{2} (M^2 + \xi h^2) R + \frac{1}{2} \partial_\mu h \partial^\mu h - \frac{\lambda}{4} (h^2 - v^2)^2 \right] . \quad (3.2)$$

We have written down only the terms relevant to our study. The constant M and the reduced Planck mass $M_p = \frac{1}{\sqrt{8\pi G}} = 2.4335 \times 10^{18}$ GeV are related by

$$M_p^2 = M^2 + \xi v^2 \quad (3.3)$$

with the Higgs field expectation value $v = 246$ GeV. We are assuming that $\hbar = c = 1$.

This theory, however, is best studied in the Einstein frame by performing the conformal transformation

$$g_{\mu\nu} \longrightarrow e^{\omega\varphi} g_{\mu\nu} \quad \text{with} \quad \omega = -\sqrt{\frac{2}{3}} \frac{1}{M_p} . \quad (3.4)$$

The fields φ and h are related by

$$(M^2 + \xi h^2) e^{\omega\varphi} = M_p^2 . \quad (3.5)$$

Therefore one can use either one of the two fields to explore the cosmological properties of the theory. We find it more convenient to keep the field φ instead of the field h as it makes the various approximations more transparent.

In terms of the field φ and up to a total derivative, the action in the Einstein frame is given by

$$S_E = \int d^4x \sqrt{-g} \left\{ -\frac{M_p^2}{2} R + \frac{3}{4} M_p^2 \omega^2 \left[1 + \frac{1}{6\xi} \frac{1}{\left(1 - \frac{M^2}{M_p^2} e^{\omega\varphi}\right)} \right] g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi - \frac{\lambda}{4} \left(\frac{M_p^2}{\xi} \right)^2 \left(1 - e^{\omega\varphi} \right)^2 \right\} . \quad (3.6)$$

We have used $M_p^2 = M^2 + \xi v^2$ in the last term. We notice that the potential terms is readily in the much sought exponential form (as in the Starobinsky model). This is precisely what one needs for inflation. However, the kinetic term for the scalar field φ is not standard. Nevertheless, in the case when

$$\frac{M^2}{M_p^2} e^{\omega\varphi} \ll 1 \quad , \quad \xi \gg 1 \quad (3.7)$$

the second term in the kinetic energy can be neglected and one arrives at the action¹

$$S_E \simeq \int d^4x \sqrt{-g} \left\{ -\frac{M_{\text{P}}^2}{2} R + \frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi - \frac{\lambda}{4} \left(\frac{M_{\text{P}}^2}{\xi} \right)^2 \left(1 - e^{\omega\varphi} \right)^2 \right\}. \quad (3.8)$$

This theory looks then very much like the Starobinsky model (2.4) with the identification $3M_{\text{S}}^2 = \frac{\lambda}{\xi^2}$. The observed cosmological data requires $\xi = 1.8 \times 10^4$ for a Higgs self-coupling² $\lambda = 15 \times 10^{-2}$ (see [12, 13]). These values are also compatible with $M_{\text{S}} \approx 10^{-5}$.

The authors of refs.[11, 12, 13] have adopted another route in studying the model in (3.2). There, a standard kinetic term was obtained by introducing a new field χ through

$$\left(\frac{d\chi}{d\varphi} \right)^2 = \frac{3}{2} M_{\text{P}}^2 \omega^2 \left[1 + \frac{1}{6\xi} \frac{1}{\left(1 - \frac{M^2}{M_{\text{P}}^2} e^{\omega\varphi} \right)} \right]. \quad (3.9)$$

The action (3.6) becomes then

$$S_E = \int d^4x \sqrt{-g} \left\{ -\frac{M_{\text{P}}^2}{2} R + \frac{1}{2} g^{\mu\nu} \partial_\mu \chi \partial_\nu \chi - U(\chi) \right\}. \quad (3.10)$$

The potential term $U(\chi)$ for the new field $\chi(x)$ is extracted from

$$U(\chi) = \frac{\lambda}{4} \left(\frac{M_{\text{P}}^2}{\xi} \right)^2 \left(1 - e^{\omega\varphi} \right)^2 \quad (3.11)$$

by first integrating (3.9) and then expressing φ in terms of χ . This results in a quite involved expression for $U(\chi)$. However, when $\frac{M^2}{M_{\text{P}}^2} e^{\omega\varphi} \ll 1$ and $\xi \gg 1$ one sees from (3.9)

that $\omega\varphi \simeq -\sqrt{\frac{2}{3}} \frac{1}{M_{\text{P}}} \chi$. In this case the actions (3.10) and (3.8) are the same after the exchange $\varphi \leftrightarrow \chi$.

4 A scalar-tensor gravity with the Higgs field

In this section, we propose another modification of the coupling of the Standard Model to gravity. The Higgs field is still non-minimally coupled to gravity. However, the Higgs kinetic term is in a non-standard form³. In this way, we obtain (in the Einstein frame) a scalar field with a canonical kinetic term moving in a potential having the desired exponential behaviour. The important feature we should emphasise here is that no approximation is needed.

We start from the action

$$S_J = \int d^4x \sqrt{-g} \left[-\frac{1}{2} (M^2 + \xi h^2) R + \frac{1}{2} \frac{\xi h^2}{(M^2 + \xi h^2)} \partial_\mu h \partial^\mu h - \frac{\lambda}{4} (h^2 - v^2)^2 \right]. \quad (4.1)$$

¹The approximation $\frac{M^2}{M_{\text{P}}^2} e^{\omega\varphi} \ll 1$ is the slow-roll condition and corresponds to $h^2 \gg \frac{M^2}{\xi}$ as can be seen from (3.5).

²The constant λ is related to the Higgs mass m_{H} and the Higgs expectation value v by the relation $\lambda = \frac{m_{\text{H}}^2}{2v^2} \simeq \frac{(126)^2}{2 \times (246)^2} \simeq 0.13$.

³Modifications of the scalar kinetic term have been used in the context of attractor cosmology and chaotic inflation. See refs.[17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28] for a selection of articles on the subject.

This means that the gauged kinetic term for the Higgs fields is modified as

$$\frac{2\xi (H^\dagger H)}{[M^2 + 2\xi (H^\dagger H)]} (D_\mu H)^\dagger (D^\mu H) . \quad (4.2)$$

Here D_μ is the usual $SU(2) \times U(1)$ gauge covariant derivative. It is important to notice that in the limit $h^2 \gg \frac{M^2}{\xi}$, the two models (4.1) and (3.2) coincide.

We reach the Einstein frame by rescaling the metric as

$$g_{\mu\nu} \longrightarrow e^{\omega\varphi} g_{\mu\nu} \quad \text{with} \quad (M^2 + \xi h^2) e^{\omega\varphi} = M_{\text{p}}^2 . \quad (4.3)$$

Keeping as our variable the scalar field φ , we arrive at the action (up to a total derivative)

$$S_{\text{E}} = \int d^4x \sqrt{-g} \left\{ -\frac{M_{\text{p}}^2}{2} R + \frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi - \frac{\lambda}{4} \left(\frac{M_{\text{p}}^2}{\xi} \right)^2 \left(1 - e^{\omega\varphi} \right)^2 \right\} . \quad (4.4)$$

The constant ω is fixed by the relation

$$\omega = -\frac{2}{M_{\text{p}}} \sqrt{\frac{\xi}{(1+6\xi)}} \simeq -\sqrt{\frac{2}{3}} \frac{1}{M_{\text{p}}} \quad (\text{for } \xi \gg 1) . \quad (4.5)$$

Again, the relation $M_{\text{p}}^2 = M^2 + \xi v^2$ has been used. The potential of the scalar field φ varies slowly (the slow-roll condition) when $e^{\omega\varphi} \ll 1$. That is, inflation takes place for a large Higgs field h as deduced from $(M^2 + \xi h^2) e^{\omega\varphi} = M_{\text{p}}^2$.

The theory defined by the action (4.4) has been obtained without any approximation. On the contrary, for the inflationary model defined by (3.8) the approximations $\frac{M^2}{M_{\text{p}}^2} e^{\omega\varphi} \ll 1$ and $\xi \gg 1$ are assumed [12, 13].

As usual, the first and second slow-roll parameters are defined by [1, 2, 3, 4, 5, 6]

$$\epsilon = \frac{M_{\text{p}}^2}{2} \left(\frac{V'}{V} \right)^2 , \quad \eta = M_{\text{p}}^2 \frac{V''}{V} , \quad (4.6)$$

where a prime stands for the derivative with respect to φ of the potential

$$V(\varphi) = \frac{\lambda}{4} \left(\frac{M_{\text{p}}^2}{\xi} \right)^2 \left(1 - e^{\omega\varphi} \right)^2 . \quad (4.7)$$

Moreover, the primordial power spectra of scalar perturbation and tensor perturbations generated during inflation are characterised by three observables: the scalar power spectrum amplitude A_s , the scalar spectral index (or tilt) n_s and the tensor-to-scalar power ratio r . In terms of the potential $V(\varphi)$, these inflationary observables are given by (see, for example, [29])

$$A_s = \frac{1}{24\pi^2 M_{\text{p}}^4} \frac{V}{\epsilon} , \quad n_s = 1 + 2\eta - 6\epsilon , \quad r = 16\epsilon . \quad (4.8)$$

Explicitly, we have

$$\begin{aligned}
A_s &= \frac{1}{192\pi^2 M_p^2 \omega^2} \frac{\lambda}{\xi^2} \frac{(1 - e^{\omega\varphi})^4}{e^{2\omega\varphi}} , \\
n_s &= 1 - 4M_p^2 \omega^2 \frac{e^{\omega\varphi} (1 + e^{\omega\varphi})}{(1 - e^{\omega\varphi})^2} , \\
r &= 32M_p^2 \omega^2 \frac{e^{2\omega\varphi}}{(1 - e^{\omega\varphi})^2} .
\end{aligned} \tag{4.9}$$

The quantities A_s , r and n_s are evaluated at $\varphi = \varphi_*$ defined through

$$N_* = \frac{1}{M_p} \int_{\varphi_{\text{end}}}^{\varphi_*} \frac{d\varphi}{\sqrt{2\epsilon}} = \frac{1}{2M_p^2 \omega^2} \left[(e^{-\omega\varphi_*} + \omega\varphi_*) - (e^{-\omega\varphi_{\text{end}}} + \omega\varphi_{\text{end}}) \right] . \tag{4.10}$$

Here N_* is the number of e-folds and φ_{end} is the value of the field at which inflation ends. This is determined by the condition $\epsilon(\varphi_{\text{end}}) = 1$ and is found to be given by

$$\varphi_{\text{end}} = -\frac{1}{\omega} \ln \left(1 - \sqrt{2} M_p \omega \right) . \tag{4.11}$$

If one uses the approximation $M_p \omega \simeq -\sqrt{\frac{2}{3}}$, as given in (4.5), then $\varphi_{\text{end}} \simeq 0.9402 M_p$.

The computation of φ_* follows from (4.10) and we found

$$\begin{aligned}
\omega\varphi_* &= W(-e^{-c}) + c , \\
c &\equiv 2M_p^2 \omega^2 N_* + (e^{-\omega\varphi_{\text{end}}} + \omega\varphi_{\text{end}}) \simeq \frac{4}{3} N_* + 1.3870 .
\end{aligned} \tag{4.12}$$

Here W is the Lambert functions satisfying $W(x)e^{W(x)} = x$. The constant $c \simeq 81.3870$ for $N_* = 60$.

Since in our case the constant c is real and we have $-e^{-1} < -e^{-c} < 0$ then $W(-e^{-c})$ could be either the principal branch W_0 or the branch W_{-1} . Using the defining relation of W one gets

$$e^{\omega\varphi_*} = -\frac{1}{W(-e^{-c})} . \tag{4.13}$$

This is what one needs for the determination of the parameters ϵ , η and A_s given in (4.9). See also [9] for a similar treatment.

As $-e^{-c}$ is of the order of -10^{-36} , the two branches of the Lambert function $W(-e^{-c})$ have, around zero, the expansions [30, 31, 32]

$$\begin{aligned}
W_0(-e^{-c}) &= -e^{-c} - (-e^{-c})^2 + \dots , \\
W_{-1}(-e^{-c}) &= -c - \ln(c) + \dots .
\end{aligned} \tag{4.14}$$

Choosing the branch W_0 leads to an extremely big value for $e^{\omega\varphi_*}$. Therefore, we take $e^{\omega\varphi_*} = -1/W_{-1}(-e^{-c})$ and to first order we have $e^{\omega\varphi_*} \simeq 1/c$.

Substituting $e^{\omega\varphi}$ by approximately $1/c$ in (4.9) and assuming that $c \gg 1$, we find that

$$A_s \simeq \frac{1}{192\pi^2 M_p^2 \omega^2} \frac{\lambda}{\xi^2} c^2 \simeq \frac{1}{72\pi^2} \frac{\lambda}{\xi^2} N_*^2 . \quad (4.15)$$

In the last expression we have taken $c \simeq \frac{4}{3} N_*$. The measured value for A_s is around [33]

$$A_s \simeq 10^{-10} e^{3.094} . \quad (4.16)$$

This fixes the value of the parameter ξ as

$$\xi \simeq 600 c \sqrt{\lambda} \simeq 800 N_* \sqrt{\lambda} . \quad (4.17)$$

Taking $N_* \simeq 60$ [33] and assuming that the Higgs self-coupling is $\lambda = 0.15$, we get the value

$$\xi \simeq 1.86 \times 10^4 . \quad (4.18)$$

In the same way and for $N_* = 60$, the quantities n_s and r are approximately given by

$$\begin{aligned} n_s &\simeq 1 - \frac{2}{N_*} \simeq 0.9667 , \\ r &\simeq \frac{12}{N_*^2} \simeq 0.0033 . \end{aligned} \quad (4.19)$$

These nicely agree with the measured values [33].

Before leaving this section, let us explore how does the field φ couple to the other fields of the Standard Model. In the unitary gauge and according to (4.2), a generic coupling between the Higgs field and any gauge field is of the form⁴

$$\sqrt{-g} \frac{\xi h^2}{(M^2 + \xi h^2)} h^2 A_\mu A_\nu g^{\mu\nu} . \quad (4.20)$$

Under the rescalings $g_{\mu\nu} \rightarrow e^{\omega\varphi} g_{\mu\nu}$, $A_\mu \rightarrow A_\mu$ with $(M^2 + \xi h^2) e^{\omega\varphi} = M_p^2$, this coupling term becomes

$$\sqrt{-g} \frac{\xi h^2}{(M^2 + \xi h^2)} h^2 A_\mu A_\nu g^{\mu\nu} \rightarrow \frac{M_p^2}{\xi} \left(1 - \frac{M^2}{M_p^2} e^{\omega\varphi}\right)^2 \sqrt{-g} A_\mu A_\nu g^{\mu\nu} . \quad (4.21)$$

We see that in the inflationary phase when $\frac{M^2}{M_p^2} e^{\omega\varphi} \ll 1$ a decoupling between the fields φ and A_μ takes place.

The Yukawa terms have not been modified in this model. A generic mass generating Yukawa interaction is still given by

$$\sqrt{-g} h (\bar{\psi}_L \chi_R + \bar{\chi}_R \psi_L) , \quad (4.22)$$

where ψ_L is a left-handed fermion and χ_R is a right-handed fermion. Under the rescalings $g_{\mu\nu} \rightarrow e^{\omega\varphi} g_{\mu\nu}$, $(\psi_L, \chi_R) \rightarrow e^{-\frac{3}{4}\omega\varphi} (\psi_L, \chi_R)$ together with $(M^2 + \xi h^2) e^{\omega\varphi} = M_p^2$, this fermionic coupling term transforms into

$$\sqrt{-g} h (\bar{\psi}_L \chi_R + \bar{\chi}_R \psi_L) \rightarrow \frac{M_p}{\sqrt{\xi}} \left(1 - \frac{M^2}{M_p^2} e^{\omega\varphi}\right)^{\frac{1}{2}} \sqrt{-g} (\bar{\psi}_L \chi_R + \bar{\chi}_R \psi_L) . \quad (4.23)$$

The interaction between the field φ and the fermions is highly suppressed during inflation.

⁴When the Standard Model is coupled to gravity, all the fermionic kinetic terms as well as the gauge kinetic terms are invariant under the local rescalings $g_{\mu\nu} \rightarrow e^{\omega\varphi} g_{\mu\nu}$, $A_\mu \rightarrow A_\mu$, $\psi_{L,R} \rightarrow e^{-\frac{3}{4}\omega\varphi} \psi_{L,R}$. The spacetime dependent Dirac γ^μ matrices are also rescaled as $\gamma^\mu \rightarrow e^{-\frac{1}{2}\omega\varphi} \gamma^\mu$.

5 Higgs-dilaton gravity

Our study could be extended to the Higgs-dilaton models of gravity [34, 35, 36, 37]. For this, we consider the action

$$S_J = \int d^4x \sqrt{-g} \left[-\frac{1}{2} (\tau \sigma^2 + \xi h^2) R + \frac{A}{2} \partial_\mu \sigma \partial^\mu \sigma + \frac{B}{2} \partial_\mu h \partial^\mu h + C \partial_\mu \sigma \partial^\mu h - \frac{\lambda}{4} (h^2 - \alpha \sigma^2)^2 - \beta \sigma^4 \right]. \quad (5.1)$$

The quantities A , B and C are functions of the two scalar fields σ and h . The Higgs-dilaton model correspond to $A = B = 1$ and $C = 0$ [34, 35, 36, 37].

As usual, we rescale the metric as

$$g_{\mu\nu} \longrightarrow e^{\omega\varphi} g_{\mu\nu} \quad \text{with} \quad (\tau \sigma^2 + \xi h^2) e^{\omega\varphi} = M_{\text{p}}^2. \quad (5.2)$$

The aim now is to find the functions A , B and C such that two of the scalar fields have canonical kinetic terms and their corresponding potential is simple. As the three fields σ , h and φ are related by the last relation, we choose to keep the two fields σ and φ . The functions A , B and C that fulfill these requirements are

$$\begin{aligned} A &= \frac{\tau \sigma^2 + \xi h^2}{M_{\text{p}}^2} + \frac{4\tau^2 (1 - \frac{2}{3}\omega^2 M_{\text{p}}^2)}{\omega^2 M_{\text{p}}^2} \frac{\sigma^2}{\tau \sigma^2 + \xi h^2}, \\ B &= \frac{4\xi^2 (1 - \frac{2}{3}\omega^2 M_{\text{p}}^2)}{\omega^2 M_{\text{p}}^2} \frac{h^2}{\tau \sigma^2 + \xi h^2}, \\ C &= \frac{4\tau \xi (1 - \frac{2}{3}\omega^2 M_{\text{p}}^2)}{\omega^2 M_{\text{p}}^2} \frac{\sigma h}{\tau \sigma^2 + \xi h^2}. \end{aligned} \quad (5.3)$$

With these expression for A , B and C , the above rescaling of the metric yields the action

$$S_E = \int d^4x \sqrt{-g} \left[-\frac{M_{\text{p}}^2}{2} R + \frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma + \frac{1}{2} \partial_\mu \varphi \partial^\mu \varphi - \frac{\lambda}{4} \left(\frac{M_{\text{p}}^2}{\xi} \right)^2 \left[1 - \frac{\xi}{M_{\text{p}}^2} \left(\alpha + \frac{\tau}{\xi} \right) \sigma^2 e^{\omega\varphi} \right]^2 - \beta (\sigma^2 e^{\omega\varphi})^2 \right]. \quad (5.4)$$

Although the first term in the expression of A breaks the global scale invariance, the resulting action in the Einstein frame is simple. It is appealing to investigate the cosmological consequences of this theory along the lines of ref.[38]. We will report on this project elsewhere. It is also worth treating the case $A = B = 1$ and $C = \text{constant}$, which preserves the global scale invariance.

6 Conclusions

There is a freedom when it comes to coupling the Standard Model of particle physics to gravity. Since the inclusion of gravity leads to non-renormalisability of the theory, the

Standard Model coupled to gravity might therefore be viewed as an effective theory. In this note we have exploited this freedom and proposed a viable effective theory for describing the inflationary regime of the early Universe. We have modified the model, advocated in ref.[12], where the Higgs field is non-minimally coupled to gravity. The guiding principle to this modification is that it results in *i*) a canonical kinetic term for the inflaton field and *ii*) a simple slow-roll potential for this field. We have indeed succeeded in meeting these two requirements without any approximation. This is achieved by modifying the Higgs kinetic term. Our model is therefore a completion of the effective theory proposed in [12]. It totally agrees with its cosmological findings as far as inflation is concerned. It is also in accordance with the predictions of the Starobinsky model of inflation (R^2 -inflation) since one goes from the theory in (2.4) to the one in (4.4) by the simple identification $3M_{\text{S}}^2 = \frac{\lambda}{\xi^2}$.

A natural question⁵ arises now of how to distinguish between the theory of Higgs inflation with a canonical kinetic term [12] from the model presented in this note? There is in fact a clear signature that could differentiate the two models. It resides in the way that the inflaton field φ couples to the other fields of the Standard Model. In Higgs inflation with the canonical kinetic term $(D_\mu H)^\dagger (D^\mu H)$, a typical coupling between the field φ and any gauge field A_μ is, in the unitary gauge, of the form

$$\sqrt{-g} h^2 A_\mu A_\nu g^{\mu\nu} \longrightarrow \frac{M_{\text{P}}^2}{\xi} \left(1 - \frac{M^2}{M_{\text{P}}^2} e^{\omega\varphi} \right) \sqrt{-g} A_\mu A_\nu g^{\mu\nu} . \quad (6.1)$$

The second expression is obtained after the rescaling of the metric. This is to be compared to (4.21) where the factor $\left(1 - \frac{M^2}{M_{\text{P}}^2} e^{\omega\varphi} \right)$ is to the power 2. The coupling of the field φ to the fermions is the same in both models and is as given in (4.23).

The coupling of the inflaton field φ to the massive gauge fields (the gauge bosons W^+ , W^- and Z) and to the fermions is relevant to the physics of the preheating that follows inflation. The details of the preheating mechanism in the Standard Model non-minimally coupled to gravity could be found in refs.[39, 40] with a summary in [13]. As a matter of fact it is precisely this preheating which distinguishes Higgs inflation from R^2 -inflation [41, 42, 43].

After the end of inflation (around $\varphi = \varphi_{\text{end}} \simeq 0.9402M_{\text{P}}$) and for small values of the inflaton field φ , the potential is expanded as

$$V(\varphi) = \frac{\lambda}{4} \left(\frac{M_{\text{P}}^2}{\xi} \right)^2 \left[\omega^2 \varphi^2 + \omega^3 \varphi^3 + \frac{7}{12} \omega^4 \varphi^4 + \dots \right] . \quad (6.2)$$

To first approximation, only the quadratic term is maintained and the inflaton field φ inevitably oscillates around the minimum of the potential. This oscillatory energy is used to create the heavy particles of the Standard Model which would then decay into light particles [39, 40]. In a cosmological context, the field φ is a function of time. This implies that the masses would also depend on time as can be seen from (4.21) and (4.23). To first order, the

⁵I thank an anonymous referee for insisting on this important point.

masses of the gauge bosons in our model are

$$\begin{aligned}
M_{W^\pm} &= \frac{g_2}{2} \frac{M_p}{\sqrt{\xi}} \left(1 - \frac{M^2}{M_p^2} e^{\omega\varphi} \right) \simeq \frac{g_2}{2} \frac{M_p}{\sqrt{\xi}} \left(\frac{\xi v^2}{M_p^2} - \frac{M^2}{M_p^2} \omega\varphi \right) , \\
M_Z &= \frac{1}{2} \sqrt{g_1^2 + g_2^2} \frac{M_p}{\sqrt{\xi}} \left(1 - \frac{M^2}{M_p^2} e^{\omega\varphi} \right) \simeq \frac{1}{2} \sqrt{g_1^2 + g_2^2} \frac{M_p}{\sqrt{\xi}} \left(\frac{\xi v^2}{M_p^2} - \frac{M^2}{M_p^2} \omega\varphi \right) . \quad (6.3)
\end{aligned}$$

Here g_1 and g_2 are the gauge coupling constants associated to the gauge groups $U(1)$ and $SU(2)$, respectively. Of course, the quantity $\frac{\xi v^2}{M_p^2}$ is negligible. On the other hand, the gauge masses of ref.[12] are given by the same expressions but with the replacement $\left(\frac{\xi v^2}{M_p^2} - \frac{M^2}{M_p^2} \omega\varphi \right) \rightarrow \left(\frac{\xi v^2}{M_p^2} - \frac{M^2}{M_p^2} \omega\varphi \right)^{1/2}$. We would expect this difference to show up in the preheating regime.

The evolution of the Universe after inflation (the preheating) is of importance in the precise determination of the number of e-foldings N_* . This in turn leads to a better estimates of the scalar spectral index n_s and the tensor-to-scalar power ratio r . The complete analyses of the preheating scenario (following the steps of [39, 40]) of the model presented here is out of the scope of this note and we hope to treat it in the near future.

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