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A concurrent micro/macro FE-model optimized with a limit analysis tool for the assessment of dry-joint masonry structures

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25 **1 Introduction**

26 Field inspections after earthquake events report how out-of-plane failure mechanisms are prone
27 to occur in historical masonry structures. Undesired consequences include the collapse of
28 buildings, human losses, and loss of societal identity (Stepinac et al., 2021; Vlachakis et al.,
29 2020). Preventive remedial interventions on the built heritage are complex to perform since a
30 sound knowledge of the structural and material features is lacking for most of the cases.
31 Scientifically based studies are less susceptible to inadequate actions and advocate for proper
32 structural analysis tools. To this aim, the literature was, in the last decades, enriched with the
33 development of analysis methods for masonry structures. A plethora of strategies can be found
34 now but seems clear that research leans towards the so-called (i) analytical and (ii) numerical
35 approaches (Ferreira et al., 2014; Ferreira et al., 2015; D'Altri et al., 2019).

36 Analytical approaches are often based on the theorems of limit analysis and through a force-
37 or displacement-based formulation (Cascini et al., 2018; Gianmarco de Felice et al., 2001).
38 These are especially suitable for a rapid seismic fragility assessment, as require few input
39 material parameters and provide good estimations on collapse load multiplier for defined
40 failure mechanisms (Giuffré 1996; D'Ayala, and Speranza 2003); however, are unable to track
41 displacement history and damage evolution. To what concerns numerical approaches, the Finite
42 Element Method (FEM) (Fortunato et al., 2017; Aşıkoğlu et al., 2019) and the Discrete Element
43 Method (DEM) (Savalle et al., 2020; Lemos, 2007; Lemos, 2019; Bui et al., 2017; Gonen et
44 al., 2021) are largely used. DEM is now well suited for masonries with both dry- and mortared
45 joints, but still requires a full representation of the blocks (masonry units) arrangement (Lemos,
46 2007). FEM allows a more versatile application as masonry can be represented either through
47 a continuous equivalent media (designated macro-modeling) or by a discrete representation of
48 units and joints (designated micro-modeling). Linear and non-linear static and dynamic
49 analyses are eligible. Nonetheless, additionally to the significant amount of data needed to

50 characterize the non-linear response of materials, the analysis can be both time-consuming and
51 computationally expensive when estimating the ultimate ductility level of the structure.

52 To cope with the prohibitive computational cost, especially when dealing with large-scale
53 structures and within full material nonlinearity, multi-scale FE methods seem a promising
54 alternative and are in between the micro- and macro- FE schemes. Classical FE² approaches,
55 i.e. the full continuum-FE methods, have clear advantages if linear elastic behavior is assumed,
56 but obtaining a micro-scale solution at each load step of a non-linear process for each Gauss
57 point may turn the problem prohibitive from a computational point of view; especially if
58 nonlinearities are assumed (Otero et al., 2015; Lourenço et al., 2020). These strategies still have
59 a higher computational cost than a FE macroscopic model. Hence full continuum-based FE²
60 approaches are seldom used for dynamic purposes and complex structural analysis (Lourenço
61 et al., 2020). The development of techniques that keep accuracy to acceptable levels and speed
62 up the processing running times is critical. Several authors tried, therefore, to address
63 simplifications on two-step frameworks.

64 The use of discrete FE-based methods at a macro-level is a promising alternative. Two-step
65 approaches based on a discrete FE-based at a macro-scale are very practical due to the decrease
66 of the number of degrees of freedom (comparing to a continuous approach) and are especially
67 useful to perform dynamic analysis. Several studies have shown the clear advantages of this
68 process since it allows a good trade-off between consumed time and results' accuracy and
69 enables the study of real scale buildings. The latter is even more clear if simplifications are
70 further assumed at both macro- or micro-scales, as observed in (Gabriele Milani et al., 2011;
71 Bertolesi et al., 2019; Sharma et al., 2021; Casolo et al., 2013; Silva et al., 2017).

72 The use of limit analysis can be also a promising alternative. Some authors used a kinematic
73 theorem of limit analysis at a macro-level to obtain the homogenized failure surfaces with a
74 very limited computational effort (Cecchi et al., 2008; G. Milani et al., 2006; de Buhan et al.,

75 1997). Such methods give a lower or upper bound estimate on the failure collapse load that can
76 be scarce in some cases. Nonetheless, limit analysis is also being used together with FE-based
77 strategies. Recently, Betti and Galano (2012) and Cundari et al. (2017) proposed similar
78 frameworks, in which the global structural analysis was achieved by non-linear static or
79 dynamic analysis aiming at the detection of the most likely collapse mechanisms. Then, at a
80 second step, an upper bound limit analysis method was applied in the identified mechanisms
81 to compute the maximum horizontal acceleration that the structure can withstand. Additionally,
82 Funari et al. (2020) developed a non-linear static analysis to identify the most prone failure
83 mechanisms and then, in a second step, aimed to refine the geometry of the failure mechanism
84 through an optimization based on limit analysis and genetic algorithm; hence exploring an
85 extensive set kinematically compatible solutions. D'Altri, et al. (2021) used limit analysis as a
86 first step towards the identification of cracked surfaces and, in the next step, a macroscopic FE
87 model was used to perform non-linear quasi-static analysis, in which the cracking zones were
88 simulated with a microscopic description. However, a full-nonlinear behavior for the whole
89 structure (even for the non-cracked zone) was assumed, which blurs the computational
90 efficiency of the procedure and especially highlights the interest over more sophisticated
91 approaches.

92 In this endeavor for a fast tool, yet able to give accurate descriptions of the structure's capacity
93 and damage evolution, one may stress the so-called concurrent multi-scale approaches. These
94 have been already applied to simulate fracture propagation in composites (Talebi et al. 2015;
95 Ghosh 2015), nanocomposite (Ren et al. 2016), but also in the study of masonry structures
96 (Lorenzo Leonetti et al., 2018; Driesen et al., 2021; Lourenço et al., 2020). A concurrent
97 multiscale approach contemplates two well-separated scales, i.e. refined and coarse domains,
98 described by a non-conforming mesh discretization and solved simultaneously. The refined
99 domain, which ensures the modeling of non-linearities in the material behavior, is adopted in

100 the regions that expect to fail, whereas, in the coarse domain, non-linearities are assumed
101 negligible.

102 Lloberas-Valls et al. (2012a) investigated several incompatible mesh connections in the
103 framework of a strongly coupled multiscale model to describe the crack growth and
104 coalescence phenomena. Their model integrated a sophisticated zoom-in procedure that
105 enables, during the loading history and based on a proper mechanical criterion, to switch from
106 a coarser to a finer discretization of the media. Similarly, Talebi et al. (2015) developed a
107 concurrent coupling scheme suitable to simulate the crack and dislocations at an atomistic
108 level. Rodrigues et al. (2018) focused on the definition of an adaptive concurrent multiscale
109 approach for the crack propagation phenomena in concrete structures. Their main novelty lies
110 in using a non-periodic RVE cell, in which the FE mesh between the refined and coarse
111 domains are independent.

112 A noteworthy study of a concurrent multiscale approach to investigate the in-plane failure of
113 masonry structures was developed by Leonetti et al. (2018). A multiscale/multidomain-based
114 computational scheme allowed to reduce the computational cost associated with a classical FE
115 micro-modeling approach. Furthermore, a recent study on the subject has shown the potential
116 of multiscale approaches applied to masonry in a bi-dimensional framework and pointed out,
117 as a future research path, the interest of limit analysis envisioned as a preliminary step for this
118 kind of procedure (Driesen et al., 2021).

119 The literature shows the potential of two-step procedures. However, the development of such
120 tools aiming at an optimal localization of non-linearities – to reduce the associated convergence
121 issues and computational cost – is still needed. Although concurrent multiscale approaches are
122 certainly a promising alternative to simulate the failure of an extensive range of materials, e.g.
123 concrete, masonry, composites, among others, its use is still limited to bi-dimensional
124 frameworks and for small-scale case studies. In this context, this paper presents an integrated

125 two-stepped procedure that was developed with the aim of enriching the literature on the field
126 of FE concurrent model. The main contribution is the possibility of conducting a three-
127 dimensional analysis of masonry structures within a low computational cost. To this aim, at a
128 first step, a limit analysis tool finds the most prone failure mechanisms. At a second step, a FE
129 concurrent multiscale approach is used to study the in-plane and out-of-plane response of
130 masonry structures. Both steps are coupled, meaning that the failure surfaces that are found
131 with limit analysis are modeled within a microscopic approach. Furthermore, and to fully
132 optimize running times, the domain that is beyond the cracking surfaces, from an a-priori given
133 characteristic length, is modeled as an elastic and orthotropic media.

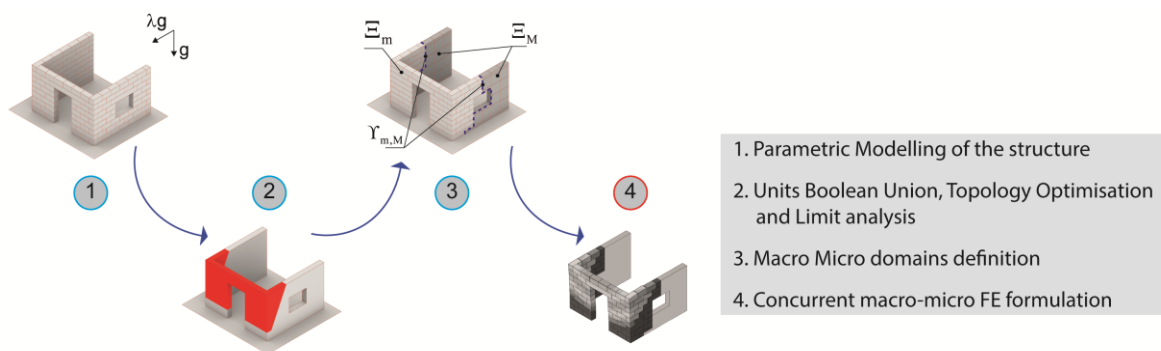
134 The paper is organized into four main sections: section 2 describes the analysis framework
135 focusing on both theoretical and numerical aspects; section 3 reports three validation examples,
136 which differ in scale, and reports the benchmark given as a case study; and finally, final remarks
137 are discussed in Section 4.

138 **2 Two-scale framework: general description**

139 A numerical framework is presented aiming an accurate description of the in- and out-of-plane
140 mechanical behavior of unreinforced masonry (URM) structures. It was formulated to require
141 a lower computational cost than full FE microscopic and macroscopic (non-linear) strategies
142 (Roca et al., 2013; Lourenço et al., 2020). The so-called concurrent approach (firstly presented
143 by Fish (2006)) is adopted together with a limit analysis tool. In this regard, the framework has
144 two sequential and coupled steps, in which a limit analysis is conducted first, and a concurrent
145 FE analysis is employed next.

146 The framework described in more detail next includes three main tasks, as given in Figure 1,
147 needed to compute the mechanical response of URM structures. The first step consists of the
148 geometric modeling of the structure via an explicit representation of both masonry units and
149 joints (micro-modeling approach). In the second step, masonry units are merged, and its

150 topology is optimized to provide a macro representation of the media. Prone in-plane and/or
 151 out-of-plane failure mechanisms are a-priori assigned, and the location of the yielding surfaces
 152 is optimized by an upper bound limit analysis theorem coupled with a heuristic solver. At this
 153 stage, the third step is conducted, in which an ad-hoc script represents the sub-structure
 154 activated by the failure mechanism through a micro-scale representation. The outer domain,
 155 i.e. the rest of the structure that is not involved in the mechanism, keeps a macro and continuous
 156 representation. Finally, the concurrent FE multiscale model can be used to perform the
 157 structural assessment of the structure through linear/non-linear quasi-static/dynamic type of
 158 analysis and within a FE environment. Further details over each step are addressed in the next
 159 sections.



160
 161 Figure 1: Schematic representation of the proposed two-step numerical strategy.

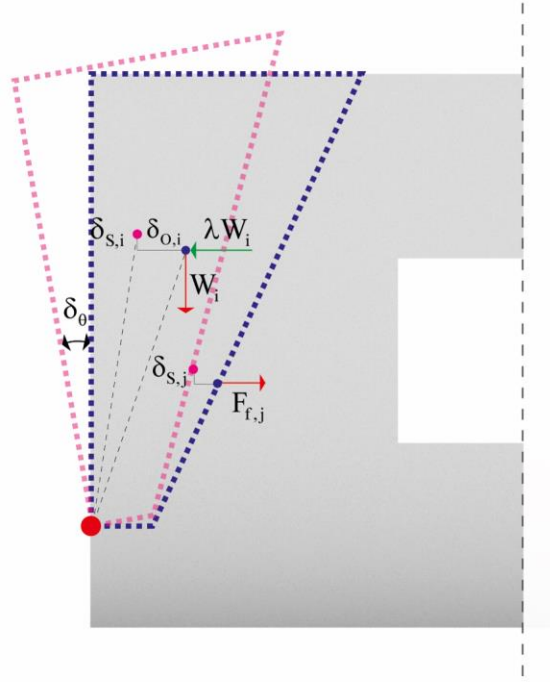
162 2.1 Parametric modelling of the structure

163 The geometric modeling of the structure is the first step of the framework (node one in Figure
 164 1). To this aim, it requires the knowledge of the masonry arrangement since a micro-modeling
 165 approach is assumed. Although a full representation of masonry units and joints can be
 166 cumbersome and time-consuming, the framework integrates a digital tool that was recently
 167 proposed by Savalle, et al. (2021). This tool allows the pre-processing of the geometry via an
 168 automatic generation of the masonry arrangement, and it was implemented in the environment
 169 offered by Rhinoceros (+ Grasshopper) through C# programming language. It includes an
 170 initial discretization of the structure into elementary parts, i.e. walls arrangement, location of

171 corners, location of T-connection, among other substructures, which can be assembled without
172 any restriction aiming to form complex three-dimensional structures. Then, by setting up the
173 dimensions of units and joints – given as user input –, the masonry pattern is positioned to
174 respect the latter structural features. Openings can be also included in the walls, and the user
175 can specify its height, length, position, and the dimension of lintels. For the sake of brevity, the
176 reader is referred to (Savalle et al., 2021) for further details.

177 **2.2 Upper-Bound Limit Analysis**

178 The geometric model defined in the first step serves as a basis to conduct the second step of
179 the framework. In this sub-step of the framework, prone failure mechanisms are pre-defined
180 and assessed through an optimization tool that integrates an Upper-Bound limit analysis
181 theorem coupled with a heuristic solver (node two in Figure 1). Therefore, the geometry of the
182 expected active failure mechanism is parametrized to find the optimal configuration. The
183 optimization problem aims at the minimization of the horizontal load multiplier, which can be
184 formulated through the principle of virtual work. Figure 2 describes an overturning mechanism
185 of a masonry wall, in which the kinematic description required to formulate the problem is
186 conditioned by one virtual rotation δ_θ (Casapulla, et al. 2014; Funari, et al. 2020). The
187 formulation of such a mechanism is addressed next, as it is the one assumed for the benchmarks
188 reported in this study. It worth stressing that the formulation resorts on a representation of the
189 media through a macro-approach – units forming the masonry prototype are merged –, in which
190 the mechanism is represented by one virtual parameter only (Figure 2). Such assumption is
191 convenient for an initial assessment of the most prone mechanism, and it is largely used in
192 classic limit analysis approaches, see for instance (Sorrentino et al., 2017; D’Ayala et al.,
193 2002).



194

195 Figure 2: Kinematic description of the overturning failure mechanism.

196 As presented in Figure 2, the external virtual work contains both the overturning as well as the
 197 stabilizing works performed by the inertial forces, whereas the internal work derives from the
 198 friction force at contact interfaces:

$$\begin{aligned}
 \delta W_{\text{ext}} &= \lambda \sum_{i=1}^n W_i \delta_{O,i} - \sum_{i=1}^n W_i \delta_{S,i} \\
 \delta W_{\text{int}} &= \sum_{j=1}^n F_{f,j} \delta_{S,j}
 \end{aligned} \tag{1}$$

200 in which w_i are the inertial forces arising from the self-weight of the i -th masonry wall and
 201 including as well the contribution of roof and floors; $\delta_{o,i}$ and $\delta_{s,i}$ are the virtual overturning
 202 or stabilizing displacements of the application point of the inertial forces (that coincides with
 203 the virtual centroid if self-weight is considered only); $F_{f,j}$ are the frictional forces computed
 204 as a weighted value of the maximum friction force F_{max} based on the inclination of the crack
 205 line, as given next (Casapulla et al., 2014):

$$F_f = F_{\text{max}} \left(1 - \frac{\alpha_c}{\alpha_b} \right) \tag{2}$$

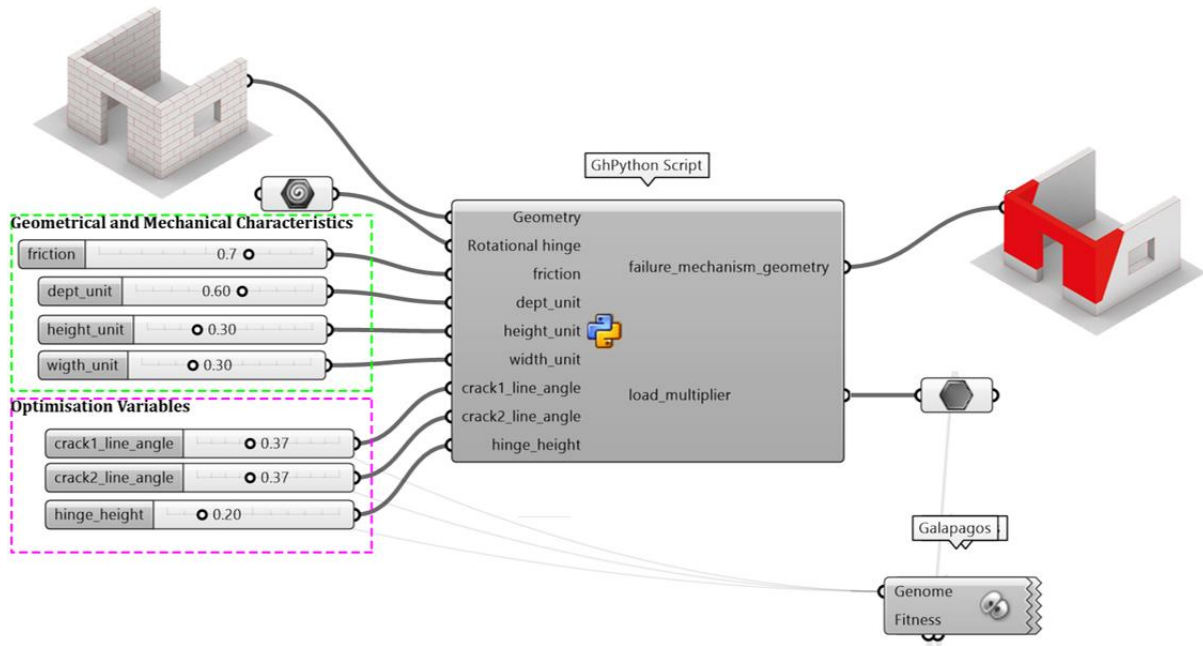
207 Here, α_b and α_c are the maximum frictional and crack angle, respectively (Funari, Mehrotra,
 208 and Lourenço 2021). The value of the horizontal load multiplier λ is obtained by solving Eq.
 209 (1) and reads:

$$210 \quad \lambda = \frac{\sum_{j=1}^n F_{f,j} \delta_{s,j} + \sum_{i=1}^n W_i \delta_{s,i}}{\sum_{i=1}^n W_i \delta_{o,i}} \quad (3)$$

211 It is worth noting that the value λ depends on the geometry of the failure mechanism, which is
 212 defined by the crack inclination α_c and the height of the rotational hinge H_h . Such variables
 213 define the set of possible λ values. At last, the optimization of the out-of-plane failure
 214 mechanism geometry is achieved by solving the following constraint minimization problem:

$$215 \quad \left\{ \min \lambda : \begin{bmatrix} \tan \alpha_{\min} \leq \tan \alpha_c \leq \tan \alpha_b \\ 0 \leq H_h \leq H_w \end{bmatrix} \right. \quad (4)$$

216 The constrained optimization problem defined in Eq. (4) was numerically implemented in a
 217 GHPython script, as depicted in Figure 3. Input data include: (i) the geometry defined in the
 218 first step (node one in Figure 1), (ii) the friction coefficient of the masonry, (iii) the rotational
 219 axis, and (iv) the geometric dimensions of masonry units. As shown in Figure 3, the variables
 220 of the optimization problem are grouped in the magenta box, i.e.: (i) the slope of the crack
 221 surfaces, and (ii) the height of the rotational axis. The solution is achieved using the GH
 222 component Galapagos (Rutten, 2013), in which a heuristic research method based on a genetic
 223 algorithm is implemented. The optimization problem finds the configuration for the critical
 224 failure mechanism and under a low processing time (few seconds). The kinematic problem
 225 used herein has theoretical background on the works of Turco, et al. (2020); Funari, et al.
 226 (2020).



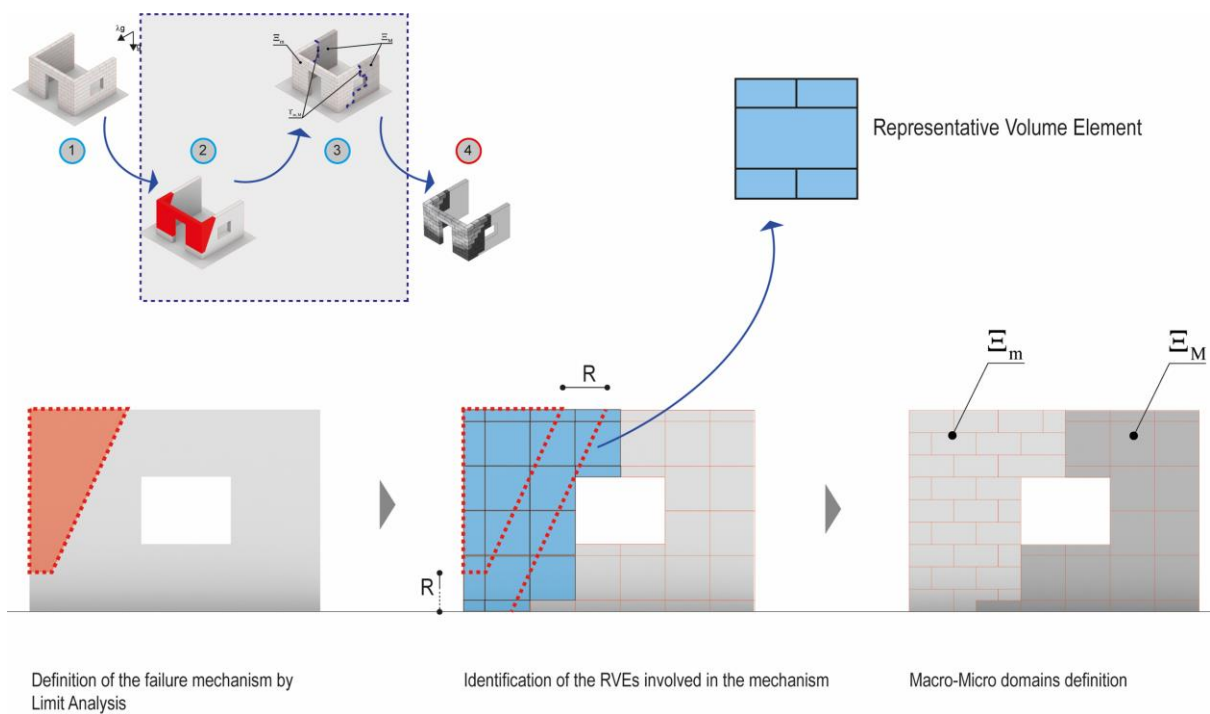
227
 228 Figure 3: Limit analysis tool implemented in Rhino+Grasshopper through GHPython scripting
 229 that finds the critical failure mechanism.

230 2.3 Macro-Micro domains definition

231 The definition of macro- and micro-domains is the third step of the framework (node three in
 232 Figure 1). It resumes being a fast procedure once the theoretical failure mechanism is found by
 233 limit analysis. The decomposition of both domains is directly defined over the failure
 234 mechanism. Although other re-meshing approaches would be plausible, the present study
 235 assumes two different scales. A finer scale, designated as micro-domain Ξ_m that is attributed
 236 to the substructure defined by the active failure mechanism, in which masonry arrangement is
 237 explicitly represented. A coarser scale, designated as macro domain Ξ_M is attributed to the rest
 238 of the structure, in which masonry is represented through a continuous and equivalent elastic
 239 media.

240 A contentious issue is the finding of the frontier between domains. Cracking tends to spread
 241 from the main surfaces failures and have an important role in the non-linear behavior of
 242 masonry and in damage-induced orthotropy. To avoid inaccurate solutions retrieved from the
 243 disregard of this cracking, the damage in the vicinity of the main failure surfaces was

244 considered by adding the scalar parameter R as input. The parameter R is a length value that
 245 extends the part of the structure being characterized with a microdomain, as presented in Figure
 246 4. The choice of R affects both the accuracy and computational time of the solution since it
 247 increases or decreases the non-linear region of the model. Therefore, a proper choice of R -value
 248 is paramount and it is recommended that it includes: (i) the epistemic error in the prediction of
 249 the hinges through limit analysis, (ii) the existence of a potential curved failure surface, in
 250 converse to the straight-type yielded surfaces assumed by the limit analysis tool, and (iii) the
 251 more diffuse failure surfaces due to the zig-zag damage (especially in sliding and flexure
 252 mechanisms) in actual masonry specimens (Restrepo Vélez et al., 2014; Bui et al., 2017;
 253 Cascini et al., 2018).



255 Figure 4: Schematic representation of the decomposition procedure into macro- and micro-
 256 domains.

257 In this regard, and as schematically described in Figure 4, the dimension of the Representative
 258 Volume Element (RVE) defines the grid that enables the domain decomposition. This resorts
 259 to be an alike strategy followed by other studies, as (Vandoren et al., 2013; Lorenzo Leonetti
 260 et al., 2018; Alessandri et al., 2015; Almeida et al., 2020; Lloberas-Valls et al., 2012; Driesen

261 et al., 2021). A classical RVE adopted for a running-bond pattern (Trovalusci et al., 2015) was
262 considered since the selected case studies follow such arrangement; needless to state that other
263 configurations can be employed. The domain decomposition of every grid region, into a micro-
264 description of the RVE, is performed if intersect the failure surface (plus the characteristic
265 length R); being the other grid elements kept as macro domain Ξ_M regions.

266 **2.4 Concurrent FE macro-micro model**

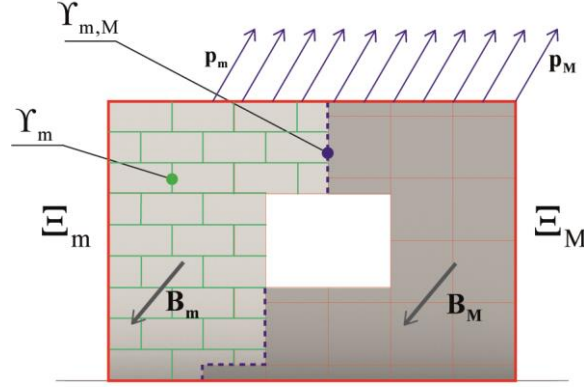
267 The limit analysis procedure performed over the structure allows identifying two subdomains
268 that have different scales of computation, i.e. the macroscopic Ξ_M and the microscopic Ξ_m
269 domains (Figure 1, node 3). Both are concurrent, meaning that together define simultaneously
270 different volumes of the structure. Towards a low computational cost, material non-linearities
271 are assigned only to the materials belonging to the micro-domain Ξ_m . On the other hand, the
272 media inside the Ξ_M domain is simulated with an equivalent linear elastic orthotropic material,
273 whose elastic properties are computed with a proper homogenization strategy to guarantee the
274 objectivity of the solution. Such hypotheses are particularly suitable for well-marked failures,
275 as the ones experienced in unreinforced masonry structures: local failure mechanisms governed
276 by out-of-plane loading due to poor connection between structural elements (Malena et al.,
277 2019; Restrepo Vélez et al., 2014).

278 **2.4.1 Variational Formulation**

279 The concurrent FE model requires a numerical solution for each scale and was implemented in
280 the FE software Abaqus (2014). A set of weak form equations are solved in a coupled manner
281 through a variational formulation for both the Ξ_M and the Ξ_m domains. Proper kinematic
282 constraints are employed at the regions where both domains meet, designated as interfaces
283 $\Upsilon_{m,M}$. Specifically, an additional internal boundary condition is associated with $\Upsilon_{m,M}$ and
284 seeks to enforce the continuity between total displacement (Lorenzo Leonetti et al., 2018;
285 Driesen et al., 2021):

$$\begin{aligned}
286 \quad \Xi_M: \quad & \int_{\Xi_M} \boldsymbol{\sigma}_M \cdot \delta \boldsymbol{\varepsilon}_M d\Xi_M - \int_{\Upsilon_{m,M}} \zeta \cdot \delta \mathbf{u}_M d\Upsilon_{m,M} = \int_{\Xi_M} \mathbf{B}_M^T \cdot \delta \mathbf{u}_M d\Xi_M + \int_{\Upsilon} \mathbf{p}_M^T \cdot \delta \mathbf{u}_M d\Upsilon \\
\Xi_m: \quad & \int_{\Xi_m} \boldsymbol{\sigma}_m \cdot \delta \boldsymbol{\varepsilon}_m d\Xi_m + \int_{\Upsilon_m} \mathbf{t} \cdot \delta \bar{\mathbf{u}}_m d\Upsilon_m + \int_{\Upsilon_{m,M}} \zeta \cdot \delta \mathbf{u}_m d\Upsilon_{m,M} = \int_{\Xi_m} \mathbf{B}_m^T \cdot \delta \mathbf{u}_m d\Xi_m + \int_{\Upsilon} \mathbf{p}_m^T \cdot \delta \mathbf{u}_m d\Upsilon \quad (5) \\
\Upsilon_{m,M}: \quad & \int_{\Upsilon_{m,M}} \delta \zeta (\mathbf{u}_m - \mathbf{u}_M) d\Upsilon_{m,M} = 0
\end{aligned}$$

287 in which \mathbf{u}_M and \mathbf{u}_m are the displacement fields belonging to the Ξ_M and Ξ_m subdomains,
288 respectively. Υ_m represent the interfaces of the micro-domain and Υ represent the boundaries
289 with applied external forces (surface tractions or nodal forces). \mathbf{B}_m and \mathbf{B}_M are the vectors
290 containing the body loads along with the three global cartesian directions, \mathbf{p}_m and \mathbf{p}_M are the
291 vectors of the surfaces loads active on the boundary, ζ is the Lagrange load multiplier of the
292 forces that control the residual interface gap across adjacent domains, \mathbf{t} is the traction force
293 acting at the interfaces within the Ξ_M domain, and $\bar{\mathbf{u}}_m$ is the displacement jump at the Υ_m
294 interfaces (Figure 5).



296 Figure 5: Schematic representation of the variational formulation of the concurrent FE
297 numerical approach.

298 Equation (5) defines the concurrent FE multiscale approach that is solved within an explicit
299 scheme available in ABAQUS (Abaqus, 2014). The static solution is obtained by dynamic
300 relaxation, using scaled masses and artificial damping. To this aim, the energy balance is
301 continuously evaluated and to guarantee that the kinetic energy of the deforming media is
302 below a small fraction of the total internal energy (1–5%) (Abaqus, 2014). The latter condition
303

304 must hold to guarantee the objectivity of the results through an explicit procedure. In this
305 context, smooth step amplitude curves and a small-time increment allow reaching appropriate
306 results.

307 **2.4.2 Micro-domain numerical model**

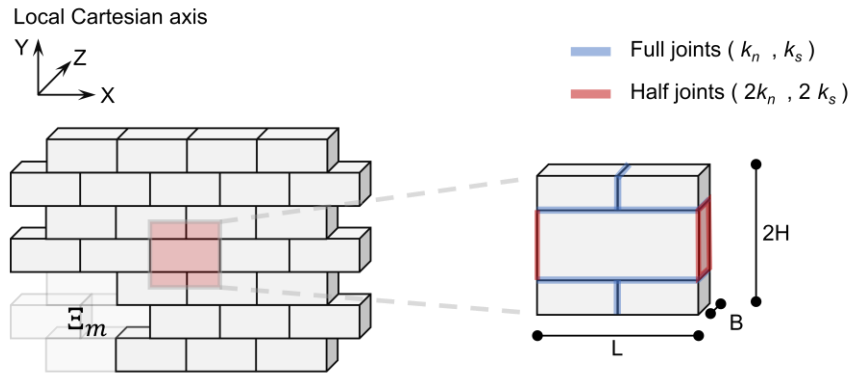
308 Masonry units are assumed to be deformable discrete blocks following an isotropic and linear
309 elastic constitutive law (\mathbf{E}_u , ν_u). Joints are represented by zero-thickness interfaces, which
310 include a non-associative plastic flow rule and a classical Mohr-Coulomb failure surface
311 criterion. Normal and tangential contact behaviors (stress-displacement laws) assume an
312 infinitesimal interpenetration between blocks. A linear relationship between the over-closure
313 displacements and the applied stress is defined by the normal k_n and tangential k_s stiffness
314 values. A friction coefficient (f) defines the plastic slipping criterion in shear within a penalty
315 approach, in which a perfectly plastic response occurs after reaching the critical shear stress.
316 For the present case study, only dry mortar type of masonry is studied and, therefore, cohesion
317 has been neglected when representing joint interfaces (in tension and shear regimes).

318 According to the distinct element method, a local damping factor is considered. The equations
319 of motion are damped to reach a force equilibrium state as quickly as possible under the applied
320 initial and boundary conditions. Damping is velocity-proportional (magnitude of the damping
321 force is proportional to the velocity of the blocks) and it was assumed equal to 0.8 in the present
322 study. The adopted FE software is Abaqus (2014) contains the latter mentioned contact
323 interface model. The constitutive law is automatically assigned to all the interfaces of the
324 micro-domain Ξ_m through the General Contact algorithm (Abaqus, 2014).

325 **2.4.3 Macro-domain numerical model**

326 The macro-domain Ξ_M represents masonry through an orthotropic linear elastic media.
327 According to the theory of elasticity, the spatial stiffness matrix for an orthotropic material is
328 defined by a 6×6 symmetric matrix, which is fully determined through nine engineering

329 constants, i.e. three elastic moduli E_{XX} , E_{YY} , E_{ZZ} , three Poisson's ratios ν_{XY} , ν_{XZ} , ν_{YZ} ,
 330 and three shear moduli G_{XY} , G_{XZ} , G_{YZ} , associated with the principal directions (Figure 6).



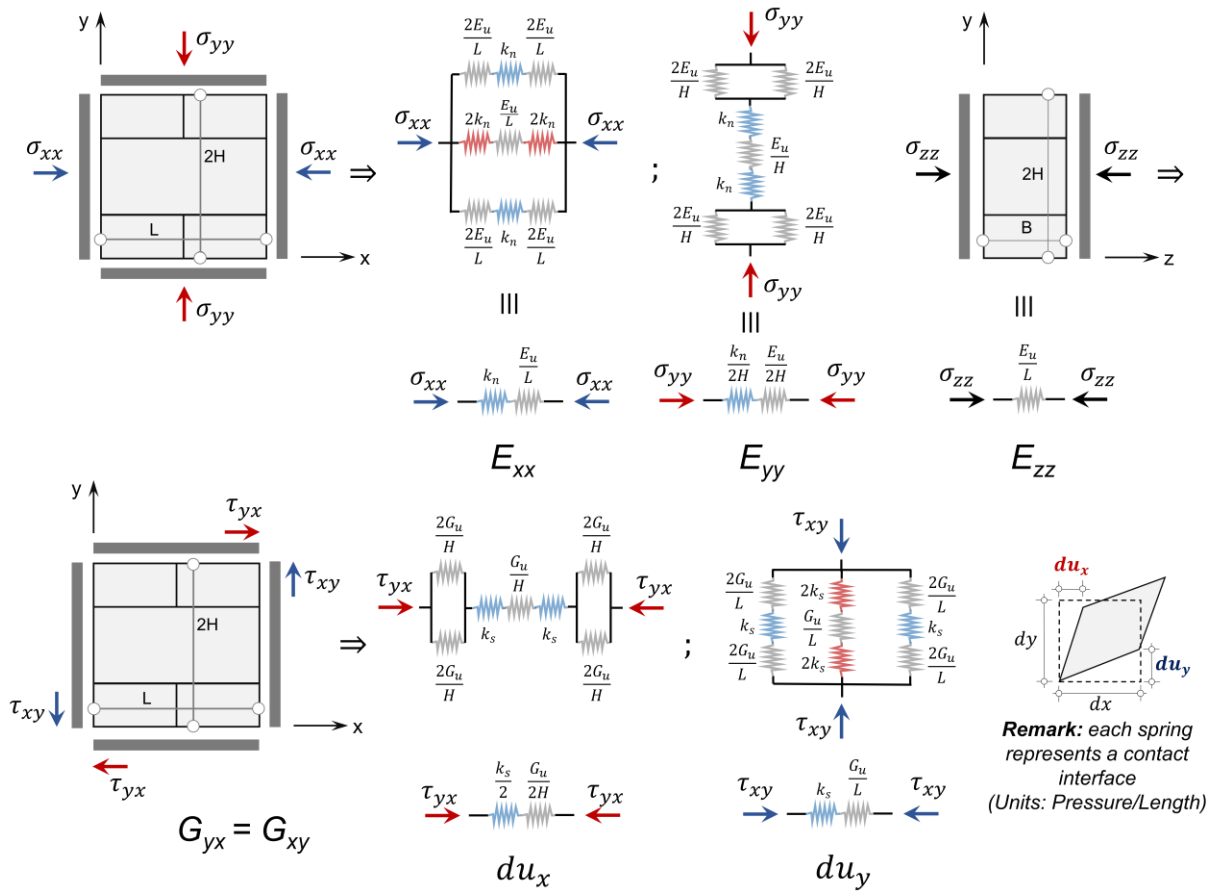
331

332 Figure 6: Representative Volume Element (RVE) for the homogenization procedure of the
 333 macro-domain Ξ_M .

334 A closed-form solution was found to define the material elasticity matrix of a running-bond
 335 dry-joint masonry. It was inspired in the works of Kouris et al. (2020) for a two-dimensional
 336 media, in which a set of equivalent springs represent the in-plane response of a mortared
 337 masonry RVE. Herein, the equivalent three-dimensional elastic response is defined by ad-hoc
 338 expressions formulated based on the representation of a set of springs to describe the masonry,
 339 as presented in Figure 7. Since contact interfaces are being used to characterize the numerical
 340 behavior of dry-mortar joints, the springs given in Figure 7 correspond to surfaces in the
 341 numerical model (unit of Pressure/Length). Parameters L , H , and B denote the unit length (X),
 342 height (Y), and width (Z), respectively; hence the RVE's height is given as $2H$ and length given
 343 by L (Figure 7). An assemblage of springs is conceived aiming at the representation of a system
 344 equivalent to a running-bond masonry RVE. Since a dry-joint masonry will be studied only,
 345 the compression range is the one that is analyzed here being the RVE subjected to compression
 346 stress σ for mode-I deformation modes. The computed elastic homogenized properties are
 347 described in terms of equivalent Young's moduli, Shear moduli, and Poisson's coefficients.

348 Figure 7 presents the latter assumptions. More detail on the formulation is given in Appendix

349 1.



350

351 Figure 7: Strategy followed to compute the equivalent elastic properties of the masonry RVE:

352 Young's moduli and shear moduli (details for the Poisson's ratio are given in Appendix A).

353 Table 1 summarizes the closed-form expression to compute the elastic properties of the

354 equivalent linear elastic orthotropic material; the reader is referred to (Kouris et al., 2020) for

355 further theoretical details within an alike procedure. The mechanic-based formulation adopted

356 has clear simplifications, but brings advantages related to the ease of computational

357 implementation. Nonetheless, among the more sophisticated models available in the literature,

358 only a few deal with dry-joint masonries, and the majority are devoted to 2D frameworks (G.

359 de Felice et al., 2010).

360 Table 1: Equivalent homogenized elastic properties for a running-bond masonry RVE
 361 (orthotropic material).

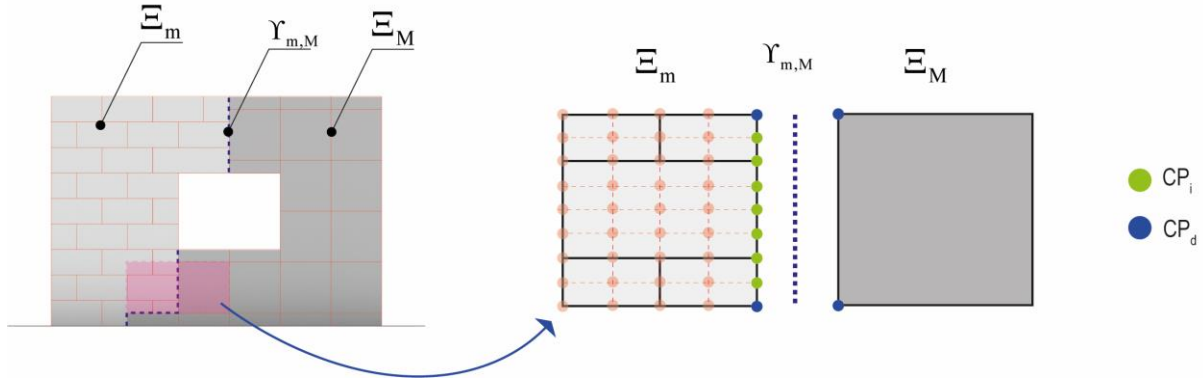
Young's modulus	Poisson's ratio	Shear modulus
$E_{xx} = \frac{Lk_n E_u}{E_b + Lk_n}$	$\nu_{xz} = \nu_{xy} = -\frac{\epsilon_{zz}}{\epsilon_{xx}} = \nu_u \frac{E_{xx}}{E_u}$	$G_{xy} = \frac{HLk_s G_u}{G_u(H + L) + 2HLk_s}$
$E_{yy} = \frac{Hk_n E_u}{E_u + Hk_n}$	$\nu_{zx} = \nu_{zy} = \nu_u$	$G_{xz} = \frac{Lk_s G_u}{G_u + 2Lk_s}$
$E_{zz} = E_u$	$\nu_{yx} = \nu_{yz} = \nu_u \frac{E_{zz}}{E_u}$	$G_{zy} = \frac{Hk_s G_u}{G_u + 2Hk_s}$

362
 363 **2.4.4 Micro and macro interfaces**

364 The concurrent FE model included two domains – macro Ξ_M and micro Ξ_m – that embody
 365 the structure numerical model and represent the mechanical behavior of masonry. These
 366 domains are assigned to different volumes and meet in different regions by sharing a common
 367 surface interface $\Upsilon_{m,M}$. Such interface $\Upsilon_{m,M}$ is characterized by two-adjointing boundaries with
 368 different scale representations for the masonry and FE mesh sizes ranging $\Delta_M / \Delta_m \approx 10$, in
 369 which Δ_M and Δ_m are the characteristic FE mesh size of the Ξ_M and Ξ_m domains,
 370 respectively.

371 Interfaces $\Upsilon_{m,M}$ must link adjoining non-conforming FE meshes. Contact points (or nodes) of
 372 each boundary are set into two families, defined as: CP_1 , which corresponds to the contact points
 373 located on the Ξ_M domain only; and the CP_d , which is the set of contact points that are paired
 374 between Ξ_M and Ξ_m domains. The continuity of the displacement between CP_d is imposed
 375 through a Lagrange multiplier functional. For CP_1 , the Lagrange multiplier can be achieved
 376 following several procedures, e.g. approximation through shape functions. Figure 8 represents
 377 this concept in the case of a two-dimensional problem for the sake of simplicity, but the
 378 interpolation occurs in two orthogonal directions because the proposed strategy is processed in
 379 the three-dimensional space. From a numerical standpoint, such kinematic conditions are

380 implemented by means of a tie constraint algorithm, which is available in Abaqus (2014).
 381 Therefore, the assumption of total displacement continuity between the two domains is
 382 guaranteed, even though the FE mesh are non-concordant.



383
 384 Figure 8: Concurrent FE model and localization of the interface $\Upsilon_{m,M}$ that link the macro- and
 385 micro-domains with the corresponding contact points.

386 **3 Two-scale framework: numerical application**

387 The numerical application of the proposed two-step framework is conducted over three case
 388 studies, which include small to large-scale structures. Aiming to validate the strategy, the
 389 results found are compared against a microscopic model, hereafter named as RMM (reference
 390 microscopic model). The RMM is one the most accurate (and computationally expensive)
 391 numerical strategies at disposal in the literature and serves, therefore, as a reference method for
 392 validation purposes.

393 The first case study is a dry-joint masonry wall with an opening and subjected to an in-plane
 394 shear load. The second case study addresses three connected dry-stone masonry walls within a
 395 U-shape plan arrangement (Smoljanović et al., 2018). It aims to explore the potential of the
 396 proposed approach when the structures are affected by coupled in-plane and out-plane
 397 mechanisms. Finally, the third case study is a large-scale monumental building, whose
 398 geometry is inspired by the Church of San Nicolò Capodimonte located in Camogli (Genova,
 399 Italy) (Funari, Mehrotra, and Lourenço 2021).

400 **3.1 Small-scale structure: URM shear wall**

401 The first case study concerns a dry-joint masonry shear wall. The wall is fixed at the base and
 402 its geometry is given in Figure 9. Two load cases are applied in a sequent manner: the self-
 403 weight is applied first, and a lateral body force (mass proportional) is applied next through an
 404 incremental load factor λ , as presented in Figure 9 (node 1). Material properties required to
 405 complete the proposed procedure are given in

406 Table 2, specifically the material density (ρ) and friction coefficient (f) for the limit analysis
 407 procedure; the Young’s modulus and the Poisson’s coefficient of the masonry units to define
 408 the overall elastic orthotropic matrix of the Ξ_M domain according to the closed-form solutions
 409 of section 2.4.3; and the normal and tangential stiffness values for the micro-macro interfaces.
 410 The dimensions of units are $0.80 \times 0.35 \times 0.40\text{m}^3$ ($L \times H \times B$).

411 Table 2: Mechanical properties adopted in the RMM and proposed CMM for the URM shear
 412 wall study.

ρ [kg m^{-3}]	f	E_u [Pa]	ν_u	k_n [Pa m^{-1}]	k_s [Pa m^{-1}]
2000	0.7	10×10^9	0.2	1×10^9	1×10^9

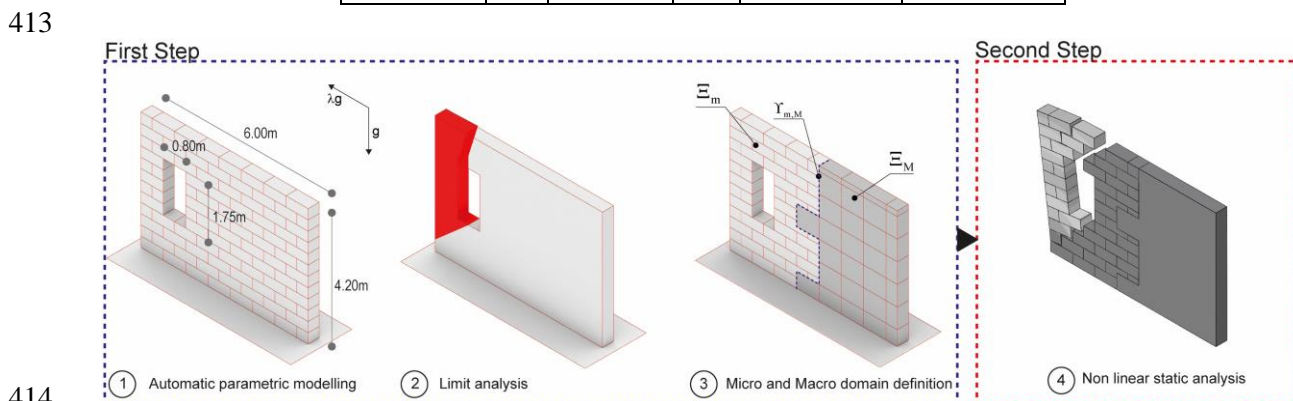
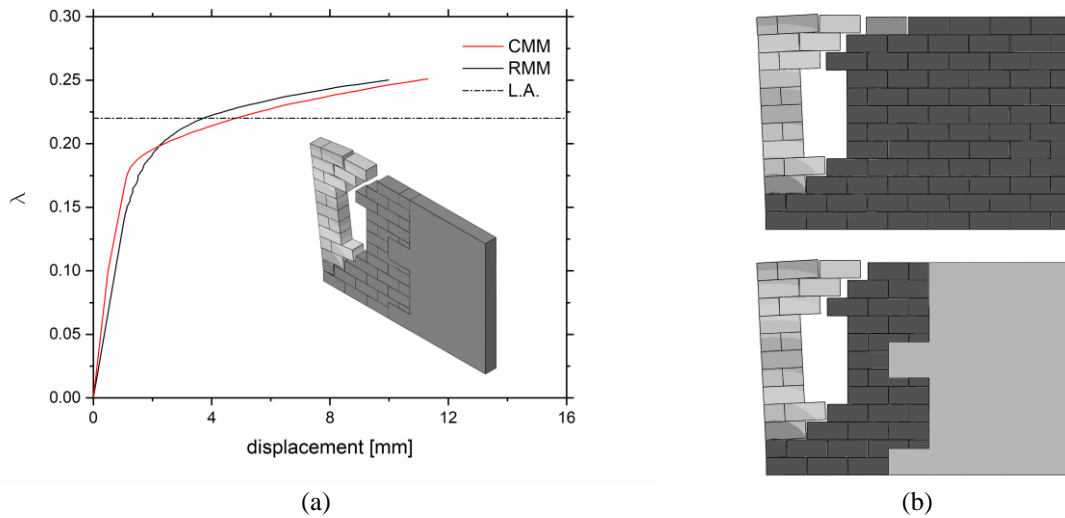


Figure 9: Proposed two-step framework applied to the URM shear wall case study.

416 The proposed strategy is employed – as addressed in section 2 –, in which the modeling of the
 417 wall was achieved by a semi-automatic parametric micro-modeling, and a limit analysis tool
 418 applied next aiming the detection of the most prone failure mechanism geometry for the given
 419 loading conditions. The limit analysis uses a heuristic procedure embedded in the Galapagos

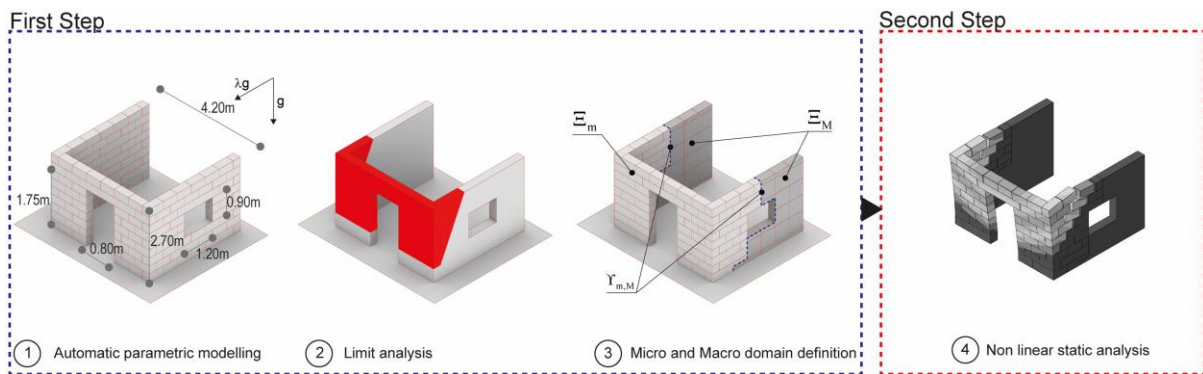
420 solver (Rutten, 2013) and converged to a load multiplier value equal to $\lambda = 0.219$ (meaning
421 an equivalent shear base force of $0.219g$, in which g is the gravitational acceleration). The
422 obtained failure mechanism, linked with the third sub-step of the proposed algorithm as
423 remarked in Figure 1, follows a re-meshing procedure to define the two non-overlapping
424 domains, i.e. the micro Ξ_m and the macro Ξ_M domains. The characteristic length R was
425 assumed to be $R = 2L$ (L is the length of the masonry unit that, for the dry-joint masonry of
426 this case study, matches the length of the RVE). Note that the failure defined by limit analysis
427 can have a jagged profile that may be caused, for instance, by the presence of openings.
428 The transfer between the first processing step, i.e. the limit analysis, with the second processing
429 step of the framework, i.e. the structural analysis by a concurrent FE model, is performed
430 through a Python script (The Python Language Reference — Python 3.9.5 Documentation,
431 2021). It allows the automatic creation of the numerical FE model within Abaqus CAE
432 environment (Abaqus, 2014), in which both domains are properly represented. Masonry units
433 that belong to the Ξ_m domain are discretized by eight-node linear hexahedral finite elements
434 (C3D8R in Abaqus (2014)); thus leading to a $\Delta_m / \kappa = 1 / 2$, in which $\kappa = \min(L, B, H)$. In
435 the Ξ_M domain, a coarser mesh (structured FE mesh with squared elements) was adopted with
436 a $\Delta_M / L = 1$, meaning that the FE size is $0.80m$ to match the length of masonry units.
437 Results of the proposed CMM and RMM are presented in Figure 10, both in terms of lateral
438 load-displacement capacity (node with maximum displacement as control node) and total
439 displacement map at collapse ($\lambda = 0.220$). The comparison of the results allows
440 demonstrating that the proposed approach ensures a good solution accuracy. Furthermore, it is
441 noteworthy to highlight that the CMM allows saving 58% of the computational time cost
442 required by the RMM.



443 Figure 10: Results obtained for the in-plane loaded masonry wall: (a) lateral load-displacement
 444 relationship; and (b) displacement map for the RMM and proposed CMM.

445 3.2 Small-to-medium scale structure: U-shaped URM walls

446 The second case study concerns a URM structure composed of three walls within a U-shaped
 447 plan arrangement. It is based on the Smoljanović, et al. (2018) works and brings more
 448 complexity than the former case study since both in- and out-of-plane co-exist. Walls are fixed
 449 at the base and the geometry of the structure is given in Figure 11. Two load cases are applied
 450 in a sequent manner: the self-weight is applied first, and a lateral body force (mass proportional)
 451 is applied next through an incremental load factor λ , as presented in Figure 11 (node 1). The
 452 lateral force is orthogonal to the façade wall and then following its out-of-plane direction.



453 Figure 11: Proposed two-step framework applied to the U-shaped URM case study.
 454

455 Material properties required to complete the proposed procedure are given in Table 3: the
 456 material density (ρ) and friction coefficient (ν) for the limit analysis procedure; the Young's
 457 modulus and Poisson's coefficient of masonry units to define the overall elastic orthotropic
 458 matrix of the Ξ_M domain according to the closed-form solutions of section 2.4.3; and the
 459 normal and tangential stiffness values for the micro-macro interfaces. The dimensions of units
 460 are $0.60 \times 0.30 \times 0.30 \text{m}^3$ ($L \times H \times B$).

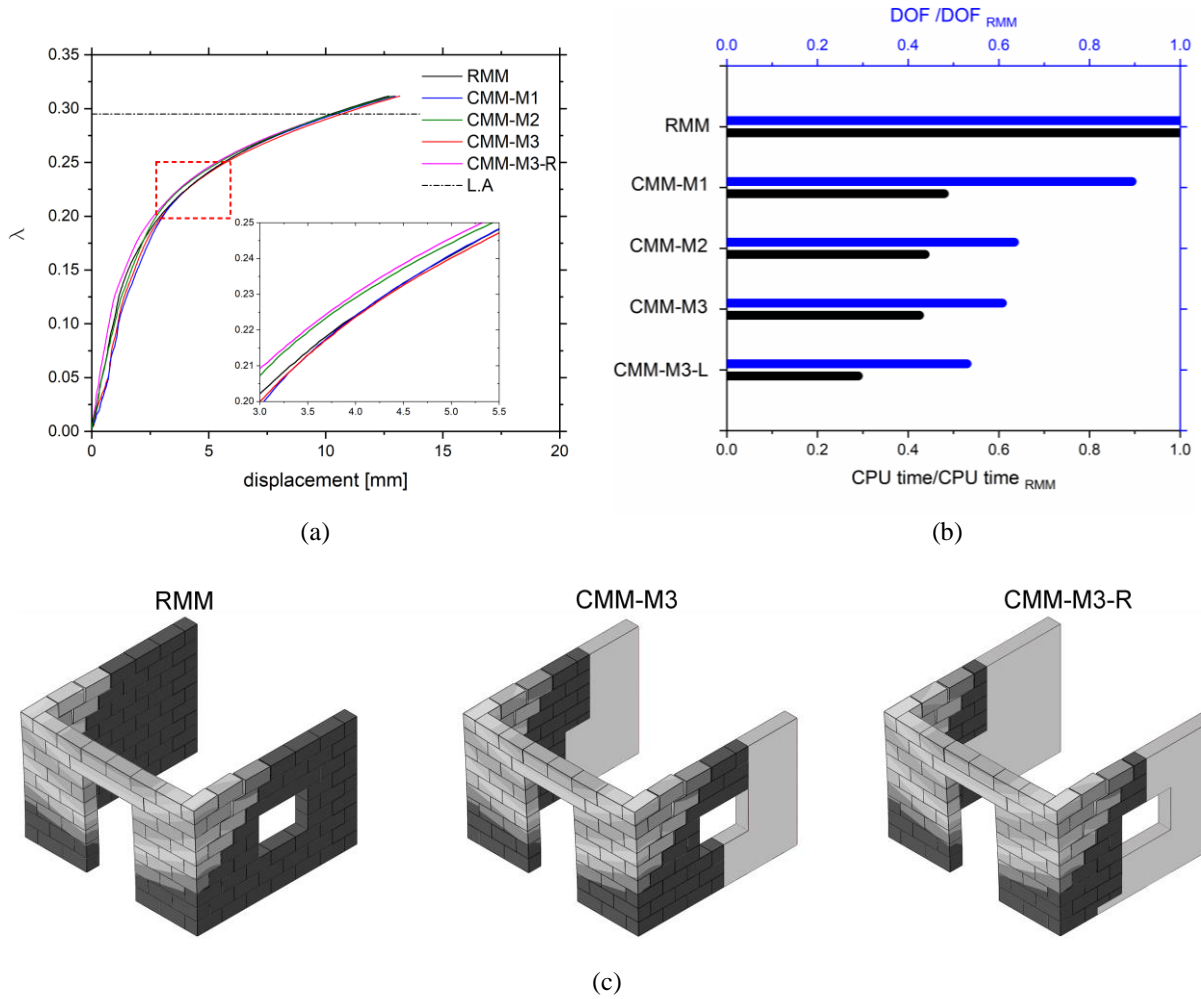
461 Table 3: Mechanical properties adopted in the RMM and proposed CMM for the U-shaped
 462 URM structure studied by Smoljanović, et al. (2018).

ρ [kg m^{-3}]	E_u [Pa]	ν_u	k_n [Pa m^{-1}]	k_s [Pa m^{-1}]	f
2000	10×10^9	0.2	1×10^9	1×10^9	0.7

463 The proposed strategy is employed and the obtained failure mechanism through limit analysis
 464 is given in Figure 11 (node 2), in which the out-of-plane mechanism of the main façade governs
 465 the collapse mode. The numerical FE concurrent model (CMM) is then developed at Abaqus
 466 CAE environment (Abaqus, 2014) by a re-meshing procedure that retrieves two non-
 467 overlapping domains, i.e. the micro Ξ_m and the macro Ξ_M domains. A characteristic length R
 468 was assumed to be $R = 2L$ (L is the length of the masonry unit that, for the dry-joint masonry
 469 of this case study, matches the length of the RVE). Masonry units that belong to the Ξ_m domain
 470 are discretized by eight-node linear hexahedral finite elements (C3D8R in Abaqus (2014));
 471 thus leading to a $\Delta_m / \kappa = 1 / 2$, in which $\kappa = \min(L, B, H)$. In the Ξ_M domain, three mesh
 472 refinements (structured FE mesh with squared elements) were evaluated to assess the trade-off
 473 between accuracy-computational achieved. To this aim, the following FE mesh ratios were
 474 adopted: (i) CMM-M1, with a finer FE mesh and size given by $\Delta_M / \kappa = 1 / 2$; (ii) CMM-M2,
 475 an in-between mesh refinement with a size given by $\Delta_M / L = 1 / 2$; and (iii) CMM-M3, with
 476 a coarser FE mesh and size given by $\Delta_M / L = 1$.

478 Furthermore, the influence of the parameter R was investigated. Note that R is a parameter
479 (units of length) that directly affect the volumes of both micro- and macro-domains (see section
480 2.4.2). Therefore, an additional model designated as CMM-M3-R was also considered: it has a
481 FE mesh size for the Ξ_M domain given by $\Delta_M / L = 1$ and an $R=L$ (half-value of the other
482 CMM models).

483 Results from the proposed CMM and RMM are presented in Figure 12. Lateral load-
484 displacement capacity curves (node with maximum displacement as control node) in Figure
485 12a show slight differences in the elastic range, yet negligible from a structural engineering
486 standpoint as are within a 5% bound. CMM model is slightly stiffer than the RMM, especially
487 in the linear range, and it may be explained due to the loss of accuracy that macro-modeling
488 offers when compared with a micro-modeling approach. Nonetheless, differences are
489 unnoticeable when plastic deformations govern the response; is noteworthy to highlight that
490 the CMM micro-domain is responsible for such deformation. Collapse occurs for a load-factor
491 around $\lambda = 0.295$ for all the studied numerical models. It is important to point out that the
492 collapse instant is defined when the kinematic energy is higher than 5% of the total energy
493 because an explicit formulation was adopted. In such a context, the results allow demonstrating
494 that the proposed approach ensures a promising solution accuracy. Furthermore, it is
495 noteworthy to highlight that the CMM allows saving 58% of the computational time cost
496 required by the RMM.



497 Figure 12: Results obtained for the U-shaped URM structure: (a) lateral load-displacement
 498 relationship; (b) computational time (CPU) and number of degrees-of-freedom (DOFs) for each
 499 numerical simulation; and (c) displacement map for the RMM and proposed CMM-M3 and
 500 CMM-M3-R.

501 Figure 12b reports a comparison including the required computational time (CPU) and the
 502 number of degrees-of-freedom (DOFs) for each numerical model. Despite CMM-M1, CMM-
 503 M2 and CMM-M3 have clear differences in the number of DOF, differences in the required
 504 CPU time are minimal and allow saving around 60% of the time in comparison with the RMM.
 505 In this regard, the approach seems to not suffer from a substantial mesh bias at the Ξ_M domain;
 506 this holds at least for FE mesh sizes with a dimension lower than the RVE size. On the other
 507 hand, Figure 12b allows doing an important finding, i.e. the parameter R has a significant

508 impact since it allows decreasing 10% of the required CPU time when compared to the CMM-
509 M3. Therefore, this parameter must be assessed with care. It allows improving the
510 computational time, but decreasing its value may also compromise the accuracy level. The
511 authors suggest a value bounded by $R = L$ to $R = 2L$.

512 The failure mechanisms obtained with the RMM, CMM-M3, and CMM-M3-R are summarized
513 in Figure 12c through total displacement maps. The proposed FE concurrent models (for both
514 R values) capture well the expected failure mechanism. At last, it is noteworthy to highlight
515 that the proposed multi-scale framework returns promising results, in terms of load capacity
516 curve and expected failure mechanism, while saving 65% of the computational time if
517 compared to an accurate micro-modeling strategy (RMM).

518 **3.3 Large-scale structure: URM church**

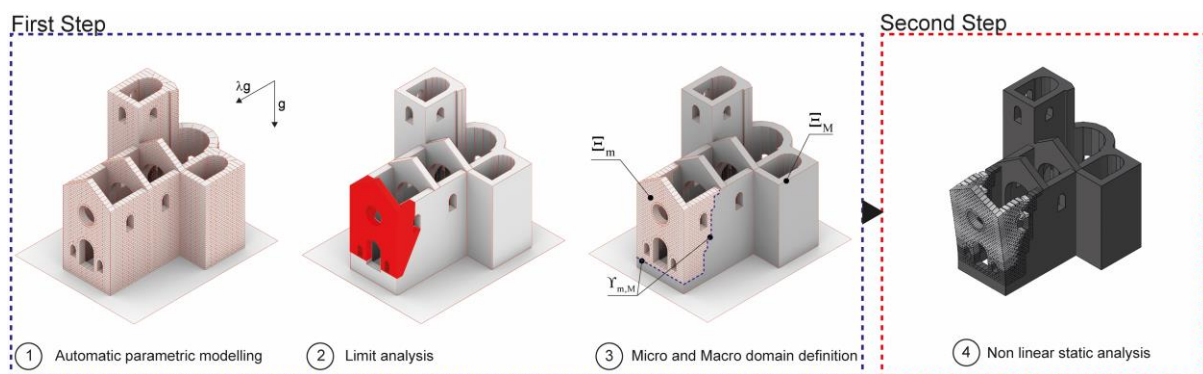
519 The last case study – and the most complex one – concerns a URM church and aims to evaluate
520 the promptness and accuracy of the two-step framework when applied for a large-scale
521 structure. The URM church is characterized by a plan consisting of a Latin Cross (Funari,
522 Mehrotra, and Lourenço 2021). The geometry of the church is given in Figure 13 and some
523 important features can be addressed: the main façade wall has a total height of 14.0m and a
524 base ranging 7.50m; the single bell tower is the tallest structural element, with a height of
525 17.0m; and the total length of the church is around 19.60m. Fixed boundary conditions are set
526 at the base of the church walls. For the structural analysis, two load cases were considered and
527 applied in a sequent manner: the self-weight is applied first, and a lateral body force (mass
528 proportional) is applied next through an incremental load factor λ , as presented in Figure 11
529 (node 1). Such lateral force, which intends to be representative of a seismic excitation, was
530 applied along the longitudinal direction of the church, as this is typically the weakest direction.
531 Material properties required to complete the proposed procedure are given in Table 4: the
532 material density (ρ) and friction coefficient (ν) for the limit analysis procedure; the Young's

533 modulus and Poisson's coefficient of the masonry units to define the overall elastic orthotropic
 534 matrix of the Ξ_M domain according to the closed-form solutions of section 2.4.3; and the
 535 normal and tangential stiffness values for the micro-macro interfaces. The dimensions of units
 536 are $0.57 \times 0.275 \times 0.90\text{m}^3$ ($L \times H \times B$) .

537 Table 4: Mechanical properties adopted in the RMM and proposed CMM for the URM
 538 church study.

ρ [kg m^{-3}]	f	E_b [Pa]	ν_b	k_n [Pa m^{-1}]	k_s [Pa m^{-1}]
2000	0.6	20×10^9	0.2	1×10^{11}	1×10^{11}

539

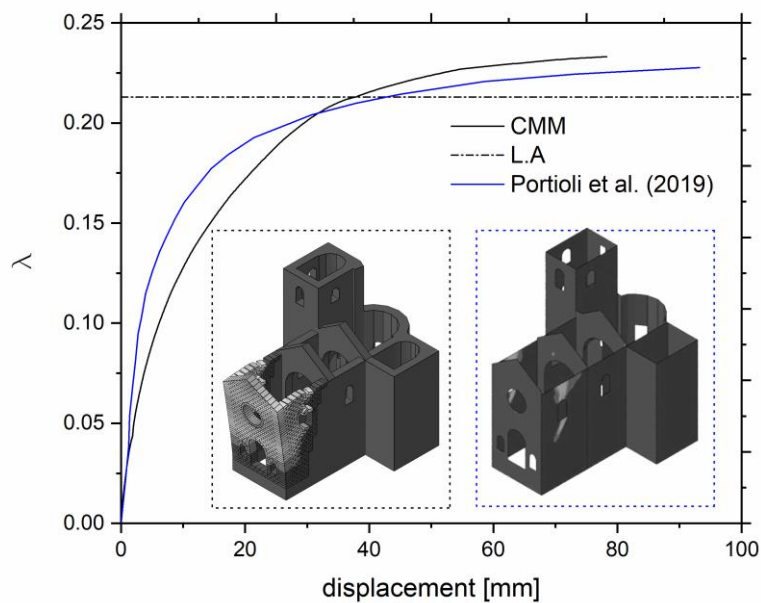


540

541 Figure 13: Proposed two-step framework applied to the URM church case study.

542 The proposed strategy is employed and the obtained failure mechanism through limit analysis
 543 is given in Figure 13 (node 2). The overturning mechanism of the gable wall of the church
 544 governs the collapse mode. Figure 13 (node 3) presents the corresponding numerical FE
 545 concurrent model (CMM) developed at Abaqus CAE environment (Abaqus, 2014) by a re-
 546 meshing procedure that retrieves the two non-overlapping domains, i.e. the micro Ξ_m and the
 547 macro Ξ_M . A characteristic length $R = 2L$ was assumed. Masonry units that belong to the
 548 Ξ_M domain are discretized by eight-node linear hexahedral finite elements (C3D8R in Abaqus
 549 (2014)); thus leading to a $\Delta_m / \kappa = 1 / 2$, in which $\kappa = \min (L, B, H)$. In the Ξ_M domain, a
 550 FE mesh refinement (structured FE mesh with squared elements) was considered with a size
 551 given by $\Delta_M / L = 1$.

552 Figure 14 summarizes the lateral load-displacement (pushover) curve found with the proposed
 553 CMM, together with the load multiplier value ($\lambda = 0.213$) of the limit analysis processed in
 554 the second sub-step (Figure 13) and with the numerical model from Malena, et al. (2019). The
 555 latter numerical model developed by Malena, et al. (2019) is based on a homogeneous
 556 macroscopic model with an elasto-plastic constitutive relation for the masonry.



557
 558 Figure 14: Results obtained for the URM church: lateral load-displacement (pushover) curve
 559 and displacement map for the proposed CMM and macroscopic model from Malena, et al.
 560 (2019).

561 The comparison of the results allows demonstrating that the proposed approach ensures a good
 562 solution accuracy, especially to what concerns the structure's load capacity. Some deviations
 563 still pose within the elastic range. The limit analysis allows predictions on the collapse load are
 564 within 8% difference. Figure 14 also gives the comparison in terms of failure mechanism. The
 565 proposed model offers a clear identification of the failure surfaces, as damage localization is
 566 directly lumped on FE interfaces. In converse, general insight over the failure mechanism is
 567 difficult to conduct with the macroscopic model used for comparison purposes; this is,
 568 however, a general disadvantage of macroscopic models that preclude a cracking-localization

569 algorithm (Clemente et al., 2006). At last, it bears noticing that the proposed CMM has, within
570 the micro-domain Ξ_m , a total of 800 masonry stone units that leads to a total CPU time of
571 75min. A comparison with a RMM was disregarded in this case study since the number of
572 masonry units would increase up to more than 5000; meaning that a prohibitive CPU time
573 would be required. No indication of CPU time required for the macroscopic model is reported
574 in Malena, et al. (2019) and, therefore, a quantitative comparison on this matter is omitted.

575 **4 Final remarks**

576 A two-step procedure was proposed aiming at the in- and out-of-plane mechanical study of
577 dry-joint masonry structures. At a first step, a semi-automatic digital tool allows the parametric
578 modeling of the structure that, together with an upper bound limit analysis tool and a heuristic
579 optimization solver, enables tracking the most prone failure mechanism. The time required to
580 process the first step is limited to a matter of seconds. At a second step, a coupled three-
581 dimensional concurrent FE model with micro- and macro-scales is assumed. A micro-modeling
582 description of the masonry is allocated to regions activated by the failure mechanism found in
583 the former step. The other regions of the domain are modeled via a macro-approach, whose
584 constitutive response is elastic and orthotropic and based on closed-form homogenized-based
585 solutions. The time required to complete the second step is conditioned by the scale of the
586 structure and type of structural analysis performed, as the modeling of the concurrent FE model
587 is automatic and takes a matter of seconds.

588 The application of the framework was achieved through non-linear quasi-static analysis on
589 three benchmarks: (i) an in-plane loaded URM shear wall; (ii) a U-shaped URM structure; and
590 (iii) a URM church. Results demonstrate the potential and advantages of the proposed
591 approach. It was able to predict, with a marginal difference (lower than 1%), the collapse load
592 value. Failure collapse modes resemble to be alike with the ones found with a microscopic FE
593 model (first two case studies) and with a literature macroscopic FE model (for the third and

594 last case study). Furthermore, the tool demonstrated that is quite attractive from a
595 computational standpoint. It allows reducing the CPU time up to 60% in a small-to-medium
596 scale structure (first and second case studies) when compared to a full microscopic FE model.
597 Eventually, it may be the only alternative to macroscopic FE models when assessing large-
598 scale structures, as micro-modeling proved to be a challenge.

599 At last, a comment on future research streams is of value. The two-step procedure is
600 computational quite attractive, robust, and allows higher levels of accuracy. This is so because
601 it is based on a sequential process in which a continuous transfer of information between scales
602 is precluded during the analysis; as required in classical multi-domain strategies that need
603 activation rules to process the macro-to-micro decomposition (L. Leonetti et al., 2018; Reccia
604 et al., 2018; Driesen et al., 2021). Nonetheless, further studies need to be carried out to validate
605 the approach in other contexts, for instance when assessing mortared masonry structures. In
606 such a context, the authors believe that future works may include: (i) the definition of a more
607 sophisticated limit analysis tool, e.g. (G. Milani, 2015; Chiozzi et al., 2017); and (ii) the
608 implementation of an interfacial contact model at the micro-domain and within the FE
609 concurrent model that can represent better the behavior of mortared joints (Lourenço et al.,
610 2020).

611 **5 Appendix**

612 This appendix details the derivation of the formulas presented in Table 1, which have been
613 formulated considering a spring's representation analogy (see Figure 7) and based on the
614 infinitesimal strain theory.

615 The Young's modulus E_{xx} , E_{yy} and E_{zz} can be obtained following the same procedure. For the
616 save of brevity, only E_{xx} component is addressed here. According to Hooke's law, the axial
617 deformation and displacement read as:

$$618 \quad E_{xx} = \frac{\sigma_{xx}}{\varepsilon_{xx}}; \quad \varepsilon_{xx} = \frac{\Delta u}{L}; \quad \Delta u = du_u + du_j \quad (\text{A.1})$$

619 In which σ_{xx} is the axial load, ε_{xx} is the axial deformation, Δu is the total displacement of the
620 RVE, du_u is the displacement component related to the unit, and du_j is displacement
621 component related to the joints, i.e. its normal displacement (interpenetration). Both contact
622 interfaces have the same applied uni-axial stress and, therefore, Equations A.1 reads as:

$$623 \quad du_u = \frac{\sigma_{xx}L}{E_u}; \quad du_j = \frac{\sigma_{xx}}{k_n}; \quad \frac{\sigma_{xx}}{L} \left(\frac{L}{E_u} + \frac{1}{k_n} \right) = \sigma_{xx} \left(\frac{k_nL + E_u}{E_u k_n L} \right) \quad (\text{A.2})$$

624 Which corresponds to the following uni-axial Young's modulus E_{xx} :

$$625 \quad E_{xx} = \frac{E_u k_n L}{k_n L + E_u} \quad (\text{A.3})$$

626 For the in-plane shear moduli, one assumes the symmetry of the shearing stress components.
627 Therefore, the in-plane shear moduli are defined as the ratio between the corresponding shear
628 stress component and relative deformation. Accordingly:

$$629 \quad G_{xy} = G_{yx} = \frac{\tau_{xy}}{\gamma_{xy}}; \quad \text{with } \gamma_{xy} = \frac{du_x}{dy} + \frac{du_y}{dx} \quad (\text{A.4})$$

630 The shearing deformation γ_{xy} ($= \gamma_{yx}$) should be computed when subjecting the RVE to a pure
631 shear mechanism (Figure 7). Recalling that du_u is the shear displacement component related
632 with the block deformation and du_j the shear displacement component related with the joint,
633 the individual shearing deformation components are defined as:

$$634 \quad \begin{aligned} \frac{du_x}{dy} &= \frac{du_u + du_j}{2H} = \frac{\tau_{yx} \left(\frac{2}{k_s} + \frac{2H}{G_u} \right)}{2H} = \tau_{yx} \frac{G_u + k_s H}{G_u k_s H} \\ \frac{du_y}{dx} &= \frac{du_u + du_j}{L} = \frac{\tau_{xy} \left(\frac{1}{k_s} + \frac{L}{G_u} \right)}{L} = \tau_{xy} \frac{G_u + k_s L}{G_u k_s L} \end{aligned} \quad (\text{A.5})$$

635 By combining Eq. A.5 with Eq. A.4, one writes that the in-plane shear modulus is computed
636 as:

$$637 \quad G_{yx} = G_{xy} = \frac{1}{\frac{1}{2} \left(\frac{G_u + k_s H}{G_u k_s H} + \frac{G_u + k_s L}{G_u k_s L} \right)} = \frac{G_u k_s H L}{G_u (H+L) + 2k_s H L} \quad (\text{A.6})$$

638 Lastly, the equivalent in-plane Poisson's ratio $\nu_{xy} = \nu_{yx}$ is demonstrated. To this aim, it bears
639 highlighting that the lateral deformation in the joints was assumed to be zero since the study

640 deals with dry-mortar masonries. Therefore, the subscript u is related to the unit only and the
641 Poisson's ratio is given as:

$$642 \quad \varepsilon_{xx,u} = \frac{\sigma_{xx}}{E_u} = \varepsilon_{xx} \frac{E_{xx}}{E_u}; \quad \varepsilon_{yy} = \varepsilon_{yy,u} = -\nu_u \times \varepsilon_{xx,u} = -\nu_u \varepsilon_{xx} \frac{E_{xx}}{E_u} \quad (\text{A.7})$$

$$643 \quad \nu_{yx} = \nu_{xy} = -\frac{\varepsilon_{yy}}{\varepsilon_{xx}} = \nu_u \frac{E_{xx}}{E_u} \quad (\text{A.8})$$

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