



**HAL**  
open science

# A concurrent micro/macro FE-model optimized with a limit analysis tool for the assessment of dry-joint masonry structures

Marco Funari, Luís Silva, Nathanael Savalle, Paulo Lourenço

## ► To cite this version:

Marco Funari, Luís Silva, Nathanael Savalle, Paulo Lourenço. A concurrent micro/macro FE-model optimized with a limit analysis tool for the assessment of dry-joint masonry structures. *International Journal for Multiscale Computational Engineering*, In press, 10.1615/IntJMultCompEng.2021040212 . hal-03587681

**HAL Id: hal-03587681**

**<https://hal.science/hal-03587681>**

Submitted on 25 Feb 2022

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.



## 25 **1 Introduction**

26 Field inspections after earthquake events report how out-of-plane failure mechanisms are prone  
27 to occur in historical masonry structures. Undesired consequences include the collapse of  
28 buildings, human losses, and loss of societal identity (Stepinac et al., 2021; Vlachakis et al.,  
29 2020). Preventive remedial interventions on the built heritage are complex to perform since a  
30 sound knowledge of the structural and material features is lacking for most of the cases.  
31 Scientifically based studies are less susceptible to inadequate actions and advocate for proper  
32 structural analysis tools. To this aim, the literature was, in the last decades, enriched with the  
33 development of analysis methods for masonry structures. A plethora of strategies can be found  
34 now but seems clear that research leans towards the so-called (i) analytical and (ii) numerical  
35 approaches (Ferreira et al., 2014; Ferreira et al., 2015; D'Altri et al., 2019).

36 Analytical approaches are often based on the theorems of limit analysis and through a force-  
37 or displacement-based formulation (Cascini et al., 2018; Gianmarco de Felice et al., 2001).  
38 These are especially suitable for a rapid seismic fragility assessment, as require few input  
39 material parameters and provide good estimations on collapse load multiplier for defined  
40 failure mechanisms (Giuffré 1996; D'Ayala, and Speranza 2003); however, are unable to track  
41 displacement history and damage evolution. To what concerns numerical approaches, the Finite  
42 Element Method (FEM) (Fortunato et al., 2017; Aşıkoğlu et al., 2019) and the Discrete Element  
43 Method (DEM) (Savalle et al., 2020; Lemos, 2007; Lemos, 2019; Bui et al., 2017; Gonen et  
44 al., 2021) are largely used. DEM is now well suited for masonries with both dry- and mortared  
45 joints, but still requires a full representation of the blocks (masonry units) arrangement (Lemos,  
46 2007). FEM allows a more versatile application as masonry can be represented either through  
47 a continuous equivalent media (designated macro-modeling) or by a discrete representation of  
48 units and joints (designated micro-modeling). Linear and non-linear static and dynamic  
49 analyses are eligible. Nonetheless, additionally to the significant amount of data needed to

50 characterize the non-linear response of materials, the analysis can be both time-consuming and  
51 computationally expensive when estimating the ultimate ductility level of the structure.

52 To cope with the prohibitive computational cost, especially when dealing with large-scale  
53 structures and within full material nonlinearity, multi-scale FE methods seem a promising  
54 alternative and are in between the micro- and macro- FE schemes. Classical FE<sup>2</sup> approaches,  
55 i.e. the full continuum-FE methods, have clear advantages if linear elastic behavior is assumed,  
56 but obtaining a micro-scale solution at each load step of a non-linear process for each Gauss  
57 point may turn the problem prohibitive from a computational point of view; especially if  
58 nonlinearities are assumed (Otero et al., 2015; Lourenço et al., 2020). These strategies still have  
59 a higher computational cost than a FE macroscopic model. Hence full continuum-based FE<sup>2</sup>  
60 approaches are seldom used for dynamic purposes and complex structural analysis (Lourenço  
61 et al., 2020). The development of techniques that keep accuracy to acceptable levels and speed  
62 up the processing running times is critical. Several authors tried, therefore, to address  
63 simplifications on two-step frameworks.

64 The use of discrete FE-based methods at a macro-level is a promising alternative. Two-step  
65 approaches based on a discrete FE-based at a macro-scale are very practical due to the decrease  
66 of the number of degrees of freedom (comparing to a continuous approach) and are especially  
67 useful to perform dynamic analysis. Several studies have shown the clear advantages of this  
68 process since it allows a good trade-off between consumed time and results' accuracy and  
69 enables the study of real scale buildings. The latter is even more clear if simplifications are  
70 further assumed at both macro- or micro-scales, as observed in (Gabriele Milani et al., 2011;  
71 Bertolesi et al., 2019; Sharma et al., 2021; Casolo et al., 2013; Silva et al., 2017).

72 The use of limit analysis can be also a promising alternative. Some authors used a kinematic  
73 theorem of limit analysis at a macro-level to obtain the homogenized failure surfaces with a  
74 very limited computational effort (Cecchi et al., 2008; G. Milani et al., 2006; de Buhan et al.,

75 1997). Such methods give a lower or upper bound estimate on the failure collapse load that can  
76 be scarce in some cases. Nonetheless, limit analysis is also being used together with FE-based  
77 strategies. Recently, Betti and Galano (2012) and Cundari et al. (2017) proposed similar  
78 frameworks, in which the global structural analysis was achieved by non-linear static or  
79 dynamic analysis aiming at the detection of the most likely collapse mechanisms. Then, at a  
80 second step, an upper bound limit analysis method was applied in the identified mechanisms  
81 to compute the maximum horizontal acceleration that the structure can withstand. Additionally,  
82 Funari et al. (2020) developed a non-linear static analysis to identify the most prone failure  
83 mechanisms and then, in a second step, aimed to refine the geometry of the failure mechanism  
84 through an optimization based on limit analysis and genetic algorithm; hence exploring an  
85 extensive set kinematically compatible solutions. D'Altri, et al. (2021) used limit analysis as a  
86 first step towards the identification of cracked surfaces and, in the next step, a macroscopic FE  
87 model was used to perform non-linear quasi-static analysis, in which the cracking zones were  
88 simulated with a microscopic description. However, a full-nonlinear behavior for the whole  
89 structure (even for the non-cracked zone) was assumed, which blurs the computational  
90 efficiency of the procedure and especially highlights the interest over more sophisticated  
91 approaches.

92 In this endeavor for a fast tool, yet able to give accurate descriptions of the structure's capacity  
93 and damage evolution, one may stress the so-called concurrent multi-scale approaches. These  
94 have been already applied to simulate fracture propagation in composites (Talebi et al. 2015;  
95 Ghosh 2015), nanocomposite (Ren et al. 2016), but also in the study of masonry structures  
96 (Lorenzo Leonetti et al., 2018; Driesen et al., 2021; Lourenço et al., 2020). A concurrent  
97 multiscale approach contemplates two well-separated scales, i.e. refined and coarse domains,  
98 described by a non-conforming mesh discretization and solved simultaneously. The refined  
99 domain, which ensures the modeling of non-linearities in the material behavior, is adopted in

100 the regions that expect to fail, whereas, in the coarse domain, non-linearities are assumed  
101 negligible.

102 Lloberas-Valls et al. (2012a) investigated several incompatible mesh connections in the  
103 framework of a strongly coupled multiscale model to describe the crack growth and  
104 coalescence phenomena. Their model integrated a sophisticated zoom-in procedure that  
105 enables, during the loading history and based on a proper mechanical criterion, to switch from  
106 a coarser to a finer discretization of the media. Similarly, Talebi et al. (2015) developed a  
107 concurrent coupling scheme suitable to simulate the crack and dislocations at an atomistic  
108 level. Rodrigues et al. (2018) focused on the definition of an adaptive concurrent multiscale  
109 approach for the crack propagation phenomena in concrete structures. Their main novelty lies  
110 in using a non-periodic RVE cell, in which the FE mesh between the refined and coarse  
111 domains are independent.

112 A noteworthy study of a concurrent multiscale approach to investigate the in-plane failure of  
113 masonry structures was developed by Leonetti et al. (2018). A multiscale/multidomain-based  
114 computational scheme allowed to reduce the computational cost associated with a classical FE  
115 micro-modeling approach. Furthermore, a recent study on the subject has shown the potential  
116 of multiscale approaches applied to masonry in a bi-dimensional framework and pointed out,  
117 as a future research path, the interest of limit analysis envisioned as a preliminary step for this  
118 kind of procedure (Driesen et al., 2021).

119 The literature shows the potential of two-step procedures. However, the development of such  
120 tools aiming at an optimal localization of non-linearities – to reduce the associated convergence  
121 issues and computational cost – is still needed. Although concurrent multiscale approaches are  
122 certainly a promising alternative to simulate the failure of an extensive range of materials, e.g.  
123 concrete, masonry, composites, among others, its use is still limited to bi-dimensional  
124 frameworks and for small-scale case studies. In this context, this paper presents an integrated

125 two-stepped procedure that was developed with the aim of enriching the literature on the field  
126 of FE concurrent model. The main contribution is the possibility of conducting a three-  
127 dimensional analysis of masonry structures within a low computational cost. To this aim, at a  
128 first step, a limit analysis tool finds the most prone failure mechanisms. At a second step, a FE  
129 concurrent multiscale approach is used to study the in-plane and out-of-plane response of  
130 masonry structures. Both steps are coupled, meaning that the failure surfaces that are found  
131 with limit analysis are modeled within a microscopic approach. Furthermore, and to fully  
132 optimize running times, the domain that is beyond the cracking surfaces, from an a-priori given  
133 characteristic length, is modeled as an elastic and orthotropic media.

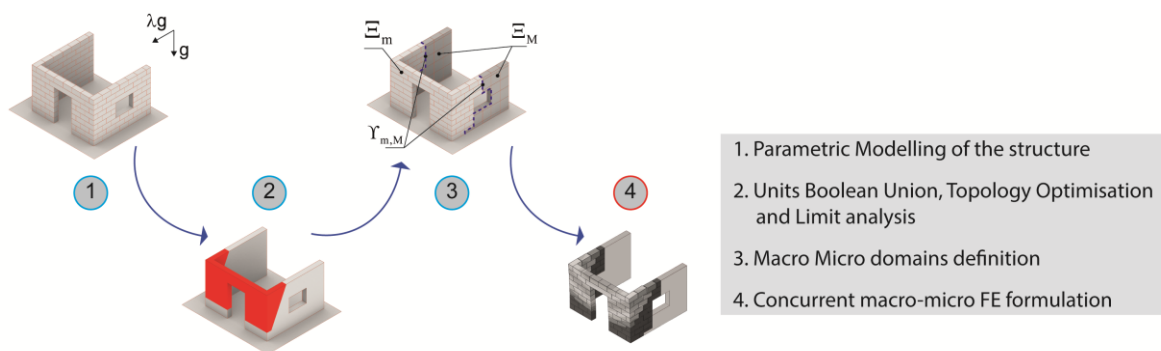
134 The paper is organized into four main sections: section 2 describes the analysis framework  
135 focusing on both theoretical and numerical aspects; section 3 reports three validation examples,  
136 which differ in scale, and reports the benchmark given as a case study; and finally, final remarks  
137 are discussed in Section 4.

## 138 **2 Two-scale framework: general description**

139 A numerical framework is presented aiming an accurate description of the in- and out-of-plane  
140 mechanical behavior of unreinforced masonry (URM) structures. It was formulated to require  
141 a lower computational cost than full FE microscopic and macroscopic (non-linear) strategies  
142 (Roca et al., 2013; Lourenço et al., 2020). The so-called concurrent approach (firstly presented  
143 by Fish (2006)) is adopted together with a limit analysis tool. In this regard, the framework has  
144 two sequential and coupled steps, in which a limit analysis is conducted first, and a concurrent  
145 FE analysis is employed next.

146 The framework described in more detail next includes three main tasks, as given in Figure 1,  
147 needed to compute the mechanical response of URM structures. The first step consists of the  
148 geometric modeling of the structure via an explicit representation of both masonry units and  
149 joints (micro-modeling approach). In the second step, masonry units are merged, and its

150 topology is optimized to provide a macro representation of the media. Prone in-plane and/or  
 151 out-of-plane failure mechanisms are a-priori assigned, and the location of the yielding surfaces  
 152 is optimized by an upper bound limit analysis theorem coupled with a heuristic solver. At this  
 153 stage, the third step is conducted, in which an ad-hoc script represents the sub-structure  
 154 activated by the failure mechanism through a micro-scale representation. The outer domain,  
 155 i.e. the rest of the structure that is not involved in the mechanism, keeps a macro and continuous  
 156 representation. Finally, the concurrent FE multiscale model can be used to perform the  
 157 structural assessment of the structure through linear/non-linear quasi-static/dynamic type of  
 158 analysis and within a FE environment. Further details over each step are addressed in the next  
 159 sections.



160  
 161 Figure 1: Schematic representation of the proposed two-step numerical strategy.

## 162 2.1 Parametric modelling of the structure

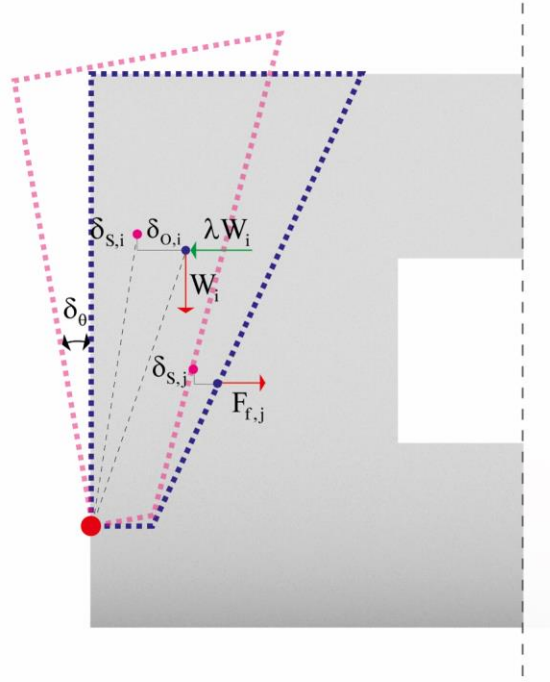
163 The geometric modeling of the structure is the first step of the framework (node one in Figure  
 164 1). To this aim, it requires the knowledge of the masonry arrangement since a micro-modeling  
 165 approach is assumed. Although a full representation of masonry units and joints can be  
 166 cumbersome and time-consuming, the framework integrates a digital tool that was recently  
 167 proposed by Savalle, et al. (2021). This tool allows the pre-processing of the geometry via an  
 168 automatic generation of the masonry arrangement, and it was implemented in the environment  
 169 offered by Rhinoceros (+ Grasshopper) through C# programming language. It includes an  
 170 initial discretization of the structure into elementary parts, i.e. walls arrangement, location of



171 corners, location of T-connection, among other substructures, which can be assembled without  
172 any restriction aiming to form complex three-dimensional structures. Then, by setting up the  
173 dimensions of units and joints – given as user input –, the masonry pattern is positioned to  
174 respect the latter structural features. Openings can be also included in the walls, and the user  
175 can specify its height, length, position, and the dimension of lintels. For the sake of brevity, the  
176 reader is referred to (Savalle et al., 2021) for further details.

## 177 **2.2 Upper-Bound Limit Analysis**

178 The geometric model defined in the first step serves as a basis to conduct the second step of  
179 the framework. In this sub-step of the framework, prone failure mechanisms are pre-defined  
180 and assessed through an optimization tool that integrates an Upper-Bound limit analysis  
181 theorem coupled with a heuristic solver (node two in Figure 1). Therefore, the geometry of the  
182 expected active failure mechanism is parametrized to find the optimal configuration. The  
183 optimization problem aims at the minimization of the horizontal load multiplier, which can be  
184 formulated through the principle of virtual work. Figure 2 describes an overturning mechanism  
185 of a masonry wall, in which the kinematic description required to formulate the problem is  
186 conditioned by one virtual rotation  $\delta_\theta$  (Casapulla, et al. 2014; Funari, et al. 2020). The  
187 formulation of such a mechanism is addressed next, as it is the one assumed for the benchmarks  
188 reported in this study. It worth stressing that the formulation resorts on a representation of the  
189 media through a macro-approach – units forming the masonry prototype are merged –, in which  
190 the mechanism is represented by one virtual parameter only (Figure 2). Such assumption is  
191 convenient for an initial assessment of the most prone mechanism, and it is largely used in  
192 classic limit analysis approaches, see for instance (Sorrentino et al., 2017; D’Ayala et al.,  
193 2002).



194

195 Figure 2: Kinematic description of the overturning failure mechanism.

196 As presented in Figure 2, the external virtual work contains both the overturning as well as the  
 197 stabilizing works performed by the inertial forces, whereas the internal work derives from the  
 198 friction force at contact interfaces:

$$\begin{aligned}
 \delta W_{\text{ext}} &= \lambda \sum_{i=1}^n W_i \delta_{O,i} - \sum_{i=1}^n W_i \delta_{S,i} \\
 \delta W_{\text{int}} &= \sum_{j=1}^n F_{f,j} \delta_{S,j}
 \end{aligned} \tag{1}$$

200 in which  $w_i$  are the inertial forces arising from the self-weight of the  $i$ -th masonry wall and  
 201 including as well the contribution of roof and floors;  $\delta_{o,i}$  and  $\delta_{s,i}$  are the virtual overturning  
 202 or stabilizing displacements of the application point of the inertial forces (that coincides with  
 203 the virtual centroid if self-weight is considered only);  $F_{f,j}$  are the frictional forces computed  
 204 as a weighted value of the maximum friction force  $F_{\text{max}}$  based on the inclination of the crack  
 205 line, as given next (Casapulla et al., 2014):

$$F_f = F_{\text{max}} \left( 1 - \frac{\alpha_c}{\alpha_b} \right) \tag{2}$$

206

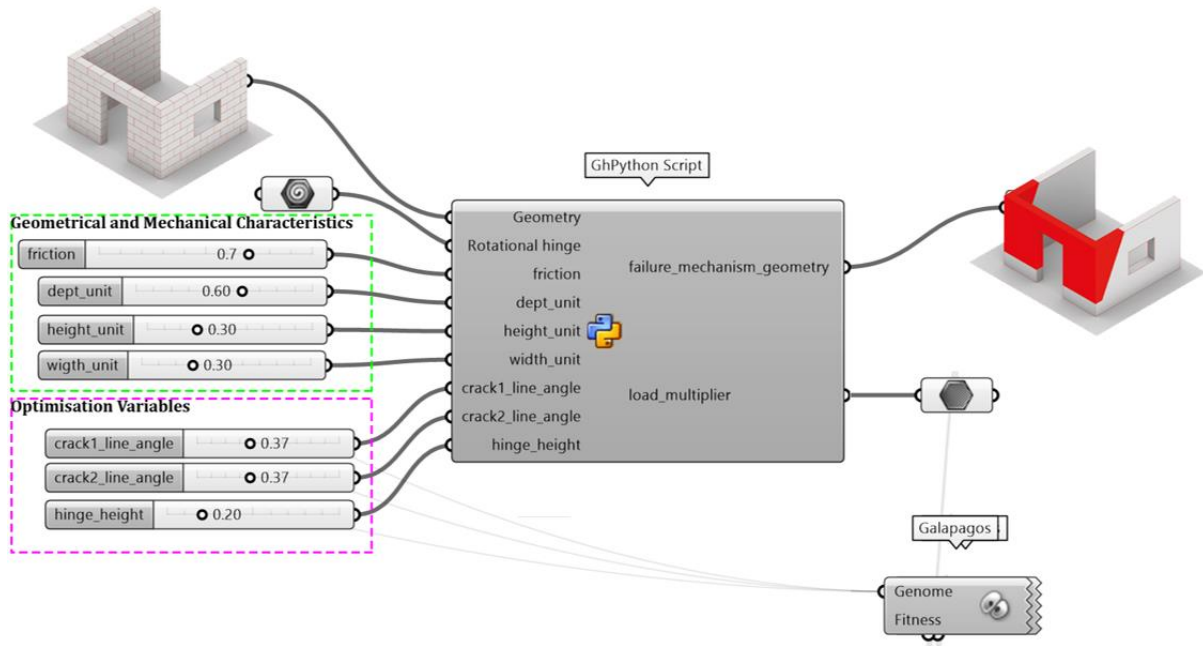
207 Here,  $\alpha_b$  and  $\alpha_c$  are the maximum frictional and crack angle, respectively (Funari, Mehrotra,  
 208 and Lourenço 2021). The value of the horizontal load multiplier  $\lambda$  is obtained by solving Eq.  
 209 (1) and reads:

$$210 \quad \lambda = \frac{\sum_{j=1}^n F_{f,j} \delta_{s,j} + \sum_{i=1}^n W_i \delta_{s,i}}{\sum_{i=1}^n W_i \delta_{o,i}} \quad (3)$$

211 It is worth noting that the value  $\lambda$  depends on the geometry of the failure mechanism, which is  
 212 defined by the crack inclination  $\alpha_c$  and the height of the rotational hinge  $H_h$ . Such variables  
 213 define the set of possible  $\lambda$  values. At last, the optimization of the out-of-plane failure  
 214 mechanism geometry is achieved by solving the following constraint minimization problem:

$$215 \quad \left\{ \min \lambda : \left[ \begin{array}{l} \tan \alpha_{\min} \leq \tan \alpha_c \leq \tan \alpha_b \\ 0 \leq H_h \leq H_w \end{array} \right] \right. \quad (4)$$

216 The constrained optimization problem defined in Eq. (4) was numerically implemented in a  
 217 GHPython script, as depicted in Figure 3. Input data include: (i) the geometry defined in the  
 218 first step (node one in Figure 1), (ii) the friction coefficient of the masonry, (iii) the rotational  
 219 axis, and (iv) the geometric dimensions of masonry units. As shown in Figure 3, the variables  
 220 of the optimization problem are grouped in the magenta box, i.e.: (i) the slope of the crack  
 221 surfaces, and (ii) the height of the rotational axis. The solution is achieved using the GH  
 222 component Galapagos (Rutten, 2013), in which a heuristic research method based on a genetic  
 223 algorithm is implemented. The optimization problem finds the configuration for the critical  
 224 failure mechanism and under a low processing time (few seconds). The kinematic problem  
 225 used herein has theoretical background on the works of Turco, et al. (2020); Funari, et al.  
 226 (2020).



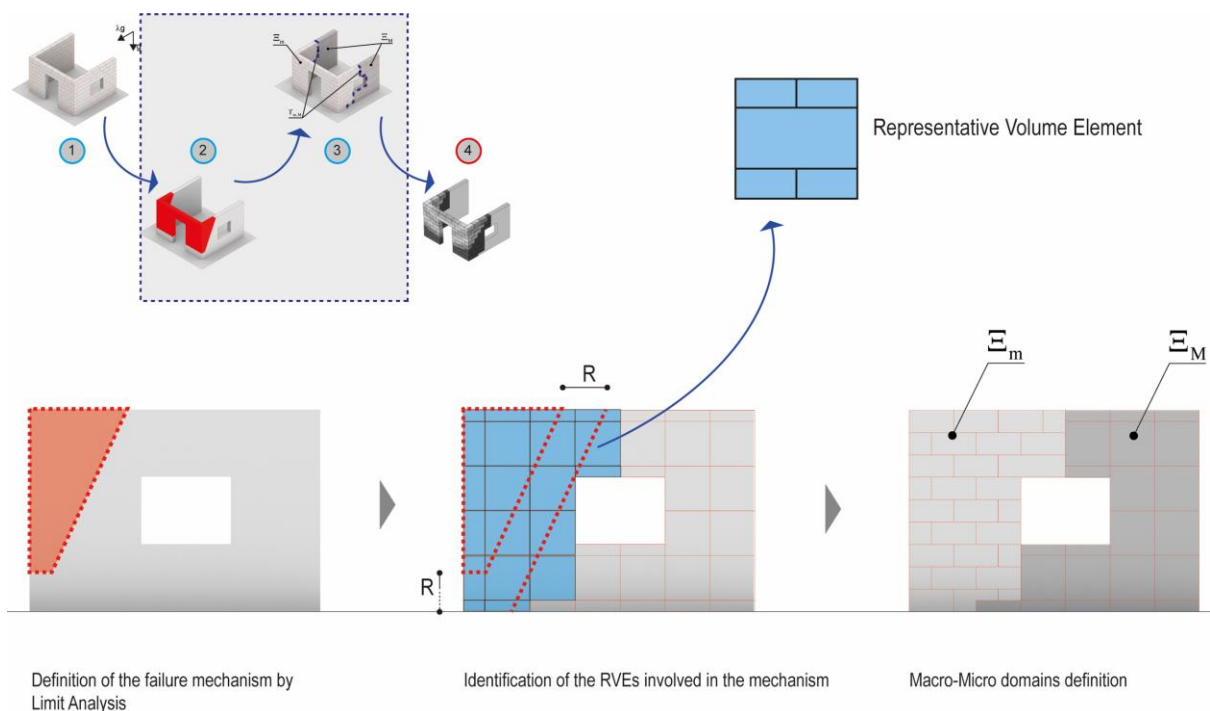
227  
 228 Figure 3: Limit analysis tool implemented in Rhino+Grasshopper through GHPython scripting  
 229 that finds the critical failure mechanism.

### 230 2.3 Macro-Micro domains definition

231 The definition of macro- and micro-domains is the third step of the framework (node three in  
 232 Figure 1). It resumes being a fast procedure once the theoretical failure mechanism is found by  
 233 limit analysis. The decomposition of both domains is directly defined over the failure  
 234 mechanism. Although other re-meshing approaches would be plausible, the present study  
 235 assumes two different scales. A finer scale, designated as micro-domain  $\Xi_m$  that is attributed  
 236 to the substructure defined by the active failure mechanism, in which masonry arrangement is  
 237 explicitly represented. A coarser scale, designated as macro domain  $\Xi_M$  is attributed to the rest  
 238 of the structure, in which masonry is represented through a continuous and equivalent elastic  
 239 media.

240 A contentious issue is the finding of the frontier between domains. Cracking tends to spread  
 241 from the main surfaces failures and have an important role in the non-linear behavior of  
 242 masonry and in damage-induced orthotropy. To avoid inaccurate solutions retrieved from the  
 243 disregard of this cracking, the damage in the vicinity of the main failure surfaces was

244 considered by adding the scalar parameter  $R$  as input. The parameter  $R$  is a length value that  
 245 extends the part of the structure being characterized with a microdomain, as presented in Figure  
 246 4. The choice of  $R$  affects both the accuracy and computational time of the solution since it  
 247 increases or decreases the non-linear region of the model. Therefore, a proper choice of  $R$ -value  
 248 is paramount and it is recommended that it includes: (i) the epistemic error in the prediction of  
 249 the hinges through limit analysis, (ii) the existence of a potential curved failure surface, in  
 250 converse to the straight-type yielded surfaces assumed by the limit analysis tool, and (iii) the  
 251 more diffuse failure surfaces due to the zig-zag damage (especially in sliding and flexure  
 252 mechanisms) in actual masonry specimens (Restrepo Vélez et al., 2014; Bui et al., 2017;  
 253 Cascini et al., 2018).



255 Figure 4: Schematic representation of the decomposition procedure into macro- and micro-  
 256 domains.

257 In this regard, and as schematically described in Figure 4, the dimension of the Representative  
 258 Volume Element (RVE) defines the grid that enables the domain decomposition. This resorts  
 259 to be an alike strategy followed by other studies, as (Vandoren et al., 2013; Lorenzo Leonetti  
 260 et al., 2018; Alessandri et al., 2015; Almeida et al., 2020; Lloberas-Valls et al., 2012; Driesen

261 et al., 2021). A classical RVE adopted for a running-bond pattern (Trovalusci et al., 2015) was  
262 considered since the selected case studies follow such arrangement; needless to state that other  
263 configurations can be employed. The domain decomposition of every grid region, into a micro-  
264 description of the RVE, is performed if intersect the failure surface (plus the characteristic  
265 length  $R$ ); being the other grid elements kept as macro domain  $\Xi_M$  regions.

## 266 **2.4 Concurrent FE macro-micro model**

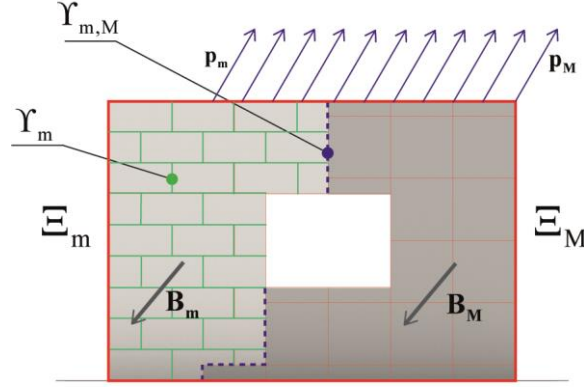
267 The limit analysis procedure performed over the structure allows identifying two subdomains  
268 that have different scales of computation, i.e. the macroscopic  $\Xi_M$  and the microscopic  $\Xi_m$   
269 domains (Figure 1, node 3). Both are concurrent, meaning that together define simultaneously  
270 different volumes of the structure. Towards a low computational cost, material non-linearities  
271 are assigned only to the materials belonging to the micro-domain  $\Xi_m$ . On the other hand, the  
272 media inside the  $\Xi_M$  domain is simulated with an equivalent linear elastic orthotropic material,  
273 whose elastic properties are computed with a proper homogenization strategy to guarantee the  
274 objectivity of the solution. Such hypotheses are particularly suitable for well-marked failures,  
275 as the ones experienced in unreinforced masonry structures: local failure mechanisms governed  
276 by out-of-plane loading due to poor connection between structural elements (Malena et al.,  
277 2019; Restrepo Vélez et al., 2014).

### 278 **2.4.1 Variational Formulation**

279 The concurrent FE model requires a numerical solution for each scale and was implemented in  
280 the FE software Abaqus (2014). A set of weak form equations are solved in a coupled manner  
281 through a variational formulation for both the  $\Xi_M$  and the  $\Xi_m$  domains. Proper kinematic  
282 constraints are employed at the regions where both domains meet, designated as interfaces  
283  $\Upsilon_{m,M}$ . Specifically, an additional internal boundary condition is associated with  $\Upsilon_{m,M}$  and  
284 seeks to enforce the continuity between total displacement (Lorenzo Leonetti et al., 2018;  
285 Driesen et al., 2021):

$$\begin{aligned}
286 \quad \Xi_M: \quad & \int_{\Xi_M} \boldsymbol{\sigma}_M \cdot \delta \boldsymbol{\varepsilon}_M d\Xi_M - \int_{\Upsilon_{m,M}} \zeta \cdot \delta \mathbf{u}_M d\Upsilon_{m,M} = \int_{\Xi_M} \mathbf{B}_M^T \cdot \delta \mathbf{u}_M d\Xi_M + \int_Y \mathbf{p}_M^T \cdot \delta \mathbf{u}_M dY \\
\Xi_m: \quad & \int_{\Xi_m} \boldsymbol{\sigma}_m \cdot \delta \boldsymbol{\varepsilon}_m d\Xi_m + \int_{\Upsilon_m} \mathbf{t} \cdot \delta \bar{\mathbf{u}}_m d\Upsilon_m + \int_{\Upsilon_{m,M}} \zeta \cdot \delta \mathbf{u}_m d\Upsilon_{m,M} = \int_{\Xi_m} \mathbf{B}_m^T \cdot \delta \mathbf{u}_m d\Xi_m + \int_Y \mathbf{p}_m^T \cdot \delta \mathbf{u}_m dY \quad (5) \\
\Upsilon_{m,M}: \quad & \int_{\Upsilon_{m,M}} \delta \zeta (\mathbf{u}_m - \mathbf{u}_M) d\Upsilon_{m,M} = 0
\end{aligned}$$

287 in which  $\mathbf{u}_M$  and  $\mathbf{u}_m$  are the displacement fields belonging to the  $\Xi_M$  and  $\Xi_m$  subdomains,  
288 respectively.  $\Upsilon_m$  represent the interfaces of the micro-domain and  $Y$  represent the boundaries  
289 with applied external forces (surface tractions or nodal forces).  $\mathbf{B}_m$  and  $\mathbf{B}_M$  are the vectors  
290 containing the body loads along with the three global cartesian directions,  $\mathbf{p}_m$  and  $\mathbf{p}_M$  are the  
291 vectors of the surfaces loads active on the boundary,  $\zeta$  is the Lagrange load multiplier of the  
292 forces that control the residual interface gap across adjacent domains,  $\mathbf{t}$  is the traction force  
293 acting at the interfaces within the  $\Xi_M$  domain, and  $\bar{\mathbf{u}}_m$  is the displacement jump at the  $\Upsilon_m$   
294 interfaces (Figure 5).



296 Figure 5: Schematic representation of the variational formulation of the concurrent FE  
297 numerical approach.

298 Equation (5) defines the concurrent FE multiscale approach that is solved within an explicit  
299 scheme available in ABAQUS (Abaqus, 2014). The static solution is obtained by dynamic  
300 relaxation, using scaled masses and artificial damping. To this aim, the energy balance is  
301 continuously evaluated and to guarantee that the kinetic energy of the deforming media is  
302 below a small fraction of the total internal energy (1–5%) (Abaqus, 2014). The latter condition  
303

304 must hold to guarantee the objectivity of the results through an explicit procedure. In this  
305 context, smooth step amplitude curves and a small-time increment allow reaching appropriate  
306 results.

#### 307 **2.4.2 Micro-domain numerical model**

308 Masonry units are assumed to be deformable discrete blocks following an isotropic and linear  
309 elastic constitutive law ( $\mathbf{E}_u$ ,  $\nu_u$ ). Joints are represented by zero-thickness interfaces, which  
310 include a non-associative plastic flow rule and a classical Mohr-Coulomb failure surface  
311 criterion. Normal and tangential contact behaviors (stress-displacement laws) assume an  
312 infinitesimal interpenetration between blocks. A linear relationship between the over-closure  
313 displacements and the applied stress is defined by the normal  $k_n$  and tangential  $k_s$  stiffness  
314 values. A friction coefficient ( $f$ ) defines the plastic slipping criterion in shear within a penalty  
315 approach, in which a perfectly plastic response occurs after reaching the critical shear stress.  
316 For the present case study, only dry mortar type of masonry is studied and, therefore, cohesion  
317 has been neglected when representing joint interfaces (in tension and shear regimes).

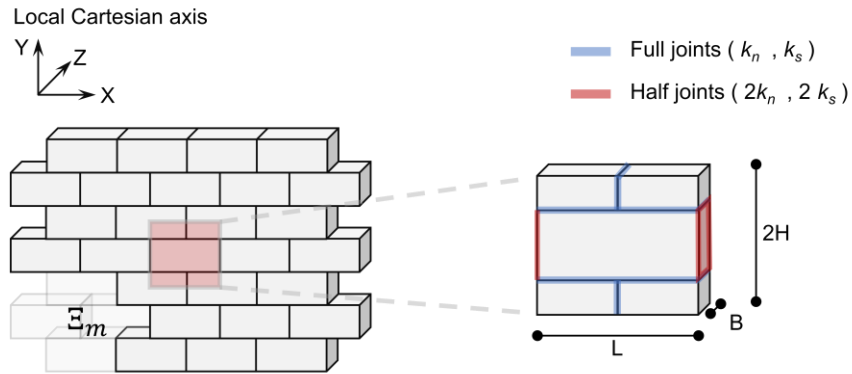
318 According to the distinct element method, a local damping factor is considered. The equations  
319 of motion are damped to reach a force equilibrium state as quickly as possible under the applied  
320 initial and boundary conditions. Damping is velocity-proportional (magnitude of the damping  
321 force is proportional to the velocity of the blocks) and it was assumed equal to 0.8 in the present  
322 study. The adopted FE software is Abaqus (2014) contains the latter mentioned contact  
323 interface model. The constitutive law is automatically assigned to all the interfaces of the  
324 micro-domain  $\Xi_m$  through the General Contact algorithm (Abaqus, 2014).

#### 325 **2.4.3 Macro-domain numerical model**

326 The macro-domain  $\Xi_M$  represents masonry through an orthotropic linear elastic media.  
327 According to the theory of elasticity, the spatial stiffness matrix for an orthotropic material is  
328 defined by a  $6 \times 6$  symmetric matrix, which is fully determined through nine engineering



329 constants, i.e. three elastic moduli  $E_{XX}$ ,  $E_{YY}$ ,  $E_{ZZ}$ , three Poisson's ratios  $\nu_{XY}$ ,  $\nu_{XZ}$ ,  $\nu_{YZ}$ ,  
 330 and three shear moduli  $G_{XY}$ ,  $G_{XZ}$ ,  $G_{YZ}$ , associated with the principal directions (Figure 6).



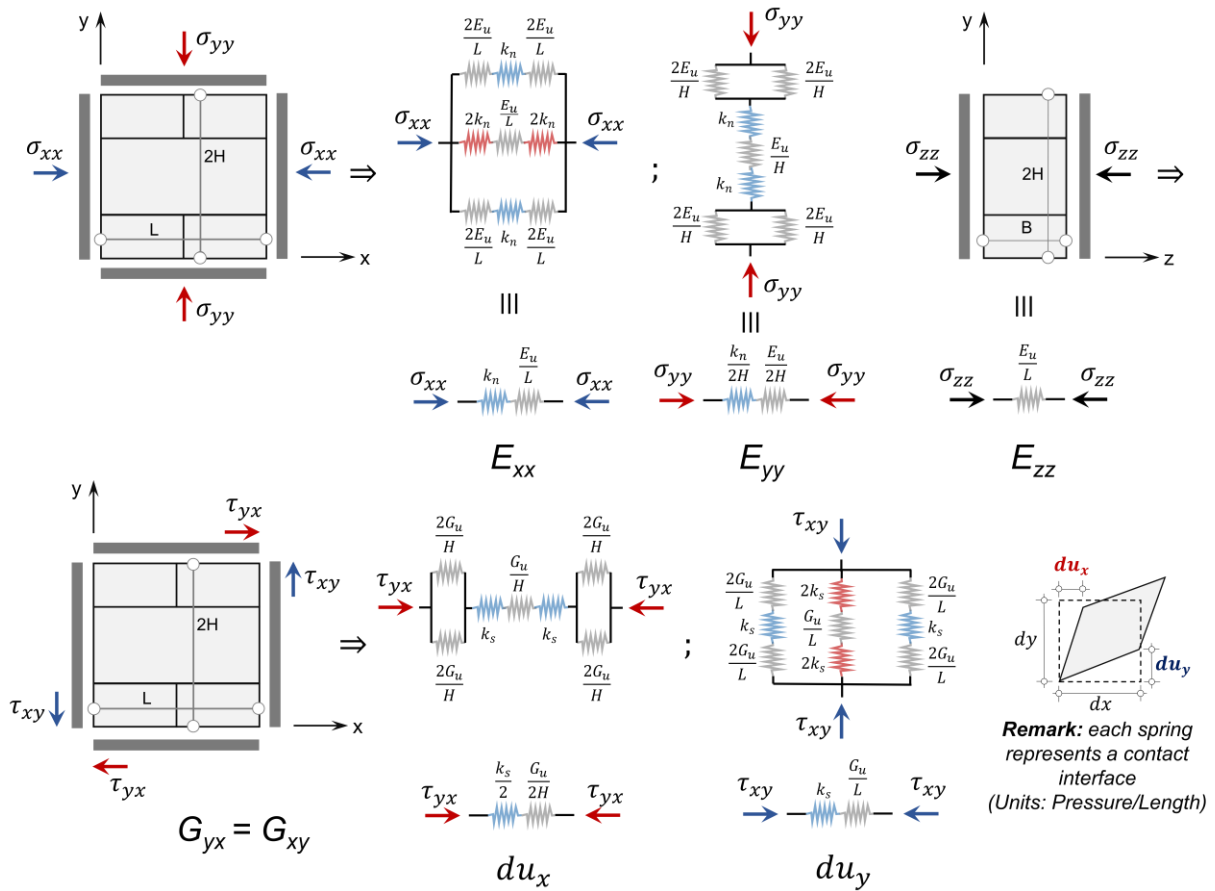
331

332 Figure 6: Representative Volume Element (RVE) for the homogenization procedure of the  
 333 macro-domain  $\Xi_M$ .

334 A closed-form solution was found to define the material elasticity matrix of a running-bond  
 335 dry-joint masonry. It was inspired in the works of Kouris et al. (2020) for a two-dimensional  
 336 media, in which a set of equivalent springs represent the in-plane response of a mortared  
 337 masonry RVE. Herein, the equivalent three-dimensional elastic response is defined by ad-hoc  
 338 expressions formulated based on the representation of a set of springs to describe the masonry,  
 339 as presented in Figure 7. Since contact interfaces are being used to characterize the numerical  
 340 behavior of dry-mortar joints, the springs given in Figure 7 correspond to surfaces in the  
 341 numerical model (unit of Pressure/Length). Parameters  $L$ ,  $H$ , and  $B$  denote the unit length ( $X$ ),  
 342 height ( $Y$ ), and width ( $Z$ ), respectively; hence the RVE's height is given as  $2H$  and length given  
 343 by  $L$  (Figure 7). An assemblage of springs is conceived aiming at the representation of a system  
 344 equivalent to a running-bond masonry RVE. Since a dry-joint masonry will be studied only,  
 345 the compression range is the one that is analyzed here being the RVE subjected to compression  
 346 stress  $\sigma$  for mode-I deformation modes. The computed elastic homogenized properties are  
 347 described in terms of equivalent Young's moduli, Shear moduli, and Poisson's coefficients.

348 Figure 7 presents the latter assumptions. More detail on the formulation is given in Appendix

349 1.



350

351 Figure 7: Strategy followed to compute the equivalent elastic properties of the masonry RVE:

352 Young's moduli and shear moduli (details for the Poisson's ratio are given in Appendix A).

353 Table 1 summarizes the closed-form expression to compute the elastic properties of the

354 equivalent linear elastic orthotropic material; the reader is referred to (Kouris et al., 2020) for

355 further theoretical details within an alike procedure. The mechanic-based formulation adopted

356 has clear simplifications, but brings advantages related to the ease of computational

357 implementation. Nonetheless, among the more sophisticated models available in the literature,

358 only a few deal with dry-joint masonries, and the majority are devoted to 2D frameworks (G.

359 de Felice et al., 2010).

360 Table 1: Equivalent homogenized elastic properties for a running-bond masonry RVE  
 361 (orthotropic material).

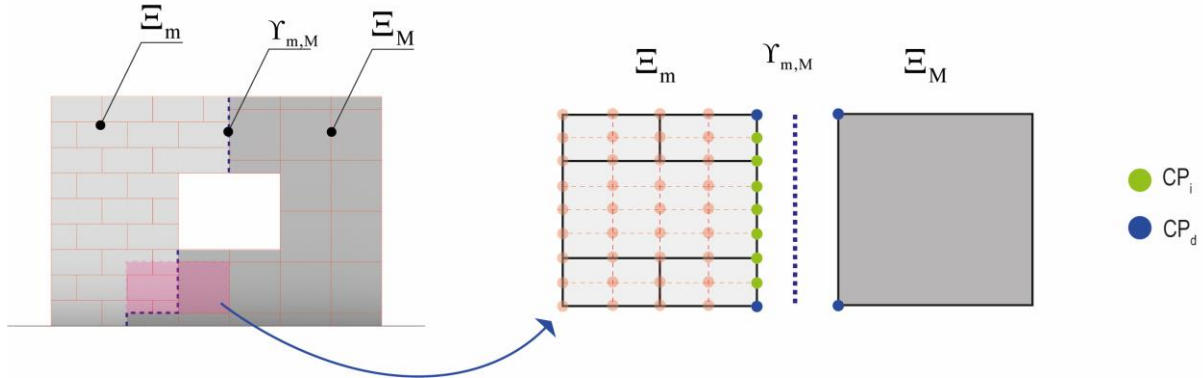
Young's modulus	Poisson's ratio	Shear modulus
$E_{xx} = \frac{Lk_n E_u}{E_b + Lk_n}$	$\nu_{xz} = \nu_{xy} = -\frac{\epsilon_{zz}}{\epsilon_{xx}} = \nu_u \frac{E_{xx}}{E_u}$	$G_{xy} = \frac{HLk_s G_u}{G_u(H + L) + 2HLk_s}$
$E_{yy} = \frac{Hk_n E_u}{E_u + Hk_n}$	$\nu_{zx} = \nu_{zy} = \nu_u$	$G_{xz} = \frac{Lk_s G_u}{G_u + 2Lk_s}$
$E_{zz} = E_u$	$\nu_{yx} = \nu_{yz} = \nu_u \frac{E_{zz}}{E_u}$	$G_{zy} = \frac{Hk_s G_u}{G_u + 2Hk_s}$

362  
 363 **2.4.4 Micro and macro interfaces**

364 The concurrent FE model included two domains – macro  $\Xi_M$  and micro  $\Xi_m$  – that embody  
 365 the structure numerical model and represent the mechanical behavior of masonry. These  
 366 domains are assigned to different volumes and meet in different regions by sharing a common  
 367 surface interface  $\Upsilon_{m,M}$ . Such interface  $\Upsilon_{m,M}$  is characterized by two-adjointing boundaries with  
 368 different scale representations for the masonry and FE mesh sizes ranging  $\Delta_M / \Delta_m \approx 10$ , in  
 369 which  $\Delta_M$  and  $\Delta_m$  are the characteristic FE mesh size of the  $\Xi_M$  and  $\Xi_m$  domains,  
 370 respectively.

371 Interfaces  $\Upsilon_{m,M}$  must link adjoining non-conforming FE meshes. Contact points (or nodes) of  
 372 each boundary are set into two families, defined as:  $CP_i$ , which corresponds to the contact points  
 373 located on the  $\Xi_M$  domain only; and the  $CP_d$ , which is the set of contact points that are paired  
 374 between  $\Xi_M$  and  $\Xi_m$  domains. The continuity of the displacement between  $CP_d$  is imposed  
 375 through a Lagrange multiplier functional. For  $CP_i$ , the Lagrange multiplier can be achieved  
 376 following several procedures, e.g. approximation through shape functions. Figure 8 represents  
 377 this concept in the case of a two-dimensional problem for the sake of simplicity, but the  
 378 interpolation occurs in two orthogonal directions because the proposed strategy is processed in  
 379 the three-dimensional space. From a numerical standpoint, such kinematic conditions are

380 implemented by means of a tie constraint algorithm, which is available in Abaqus (2014).  
 381 Therefore, the assumption of total displacement continuity between the two domains is  
 382 guaranteed, even though the FE mesh are non-concordant.



383 Figure 8: Concurrent FE model and localization of the interface  $\Gamma_{m,M}$  that link the macro- and  
 384 micro-domains with the corresponding contact points.  
 385

### 386 **3 Two-scale framework: numerical application**

387 The numerical application of the proposed two-step framework is conducted over three case  
 388 studies, which include small to large-scale structures. Aiming to validate the strategy, the  
 389 results found are compared against a microscopic model, hereafter named as RMM (reference  
 390 microscopic model). The RMM is one the most accurate (and computationally expensive)  
 391 numerical strategies at disposal in the literature and serves, therefore, as a reference method for  
 392 validation purposes.

393 The first case study is a dry-joint masonry wall with an opening and subjected to an in-plane  
 394 shear load. The second case study addresses three connected dry-stone masonry walls within a  
 395 U-shape plan arrangement (Smoljanović et al., 2018). It aims to explore the potential of the  
 396 proposed approach when the structures are affected by coupled in-plane and out-plane  
 397 mechanisms. Finally, the third case study is a large-scale monumental building, whose  
 398 geometry is inspired by the Church of San Nicolò Capodimonte located in Camogli (Genova,  
 399 Italy) (Funari, Mehrotra, and Lourenço 2021).

400 **3.1 Small-scale structure: URM shear wall**

401 The first case study concerns a dry-joint masonry shear wall. The wall is fixed at the base and  
 402 its geometry is given in Figure 9. Two load cases are applied in a sequent manner: the self-  
 403 weight is applied first, and a lateral body force (mass proportional) is applied next through an  
 404 incremental load factor  $\lambda$ , as presented in Figure 9 (node 1). Material properties required to  
 405 complete the proposed procedure are given in

406 Table 2, specifically the material density ( $\rho$ ) and friction coefficient ( $f$ ) for the limit analysis  
 407 procedure; the Young’s modulus and the Poisson’s coefficient of the masonry units to define  
 408 the overall elastic orthotropic matrix of the  $\Xi_M$  domain according to the closed-form solutions  
 409 of section 2.4.3; and the normal and tangential stiffness values for the micro-macro interfaces.  
 410 The dimensions of units are  $0.80 \times 0.35 \times 0.40\text{m}^3$  ( $L \times H \times B$ ).

411 Table 2: Mechanical properties adopted in the RMM and proposed CMM for the URM shear  
 412 wall study.

$\rho$ [ $\text{kg m}^{-3}$ ]	$f$	$E_u$ [Pa]	$\nu_u$	$k_n$ [ $\text{Pa m}^{-1}$ ]	$k_s$ [ $\text{Pa m}^{-1}$ ]
2000	0.7	$10 \times 10^9$	0.2	$1 \times 10^9$	$1 \times 10^9$

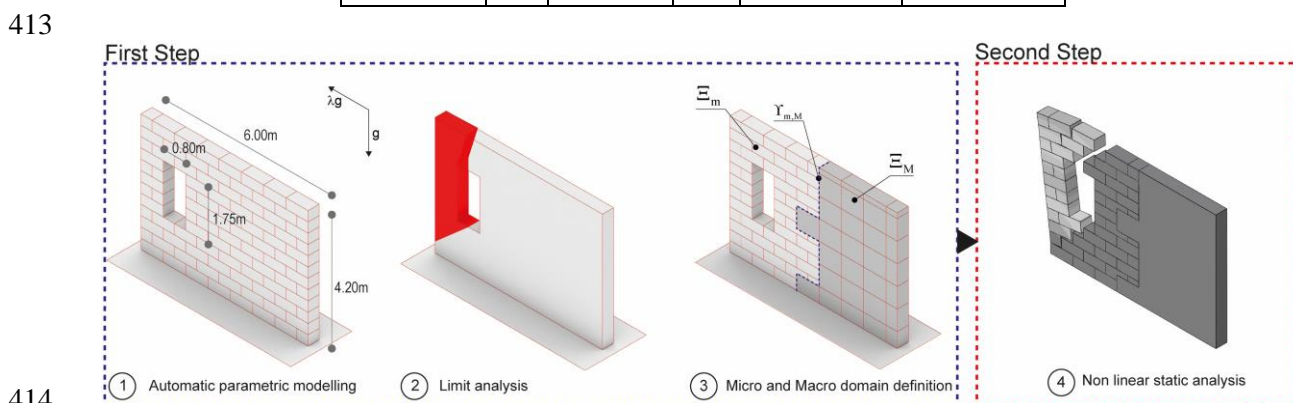
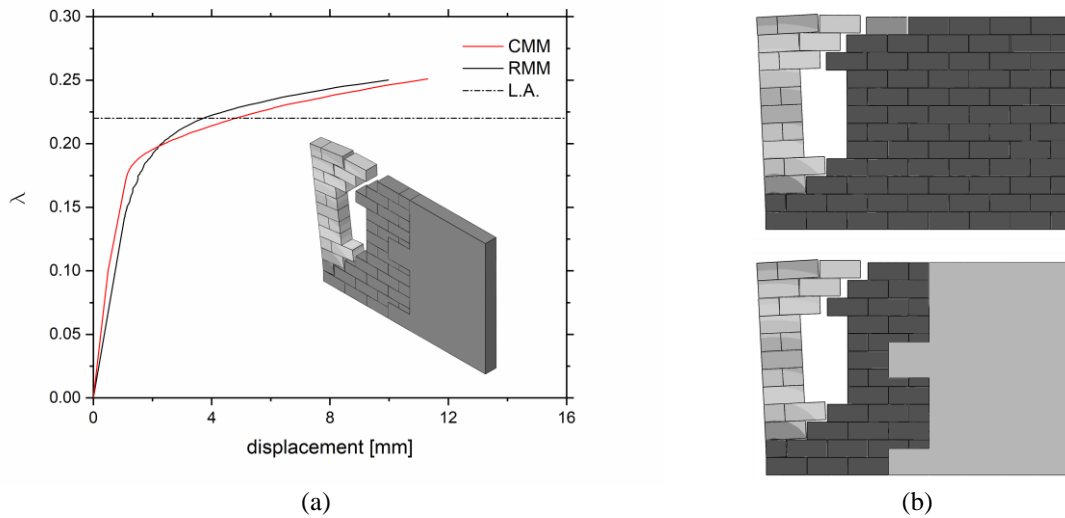


Figure 9: Proposed two-step framework applied to the URM shear wall case study.

416 The proposed strategy is employed – as addressed in section 2 –, in which the modeling of the  
 417 wall was achieved by a semi-automatic parametric micro-modeling, and a limit analysis tool  
 418 applied next aiming the detection of the most prone failure mechanism geometry for the given  
 419 loading conditions. The limit analysis uses a heuristic procedure embedded in the Galapagos

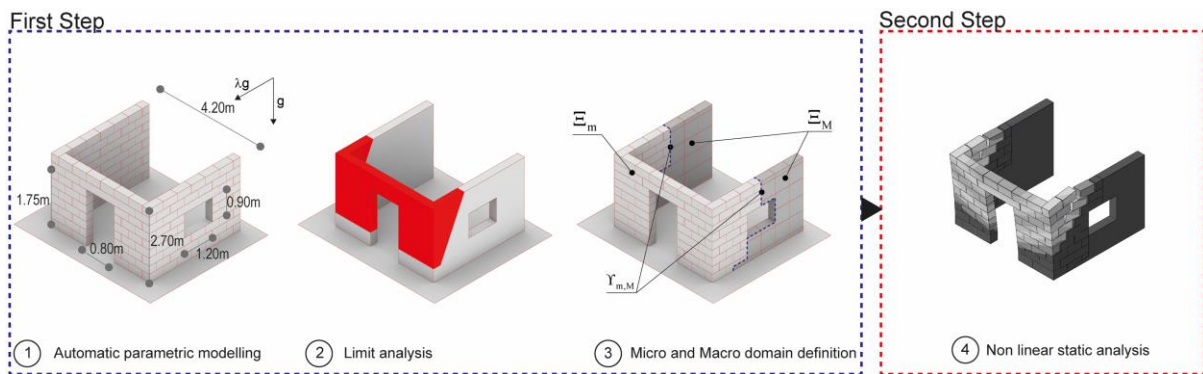
420 solver (Rutten, 2013) and converged to a load multiplier value equal to  $\lambda = 0.219$  (meaning  
421 an equivalent shear base force of  $0.219g$ , in which  $g$  is the gravitational acceleration). The  
422 obtained failure mechanism, linked with the third sub-step of the proposed algorithm as  
423 remarked in Figure 1, follows a re-meshing procedure to define the two non-overlapping  
424 domains, i.e. the micro  $\Xi_m$  and the macro  $\Xi_M$  domains. The characteristic length  $R$  was  
425 assumed to be  $R = 2L$  ( $L$  is the length of the masonry unit that, for the dry-joint masonry of  
426 this case study, matches the length of the RVE). Note that the failure defined by limit analysis  
427 can have a jagged profile that may be caused, for instance, by the presence of openings.  
428 The transfer between the first processing step, i.e. the limit analysis, with the second processing  
429 step of the framework, i.e. the structural analysis by a concurrent FE model, is performed  
430 through a Python script (The Python Language Reference — Python 3.9.5 Documentation,  
431 2021). It allows the automatic creation of the numerical FE model within Abaqus CAE  
432 environment (Abaqus, 2014), in which both domains are properly represented. Masonry units  
433 that belong to the  $\Xi_m$  domain are discretized by eight-node linear hexahedral finite elements  
434 (C3D8R in Abaqus (2014)); thus leading to a  $\Delta_m / \kappa = 1 / 2$ , in which  $\kappa = \min (L, B, H)$ . In  
435 the  $\Xi_M$  domain, a coarser mesh (structured FE mesh with squared elements) was adopted with  
436 a  $\Delta_M / L = 1$ , meaning that the FE size is 0.80m to match the length of masonry units.  
437 Results of the proposed CMM and RMM are presented in Figure 10, both in terms of lateral  
438 load-displacement capacity (node with maximum displacement as control node) and total  
439 displacement map at collapse ( $\lambda = 0.220$ ). The comparison of the results allows  
440 demonstrating that the proposed approach ensures a good solution accuracy. Furthermore, it is  
441 noteworthy to highlight that the CMM allows saving 58% of the computational time cost  
442 required by the RMM.



443 Figure 10: Results obtained for the in-plane loaded masonry wall: (a) lateral load-displacement  
 444 relationship; and (b) displacement map for the RMM and proposed CMM.

### 445 3.2 Small-to-medium scale structure: U-shaped URM walls

446 The second case study concerns a URM structure composed of three walls within a U-shaped  
 447 plan arrangement. It is based on the Smoljanović, et al. (2018) works and brings more  
 448 complexity than the former case study since both in- and out-of-plane co-exist. Walls are fixed  
 449 at the base and the geometry of the structure is given in Figure 11. Two load cases are applied  
 450 in a sequent manner: the self-weight is applied first, and a lateral body force (mass proportional)  
 451 is applied next through an incremental load factor  $\lambda$ , as presented in Figure 11 (node 1). The  
 452 lateral force is orthogonal to the façade wall and then following its out-of-plane direction.



453 Figure 11: Proposed two-step framework applied to the U-shaped URM case study.  
 454

455 Material properties required to complete the proposed procedure are given in Table 3: the  
 456 material density ( $\rho$ ) and friction coefficient ( $\nu$ ) for the limit analysis procedure; the Young's  
 457 modulus and Poisson's coefficient of masonry units to define the overall elastic orthotropic  
 458 matrix of the  $\Xi_M$  domain according to the closed-form solutions of section 2.4.3; and the  
 459 normal and tangential stiffness values for the micro-macro interfaces. The dimensions of units  
 460 are  $0.60 \times 0.30 \times 0.30 \text{m}^3$  ( $L \times H \times B$ ).

461 Table 3: Mechanical properties adopted in the RMM and proposed CMM for the U-shaped  
 462 URM structure studied by Smoljanović, et al. (2018).

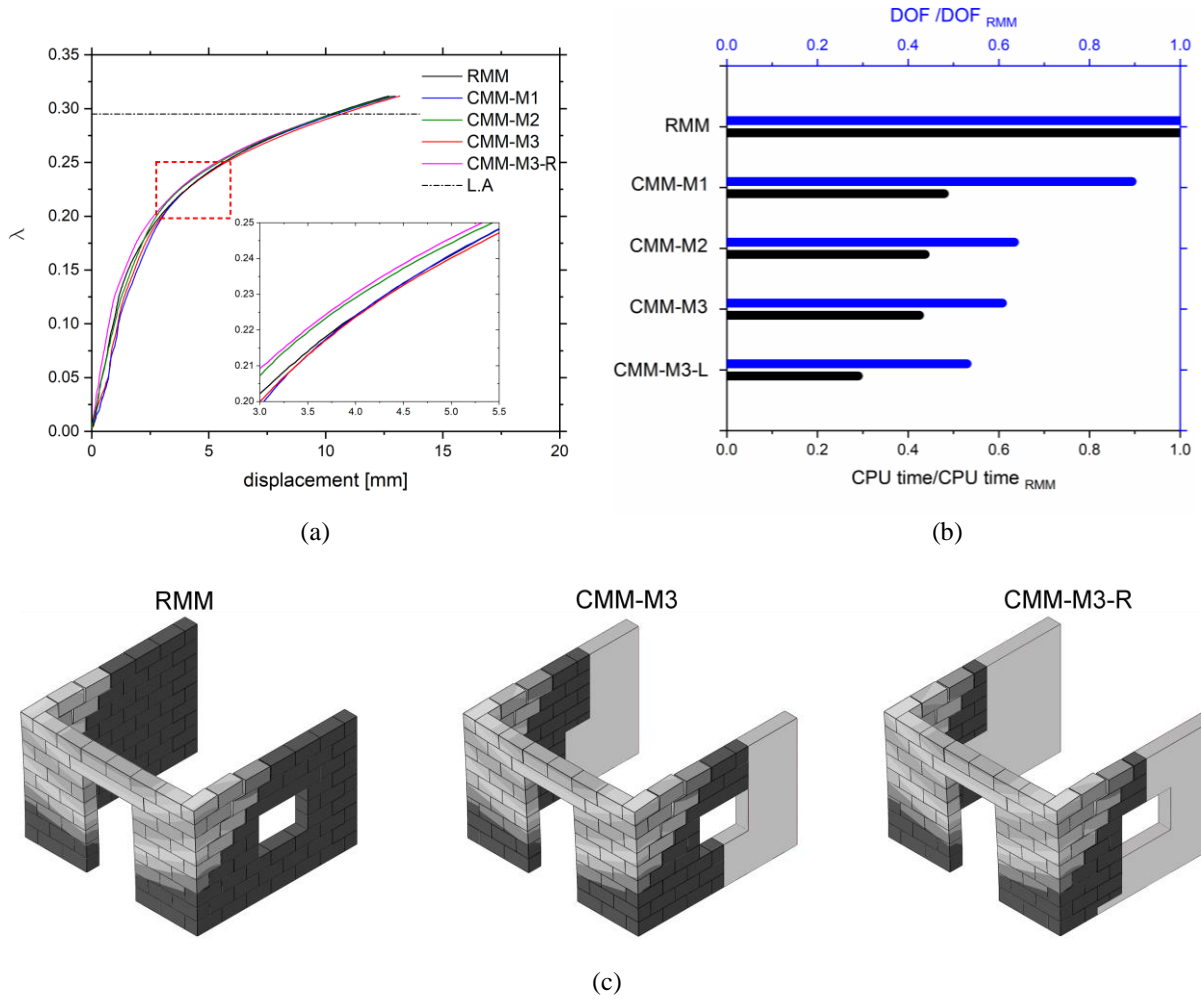
$\rho$ [ $\text{kg m}^{-3}$ ]	$E_u$ [Pa]	$\nu_u$	$k_n$ [ $\text{Pa m}^{-1}$ ]	$k_s$ [ $\text{Pa m}^{-1}$ ]	$f$
2000	$10 \times 10^9$	0.2	$1 \times 10^9$	$1 \times 10^9$	0.7

463 The proposed strategy is employed and the obtained failure mechanism through limit analysis  
 464 is given in Figure 11 (node 2), in which the out-of-plane mechanism of the main façade governs  
 465 the collapse mode. The numerical FE concurrent model (CMM) is then developed at Abaqus  
 466 CAE environment (Abaqus, 2014) by a re-meshing procedure that retrieves two non-  
 467 overlapping domains, i.e. the micro  $\Xi_m$  and the macro  $\Xi_M$  domains. A characteristic length  $R$   
 468 was assumed to be  $R = 2L$  ( $L$  is the length of the masonry unit that, for the dry-joint masonry  
 469 of this case study, matches the length of the RVE). Masonry units that belong to the  $\Xi_m$  domain  
 470 are discretized by eight-node linear hexahedral finite elements (C3D8R in Abaqus (2014));  
 471 thus leading to a  $\Delta_m / \kappa = 1 / 2$ , in which  $\kappa = \min(L, B, H)$ . In the  $\Xi_M$  domain, three mesh  
 472 refinements (structured FE mesh with squared elements) were evaluated to assess the trade-off  
 473 between accuracy-computational achieved. To this aim, the following FE mesh ratios were  
 474 adopted: (i) CMM-M1, with a finer FE mesh and size given by  $\Delta_M / \kappa = 1 / 2$ ; (ii) CMM-M2,  
 475 an in-between mesh refinement with a size given by  $\Delta_M / L = 1 / 2$ ; and (iii) CMM-M3, with  
 476 a coarser FE mesh and size given by  $\Delta_M / L = 1$ .



478 Furthermore, the influence of the parameter  $R$  was investigated. Note that  $R$  is a parameter  
479 (units of length) that directly affect the volumes of both micro- and macro-domains (see section  
480 2.4.2). Therefore, an additional model designated as CMM-M3-R was also considered: it has a  
481 FE mesh size for the  $\Xi_M$  domain given by  $\Delta_M / L = 1$  and an  $R=L$  (half-value of the other  
482 CMM models).

483 Results from the proposed CMM and RMM are presented in Figure 12. Lateral load-  
484 displacement capacity curves (node with maximum displacement as control node) in Figure  
485 12a show slight differences in the elastic range, yet negligible from a structural engineering  
486 standpoint as are within a 5% bound. CMM model is slightly stiffer than the RMM, especially  
487 in the linear range, and it may be explained due to the loss of accuracy that macro-modeling  
488 offers when compared with a micro-modeling approach. Nonetheless, differences are  
489 unnoticeable when plastic deformations govern the response; is noteworthy to highlight that  
490 the CMM micro-domain is responsible for such deformation. Collapse occurs for a load-factor  
491 around  $\lambda = 0.295$  for all the studied numerical models. It is important to point out that the  
492 collapse instant is defined when the kinematic energy is higher than 5% of the total energy  
493 because an explicit formulation was adopted. In such a context, the results allow demonstrating  
494 that the proposed approach ensures a promising solution accuracy. Furthermore, it is  
495 noteworthy to highlight that the CMM allows saving 58% of the computational time cost  
496 required by the RMM.



497 Figure 12: Results obtained for the U-shaped URM structure: (a) lateral load-displacement  
 498 relationship; (b) computational time (CPU) and number of degrees-of-freedom (DOFs) for each  
 499 numerical simulation; and (c) displacement map for the RMM and proposed CMM-M3 and  
 500 CMM-M3-R.

501 Figure 12b reports a comparison including the required computational time (CPU) and the  
 502 number of degrees-of-freedom (DOFs) for each numerical model. Despite CMM-M1, CMM-  
 503 M2 and CMM-M3 have clear differences in the number of DOF, differences in the required  
 504 CPU time are minimal and allow saving around 60% of the time in comparison with the RMM.  
 505 In this regard, the approach seems to not suffer from a substantial mesh bias at the  $\Xi_M$  domain;  
 506 this holds at least for FE mesh sizes with a dimension lower than the RVE size. On the other  
 507 hand, Figure 12b allows doing an important finding, i.e. the parameter R has a significant

508 impact since it allows decreasing 10% of the required CPU time when compared to the CMM-  
509 M3. Therefore, this parameter must be assessed with care. It allows improving the  
510 computational time, but decreasing its value may also compromise the accuracy level. The  
511 authors suggest a value bounded by  $R = L$  to  $R = 2L$ .

512 The failure mechanisms obtained with the RMM, CMM-M3, and CMM-M3-R are summarized  
513 in Figure 12c through total displacement maps. The proposed FE concurrent models (for both  
514 R values) capture well the expected failure mechanism. At last, it is noteworthy to highlight  
515 that the proposed multi-scale framework returns promising results, in terms of load capacity  
516 curve and expected failure mechanism, while saving 65% of the computational time if  
517 compared to an accurate micro-modeling strategy (RMM).

### 518 **3.3 Large-scale structure: URM church**

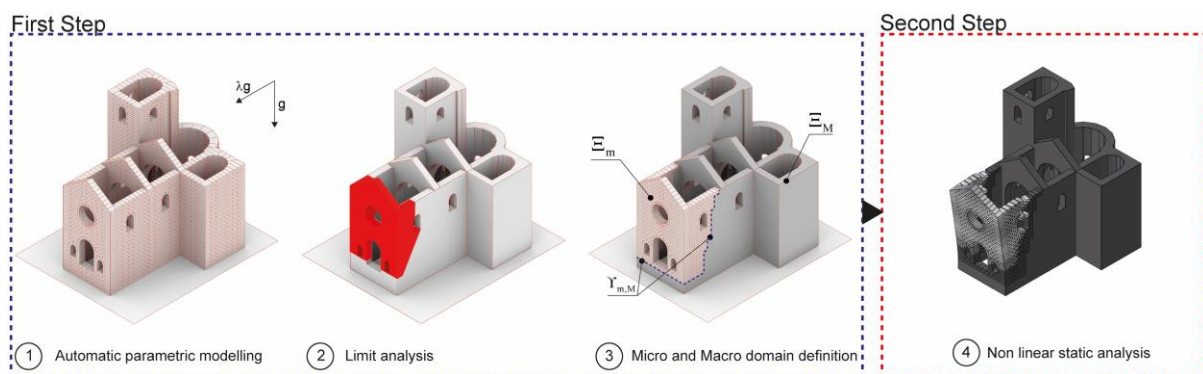
519 The last case study – and the most complex one – concerns a URM church and aims to evaluate  
520 the promptness and accuracy of the two-step framework when applied for a large-scale  
521 structure. The URM church is characterized by a plan consisting of a Latin Cross (Funari,  
522 Mehrotra, and Lourenço 2021). The geometry of the church is given in Figure 13 and some  
523 important features can be addressed: the main façade wall has a total height of 14.0m and a  
524 base ranging 7.50m; the single bell tower is the tallest structural element, with a height of  
525 17.0m; and the total length of the church is around 19.60m. Fixed boundary conditions are set  
526 at the base of the church walls. For the structural analysis, two load cases were considered and  
527 applied in a sequent manner: the self-weight is applied first, and a lateral body force (mass  
528 proportional) is applied next through an incremental load factor  $\lambda$ , as presented in Figure 11  
529 (node 1). Such lateral force, which intends to be representative of a seismic excitation, was  
530 applied along the longitudinal direction of the church, as this is typically the weakest direction.  
531 Material properties required to complete the proposed procedure are given in Table 4: the  
532 material density ( $\rho$ ) and friction coefficient ( $\nu$ ) for the limit analysis procedure; the Young's

533 modulus and Poisson's coefficient of the masonry units to define the overall elastic orthotropic  
 534 matrix of the  $\Xi_M$  domain according to the closed-form solutions of section 2.4.3; and the  
 535 normal and tangential stiffness values for the micro-macro interfaces. The dimensions of units  
 536 are  $0.57 \times 0.275 \times 0.90\text{m}^3$  ( $L \times H \times B$ ) .

537 Table 4: Mechanical properties adopted in the RMM and proposed CMM for the URM  
 538 church study.

$\rho$ [ $\text{kg m}^{-3}$ ]	$f$	$E_b$ [Pa]	$\nu_b$	$k_n$ [ $\text{Pa m}^{-1}$ ]	$k_s$ [ $\text{Pa m}^{-1}$ ]
2000	0.6	$20 \times 10^9$	0.2	$1 \times 10^{11}$	$1 \times 10^{11}$

539

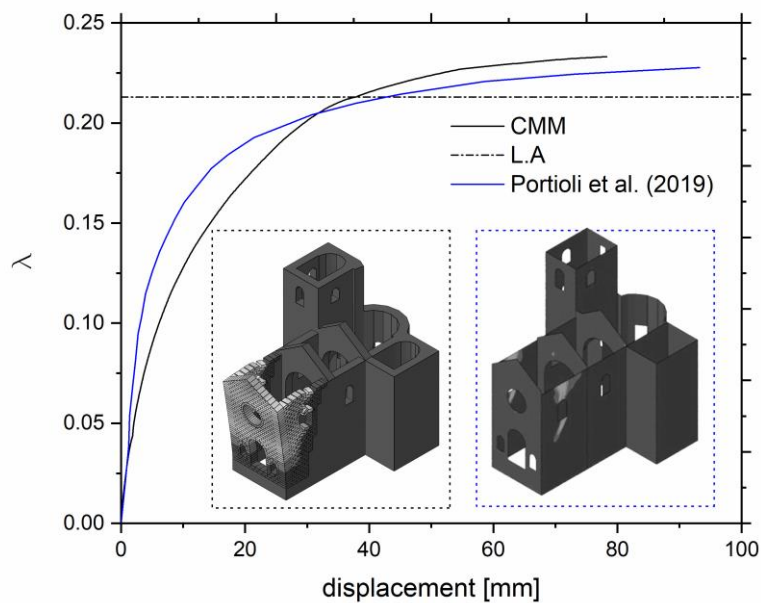


540

541 Figure 13: Proposed two-step framework applied to the URM church case study.

542 The proposed strategy is employed and the obtained failure mechanism through limit analysis  
 543 is given in Figure 13 (node 2). The overturning mechanism of the gable wall of the church  
 544 governs the collapse mode. Figure 13 (node 3) presents the corresponding numerical FE  
 545 concurrent model (CMM) developed at Abaqus CAE environment (Abaqus, 2014) by a re-  
 546 meshing procedure that retrieves the two non-overlapping domains, i.e. the micro  $\Xi_m$  and the  
 547 macro  $\Xi_M$  . A characteristic length  $R = 2L$  was assumed. Masonry units that belong to the  
 548  $\Xi_M$  domain are discretized by eight-node linear hexahedral finite elements (C3D8R in Abaqus  
 549 (2014)); thus leading to a  $\Delta_m / \kappa = 1 / 2$  , in which  $\kappa = \min (L, B, H)$  . In the  $\Xi_M$  domain, a  
 550 FE mesh refinement (structured FE mesh with squared elements) was considered with a size  
 551 given by  $\Delta_M / L = 1$  .

552 Figure 14 summarizes the lateral load-displacement (pushover) curve found with the proposed  
 553 CMM, together with the load multiplier value ( $\lambda = 0.213$ ) of the limit analysis processed in  
 554 the second sub-step (Figure 13) and with the numerical model from Malena, et al. (2019). The  
 555 latter numerical model developed by Malena, et al. (2019) is based on a homogeneous  
 556 macroscopic model with an elasto-plastic constitutive relation for the masonry.



557 Figure 14: Results obtained for the URM church: lateral load-displacement (pushover) curve  
 558 and displacement map for the proposed CMM and macroscopic model from Malena, et al.  
 559 (2019).

561 The comparison of the results allows demonstrating that the proposed approach ensures a good  
 562 solution accuracy, especially to what concerns the structure's load capacity. Some deviations  
 563 still pose within the elastic range. The limit analysis allows predictions on the collapse load are  
 564 within 8% difference. Figure 14 also gives the comparison in terms of failure mechanism. The  
 565 proposed model offers a clear identification of the failure surfaces, as damage localization is  
 566 directly lumped on FE interfaces. In converse, general insight over the failure mechanism is  
 567 difficult to conduct with the macroscopic model used for comparison purposes; this is,  
 568 however, a general disadvantage of macroscopic models that preclude a cracking-localization

569 algorithm (Clemente et al., 2006). At last, it bears noticing that the proposed CMM has, within  
570 the micro-domain  $\Xi_m$ , a total of 800 masonry stone units that leads to a total CPU time of  
571 75min. A comparison with a RMM was disregarded in this case study since the number of  
572 masonry units would increase up to more than 5000; meaning that a prohibitive CPU time  
573 would be required. No indication of CPU time required for the macroscopic model is reported  
574 in Malena, et al. (2019) and, therefore, a quantitative comparison on this matter is omitted.

#### 575 **4 Final remarks**

576 A two-step procedure was proposed aiming at the in- and out-of-plane mechanical study of  
577 dry-joint masonry structures. At a first step, a semi-automatic digital tool allows the parametric  
578 modeling of the structure that, together with an upper bound limit analysis tool and a heuristic  
579 optimization solver, enables tracking the most prone failure mechanism. The time required to  
580 process the first step is limited to a matter of seconds. At a second step, a coupled three-  
581 dimensional concurrent FE model with micro- and macro-scales is assumed. A micro-modeling  
582 description of the masonry is allocated to regions activated by the failure mechanism found in  
583 the former step. The other regions of the domain are modeled via a macro-approach, whose  
584 constitutive response is elastic and orthotropic and based on closed-form homogenized-based  
585 solutions. The time required to complete the second step is conditioned by the scale of the  
586 structure and type of structural analysis performed, as the modeling of the concurrent FE model  
587 is automatic and takes a matter of seconds.

588 The application of the framework was achieved through non-linear quasi-static analysis on  
589 three benchmarks: (i) an in-plane loaded URM shear wall; (ii) a U-shaped URM structure; and  
590 (iii) a URM church. Results demonstrate the potential and advantages of the proposed  
591 approach. It was able to predict, with a marginal difference (lower than 1%), the collapse load  
592 value. Failure collapse modes resemble to be alike with the ones found with a microscopic FE  
593 model (first two case studies) and with a literature macroscopic FE model (for the third and

594 last case study). Furthermore, the tool demonstrated that is quite attractive from a  
595 computational standpoint. It allows reducing the CPU time up to 60% in a small-to-medium  
596 scale structure (first and second case studies) when compared to a full microscopic FE model.  
597 Eventually, it may be the only alternative to macroscopic FE models when assessing large-  
598 scale structures, as micro-modeling proved to be a challenge.

599 At last, a comment on future research streams is of value. The two-step procedure is  
600 computational quite attractive, robust, and allows higher levels of accuracy. This is so because  
601 it is based on a sequential process in which a continuous transfer of information between scales  
602 is precluded during the analysis; as required in classical multi-domain strategies that need  
603 activation rules to process the macro-to-micro decomposition (L. Leonetti et al., 2018; Reccia  
604 et al., 2018; Driesen et al., 2021). Nonetheless, further studies need to be carried out to validate  
605 the approach in other contexts, for instance when assessing mortared masonry structures. In  
606 such a context, the authors believe that future works may include: (i) the definition of a more  
607 sophisticated limit analysis tool, e.g. (G. Milani, 2015; Chiozzi et al., 2017); and (ii) the  
608 implementation of an interfacial contact model at the micro-domain and within the FE  
609 concurrent model that can represent better the behavior of mortared joints (Lourenço et al.,  
610 2020).

## 611 **5 Appendix**

612 This appendix details the derivation of the formulas presented in Table 1, which have been  
613 formulated considering a spring's representation analogy (see Figure 7) and based on the  
614 infinitesimal strain theory.

615 The Young's modulus  $E_{xx}$ ,  $E_{yy}$  and  $E_{zz}$  can be obtained following the same procedure. For the  
616 save of brevity, only  $E_{xx}$  component is addressed here. According to Hooke's law, the axial  
617 deformation and displacement read as:

$$618 \quad E_{xx} = \frac{\sigma_{xx}}{\varepsilon_{xx}}; \quad \varepsilon_{xx} = \frac{\Delta u}{L}; \quad \Delta u = du_u + du_j \quad (\text{A.1})$$

619 In which  $\sigma_{xx}$  is the axial load,  $\varepsilon_{xx}$  is the axial deformation,  $\Delta u$  is the total displacement of the  
620 RVE,  $du_u$  is the displacement component related to the unit, and  $du_j$  is displacement  
621 component related to the joints, i.e. its normal displacement (interpenetration). Both contact  
622 interfaces have the same applied uni-axial stress and, therefore, Equations A.1 reads as:

$$623 \quad du_u = \frac{\sigma_{xx}L}{E_u}; \quad du_j = \frac{\sigma_{xx}}{k_n}; \quad \frac{\sigma_{xx}}{L} \left( \frac{L}{E_u} + \frac{1}{k_n} \right) = \sigma_{xx} \left( \frac{k_nL + E_u}{E_u k_n L} \right) \quad (\text{A.2})$$

624 Which corresponds to the following uni-axial Young's modulus  $E_{xx}$ :

$$625 \quad E_{xx} = \frac{E_u k_n L}{k_n L + E_u} \quad (\text{A.3})$$

626 For the in-plane shear moduli, one assumes the symmetry of the shearing stress components.  
627 Therefore, the in-plane shear moduli are defined as the ratio between the corresponding shear  
628 stress component and relative deformation. Accordingly:

$$629 \quad G_{xy} = G_{yx} = \frac{\tau_{xy}}{\gamma_{xy}}; \quad \text{with } \gamma_{xy} = \frac{du_x}{dy} + \frac{du_y}{dx} \quad (\text{A.4})$$

630 The shearing deformation  $\gamma_{xy}$  ( $= \gamma_{yx}$ ) should be computed when subjecting the RVE to a pure  
631 shear mechanism (Figure 7). Recalling that  $du_u$  is the shear displacement component related  
632 with the block deformation and  $du_j$  the shear displacement component related with the joint,  
633 the individual shearing deformation components are defined as:

$$634 \quad \begin{aligned} \frac{du_x}{dy} &= \frac{du_u + du_j}{2H} = \frac{\tau_{yx} \left( \frac{2}{k_s} + \frac{2H}{G_u} \right)}{2H} = \tau_{yx} \frac{G_u + k_s H}{G_u k_s H} \\ \frac{du_y}{dx} &= \frac{du_u + du_j}{L} = \frac{\tau_{xy} \left( \frac{1}{k_s} + \frac{L}{G_u} \right)}{L} = \tau_{xy} \frac{G_u + k_s L}{G_u k_s L} \end{aligned} \quad (\text{A.5})$$

635 By combining Eq. A.5 with Eq. A.4, one writes that the in-plane shear modulus is computed  
636 as:

$$637 \quad G_{yx} = G_{xy} = \frac{1}{\frac{1}{2} \left( \frac{G_u + k_s H}{G_u k_s H} + \frac{G_u + k_s L}{G_u k_s L} \right)} = \frac{G_u k_s H L}{G_u (H + L) + 2 k_s H L} \quad (\text{A.6})$$

638 Lastly, the equivalent in-plane Poisson's ratio  $\nu_{xy} = \nu_{yx}$  is demonstrated. To this aim, it bears  
639 highlighting that the lateral deformation in the joints was assumed to be zero since the study



640 deals with dry-mortar masonries. Therefore, the subscript u is related to the unit only and the  
641 Poisson's ratio is given as:

$$642 \quad \varepsilon_{xx,u} = \frac{\sigma_{xx}}{E_u} = \varepsilon_{xx} \frac{E_{xx}}{E_u}; \quad \varepsilon_{yy} = \varepsilon_{yy,u} = -\nu_u \times \varepsilon_{xx,u} = -\nu_u \varepsilon_{xx} \frac{E_{xx}}{E_u} \quad (\text{A.7})$$

$$643 \quad \nu_{yx} = \nu_{xy} = -\frac{\varepsilon_{yy}}{\varepsilon_{xx}} = \nu_u \frac{E_{xx}}{E_u} \quad (\text{A.8})$$

## 644 **6 References**

645 Abaqus, V., 6.14 Documentation, *Dassault Systemes Simulia Corporation*, vol. **651**, no. 6.2,  
646 2014.

647 Alessandri, C., Garutti, M., Mallardo, V. and Milani, G., Crack Patterns Induced by Foundation  
648 Settlements: Integrated Analysis on a Renaissance Masonry Palace in Italy, *International*  
649 *Journal of Architectural Heritage*, vol. **9**, no. 2, pp. 111–29, 2015. DOI:  
650 10.1080/15583058.2014.951795

651 Almeida, F. P. A. and Lourenço, P. B., Three-Dimensional Elastic Properties of Masonry by  
652 Mechanics of Structure Gene, *International Journal of Solids and Structures*, vol. **191–**  
653 **192**, no. May, pp. 202–11, 2020. DOI: 10.1016/j.ijsolstr.2019.12.009

654 Aşikoğlu, A., Avşar, Ö., Lourenço, P. B. and Silva, L. C., Effectiveness of Seismic Retrofitting  
655 of a Historical Masonry Structure: Kütahya Kurşunlu Mosque, Turkey, *Bulletin of*  
656 *Earthquake Engineering*, vol. **17**, no. 6, pp. 3365–95, from  
657 <https://doi.org/10.1007/s10518-019-00603-6>, 2019. DOI: 10.1007/s10518-019-00603-6

658 Bertolesi, E., Silva, L. C. and Milani, G., Validation of a Two-Step Simplified Compatible  
659 Homogenisation Approach Extended to out-Plane Loaded Masonries, *International*  
660 *Journal of Masonry Research and Innovation*, vol. **4**, p. 265, January 1, 2019. DOI:  
661 10.1504/IJMRI.2019.10019407

662 Betti, M. and Galano, L., Seismic Analysis of Historic Masonry Buildings: The Vicarious  
663 Palace in Pescia (Italy), *Buildings*, vol. **2**, no. 2, pp. 63–82, April 2012. DOI:  
664 10.3390/buildings2020063

665 Buhan, P. de and Felice, G. de, A Homogenisation Approach to the Ultimate Strength of Brick  
666 Masonry, *Journal of the Mechanics and Physics of Solids*, vol. **45**, no. 7, pp. 1085–1104,  
667 July 1997. DOI: 10.1016/S0022-5096(97)00002-1

668 Bui, T. T., Limam, A., Sarhosis, V. and Hjjaj, M., Discrete Element Modelling of the In-Plane  
669 and out-of-Plane Behaviour of Dry-Joint Masonry Wall Constructions, *Engineering*  
670 *Structures*, vol. **136**, no. October, pp. 277–94, 2017. DOI:  
671 10.1016/j.engstruct.2017.01.020

672 Casapulla, C., Cascini, L., Portioli, F. and Landolfo, R., 3D Macro and Micro-Block Models  
673 for Limit Analysis of out-of-Plane Loaded Masonry Walls with Non-Associative  
674 Coulomb Friction, *Meccanica*, vol. **49**, no. 7, pp. 1653–78, from  
675 <https://doi.org/10.1007/s11012-014-9943-8>, 2014. DOI: 10.1007/s11012-014-9943-8

676 Cascini, L., Gagliardo, R. and Portioli, F., LiABlock\_3D: A Software Tool for Collapse  
677 Mechanism Analysis of Historic Masonry Structures, *International Journal of*  
678 *Architectural Heritage*, pp. 1–20, from <https://doi.org/10.1080/15583058.2018.1509155>,  
679 September 5, 2018. DOI: 10.1080/15583058.2018.1509155

680 Casolo, S. and Milani, G., Simplified Out-of-Plane Modelling of Three-Leaf Masonry Walls  
681 Accounting for the Material Texture, *Construction and Building Materials*, vol. **40**, pp.  
682 330–51, March 2013. DOI: 10.1016/j.conbuildmat.2012.09.090

683 Cecchi, A. and Milani, G., A Kinematic FE Limit Analysis Model for Thick English Bond  
684 Masonry Walls, *International Journal of Solids and Structures*, vol. **45**, no. 5, pp. 1302–  
685 31, 2008. DOI: 10.1016/j.ijsolstr.2007.09.019

686 Chiozzi, A., Milani, G. and Tralli, A., A Genetic Algorithm NURBS-Based New Approach for  
687 Fast Kinematic Limit Analysis of Masonry Vaults, *Computers & Structures*, vol. **182**, pp.  
688 187–204, 2017. DOI: 10.1016/j.compstruc.2016.11.003

689 Clemente, R., Roca, P., and Cervera, M., Damage Model with Crack Localization–Application

690 to Historical Buildings, *Structural Analysis of Historical Constructions*, New Delhi, India,  
691 pp. 1125–34, 2006.

692 Cundari, G. A., Milani, G. and Failla, G., Seismic Vulnerability Evaluation of Historical  
693 Masonry Churches: Proposal for a General and Comprehensive Numerical Approach to  
694 Cross-Check Results, *Engineering Failure Analysis*, vol. **82**, pp. 208–28, from  
695 <http://www.sciencedirect.com/science/article/pii/S1350630717303187>, 2017. DOI:  
696 <https://doi.org/10.1016/j.engfailanal.2017.08.013>

697 D’Altri, A. M., Sarhosis, V., Milani, G., Rots, J., Cattari, S., Lagomarsino, S., Sacco, E., Tralli,  
698 A., Castellazzi, G. and Miranda, S. de, Modeling Strategies for the Computational  
699 Analysis of Unreinforced Masonry Structures: Review and Classification, *Archives of*  
700 *Computational Methods in Engineering*, 2019. DOI: 10.1007/s11831-019-09351-x

701 D’Ayala, D. and Speranza, E., Definition of Collapse Mechanisms and Seismic Vulnerability  
702 of Historic Masonry Buildings, *Earthquake Spectra*, vol. **19**, no. 3, pp. 479–509, August  
703 31, 2003. DOI: 10.1193/1.1599896

704 D’Ayala, D., and Speranza, E., An Integrated Procedure for the Assessment of Seismic  
705 Vulnerability of Historic Buildings, *12th European Conference on Earthquake*  
706 *Engineering*, October 16, 2002.

707 Driesen, C., Degée, H. and Vandoren, B., Efficient Modeling of Masonry Failure Using a  
708 Multiscale Domain Activation Approach, *Computers and Structures*, vol. **251**, p. 106543,  
709 July 2021. DOI: 10.1016/j.compstruc.2021.106543

710 Felice, G. de, Amorosi, A. and Malena, M., Elasto-Plastic Analysis of Block Structures through  
711 a Homogenization Method, *International Journal for Numerical and Analytical Methods*  
712 *in Geomechanics*, vol. **34**, no. 3, pp. 221–47, February 2010. DOI: 10.1002/nag.799

713 Felice, Gianmarco de and Giannini, R., Out-of-Plane Seismic Resistance of Masonry Walls,  
714 *Journal of Earthquake Engineering*, vol. **5**, no. 2, pp. 253–71, accessed May 22, 2018,

715 from <https://doi.org/10.1080/13632460109350394>, 2001. DOI:  
716 10.1080/13632460109350394

717 Ferreira, T. M., Costa, A. A. and Costa, A., Analysis of the Out-Of-Plane Seismic Behavior of  
718 Unreinforced Masonry: A Literature Review, *International Journal of Architectural*  
719 *Heritage*, vol. **9**, no. 8, pp. 949–72, November 10, 2014. DOI:  
720 10.1080/15583058.2014.885996

721 Ferreira, T. M., Costa, A. A., Vicente, R. and Varum, H., A Simplified Four-Branch Model for  
722 the Analytical Study of the out-of-Plane Performance of Regular Stone URM Walls,  
723 *Engineering Structures*, vol. **83**, pp. 140–53, January 2015. DOI:  
724 10.1016/j.engstruct.2014.10.048

725 Fish, J., Bridging the Scales in Nano Engineering and Science, *Journal of Nanoparticle*  
726 *Research*, vol. **8**, no. 5, pp. 577–94, 2006.

727 Fortunato, G., Funari, M. F. and Lonetti, P., Survey and Seismic Vulnerability Assessment of  
728 the Baptistery of San Giovanni in Tumba (Italy), *Journal of Cultural Heritage*, 2017. DOI:  
729 10.1016/j.culher.2017.01.010

730 Funari, M. F., Spadea, S., Ciantia, M., Lonetti, P., and Greco, F., Visual Programming for the  
731 Structural Assessment of Historic Masonry Structures, *REHABEND*, 2020.

732 Funari, Marco F, Mehrotra, A. and Lourenço, P. B., A Tool for the Rapid Seismic Assessment  
733 of Historic Masonry Structures Based on Limit Analysis Optimisation and Rocking  
734 Dynamics, *Applied Sciences (Switzerland)*, vol. **11**, no. 3, pp. 1–22, February 2021. DOI:  
735 10.3390/app11030942

736 Funari, Marco F, Spadea, S., Lonetti, P., Fabbrocino, F. and Luciano, R., Visual Programming  
737 for Structural Assessment of Out-of-Plane Mechanisms in Historic Masonry Structures,  
738 *Journal of Building Engineering*, vol. **31**, September 2020. DOI:  
739 10.1016/j.job.2020.101425

740 Ghosh, S., Adaptive Hierarchical-Concurrent Multiscale Modeling of Ductile Failure in  
741 Heterogeneous Metallic Materials, *JOM*, vol. **67**, no. 1, pp. 129–42, January 2015. DOI:  
742 10.1007/s11837-014-1193-7

743 Giuffré, A., A Mechanical Model for Statics and Dynamics of Historical Masonry Buildings,  
744 in *Protection of the Architectural Heritage Against Earthquakes*, Springer Vienna, pp.  
745 71–152, 1996.

746 Gonen, S., Pulatsu, B., Erdogmus, E., Karaesmen, E. and Karaesmen, E., Quasi-Static  
747 Nonlinear Seismic Assessment of a Fourth Century A.D. Roman Aqueduct in Istanbul,  
748 Turkey, *Heritage*, vol. **4**, no. 1, pp. 401–21, 2021. DOI: 10.3390/heritage4010025

749 Kouris, L. A. S., Bournas, D. A., Akintayo, O. T., Konstantinidis, A. A. and Aifantis, E. C., A  
750 Gradient Elastic Homogenisation Model for Brick Masonry, *Engineering Structures*, vol.  
751 **208**, p. 110311, April 2020. DOI: 10.1016/j.engstruct.2020.110311

752 Lemos, J. V., Discrete Element Modeling of Masonry Structures, *International Journal of*  
753 *Architectural Heritage*, vol. **1**, no. 2, pp. 190–213, May 31, 2007. DOI:  
754 10.1080/15583050601176868

755 Lemos, J. V., Discrete Element Modeling of the Seismic Behavior of Masonry Construction,  
756 *Buildings*, vol. **9**, no. 2, 2019. DOI: 10.3390/buildings9020043

757 Leonetti, L., Trovalusci, P. and Cechi, A., A Multiscale/Multidomain Model for the Failure  
758 Analysis of Masonry Walls: A Validation with a Combined FEM/DEM Approach,  
759 *International Journal for Multiscale Computational Engineering*, vol. **16**, no. 4, pp. 325–  
760 43, 2018. DOI: 10.1615/IntJMultCompEng.2018026988

761 Leonetti, Lorenzo, Greco, F., Trovalusci, P., Luciano, R. and Masiani, R., A Multiscale  
762 Damage Analysis of Periodic Composites Using a Couple-Stress/Cauchy Multidomain  
763 Model: Application to Masonry Structures, *Composites Part B: Engineering*, vol. **141**, pp.  
764 50–59, May 2018. DOI: 10.1016/j.compositesb.2017.12.025

765 Lloberas-Valls, O., Rixen, D. J., Simone, A. and Sluys, L. J., On Micro-to-Macro Connections  
766 in Domain Decomposition Multiscale Methods, *Computer Methods in Applied Mechanics*  
767 *and Engineering*, vol. **225–228**, pp. 177–96, accessed August 31, 2017, from  
768 <http://linkinghub.elsevier.com/retrieve/pii/S0045782512000989>, June 2012. DOI:  
769 10.1016/j.cma.2012.03.022

770 Lourenço, P. B. and Silva, L. C., Computational Applications in Masonry Structures: From the  
771 Meso-Scale to the Super-Large/Super-Complex, *International Journal for Multiscale*  
772 *Computational Engineering*, vol. **18**, no. 1, pp. 1–30, 2020. DOI:  
773 10.1615/IntJMultCompEng.2020030889

774 Malena, M., Portioli, F., Gagliardo, R., Tomaselli, G., Cascini, L. and Felice, G. de, Collapse  
775 Mechanism Analysis of Historic Masonry Structures Subjected to Lateral Loads: A  
776 Comparison between Continuous and Discrete Models, *Computers and Structures*, vol.  
777 **220**, pp. 14–31, 2019. DOI: 10.1016/j.compstruc.2019.04.005

778 Maria D’Altri, A., Presti, N. Lo, Grillanda, N., Castellazzi, G., Miranda, S. de and Milani, G.,  
779 A Two-Step Automated Procedure Based on Adaptive Limit and Pushover Analyses for  
780 the Seismic Assessment of Masonry Structures, *Computers and Structures*, vol. **252**, p.  
781 106561, August 2021. DOI: 10.1016/j.compstruc.2021.106561

782 Milani, G., Upper Bound Sequential Linear Programming Mesh Adaptation Scheme for  
783 Collapse Analysis of Masonry Vaults, *Advances in Engineering Software*, vol. **79**, pp. 91–  
784 110, January 2015. DOI: 10.1016/J.ADVENGSOFT.2014.09.004

785 Milani, G., Lourenço, P. B. and Tralli, A., Homogenised Limit Analysis of Masonry Walls,  
786 Part II: Structural Examples, *Computers & Structures*, vol. **84**, no. 3–4, pp. 181–95,  
787 January 2006. DOI: 10.1016/j.compstruc.2005.09.004

788 Milani, Gabriele and Venturini, G., Automatic Fragility Curve Evaluation of Masonry  
789 Churches Accounting for Partial Collapses by Means of 3D FE Homogenized Limit

790 Analysis, *Computers & Structures*, vol. **89**, no. 17–18, pp. 1628–48, September 2011.  
791 DOI: 10.1016/j.compstruc.2011.04.014

792 Otero, F., Oller, S., Martínez, X. and Salomón, O., Numerical Homogenization for Composite  
793 Materials Analysis. Comparison with Other Micro Mechanical Formulations, *Composite*  
794 *Structures*, vol. **122**, pp. 405–16, accessed August 28, 2017, from  
795 <http://linkinghub.elsevier.com/retrieve/pii/S0263822314006102>, April 2015. DOI:  
796 10.1016/j.compstruct.2014.11.041

797 Reccia, E., Leonetti, L., Trovalusci, P. and Cecchi, A., A Multiscale/Multidomain Model for  
798 the Failure Analysis of Masonry Walls: A Validation with a Combined FEM/DEM  
799 Approach, *International Journal for Multiscale Computational Engineering*, vol. **16**, no.  
800 4, pp. 325–43, 2018. DOI: 10.1615/IntJMultCompEng.2018026988

801 Ren, X., Chaurasia, A. K. and Seidel, G. D., Concurrent Multiscale Modeling of Coupling  
802 between Continuum Damage and Piezoresistivity in CNT-Polymer Nanocomposites,  
803 *International Journal of Solids and Structures*, vol. **96**, pp. 340–54, October 2016. DOI:  
804 10.1016/j.ijsolstr.2016.05.018

805 Restrepo Vélez, L. F., Magenes, G. and Griffith, M. C., Dry Stone Masonry Walls in Bending-  
806 Part I: Static Tests, *International Journal of Architectural Heritage*, vol. **8**, no. 1, pp. 1–  
807 28, 2014. DOI: 10.1080/15583058.2012.663059

808 Roca, P., Cervera, M., Pelà, L., Clemente, R. and Chiumenti, M., Continuum FE Models for  
809 the Analysis of Mallorca Cathedral, *Engineering Structures*, vol. **46**, pp. 653–70, January  
810 2013. DOI: 10.1016/j.engstruct.2012.08.005

811 Rodrigues, E. A., Manzoli, O. L., Bitencourt, L. A. G., Bittencourt, T. N. and Sánchez, M., An  
812 Adaptive Concurrent Multiscale Model for Concrete Based on Coupling Finite Elements,  
813 *Computer Methods in Applied Mechanics and Engineering*, vol. **328**, pp. 26–46, January  
814 2018. DOI: 10.1016/j.cma.2017.08.048

815 Rutten, D., Galapagos: On the Logic and Limitations of Generic Solvers, *Architectural Design*,  
816 vol. **83**, no. 2, pp. 132–35, March 2013. DOI: 10.1002/ad.1568

817 Savalle, N., Mousavian, E., Colombo, C., and Lourenço, P. B., FAST GENERATIVE TOOL  
818 FOR MASONRY STRUCTURES GEOMETRIES, *14th Canadian Masonry Symposium*,  
819 Montréal, Canada, pp. 1–11, 2021.

820 Savalle, N., Vincens, É. and Hans, S., Experimental and Numerical Studies on Scaled-down  
821 Dry-Joint Retaining Walls: Pseudo-Static Approach to Quantify the Resistance of a Dry-  
822 Joint Brick Retaining Wall, *Bulletin of Earthquake Engineering*, vol. **18**, no. 2, pp. 581–  
823 606, January 2020. DOI: 10.1007/s10518-019-00670-9

824 Sharma, S., Silva, L. C., Graziotti, F., Magenes, G. and Milani, G., Modelling the Experimental  
825 Seismic Out-of-Plane Two-Way Bending Response of Unreinforced Periodic Masonry  
826 Panels Using a Non-Linear Discrete Homogenized Strategy, *Engineering Structures*, vol.  
827 **242**, no. December 2020, from <https://doi.org/10.1016/j.engstruct.2021.112524>, 2021.  
828 DOI: 10.1016/j.engstruct.2021.112524

829 Silva, L. C., Lourenço, P. B. and Milani, G., Nonlinear Discrete Homogenized Model for Out-  
830 of-Plane Loaded Masonry Walls, *Journal of Structural Engineering*, vol. **143**, no. 9, p.  
831 4017099, from [https://doi.org/10.1061/\(ASCE\)ST.1943-541X.0001831](https://doi.org/10.1061/(ASCE)ST.1943-541X.0001831), September 1,  
832 2017. DOI: 10.1061/(ASCE)ST.1943-541X.0001831

833 Smoljanović, H., Živaljić, N., Nikolić, Ž. and Munjiza, A., Numerical Analysis of 3D Dry-  
834 Stone Masonry Structures by Combined Finite-Discrete Element Method, *International*  
835 *Journal of Solids and Structures*, vol. **136–137**, pp. 150–67, April 2018. DOI:  
836 10.1016/j.ijsolstr.2017.12.012

837 Sorrentino, L., D’Ayala, D., Felice, G. de, Griffith, M. C., Lagomarsino, S. and Magenes, G.,  
838 Review of Out-of-Plane Seismic Assessment Techniques Applied To Existing Masonry  
839 Buildings, *International Journal of Architectural Heritage*, vol. **11**, no. 1, pp. 2–21, from



840 <https://doi.org/10.1080/15583058.2016.1237586>, January 2, 2017. DOI:  
841 10.1080/15583058.2016.1237586

842 Stepinac, M., Lourenço, P. B., Atalić, J., Kišiček, T., Uroš, M., Baniček, M. and Šavor Novak,  
843 M., Damage Classification of Residential Buildings in Historical Downtown after the  
844 ML5.5 Earthquake in Zagreb, Croatia in 2020, *International Journal of Disaster Risk*  
845 *Reduction*, vol. **56**, p. 102140, April 2021. DOI: 10.1016/j.ijdr.2021.102140

846 Talebi, H., Silani, M. and Rabczuk, T., Concurrent Multiscale Modeling of Three Dimensional  
847 Crack and Dislocation Propagation, *Advances in Engineering Software*, vol. **80**, no. C, pp.  
848 82–92, February 2015. DOI: 10.1016/j.advengsoft.2014.09.016

849 The Python Language Reference — Python 3.9.5 Documentation, n.d.

850 Trovalusci, P., Ostoja-Starzewski, M., Bellis, M. L. De and Murralli, A., Scale-Dependent  
851 Homogenization of Random Composites as Micropolar Continua, *European Journal of*  
852 *Mechanics - A/Solids*, vol. **49**, pp. 396–407, 2015. DOI:  
853 <https://doi.org/10.1016/j.euromechsol.2014.08.010>

854 Turco, C., Funari, M. F., Spadea, S., Ciantia, M., and Lourenço, P. B., A Digital Tool Based  
855 on Genetic Algorithms and Limit Analysis for the Seismic Assessment of Historic  
856 Masonry Buildings, *Procedia Structural Integrity*, vol. **28**, 2020.

857 Vandoren, B., Proft, K. De, Simone, A. and Sluys, L. J., Mesoscopic Modelling of Masonry  
858 Using Weak and Strong Discontinuities, *Computer Methods in Applied Mechanics and*  
859 *Engineering*, vol. **255**, pp. 167–82, March 2013. DOI: 10.1016/j.cma.2012.11.005

860 Vlachakis, G., Vlachaki, E. and Lourenço, P. B., Learning from Failure: Damage and Failure  
861 of Masonry Structures, after the 2017 Lesvos Earthquake (Greece), *Engineering Failure*  
862 *Analysis*, vol. **117**, p. 104803, November 2020. DOI: 10.1016/j.engfailanal.2020.104803

863