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A micro-mechanical model for the fibrous tissues of vocal folds

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Abstract

Composed of collagen, elastin and muscular fibrous networks, vocal folds are soft laryngeal multi-layered tissues owning remarkable vibro-mechanical performances. However, the impact of their histological features on their overall mechanical properties still remains elusive. Thereby, this study presents a micro-mechanical hyperelastic model able to describe the 3D fibrous architecture and the surrounding matrices of the vocal-fold sublayers, and to predict their mechanical behavior. For each layer, the model parameters were identified using available histo-mechanical data, including their quasi-static response for key physiological loading paths, i.e., longitudinal tension, transverse compression and longitudinal shear. Regardless of the loading path, it is shown how macroscale nonlinear, anisotropic tissue responses are inherited from the fiber scale. Scenarios of micro-mechanisms are predicted, highlighting the major role of 3D fiber orientation in tension, steric hindrance in compression, and matrix contribution in shear. Finally, combining these predictions to vibrating hyperelastic Timoshenko beam’s theory, the impact of the fibrous architecture of the upper layers on vocal-fold vibratory properties is emphasized.

Keywords:
Vocal folds, 3D fibrous architecture, Micro-mechanical model, Anisotropy, Multiaxial loading, Vibratory properties

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1. Introduction

Human vocal folds possess a complex lamellar structure with two principal layers: the lamina propria, i.e., a loose connective tissue, and the vocalis or inferior thyroarytenoid muscle. Each layer is a soft material with architectured networks of collagen, elastin and/or skeletal muscle fibers (Fig. 1; [44, 14, 3]). Clinical observations clearly support the major role played by such fibrous microstructure in the vocal-fold vibrations: in cases of benign or cancerous lesions, alterations of the fiber-scale arrangement of the lamina propria systematically induce a vibratory dysfunction [27, 28, 41]; with aging, a loss of elastin fibers, fibrosis or muscle atrophy together with vocal and perceptual changes such as hoarseness, low pitch and breathiness has also been observed [89, 88, 84]. However, to date, the acquired knowledge is still not sufficient to understand the relationship between the microstructural specifications of vocal folds and their macroscale performances.

This is mainly ascribed to their challenging experimental multiscale characterization. Despite the considerable progress made in 3D micro-imaging [43, 52, 18, 73, 51, 32, 3, 48], vocal folds, along with their fibrous architectures, are hardly observable in vivo [71, 23]. Although a large biomechanical database has been collected on excised vocal folds over the last twenty years [17, 87, 49, 74, 16, 20], the 3D microscale rearrangement of the loaded tissues is still to be explored. Conversely, the development of macroscopic (tissue scale) or micro-mechanical (fiber scale) models of phonation is a promising alternative to gain an in-depth understanding of the vocal-fold biomechanics:
Figure 1: Human vocal-fold histology. (left) Idealised scheme of one fold, with focus on the fold sublayers and fibrous microstructure: ① Epithelium, ② Lamina propria, ③ Vocalis muscle. (right) Corresponding 2D histological photomicrographs prepared with HES staining: collagen fibers (yellow-orange); cytoplasms, striated muscular and elastin fibers (pink); nuclei (black-purple). Adapted from [3].

• In macroscopic approaches, phenomenological exponential and power-law functions are commonly proposed to describe the stress-strain responses typically observed when deforming soft biological tissues [6, 45, 46, 58, 59, 72]. Doing so, however, the model parameters can hardly be related to the material intrinsic structure and mechanics, and need to be adjusted according to the applied loading path. This first approach is robust and adequate in absence of tissue histo-mechanical data.

• Microstructure-based formulations are instead inspired from histological evidence, and conceived to correlate the model input parameters to the physical and structural properties of the tissue (e.g., cells, fibers and surrounding matrix). To name a few, the shape, concentration, orientation and tortuosity of fibers are the relevant structural parameters that are commonly considered. Therefrom, to determine the macroscale mechanical behavior of the tissue, homogenization techniques [61, 62, 63, 67, 69, 68, 78, 4], energetic approaches [75, 77, 79, 82, 81], statistical descriptions [18, 33, 56, 47] or variational considerations [22, 31, 54, 66, 70] are used. Regardless, the
identification and validation of these formulations with multiscale experimental data remains a challenging task.

By contrast with many other soft tissues (e.g., arteries, heart, skin), the majority of the theoretical approaches adopted to model the vocal-fold mechanical properties still rely on macroscopic formulations. Since 2010, a few authors have proposed micro-mechanical models for the vocal-fold tissues, opening a new insight into voice biomechanics [49, 75]. These models allow to predict the tensile behavior of the lamina propria, but their relevance was not assessed for other important biomechanical loadings such as transverse compression and longitudinal shear [37, 92, 97]. Furthermore, theoretical formulations still need: (i) to be fed up with 3D microstructural descriptors of human vocal-fold sublayers and to account for the fiber-to-fiber mechanical interactions likely to occur within such dense media [26], (ii) to be extended to the specific micro-arrangement of the vocalis muscle.

Within this context, the present work proposes a micro-mechanical model able to reproduce the nonlinear anisotropic mechanical properties of vocal-fold layers (i.e., lamina propria, vocalis) subjected to multiaxial finite strains, from the knowledge of their 3D fibrous architecture. It combines the use of ex vivo database acquired on human vocal-fold microstructures over the past ten years, with a recent study on their finite strain macroscale mechanics in longitudinal tension, as well as transverse compression and longitudinal shear [20]. The paper is structured as follows. Section 2 introduces an improvement of the theoretical formulation firstly proposed and validated in the context of vascular biomechanics [4, 5]. The model identification procedure is described in Section 3. Section 4 presents the vocal-fold multiscale predictions and a micro-parametrical study aimed to investigate the effect of the tissue’s 3D fibrous orientation changes on its vibro-mechanical response.

2. Micro-mechanical model

2.1. Experimental observations and assumptions

In line with histological evidence (Fig. 1; [3, 64, 40, 50, 75]), both the lamina propria and the vocalis can be conceived as 3D incompressible composite structures made of a gel-like matrix reinforced by a network of fibers. Furthermore, each fiber can in turn be seen as a bundle of quasi-aligned (myo)fibrils with wavy shapes and preferred orientations at rest:
• The *lamina propria* is made of cells and an extracellular matrix (ECM) comprising amorphous ground substances (*e.g.*, hyaluronic acid), entangled fibrous networks of collagen (mainly Type I and III) and elastin [73]. Collagen plays a key role in the mechanics of soft tissues and is, by weight, the most abundant fibrous protein in the human *lamina propria*, representing at least 50% of the total proteins (less than 10% for elastin [38, 39, 93]). The *lamina propria* is finally known to be arranged in three sublayers with distinct fibers’ type, density and arrangement, albeit very challenging to model due to the lack of available quantitative topological descriptors. In the following, the multilayered tissue of the lamina propria is therefore simplified to a one-layered structure. The fibrous network of its ECM is assumed to include a single population of collagen fibers, *i.e.*, fiber bundles of collagen fibrils, embedded into a surrounding matrix gathering the other neighboring tissue components (cells, elastin, ground substances).

• The *vocalis* is primarily made of muscle fibers (also called “muscle cells” or “rhabdomyocytes”), grouped into fiber bundles (or *fasciculi*) and wrapped together by connective tissue sheaths (Fig. 1; [3]). This muscle ECM is dominated by collagen in terms of mass, and is organized into three interconnected levels: the *epimysium*, surrounding the whole muscles, the *perimysium*, surrounding fascicles, and the *endomysium*, surrounding individual muscle fibers [10, 98]. Collagen has been recently shown to be a major load-bearing component in the finite strain passive response of skeletal muscles [98]. Therefore, in the following, each individual muscle fiber is conceived as a myofibrils bundle surrounded by a sheath of collagen fibers (*i.e.*, bundles of collagen fibrils). The other constituents of the skeletal muscle ECM (elastin, proteoglycans, glycoproteins) will constitute the matrix of the micro-mechanical model [24].

2.2. Idealization of the vocal-fold layers’ fibrous architectures

Regardless of the considered vocal-fold layer (*i.e.*, *lamina propria* or *vocalis*), its microstructure is idealized by the periodic repetition of a Representative Elementary Volume (REV) inspired from that already proposed for rubber-like materials [1, 11, 7] or self-entangled superelastic wires [86], as sketched in Fig. 2a. In the undeformed configuration $C_0$ (resp. deformed
Figure 2: Idealised geometry of the vocal-fold sublayers. a) REV in the undeformed configuration $C_0$: one node of periodicity $C_0$ (blue) and a 4-bar truss (brown) embedded in a soft isotropic matrix (green). The dotted lines illustrate the possible steric interactions of $C_0$ with the neighboring nodes. b) Correspondence fiber-bar for i) lamina propria and ii) vocalis, conceived as fiber bundles of collagen (orange) and/or muscular fibrils (pink).
configuration \( C \), the REVs can be seen as prisms with a truss of \( N = 4 \) bars, of identical initial length \( \ell_0 \) (resp. actual lengths \( \ell_i \)), embedded in a matrix and connected to a central node \( C_0 \) (resp. \( c_0 \)) and to the nodes \( C_i \) (resp. \( c_i \)), \( i \in [1, \ldots, N] \) of corresponding neighboring REVs at their extremities. The initial (resp. actual) orientation of each bar \( i \) is denoted by the direction of its unit vector \( E_i = C_0C_i / \|C_0C_i\| = \sin \theta_{0i} \cos \phi_{0i} e_x + \sin \theta_{0i} \sin \phi_{0i} e_y + \cos \theta_{0i} e_z \) (resp. \( e_i = c_0c_i / \|c_0c_i\| = \sin \theta_i \cos \phi_i e_x + \sin \theta_i \sin \phi_i e_y + \cos \phi_i e_z \)) where \( e_x, e_y \) and \( e_z \) lie along the medio-lateral, infero-superior and antero-posterior anatomical directions (Fig. 1). In the following, considering the typical orthogonal symmetry planes previously found in the vocal-fold fibrous architectures at rest [49, 3], we further assume that the initial angles of the fibrous networks do not depend on the bar \( i \): \( \forall i, \theta_{0i} = \theta_0 \) and \( \phi_{0i} = \pm \phi_0 \mp k\pi, k \in [0, 1] \).

Therefrom, as illustrated in Fig. 2b, each bar \( i \) represents the chord of a wavy (collagen or muscle) fiber bundle \( i \). Additionally, each fiber bundle \( i \) is considered as an assembly of \( n \) identical and parallel fibrils of equal waviness and length. Furthermore, each fibril \( j \) is defined by its initial diameter \( d_0 \), length \( \ell_{0j} \) (resp. actual \( \ell_j \)), and tortuosity \( \zeta_0 = \ell_0^f / \ell_0 \) (resp. \( \zeta_j = \ell_j^f / \ell_j \)). A monomodal sinusoidal function is used to describe each fibril’s initial waveform of amplitude \( R_0 \) and spatial periodicity \( H_0 \) [22, 55, 75], i.e., with \( v(u) = R_0 \sin \left( \frac{2\pi}{H_0} u \right) \) in the reference frame \((u_j, v_j)\) of fibril \( j \) sketched in Fig. 2, where \( u \) and \( v \) are the abscissa and ordinate in \((u_j, v_j)\), respectively. We assumed the bars to contain 10 typical sinusoidal periods between nodes. This arbitrary choice is not a restriction. Therewith:

- In the lamina propria (Fig. 2b-i), the volume fraction of fibrils in the REV is \( \Phi = V_f / V_{REV} \), where \( V_{REV} = 4\ell_0^3 \sin^2 \theta_0 \cos \theta_0 \sin \phi_0 \) is the volume of the REV and where \( V_f = \pi Nn(\ell_0^f / 4\ell_0) \int_0^{\ell_0} \sqrt{1 + \left( \frac{2\pi R_0}{H_0} \cos \left( \frac{2\pi}{H_0} u \right) \right)^2} \) du.

- In the vocalis (Fig. 2b-ii), even if collagen and muscle fibers are referred to the same bar (with the same mean orientation \((\theta_i, \phi_i)\) ), each family of fibrils is characterized by distinct geometrical parameters, labeled with subscript \( c \) (resp. \( m \)) for collagen (resp. muscular) fibrils. Thus the volume fraction of fibrils is decomposed as follows: \( \Phi = \Phi_c + \Phi_m \), where \( \Phi_c \) and \( \Phi_m \) are the volume fractions of collagen
and myofibrils, respectively, with waveform parameters $H_{0k}, R_{0k}$ and diameters $d_{0k}, k \in \{c, m\}$.

2.3. Micro-mechanical behavior of the constitutive materials

Matrix – Independently of the vocal-fold sublayer, the gel-like matrix is conceived as a soft isotropic, hyperelastic and incompressible material, as a first reasonable approximation. The corresponding mechanical behavior is described by a simple neo-Hookean model [96], characterized by a volumetric strain-energy function $W = 0.5 \mu (1 - \Phi) (I_1 - 3)$, where $\mu$ is the shear modulus of the matrix, and $I_1 = \text{tr}(B)$ where $B = F \cdot F^T$ ($F$ being the macroscopic transformation gradient).

Fibrils

Figure 3: (a) Strain-tension curve at the fibril scale. (b) Strain-repulsion force curve modeling fiber-to-fiber interactions.

Tension – As sketched in Fig. 3 (a), when stretched with a strain $\varepsilon_j = \ln \frac{\ell_j}{\ell_0}$, the mechanics of wavy fibrils is firstly dominated by their progressive unfolding (regimes (i) and (ii)), up to a critical strain $\varepsilon_c = \ln \xi_0$ once fully unfolded ($\ell_j = \ell^f_0$). Past this threshold, fibrils behave as straight elastic rods showing a quasi-linear tensile response (regime (iii)) with a Young
modulus $E_f$ [21, 34]. These two regimes, as well as the transition in between, are well described by the following constitutive relation [80]:

$$t_j = \frac{\pi d_0^2}{4} \left[ E_{eq0} \varepsilon_j + \frac{E_f - E_{eq0}}{2} \left( \varepsilon_j + \sqrt{(\varepsilon_j - \ln \xi_0)^2 + \alpha^2} - \sqrt{\ln^2 \xi_0 + \alpha^2} \right) \right] e_i$$

(1)

where $t_j$ is the tension along the fibril $j$. In this last equation, $\alpha$ is a parameter that ensures a smooth transition between the two aforementioned regimes. Additionally, $E_{eq0}$ is the initial tangent modulus measured on $(t_j, \varepsilon_j)$ curves. Its expression can be analytically obtained [83]:

$$E_{eq0} = E_f \langle \cos \beta_0 \rangle / \left[ \langle \cos^2 \beta_0 \rangle + 16 \langle v^2 \rangle / d_0^2 \right]$$

(2)

where $\langle \cdot \rangle = \frac{1}{\ell_0} \int_0^{\ell_0} \cdot du$, $\beta_0 = \arctan \left( 2\pi \frac{R_0}{\ell_0} \cos \frac{2\pi}{\ell_0} u \right)$ and $\langle v^2 \rangle = R_0^2 / 2$.

In the particular case of the vocalis (see Fig. 2b-ii), note that the tension force $t_{jk}$ within each type of fibrillar networks (collagen or muscular) is characterized by distinct fibril’s Young modulus $E_{fk}$ and toe-region parameter $\alpha_k$.

**Compression** – The stiffening regime (ii) - (iii) occurring in tension is not prone to occur during the compression of fibrils. Instead, fibrils should rather increase their crimping. To account for this tension-compression asymmetry, from Eq. (1), we simply assumed that fibrils compression follows the same tendency detailed in tension during regime (i) solely:

$$t_j = \frac{\pi d_0^2}{4} E_{eq0} \varepsilon_j e_j$$

(3)

**Fibers** – Between nodes, fiber bundles can be seen as wavy beams of parallel fibrils (Fig. 2b), whose transverse shear interactions are significantly weaker than their longitudinal tensile behavior. Therefrom, the tension-compression force $T_i$ in the fiber is simply expressed (with $N = 4$) as:

$$T_i = \sum_{j=1}^n t_j = n t_i = \frac{\Phi V_{REV}}{\pi d_0^2 \beta_0} t_i$$

(4)
In the case of two fibrils families (vocalis), this generic equation comes:

\[ T_i = \frac{V_{REV}}{\pi} \left( \frac{\Phi_c}{d_{0c}^2 \ell_{0c}^2} t_{1c} + \frac{\Phi_m}{d_{0m}^2 \ell_{0m}^2} t_{1m} \right) \]  

(5)

Fiber-to-fiber interactions: steric hindrance – When fibrous networks are subjected to mechanical loading, the resulting deformation does not solely arise from the (un)folding of fiber bundles, which may also rotate and get closer. These motions are physically constrained by steric hindrance, so that they generate fiber-to-fiber interactions [26]. These constraints are not taken into account in the standard eight chains model [1] whereas they should alter the REV deformation micro-mechanisms and thus its macroscale properties. Therefore, in this first approach, we added repulsive forces between neighbored unconnected nodes of the truss to take into account such steric interaction forces. More precisely, in the deformed configuration \( C \), once the relative distance \( \delta_q \) between the two unconnected nodes \( C_0 \) and \( C_q \) (\( q \neq 0 \)) exceeds a critical value \( \delta_c \), a repulsive force is activated to mimic the contact interactions between the concerned fibers. By periodicity, this comes to mimic the interactions between \( C_0 \) and its 5 neighboring nodes, as illustrated in Fig. 2 (a) (see dotted lines). The resulting \( M = 5 \) fiber-to-fiber interaction forces are noted \( R_q \) and expressed using the power-law-based function [85, 91]:

\[ R_q = R_q e_q = \beta H(\varepsilon_q) \varepsilon_q^{\varsigma} e_q \]

(6)

where \( \varepsilon_q = \ln \left( \frac{\delta_q}{\delta_c} \right) \), \( e_q = c_q c_0 / \| c_q c_0 \| \), \( H(\cdot) \) is the Heaviside function, and where \( \beta \) and \( \varsigma \) are interaction coefficients. Over small deformations and linear elasticity, \( \varsigma \) should be equal to 2 in accordance with the Hertzian contact theory [85, 91]. In the present study, dealing with hyperelastic bars and large transformations, \( \varsigma \) was not fixed. A typical strain-repulsion force curve is shown in Fig. 3(b).

In the case of the vocalis, only one coefficient of interaction \( \beta \) and one length of interaction \( \delta_c \) are defined for the two families of fibrils since steric interactions are considered at the fiber scale solely.

2.4. Macro-mechanical behavior of the overall composites

Using the approach developed for aortic tissues [4, 5], the macroscale mechanical behavior of the lamina propria and the vocalis tissues can be
determined. Regardless of the considered layer, its macroscopic Cauchy stress tensor $\sigma$ is expressed as:

$$\sigma = -p\delta + \sigma_m + \sigma_f + \sigma_s$$  \hspace{1cm} (7)

where $p$ is the incompressibility pressure, $\delta$ the identity tensor, $\sigma_m$ and $\sigma_f$ represent the stress contribution of the matrix and the fibrous network, respectively, and where the stress contribution $\sigma_s$ is induced by steric interactions.

**Matrix** – Under the previous assumptions, $\sigma_m$ can be written as:

$$\sigma_m = F \cdot \left( \frac{\partial W}{\partial F} \right)^T$$  \hspace{1cm} (8)

**Overall fibrous network** – Using the homogenization method for trusses of hyperelastic bars [15], the bar $\sigma_f$ and the steric $\sigma_s$ tensors can be expressed as:

$$\sigma_f = \frac{1}{V_{REV}} \sum_{i=1}^{N} T_i \otimes \ell_i e_i = \frac{\Phi}{\pi d_0^2 \xi_0^2} \sum_{i=1}^{N} t_i \lambda_i e_i \otimes e_i$$  \hspace{1cm} (9)

and

$$\sigma_s = \frac{\Phi}{\pi d_0^2 \xi_0^2} \sum_{q=1}^{M} R_q \delta_q^* e_q \otimes e_q$$  \hspace{1cm} (10)

for the lamina propria, with $\lambda_i = \frac{\ell_i}{\ell_0}$ and $\delta_q^* = \delta_q / \ell_0$, and:

$$\sigma_f = \frac{\Phi_c}{\pi d_0^2 \xi_0^2} \sum_{i=1}^{N} t_i \lambda_i e_i \otimes e_i + \frac{\Phi_m}{\pi d_0^2 \xi_0^2} \sum_{i=1}^{N} t_{im} \lambda_i e_i \otimes e_i$$  \hspace{1cm} (11)

and

$$\sigma_s = \left( \frac{\Phi_c}{\pi d_0^2 \xi_0^2} + \frac{\Phi_m}{\pi d_0^2 \xi_0^2} \right) \sum_{q=1}^{M} R_q \delta_q^* e_q \otimes e_q$$  \hspace{1cm} (12)

for the vocalis. Thus, the overall response of the lamina propria (resp. vocalis) depends on 12 (resp. 18) input parameters to be determined at rest:
6 (resp. 10) histological parameters: the fibrils diameter $d_0$ (resp. $d_{0k}$), their waviness amplitude $R_0$ (resp. $R_{0k}$), spatial periodicity $H_0$ (resp. $H_{0k}$) from which the tortuosity $\xi_0$ (resp. $\xi_{0k}$) can be estimated, the fibrils volume fraction $\Phi$ (resp. $\Phi_k$) and initial 3D fiber orientation $(\theta_0, \varphi_0)$.

6 (resp. 8) mechanical parameters: the fibrils Young’s modulus $E_f$ (resp. $E_{fk}$), the matrix shear modulus $\mu$, the transition parameter $\alpha$ (resp. $\alpha_k$) and the interaction coefficients $\beta$, $\varsigma$ and $\delta_c$ related to the steric effects.

3. Model identification

The identification of the histo-mechanical parameters of the model was performed by adjusting its predictions to biomechanical data recently acquired on human vocal-fold tissues [20]. To do so, a representative set of lamina propria and vocalis sublayers (hereinafter noted as LP$_i$ and V$_i$, $i \in [1, 2]$) was selected from the reported database. Each sample was sequentially subjected to a series of finite-strain and cyclic physiological loadings, i.e., longitudinal tension, transverse compression, and longitudinal shear. Only the first load-unload sequence of each test was considered, as displayed in Fig. 4 for samples (LP$_1$, V$_1$), and Supplementary Fig. S1 for samples (LP$_2$, V$_2$).

3.1. Simulated mechanical tests

To reproduce each experimental loading path, the REVs were subjected to the following loading conditions:

- Simple tension along the longitudinal direction $e_z$ of the vocal folds, i.e., with $F = F_{xx} e_x \otimes e_x + F_{yy} e_y \otimes e_y + F_{zz} e_z \otimes e_z$ and $\sigma = \sigma_{zz} e_z \otimes e_z$. The component $F_{zz}$ was controlled, whereas $F_{yy}$ was computed to ensure the transverse stress-free state condition $\sigma_{xx} = \sigma_{yy} = 0$. The component $F_{xx} = 1/F_{yy} F_{zz}$ was determined by the incompressibility condition. The hybrid conditions also allowed the pressure $p$ to be determined.

- A similar procedure was employed for simple compression along the transverse direction $e_x$. 

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• To simulate simple shear in the “longitudinal” plane \((e_z, e_x)\), the REVs were subjected to \(F = \delta + \gamma_{zx} e_z \otimes e_x\) and \(p = 0\), where \(\gamma_{zx}\) is the imposed shear strain.

Regardless of the loading path, the stress predictions of the model were expressed using the first Piola-Kirchoff stress tensor \(P = \sigma \cdot F^{-T}\).

3.2. Optimization procedure

In order to fit macroscale stress-strain responses, the following protocol was adopted to obtain optimized sets of histo-mechanical parameters:

(i) As far as possible, all input histological parameters were determined from microstructural analyses collected on the unloaded samples and observed using 2D standard optical microscopy [20]. Alternatively, the remaining histo-mechanical parameters were initialized and bounded within a range of physiological values determined from the literature.

(ii) A least-squared approach was used to minimize the discrepancies between theoretical and experimental stress tensors. To do so, a non-linear constraint optimization process was applied, as in [4]. For each tested sample, this procedure accounts for the three mechanical loading conditions the sample was subjected to. Furthermore, as the proposed model is purely hyperelastic, it cannot reproduce the hysteresis observed experimentally [20]. The optimization procedure was accordingly adjusted to experimental “neutral” stress-strain curves, lying in between the loading and unloading paths.

4. Results and discussion

4.1. Histo-mechanical parameters: choice of initial guesses and optimized values

In order to obtain the optimized histological parameters reported in Tables 1 and 2 for the lamina propria samples and in Tables 3 and 4 for the vocalis ones, initial guess corridors discussed hereafter were initially used.
Table 1: Set of histological parameters identified for lamina propria samples, LP. Gray-colored columns refer to quantities computed as a function of the determined histological parameters.

<table>
<thead>
<tr>
<th>Sample</th>
<th>( \theta_0 (\degree) )</th>
<th>( \varphi_0 (\degree) )</th>
<th>( H_0 (\mu m) )</th>
<th>( R_0 (\mu m) )</th>
<th>( d_0 (\mu m) )</th>
<th>( \Phi )</th>
<th>( \xi_0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>LP_1</td>
<td>16</td>
<td>83</td>
<td>42</td>
<td>5</td>
<td>0.4</td>
<td>0.46</td>
<td>1.129</td>
</tr>
<tr>
<td>LP_2</td>
<td>16</td>
<td>83</td>
<td>42.5</td>
<td>5</td>
<td>0.4</td>
<td>0.48</td>
<td>1.126</td>
</tr>
</tbody>
</table>

Table 2: Set of mechanical parameters identified for lamina propria samples, LP.

<table>
<thead>
<tr>
<th>Sample</th>
<th>( E_f ) (MPa)</th>
<th>( \mu ) (Pa)</th>
<th>( \alpha )</th>
<th>( \beta ) (N)</th>
<th>( \varsigma )</th>
<th>( \delta_c (\mu m) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>LP_1</td>
<td>847</td>
<td>330</td>
<td>( 4.4 \times 10^{-3} )</td>
<td>( 2 \times 10^{-4} )</td>
<td>3</td>
<td>66</td>
</tr>
<tr>
<td>LP_2</td>
<td>847</td>
<td>290</td>
<td>( 4.3 \times 10^{-3} )</td>
<td>( 4 \times 10^{-4} )</td>
<td>3</td>
<td>65.7</td>
</tr>
</tbody>
</table>

4.1.1. Lamina propria

Collagen volume fraction \( \Phi \): According to the literature, \( \Phi \) is reported to vary between 0.15 and 0.55 depending on the tissue depth \( e_x \) [14, 39, 75, 93]. Note that the upper limit may appear underestimated when analyzing the micrographs of the chosen samples LP zoomed in their deepest sublayers (Fig. 1), but it is relevant when averaged over the whole sample thickness. Within this range of admissible values, the optimization led to \( \Phi \approx 0.47 \) for both LP samples.

Collagen fibril’s diameter \( d_0 \): \( d_0 \) is known to range between 10 nm (collagen Type III) and 500 nm (Type I) [2, 30, 34, 35, 101]. By imposing these physiological boundaries, the identification procedure conducted to \( d_0 = 400 \) nm for each sample.

Fibril’s sine waveform parameters \( (H_0 ; R_0) \): The spatial period and amplitude of wavy collagen fibrils at rest were bounded within the corridors \( (10–70 \) \( \mu m \); \( 1–10 \) \( \mu m \)) respectively [3, 75]. The optimization process led to similar values for both samples, close to \( (42 \) \( \mu m \); \( 5 \) \( \mu m \)), implying an initial tortuosity \( \xi_0 \) of about 1.13.

Network 3D orientation \( (\theta_0 ; \varphi_0) \): The 3D angular distribution of collagen fibrils in the lamina propria was extrapolated from recent 3D CT images obtained on a single unloaded sample [3] for which \( (\theta_0 ; \varphi_0) = (30^\circ ; 39^\circ) \), showing a pronounced preferred orientation along \( e_z \) and a slight orthotropy in the perpendicular plane. To account for the inter-sample variability, previous values were let free to vary within
the range (0–50°; 20–90°). The optimization process led to \((\theta_0 ; \varphi_0) \approx (16° ; 83°)\) for both samples, a nearly 2D network in the plane \((\mathbf{e}_z, \mathbf{e}_y)\).

It should be noticed that a quasi-plane network \((i.e., \theta_0 \leq 35° \text{ and } \varphi_0 \geq 80°)\) was required by the model to properly reproduce the lamina propria’s shear response.

**Collagen fibril’s Young modulus** \(E_f\): Although very challenging, the mechanical properties of a single collagen (Type I) fibril have already been investigated using Atomic Force Microscopy, Micro Electro Mechanical Systems technology and X-ray diffraction [13, 34, 60, 90, 101]. In aqueous media, the tangent modulus measured in longitudinal tension, \(E_f\), is ranging from 1 MPa at small strains up to an asymptotic value of 1 GPa at finite strains, where a linear stress-strain regime is achieved. Imposing these boundaries, the optimization process yielded to \(E_f \approx 850\) MPa for both LPi samples.

**Matrix shear modulus** \(\mu\): This parameter was first initialized by the shear modulus of hyaluronic acid \(\mu_{HA} \approx 20-50\) Pa [42], \(i.e.,\) the major and most abundant component of the ground substance within the lamina propria [17, 29, 27, 28, 36, 38, 40, 42]. In order to account for other components within the matrix \((e.g.,\) cells, elastin\)), \(\mu\) was let free to vary up to 1.5 MPa, \(i.e.,\) the estimated Young’s modulus of isolated elastin fibers [100, 101]. The optimization finally yielded to \(\mu \approx \mathcal{O}(10^2)\) Pa for both samples.

**Transition parameter** \(\alpha\): Initially not fixed, the optimization yielded to \(\alpha \approx \mathcal{O}(10^{-3})\) for both samples. This order of magnitude was further confirmed by comparing the tension \(t_j\) predicted by the micromechanical model when a fibril \((d_0, H_0, R_0, E_f)\) is stretched to that predicted by the FE simulation (not shown here) of the stretching of a corrugated elastic beam with identical properties.

**Interaction coefficient** \(\beta, \zeta\) and \(\delta_c\): \(\beta, \zeta\) and \(\delta_c\) were freely adjusted during the optimization process, respectively yielding to \(\approx \mathcal{O}(10^{-4})\) N, \(\approx 3\) and \(\approx 66\) \(\mu m\) for both samples. To our knowledge, contact forces endured by entangled collagen fibers in soft living tissues are not documented. This is probably ascribable to experimental limitations. However, the reaction forces determined on collagen fibrils using transverse nano-indentation were recently reported [2], showing an
amplitude of $\mathcal{O}(10^2) \text{pN}$, i.e., of the same order of magnitude of the predicted reaction forces $R_q$ between fibers during transverse compression (see Fig. 6). It is also worth noting that interaction lengths $\delta_c$ remain rather close to the fiber’s characteristic “encumbrance”, i.e., $\mathcal{O}(2R_0 + D_0)$, where typical values of the collagen fiber bundle diameter $D_0$ range between 1 and 20 $\mu$m [10, 30].

4.1.2. Vocalis

<table>
<thead>
<tr>
<th>Sample</th>
<th>$\theta_0$ (°)</th>
<th>$\varphi_0$ (°)</th>
<th>$H_{0k}$ (µm)</th>
<th>$R_{0k}$ (µm)</th>
<th>$d_{0k}$ (µm)</th>
<th>$\Phi_k$</th>
<th>$\zeta_{0k}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_{1c}$</td>
<td>33</td>
<td>70</td>
<td>28</td>
<td>6.4</td>
<td>0.4</td>
<td>0.10</td>
<td>1.4</td>
</tr>
<tr>
<td>$V_{1m}$</td>
<td>33</td>
<td>70</td>
<td>1350</td>
<td>130</td>
<td>1</td>
<td>0.70</td>
<td>1.08</td>
</tr>
<tr>
<td>$V_{2c}$</td>
<td>28</td>
<td>67</td>
<td>30</td>
<td>5.5</td>
<td>0.4</td>
<td>0.12</td>
<td>1.28</td>
</tr>
<tr>
<td>$V_{2m}$</td>
<td>28</td>
<td>67</td>
<td>1620</td>
<td>90</td>
<td>1</td>
<td>0.70</td>
<td>1.03</td>
</tr>
</tbody>
</table>

Table 3: Set of histological parameters identified for vocalis samples, $V_i$. $k \in \{c, m\}$ labels collagen or muscular fibrils accordingly. Gray-colored columns refer to quantities computed as a function of the determined histological parameters.

<table>
<thead>
<tr>
<th>Sample</th>
<th>$E_{fk}$ (MPa)</th>
<th>$\mu$ (Pa)</th>
<th>$\alpha_k$</th>
<th>$\beta$ (N)</th>
<th>$\zeta$</th>
<th>$\delta_c$ (µm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_{1c}$</td>
<td>847</td>
<td>900</td>
<td>$4.4 \times 10^{-3}$</td>
<td>$2.2 \times 10^{-4}$</td>
<td>3</td>
<td>367</td>
</tr>
<tr>
<td>$V_{1m}$</td>
<td>0.05</td>
<td>900</td>
<td>$1.1 \times 10^{-2}$</td>
<td>$2.2 \times 10^{-4}$</td>
<td>3</td>
<td>367</td>
</tr>
<tr>
<td>$V_{2c}$</td>
<td>847</td>
<td>980</td>
<td>$4.4 \times 10^{-3}$</td>
<td>$7.6 \times 10^{-5}$</td>
<td>3</td>
<td>360</td>
</tr>
<tr>
<td>$V_{2m}$</td>
<td>0.05</td>
<td>980</td>
<td>$2.7 \times 10^{-2}$</td>
<td>$7.6 \times 10^{-5}$</td>
<td>3</td>
<td>360</td>
</tr>
</tbody>
</table>

Table 4: Set of mechanical parameters identified for vocalis samples $V_i$. $k \in \{c, m\}$ labels collagen or muscular fibrils accordingly.

For collagen sheaths, the parameters $d_{0c}$, $E_{fc}$, $\alpha_c$ were respectively set equal to the optimized values $d_0$, $E_f$, $\alpha$ previously obtained for the lamina propria (see Tables 1 and 2). Furthermore, the equivalent myofibrils diameter was set to $d_0 = 1$ $\mu$m which is in agreement with measurements performed on $V_i$ micrographs [20] and with other available data [89, 21, 57, 76, 19, 3].

Volume fractions $\Phi_k$: Consistently with the values measured for the considered samples [20], myofibrils’ (resp. collagen fibrils’) volume fraction $\Phi_m$ (resp. $\Phi_c$) in the vocalis were found within the range 0.60 to 0.80 (resp. 0.05 to 0.15). If little is reported regarding $\Phi_m$ for skeletal
muscles, our measurements for $\Phi_c$ seem consistent with other values collected on rabbit tissues [98], varying from 0.10 to 0.25. Therefrom, within these ranges of admissible values, the optimization led to $\Phi_m \approx 0.70$ (resp. $\Phi_c \approx 0.10$) for both $V_i$ samples.

**Muscular and collagen fibril's sine waveform parameters** ($H_{0k} ; R_{0k}$) : Spatial period and amplitude of wavy myofibrils ($H_{0m} ; R_{0m}$) were bounded within narrow corridors (1 300 – 1 700 $\mu$m ; 70 – 140 $\mu$m) [3]. The optimization procedure led to distinct values for $V_i$ samples, yielding to tortuosity $\xi_0$ of 1.08 and 1.03 respectively, showing rather straight fibrils at rest in both cases in accordance with Fig. 1. Regarding collagen fibrils’ tortuosity, physiological bounds were kept as set in *lamina propria*, which yielded to ($H_{0c} ; R_{0c}$) $\approx$ (30 $\mu$m ; 6 $\mu$m) after optimization. Thus, the model identification suggests that collagen fibrils covering each myofibrils bundle in the *vocalis* at rest are wavier that those laying in the *lamina propria* (from 15 up to 23 %). This result is in agreement with previous micrographs, showing a double wavelength for the collagen sheath, due to a first muscular waviness entwined with a second-degree collagen crimping.

**Network 3D orientation** ($\theta_0 ; \varphi_0$) : As for the *lamina propria*-case, initial orientation of the muscular network was determined extrapolating 3D descriptors from *vocalis* X-ray images [3], i.e., ($\theta_0 ; \varphi_0$) $\approx$ (33° ; 53°). Final model identification led to ($\theta_0 ; \varphi_0$) $\approx$ (30° ; 68°) for both samples.

**Myofibril's Young modulus** $E_{fm}$ : The "passive" longitudinal Young modulus of frog and rabbit myofibrils [65, 98] is reported to range from 1 to 35 kPa below 10% strains, rising up to $\approx$ 60 kPa at finite strains. Within this range of admissible values, the optimization process yielded to $E_{fm} \approx 50$ kPa for both $V_i$ samples.

**Matrix shear modulus** $\mu$ : *Vocalis* ground substance composition was assumed to be close to that of the *lamina propria*. Accordingly, *vocalis* matrix shear modulus $\mu$ was calibrated on HA data, as detailed for the *lamina propria*-case. This conducted to $\mu \approx \mathcal{O}(10^3)$ Pa for both samples.

**Parameter** $\alpha_m$ : Optimization yielded to $\alpha_m \approx \mathcal{O}(10^{-2})$ for both samples.
**Interaction coefficient ρ, ζ and δ:** As for the lamina propria-case, no constraints were applied to the three parameters during the model identification, yielding to values of $\rho \approx O(10^{-4})$ and $O(10^{-5})$ N for each $V_i$ sample, $\zeta \approx 3$ and $\delta_i \approx 360 \mu$m for both samples.

### 4.2. Macro and microscale predictions

A comparison between the model macroscale predictions and corresponding experimental data acquired is illustrated in Fig. 4, for both sub-layers (LP1 and V1, results obtained for LP2 and V2 are reported in Fig. S1) and for three loading modes, i.e., tension, compression, shear. For each case, the stress-strain response of a homogeneous isotropic neo-hookean material with the same mechanical properties of the corresponding matrix was superimposed to better understand the contribution of each phase to the overall composite behavior. The strain-induced evolution of microscale descriptors is displayed in Figs. 5 and 6.

- **Longitudinal tension** – Regarding the lamina propria, the model fairly well captures its macroscale nonlinear stress-strain response with a typical J-shape strain-hardening (Fig. 4a). The stress contribution predicted for the matrix-equivalent medium is negligible compared to the overall response of the composite sublayer. This emphasizes the major mechanical role of the collagen fibrous network to the longitudinal tensile behavior of the lamina propria, which is supported by predictions of lower-scale mechanisms: as shown in Fig. 5a, collagen fibrils are permanently stretched during the load, and gradually unfolded. The tensile response is coupled with a noticeable rotation and progressive alignment towards the load direction $e_z$, so that angles $\theta_i$ decrease down to $10^\circ$ at $\varepsilon_{zz} \approx 0.10$. These micro-mechanisms result in large deformations of the REV, highly stretched along $e_z$, shrunken along $e_y$ (e.g., with $\varepsilon_{yy} \approx -0.60$ for LP1 at $\varepsilon_{zz} = 0.10$), but also expanded along $e_x$ (e.g., with $\varepsilon_{xx} \approx 0.50$), thereby exhibiting an auxetic behavior. Conversely, these transverse deformations would be close to $\varepsilon_{yy} = \varepsilon_{xx} \approx -0.05$ without any fibers ($\Phi = 0$), i.e., with an incompressible isotropic material (see Fig. 5a, in green). The predicted auxeticity of the lamina propria is ascribable to a strong coupling between the high anisotropy of its collagen network and the incompressibility constraint.
Figure 4: Macroscopic stress-strain curves of vocal-fold sublayers under multiaxial loadings. Experimental data vs. model predictions obtained for lamina propria sample LP1 (left, in red) and vocalis sample V1 (right, in blue): (a) longitudinal tension, (b) transverse compression, (c) longitudinal shear.
Figure 5: Strain-induced evolution of multiscale descriptors predicted for (a) lamina propria LP₁, and (b) vocalis V₁ during tension along eₜ : (top left) macroscopic loading paths; (bottom) stereographic projection of orientation vectors eᵢ from initial to final state, i ∈ [1..4]; (top right) strain-variation of the fibril chord εᵢ, and corresponding tension tᵢ.
Figure 6: Strain-induced evolution of multiscale descriptors for *lamina propria* LP<sub>1</sub> during (a) compression along \( \mathbf{e}_x \) and (b) shear in the plane \((\mathbf{e}_x, \mathbf{e}_z)\): (top left) macroscopic loading paths; (bottom) stereographic projection of orientation vectors \( \mathbf{e}_i \) from initial to final state, \( i \in [1..4] \); (top right) strain-variation of the fibril chord \( \varepsilon_i \) and corresponding interactions \( R_q \) and tension \( t_i \).
Concerning the *vocalis*, the macroscale tensile behavior is similarly rather well predicted by the model (Fig. 4a), whereas the involved micro-mechanisms slightly differ due to histo-mechanical discrepancies between the tissues. The less pronounced nonlinear response of the *vocalis* is attributable both to a lower initial tortuosity $\zeta_0$ and to the negligible stiffness of muscular fibers compared to that of collagen fibers. Note that Fig. 5b shows the key role played by the sheaths of collagen fibers surrounding muscle fibers, as recently reported by Ward *et al.*[98], in the definition of the tissue passive tensile properties. Still, a minor volume fraction of collagen leads to lower orders of magnitude stress levels at the macroscale, compared to those displayed by the *lamina propria*.

• **Transverse compression** – The nonlinear response and strain hardening of the *lamina propria* is additionally well reproduced under transverse compression along $e_x$, *i.e.*, applied perpendicularly to the main fiber orientation. However, in this case, the macroscale nonlinearity is not inherited from the coupled unfolding/rotation of the collagenous network: indeed, Fig. 6a shows that the fibrils’ end-to-ends are barely stretched ($\forall \varepsilon_{xx}, \varepsilon_i \approx 0, t_i \approx 0$ and $\zeta_i \approx \zeta_0$). Fibers rotations in the $(e_y, e_z)$ plane are also moderate, *e.g.*, angles $\theta_i$ (resp. $\varphi_i$) of about 5° (resp. 2°) at $\varepsilon_{xx} \approx -0.25$. Auxetic effects along the $e_z$-direction are still predicted, albeit less marked than in longitudinal tension. The origin of the nonlinearity exhibited at the tissue scale should rather be sought in the steric interactions: Fig. 6a shows non-zero microscopic repulsion forces $R_k$ predicted along the load direction, while interaction forces in other $e_q$-directions (see Fig. 2) – here noted $R_l$ and $R_m$ – are not triggered. More generally, among all the modeling cases considered in the present study, steric interactions were activated under compressive loading exclusively. Furthermore, Fig. 4b displays what macroscale predictions would be when neglecting steric hindrance effects ($\beta = 0$, see dotted lines), showing a quasi-linear mechanical behavior very close to that of the matrix. By contrast with the trends obtained in longitudinal tension, this highlights the major mechanical contribution of the matrix under compression at low strains, and its strong attenuation once steric interactions are triggered (for $\varepsilon_{xx} \approx -0.05$). Similar results are obtained for *vocalis* sample $V_1$, even though a faster fiber recruitment
and unfolding are predicted in that case.

- *Longitudinal shear* – Leaving aside the experimental artifacts at the end of the load (notably for the *vocalis*) [20], the model yields to rather good theoretical predictions of both sublayers’ shear responses, capturing the quasi-constant strain hardening exhibited in Fig. 4c. The predicted mechanical behavior is very close to that of the sole matrix. This is ascribable to the mechanisms evidenced at the microscale and illustrated in Fig. 6b for LP₁: fibers rotation is negligible, and their unfolding is limited. More specifically, fibers \( i = \{1, 2\} \) are slightly stretched, while fibers \( i = \{3, 4\} \) are compressed \((\forall i, |\epsilon_i| < 0.02)\) so that they all remain crimped during the load – the complete unfolding of the fibers being predicted for \( \gamma_{\text{zx}} \approx 3 \).

### 4.3. Effect of fiber orientation on the monotonic and vibratory properties

![Figure 7: (a) Panel of the three REVs in the undeformed configuration \( C_0 \), proposed to gauge the effect of 3D fiber orientations on the *lamina propria* mechanics. (b) Predicted stress-strain responses obtained for each REV subjected to longitudinal tension, transverse compression and longitudinal shear (from left to right), along with reference experimental data (LP₁).](image)

The identified model is used in this section to further investigate the effect of fiber orientation on the mechanics of the *lamina propria*, subjected
both to quasi-static monotonic and vibratory loading. More specifically, a
major focus was drawn to the degree of alignment and symmetry along
the antero-posterior direction \( e_2 \) by studying the mechanics of REVs own-
ing identical histo-mechanical properties (see Tables 1 and 2), albeit dif-
ferent initial fiber orientations \( (\theta_0; \varphi_0) \). As illustrated in Fig. 7a, three
distinct 3D configurations were selected, differentiated two-by-two by a
single angular parameter: (i) a first reference quasi-planar and highly ori-
eted orthotropic structure REV\(_1\), resulting from the previous model iden-
tification (sample LP\(_1\)); (ii) a second configuration with a high orthotropic
fiber orientation along \( e_2 \), albeit equilibrated (i.e., with in-plane tetratropy
in the \( (e_x, e_y) \) plane simulating highly oriented fibrous structures but with
transverse isotropy), noted REV\(_2\) – this configuration corresponds to LP\(_1\)
optimal set of histo-mechanical parameters but \( \varphi_0 = 45^\circ \); (iii) a third equi-
librated structure with a moderate fiber alignment along \( e_2 \), noted REV\(_3\)
– this last configuration is less orientated than REV\(_2\) and tends toward
the rotational symmetry of the antero-posterior axis, being a standard as-
sumption to model vocal-fold tissues so far (e.g., [8, 50, 99]).

- **Monotonic loading** – The predicted stress-strain curves in longitudi-
nal tension, transverse compression and longitudinal shear for the
three structures are compared in Fig. 7b. Globally, strongly oriented
structures exhibit stiffer responses in longitudinal tension, whereas
less oriented configurations offer a higher resistance in longitudinal
shear and transverse compression. In particular, comparing the re-
sponses of REV\(_1\) and REV\(_2\) allows to estimate the impact of the mi-
crostructural symmetry : despite very close \( \theta_0 \)-values, their multi-
axial responses differ drastically. In tension, structure REV\(_1\) shows
lower stress levels and tangent moduli : the structure is required
to open along \( e_x \) before enabling fibers to work in tension along
\( e_2 \). Conversely, a transverse isotropic structure limits fibers’ rotation
along the load direction, thus leading to an earlier fiber recruitment,
which stiffens the macroscopic tissue response (see Supplementary
Videos V1, V2 and V3). Likewise, the response of REV\(_1\) is softer in
compression : the contraction and repulsive forces due to steric hin-
drance along \( e_x \) being induced after sufficient in-plane rotations of
fibers and global opening of the structure along \( e_y \) (see Supplemen-
tary Videos). Similar qualitative trends are exhibited in shear, even if
the fiber rotation is negligible in this case : fiber recruitment is faster
for REV₂, inducing higher stress levels at the macroscale (see Supplementary Videos). In conclusion, a rather small change in the initial orientation of the fibrous network may result in a combination of nonlinear micro-mechanisms of deformation (e.g., fibers’ rotations, uncrimping, tension, compression and steric interactions), which in turn would strongly impact the multiaxial behavior of the lamina propria. Finally, this study points out that an orthotropic configuration is more suited than a transverse isotropic structure to reproduce the physiological loading conditions the lamina propria is subjected to in vivo.

- **Vibratory loading** – To better highlight the impact of these fiber-scale micro-mechanisms on vocal-fold vibrations, a simplified analysis of the lamina propria vibratory properties is here tackled resorting to beam theory. Various beam models are used in the literature to semi-quantitatively describe the vocal-fold vibration [25, 9, 95, 102, 103], albeit with a poor correlation to the tissue’s microstructural specifications. The approach proposed by Kelleher *et al.* [50] is here adopted using the present micro-mechanical model to compute the natural frequencies of the lamina propria subjected to transverse vibrations. Thus, as a first approximation, the lamina propria is considered as a pinned-pinned Timoshenko beam of rectangular cross-section (initial length \( L_0 \), width \( w_{0y} \), thickness \( w_{0x} \)), with incompressible, anisotropic and hyperelastic properties. When subjected to a tensile strain \( \varepsilon_{zz} \) along the antero-posterior direction \( e_z \), the corresponding frequency of the \( n^{th} \) mode of vibration of the beam, noted \( f_n \), can be expressed as follows [50]:

\[
 f_n(\varepsilon_{zz}) = \frac{n^2 \pi^2}{L^2} \sqrt{\frac{E_I I}{\zeta \rho A} \left( 1 + \frac{2}{n^2 \pi^2 E_I I} + \frac{\theta}{k_s G_I A} \right)},
\]

with

\[
 \zeta(\varepsilon_{zz}) = 1 + n^2 \pi^2 \left( \frac{I}{AL^2} \left( 1 + \frac{\theta}{k_s G_I A} \right) + \frac{E_I I}{k_s G_I A L^2} \right),
\]

and where \( L = L_0 e^{\varepsilon_{zz}} \) is the beam length, \( A = w_x w_y \) its cross-section \((w_x = w_{0x} e^{\varepsilon_{xx}}, w_y = w_{0y} e^{\varepsilon_{yy}})\), \( \theta = A \sigma_{zz} \) the applied longitudinal tension, \( \rho \) the tissue density, \( k_s \) the Timoshenko’s shear correction coefficient \((k_s = 5/6\) for a beam of rectangular cross-section)
Figure 8: Theoretical predictions of lamina propria’s vibratory properties as functions of the applied tensile strain $\varepsilon_{zz}$. (a) Tangent longitudinal modulus $E_t$, as derived from the predicted nominal stress $P_{zz}$ (REV1, see inset). (b) Tangent transverse shear moduli $G_{tx}$ and $G_{ty}$ (REV1). (c) First three modal frequencies in the $(e_x, e_z)$- and $(e_y, e_z)$-planes of vibration (REV1). (d) Fundamental frequency $f_0$ obtained for the three structures REV1, REV2 and REV3. Blue-shaded area: human voice $f_0$ range.
and where $E_t$ is the longitudinal tangent modulus predicted by the 

micro-mechanical model at $\varepsilon_{zz}$ (see Fig. 8a in the case of REV$_1$). Accounting for the orthotropic properties of the lamina propria, two planes of vibration $(e_x, e_z)$ and $(e_y, e_z)$ were considered. The beam’s second moment of inertia $I = w_x w_y^3/12$ (resp. $w_y w_x^3/12$) and its predicted tangent shear modulus $G_t = G_{txz}$ (resp. $G_{tyz}$) were varied accordingly (see Fig. 8b in the case of REV$_1$). Choosing $\rho = 1040 \text{ kg.m}^{-3}$, $L_0 = 17 \text{ mm}$ and $w_{0x} = w_{0y} = 1.5 \text{ mm}$ [20, 50], natural frequencies of the first three modes of vibration ($n = 1..3$) were derived for REV$_1$ – the latter being expected to account for 99% of the energy at the onset of vocal-fold self-oscillation [50]. Corresponding predictions $f_1 < f_2 < f_3$ are displayed in Fig. 8c for both planes of vibration in the case of structure REV$_1$. They drastically increase with the applied pre-strain $\varepsilon_{zz}$, this being mainly ascribed to a stiffening of $E_t$ (see Fig. 8a). Additionally, the marked orthotropy of REV$_1$ induces a significant difference for $f_2$ and $f_3$ depending on the vibration plane. Note that $f_1$ is also referred to as $f_0$, it being the expected acoustical fundamental frequency during phonation to a first approximation. Comparison of $f_0$ predictions for structures REV$_1$, REV$_2$ and REV$_3$ is analyzed in Fig. 8d. Regardless of the considered case, a nonlinear evolution of $f_0$ with the applied strain is observed for all frequencies, in quantitative agreement with typical in vivo $f_0$ data (50 – 1500 Hz) [12, 53] and with former theoretical works based on the gold-standard Gasser-Ogden-Holzapfel constitutive model [50, 94]. More importantly, the induced effect of the initial fiber orientation on $f_0$ is clearly evidenced.

5. Conclusion

In this study we proposed an idealized description of the 3D fibrous architecture of human vocal-fold sublayers, i.e., the lamina propria and the vocalis, based on histological evidence. Therefrom, a new 3D micro-mechanical model of vocal-fold tissues was proposed and identified on four experimental targets, i.e., two lamina propria and two vocalis samples. From a single set of histo-mechanical parameters per sample, this model is able not only to reproduce its nonlinear and anisotropic behavior under key physiological loadings (tension, compression, shear), but also to predict the underlying deformation micro-mechanisms at the fiber scale i.e., a complex
and path-dependent combination of 3D fibers’ rotations, matrix/fibers deformations and interactions that were analyzed to clarify the inherited macroscopic tendencies. More particularly, this study highlights the major role of 3D fiber orientation in longitudinal tension, steric hindrance in transverse compression and matrix contribution in longitudinal shear. We also showed that a slight change of the histo-mechanical configuration may drastically alter both the tissue monotonic and vibratory properties, which in turn would affect vocal folds vibration in vivo.

Further developments are needed to improve the proposed micro-mechanical model. Although challenging to identify experimentally, the introduction of microscale time-dependent phenomena would be needed to account for viscoelastic effects in the vocal-fold behavior, such as its strain-rate sensitivity and the stress hysteretic response typically measured during cyclic loading. This work is planned. The mechanical contribution of “active” myofibrils should be also modeled, so as to better understand the relative contribution of the *vocalis* and the *lamina propria* in the *f₀* regulation. Finally, damage mechanisms could be studied by modifying the constitutive laws at the fiber scale.

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Conflict of interest statement

The authors declare no conflict of interest.

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APPENDIX
Figure 9: Supplementary Fig S1. Macroscopic stress-strain responses of vocal-fold sub-layers under multiaxial loadings. Experimental data vs. model predictions obtained for lamina propria sample LP_2 (left, in red) and vocalis sample V_2 (right, in blue): (a) longitudinal tension, (b) transverse compression, (c) longitudinal shear.