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Hamidreza Karami, Mohammad Azadifar, Amirhossein Mostajabi, Marcos Rubinstein, Farhad Rachidi. Numerical and Experimental Validation of Electromagnetic Time Reversal for Geolocation of Lightning Strikes. IEEE Transactions on Electromagnetic Compatibility, 2020, 62 (5), pp.2156-2163. 10.1109/TEMC.2019.2957531 . hal-03586384

HAL Id: hal-03586384

<https://hal.science/hal-03586384>

Submitted on 23 Feb 2022

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Numerical and Experimental Validation of Electromagnetic Time Reversal for Geolocation of Lightning Strikes

Hamidreza Karami, Mohammad Azadifar, *Member, IEEE*, Amirhossein Mostajabi, Marcos Rubinstein, *Fellow, IEEE*, and Farhad Rachidi, *Fellow, IEEE*

Abstract— We implement an Electromagnetic Time Reversal Technique (EMTR) to locate lightning return strokes. The Two-Dimensional Finite Difference Time Domain (2D-FDTD) is employed to simulate the EMTR process in both, the forward-time and the backward-time phases. Scatterers are included in the computational domain to emulate the presence of objects. Three possible criteria to find the optimum time slice of the EMTR process that includes the maximum peak field, maximum peak energy, and last local minimum of entropy are tested and it is found that only the entropy criterion can successfully locate the lightning discharge. Our analysis shows that the EMTR process in both, using an unchanged and a simplified medium for the backward-time works reasonably well even with only two sensors. Furthermore, we validated the proposed method via experimental results using waveforms recorded at two sensors at distances of 14.7 km and 380 km from the Säntis Tower. The results demonstrate that the EMTR back-propagation process leads to a refocusing of the radiated energy at the location of the Säntis Tower. The ambiguity in the obtained location when only two sensors are used can be resolved either by using an additional sensor or through a more accurate modelling of the terrain.

Index Terms—Electromagnetic Time Reversal, Unchanged and simplified Media, Entropy, Experimental Validation, Finite Difference Time Domain, Lightning Return Stroke.

I. INTRODUCTION

Lightning geolocation is of particular interest for numerous applications, such as thunderstorm forecasting and tracking in real-time, power utility protection, lightning warning applications, risk assessment, etc. [1]. Lightning Location Systems (LLSs) use electric and/or magnetic field waveforms radiated by the lightning discharge from the VLF to the VHF frequency bands. The performance of LLSs is measured by their detection efficiency, location accuracy and the accuracy of their peak current estimation. Two common techniques are widely used by LLSs: Time of Arrival (ToA) and Magnetic Direction Finding (MDF) [2], [3].

The focusing property of time reversal has been successfully applied to acoustics by Fink and coworkers [4] and later on to electromagnetics [5]. Electromagnetic Time Reversal (EMTR)

has been applied to Microwave Imaging [6], Radar Imaging [7], locating lightning discharges [8]–[11] and locating faults in power networks [12].

EMTR was applied for the first time by Mora et al. [8] to locate lightning discharges, although only by simulation. In order to obtain the location of the lightning source, they removed the singularity produced by the $1/R$ distance dependence of the radiation field in the time reversed electric field and determined the point at which the wave-fronts from all the sensor locations are in-phase. The developed method would need at least 3 sensors to locate the discharge. They used numerical calculations for a 4-sensor configuration to validate their algorithm. Later on, Lugin et al. [9] modified the model proposed by Mora et al. to include the effect of propagation over lossy ground.

More recently, Wang et al. [13] used EMTR to locate VHF sources during the course of a lightning flash. They proposed the maximum norm of the synthesized signal as a criterion to locate the radiation source. Further, Wang et al. proposed an improved broadband VHF lightning observation system using EMTR in the frequency domain [14].

To the best of the authors' knowledge, all the above-mentioned methods use the maximum electric field criterion to locate the lightning discharge. The limitation of this technique (based on the removal of the $1/R$ dependence in the reverse time) is that it can only be applied to the case of a flat ground, and is based on a plane wave approximation of lightning electromagnetic fields [8]. In this paper, we propose and implement an EMTR technique to locate lightning discharges, which is based on the concept of entropy introduced by Wiggins [15]. We use a Two-Dimensional Finite Difference Time Domain (2D-FDTD) method to calculate the fields associated with the lightning strike in the forward and the backward time frames. Unlike former studies which removed the $1/R$ dependence of the field amplitude, a full wave model is used for the wave propagation in both the forward and the backward phases. Objects such as mountains are modeled as well. The entropy criterion is tested against other criteria like the maximum electric field amplitude and maximum energy.

We demonstrate that the proposed scenario could work with reasonable accuracy both in the forward-time and the backward-time phases. Finally, experimental data gathered at the Säntis Tower site are used as ground truth to further validate

the EMTR method.

The rest of the paper is organized as follows. Section II provides a general overview of the EMTR radiation source location method. Section III is devoted to Numerical Simulations. In Section IV, we present an experimental validation of the geolocation of lightning strikes.

II. PRINCIPLE OF EMTR

Maxwell's equations are known to be time reversal invariant in the soft sense [16]. Let us consider Maxwell's equations in the time reversed regime by setting $t \rightarrow -t$:

$$\begin{aligned} \nabla \cdot (\varepsilon(\vec{r}) \vec{E}(\vec{r}, -t)) &= \rho(\vec{r}, -t) \\ \nabla \cdot (\mu(\vec{r}) (-\vec{H}(\vec{r}, -t))) &= 0 \\ \nabla \times \vec{E}(\vec{r}, -t) &= -\mu(\vec{r}) \frac{\partial(-\vec{H}(\vec{r}, -t))}{\partial t} \\ \nabla \times (-\vec{H}(\vec{r}, -t)) &= \varepsilon(\vec{r}) \frac{\partial \vec{E}(\vec{r}, -t)}{\partial t} + (-\vec{J}(\vec{r}, -t)) \end{aligned} \quad (1)$$

It can be seen that Maxwell's equations are time reversal invariant under the condition of setting $\vec{H}(\vec{r}, t) \rightarrow -\vec{H}(\vec{r}, -t)$ and $\vec{J}(\vec{r}, t) \rightarrow -\vec{J}(\vec{r}, -t)$, that is to say, when the direction of time is reversed, the velocity of the charges changes sign and, as a result, the sign of the electrical current and its magnetic field must also be reversed.

Experimental and/or computational EMTR enables us to reconstruct an unknown source from its known effects [17] by refocusing a wave back to its source. Three steps must be taken to locate sources via the EMTR approach:

- 1) The electromagnetic field from the source is measured or calculated (forward-time) at one or more locations.
- 2) The obtained electric field waveforms are time-reversed and back-injected into an unchanged or a simplified medium, using numerical electromagnetic simulations (backward-time). *Unchanged medium* refers to the case in which the backward-time medium corresponds exactly to the forward-time medium. *Simplified medium*, on the other hand, refers to the case in which the backward-time medium is not exactly the same as in the forward time. For example, when some or all the scatterers are removed from the medium in the backward phase.
- 3) A criterion to detect and locate the source is applied within the backward-time phase such as the maximum amplitude of the total electric field or maximum energy.

A discussion here is in order on the selection of the criterion in step 3) above. In [8], [9], [13], [14], only the radiation term of the wave is considered and its 1/R distance dependence in the backward-time phase is removed in order to deal with the singularity at the source location. This leads to back-propagation with a constant amplitude. Constructive interference of the wave-fronts from different sensors ensures a maximum in the amplitude of the total electric field and its

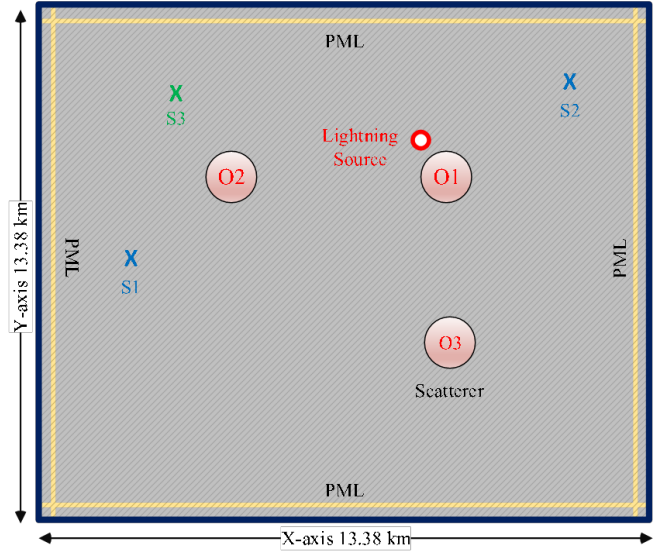


Fig. 1. Geometry of the problem. Red circle, blue crosses, and color gradient circles represent, respectively, the lightning source, field sensors, and scatterers. The green cross corresponds to the third sensor.

TABLE I
COORDINATES OF FIG. 1

Item	X (m)	Y(m)
Source Location	8035	8679
<i>S1</i>	3370	6670
<i>S2</i>	9670	9670
<i>S3</i>	4170	9170
<i>O1</i>	8170	8370
<i>O2</i>	5170	8370
<i>O3</i>	8170	5370

energy at the source location. This criterion has been used in the above-mentioned studies to locate the source.

However, the assumption of 1/R propagation reduction is only valid for the far-field region and it cannot be applied in the full-wave solution of problems including scatterers.

The concept of minimum entropy was introduced by Wiggins [15] and it was later applied to various problems such as landmine detection [18] and breast cancer detection [6]. We can compute the entropy at each instant of time by way of the following formula [6]:

$$R(E_z^n) = \frac{\left[\sum_j \sum_k (E_z^n(j, k))^2 \right]^2}{\sum_j \sum_k (E_z^n(j, k))^4} \quad (2)$$

in which $R(E_z^n)$ is the entropy associated with the electric field and $E_z^n(j, k)$ is the z component of the electric field at each grid cell at each instant of time. The parameters j , k , and n denote, respectively, the index along the x-axis, the index along the y-axis, and the number of time steps. The minimum entropy yields the focal spots in an arbitrary 2D image [15]. Here we used the entropy criterion to detect the focal spots of an image corresponding to the electric field distribution calculated by the FDTD method. In the forward time, the first local minimum corresponds to the source. As a result, in the backward time, the last local minimum should correspond to the location of the

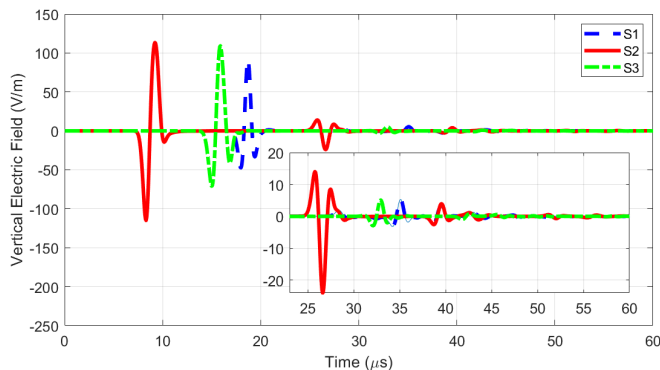


Fig. 2. Waveforms acquired in the forward-time phase by sensors S1, S2, and S3. The figure in the inset shows an expanded view of the signals reflected from the scatterers which appear at about 25 microseconds.

source. It should be noted that the length of the time window is dependent on the dimension of the searching area and the relative locations of the sensors and scatterers. This window should be large enough to ensure that the direct wave and reflected waves (from scatterers) would reach the sensor location. In other words, the last local minimum of (2) provides the optimum time slice at which the back-injected time-reversed waves will refocus back to the source. The source location can be determined by identifying the maximum amplitude of the field at this time slice.

III. NUMERICAL SIMULATION AND PROBLEM DEFINITION

We use a Two-Dimensional Finite Difference Time Domain (2D-FDTD) method to calculate the field in the forward and backward-time phases. Herein, we consider various scenarios to investigate the following points:

- Number of required sensors.
- Proper criteria to obtain an optimum focusing time and location.
- The effect of unchanged vs. simplified medium in the backward-time simulations.

A. Two-Sensor Scenario, Unchanged Medium

The geometry of the problem is shown in Fig. 1, in which the lightning source is represented by a red circle, each one of the two observation points (sensors) is represented by an x, and three scatterers are depicted as color-gradient-filled circles. A third sensor, S3, is shown in the figure as a green x and it will be used in the further scenarios. The gradient-filled circles that represent the scatterers have a radius of 250 m. They represent mountains in the computational domain. The conductivity of the objects is assumed to be 0.05 S/m and their relative permittivity is set to 10. Perfectly Matched Layers (PML) with a depth of 10 mesh cells are deployed as boundary conditions. Equally spaced cells with a length of 30 m are used to mesh the solution space. The lightning is considered as a z-axis dipole current source excited by a Gaussian pulse with a bandwidth of 2 MHz. Table I provides the coordinates of the locations of the objects, sensors and the lightning strike.

Fig. 2 shows plots of the electric field waveforms at sensors S1, S2, and S3 in the forward-time phase of EMTR. As

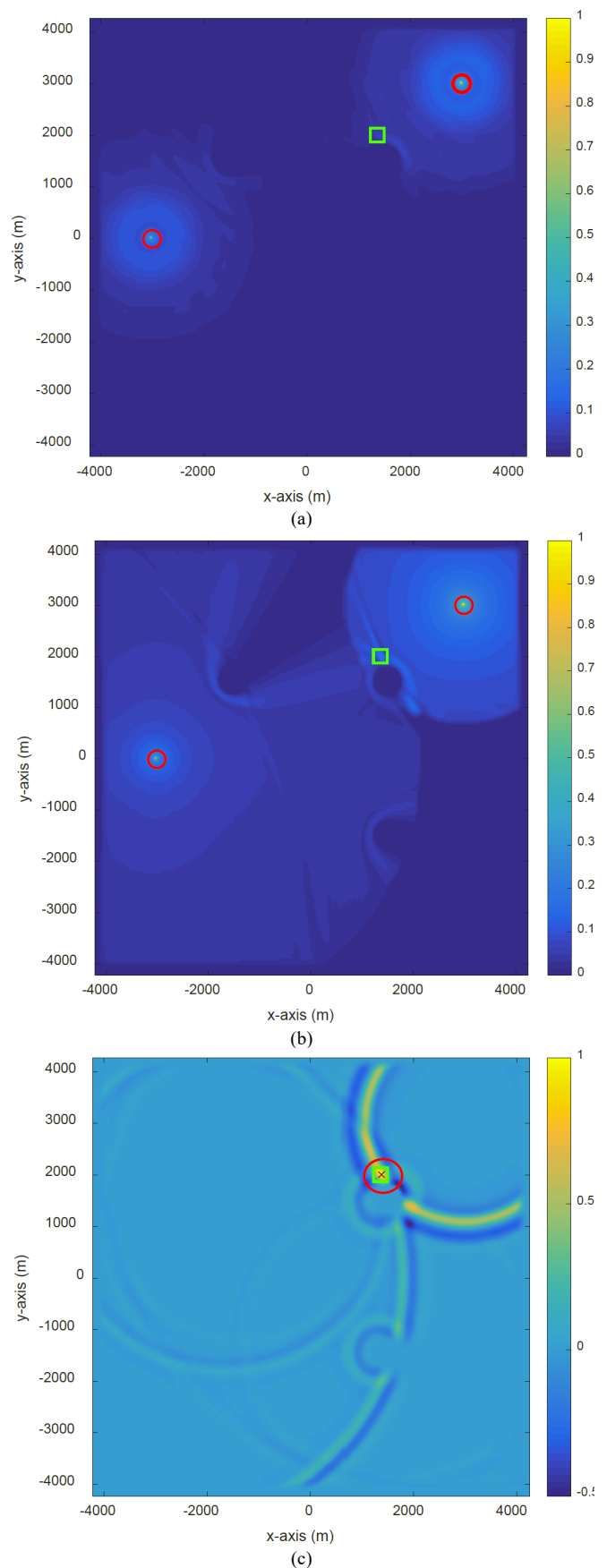


Fig. 3. Source locations (red circles) obtained using a) the normalized maximum amplitude of the energy over all the time steps, b) the normalized maximum amplitude of the total field over all the time steps, and c) the last local minimum entropy. The green square shows the actual lightning source location.

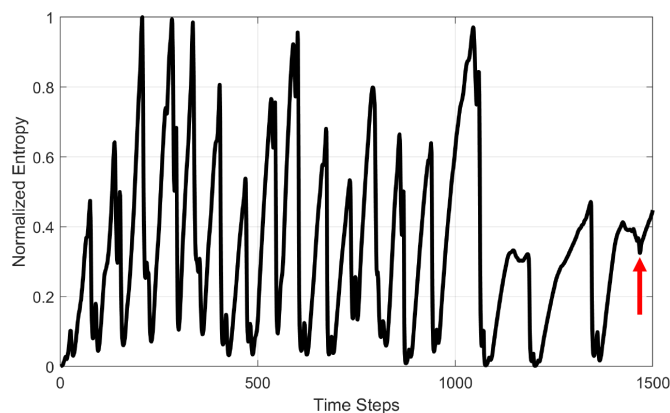


Fig. 4. Entropy criterion used to determine the optimal time at which all the time-reversed back-injected waves focus at the source location. The optimal time corresponds to the last local minimum, shown by the red arrow.

described in Section II, in the second step of the EMTR technique, the signals recorded at the sensors are time reversed and back-injected into the same medium of Fig. 1 (unchanged media). Note that we are only using the first two sensors in this scenario. The maximum amplitude of the electric field, the maximum amplitude of the energy, and the last local minimum of the entropy will now be tested to obtain the optimum focusing time and location of the original radiation source. Fig. 3 shows results obtained using the three above-mentioned criteria. In this paper, the colors in all the figures represent the normalized intensity of the considered quantity. The red-circles show the locations of the source that were obtained from the EMTR method. The correct location of the lightning strike is represented by a green square. The maximum electric field amplitude and the maximum amplitude of the energy are found to be unable to locate the lightning strike point. They erroneously identify the locations of the field sensors as the lightning strike location (Fig. 3a and 3b). As shown in these figures, the maximum amplitude of the electric field and the maximum amplitude of the energy occur at the location of the sensors, which are not at the correct source location. This is only due to the propagation with $1/R$ attenuation. To cope with this problem, Mora et al. [8] and Lugrin et al. [9] removed the $1/R$ attenuation dependence in the back-propagation step. However, this technique is only applicable when the terrain is flat and when no scatterers are present. Fig. 3c shows the electric field distribution in the computational domain at the optimum time slice obtained by the entropy method [19], [20]. The last local minimum entropy criterion, as shown in Fig. 4, allows to identify the time at which all the time-reversed back-injected waves focus at the source location. In the proposed EMTR process, the number of both forward and backward time steps is chosen to be equal. Our analysis shows that the obtained focal spot is independent of the adopted number of time steps. The lightning source can be located in this case with an error lower than the mesh cell size (30 m) of the computational domain. It should be noted that in the following parts of the paper, we only consider the entropy criterion.

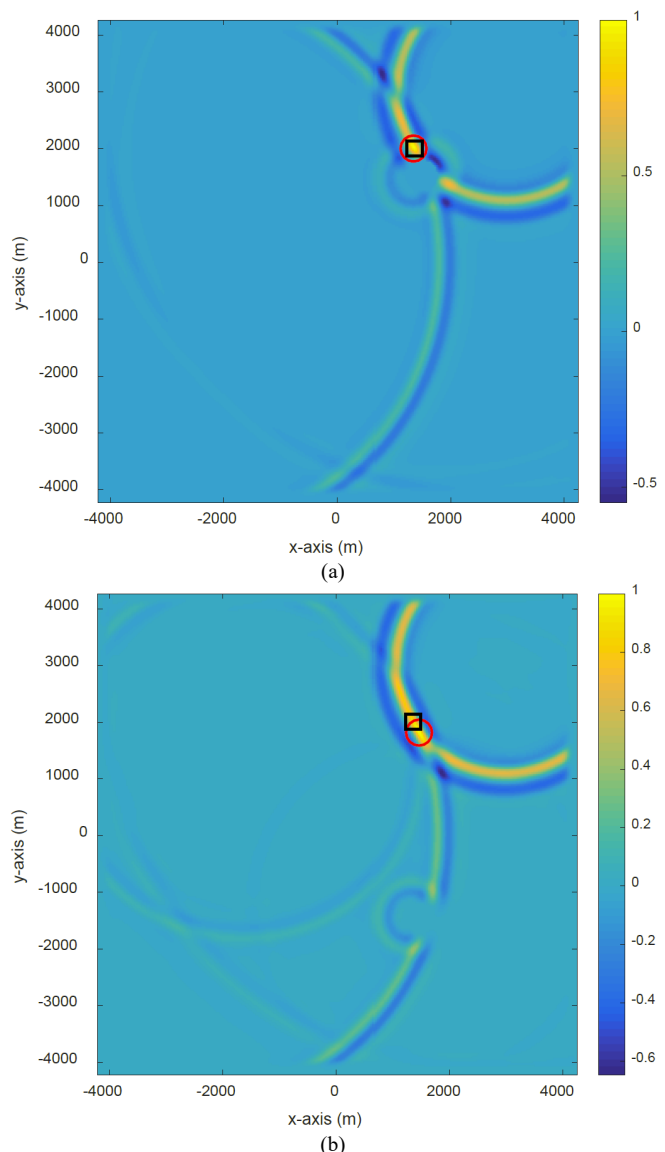


Fig. 5. Evaluation of the EMTR method with unmatched media, a) only the first scatterer is considered, b) only the first scatterer is removed. The black rectangle shows the exact source location and the red circle shows the estimated source location.

B. Two-Sensor Scenario, Simplified Backward-Time Medium

In numerous practical applications, the exact knowledge of the medium in which the wave is propagating is missing or it is too computationally burdensome to be considered. Therefore, we evaluate the efficiency of the EMTR method by changing the medium in the backward-time phase. In this respect, we consider two cases with respect to the geometry of Fig. 1: a) only the first scatterer is present in the backward-time phase, b) only the first scatterer is removed in the backward-time phase.

The obtained result, presented in Fig. 5, shows an acceptable location error of 30 and 240 m for cases a and b, respectively. Hence, it can be concluded that the accuracy of the EMTR technique may be acceptable even if the back-propagation medium is somewhat different from that of the forward-propagation phase. Also, as expected, more information on the background medium and present objects will lead to a better

performance of the EMTR technique. More in-depth investigations are needed to evaluate the effect of adopting simplified media on the performance of the EMTR technique.

C. Three-Sensor Scenario, Simplified vs Unchanged Media

We will now include the third sensor (S3), shown in green in Fig. 1. The other parameters of the medium and the solution method are similar to the one presented in section III-A. The result of the forward-time phase is depicted in Fig. 2. We consider two scenarios for the backward propagation: a) unchanged medium, b) simplified backward-time medium by removing all the scatterers. The last local minimum entropy criterion was applied here to obtain the optimum time slice. Figs. 6a and 6b present the obtained solution by EMTR in the unchanged and simplified media cases, respectively. The obtained location error for the unchanged medium case and for the simplified medium case are less than 30 m and 90 m, respectively. Our results demonstrate that by adding an extra sensor, the efficiency of EMTR for the simplified backward-time medium scenario is improved considerably and the chance of ambiguity is reduced.

IV. EXPERIMENTAL VALIDATION

In this section, we validate the EMTR process using experimental lightning data. Section IV-A describes the measurement stations and recorded waveforms. In section IV-B, we present results of the application of the EMTR technique and its validation.

A. Säntis Observation Site

The Säntis Tower was instrumented in May 2010 to measure the channel-base currents of lightning discharges striking the tower [21]. The tower is 124-m tall and it sits at the top of the 2502-m-tall Säntis Mountain. The Säntis Mountain is located in the northeastern part of Switzerland in the Appenzell region (47°14'57"N, 9°20'32"E).

An electric field measurement station was installed in Herisau (47°25'45.89"N, 9°03'57.36"E), about 14.7 km away from the Säntis Tower [22] on July 23, 2014. In addition, vertical electric fields were also measured by ALDIS in Neudorf (48°33'04.21"N, 14°03'35.15"E), Northern Austria, some 380 km from the tower [22], [23]. An over-the-Internet triggering scheme over TCP/IP was used to trigger the electric field sensors. Fig. 7 shows the geometrical location of the electric field sensors and the lightning current observation point. An upward negative lightning flash with 5 return strokes was recorded on October 21, 2014 at 20:23:22 along with its associated vertical electric fields at 14.7 and 380 km from the Säntis Tower (more details on the features of this flash can be found in [24]). Fig. 8 shows the channel-base current and electric field waveforms of one of the return strokes of this flash. The disturbances at far electric field corresponds to interaction of lightning electromagnetic pulse with ionosphere [25].

B. Application of EMTR to the Experimental Results

The electric field waveforms recorded by the sensors at

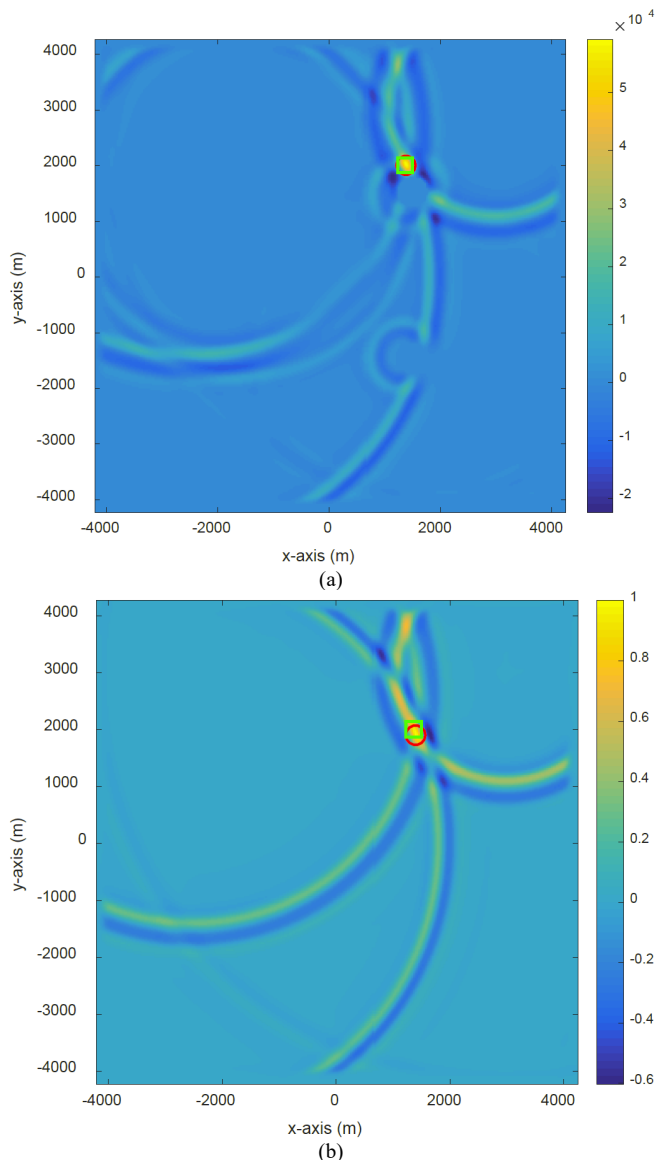


Fig. 6. Lightning localization using three field sensors, a) unchanged media, b) all scatterers are removed.

Herisau and Neudorf were time-reversed and back-injected into the solution space. The 2D-FDTD method mentioned in Section III was applied to calculate the fields in the backward time phase. The presence of mountainous terrain was ignored. Hence, we are dealing with a simplified medium scenario for the backward-time stage which, according to our investigation in Section III, is the worst-case scenario. Fig. 9 shows the obtained solution by way of the EMTR method using the last local minimum criterion. It can be observed that there is an ambiguity in the obtained focusing point. This is of course an expected result given the symmetry of the back-propagation simulation with two sensors and no scatterers. We argue that this ambiguity could be readily removed either by including the mountainous terrain, which introduces the necessary asymmetry (unchanged medium, II-A) in the backward-time phase solution or by adding another sensor (three sensors, III-C). The obtained error using the 2-sensor scenario (simplified medium) is below 260 m.

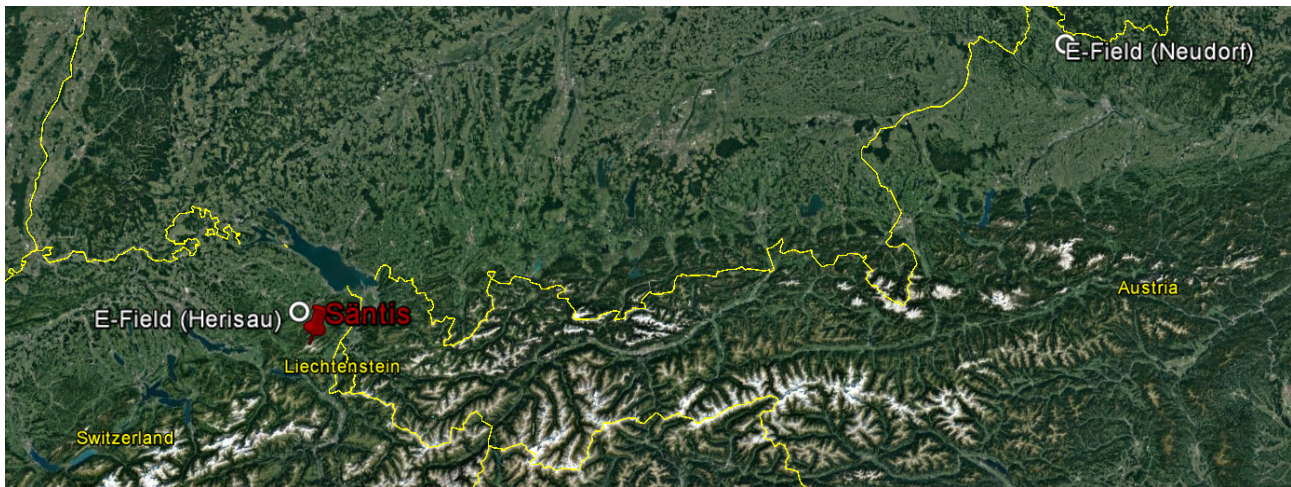


Fig. 7. Geographical location of lightning current observation site and deployed field sensors.

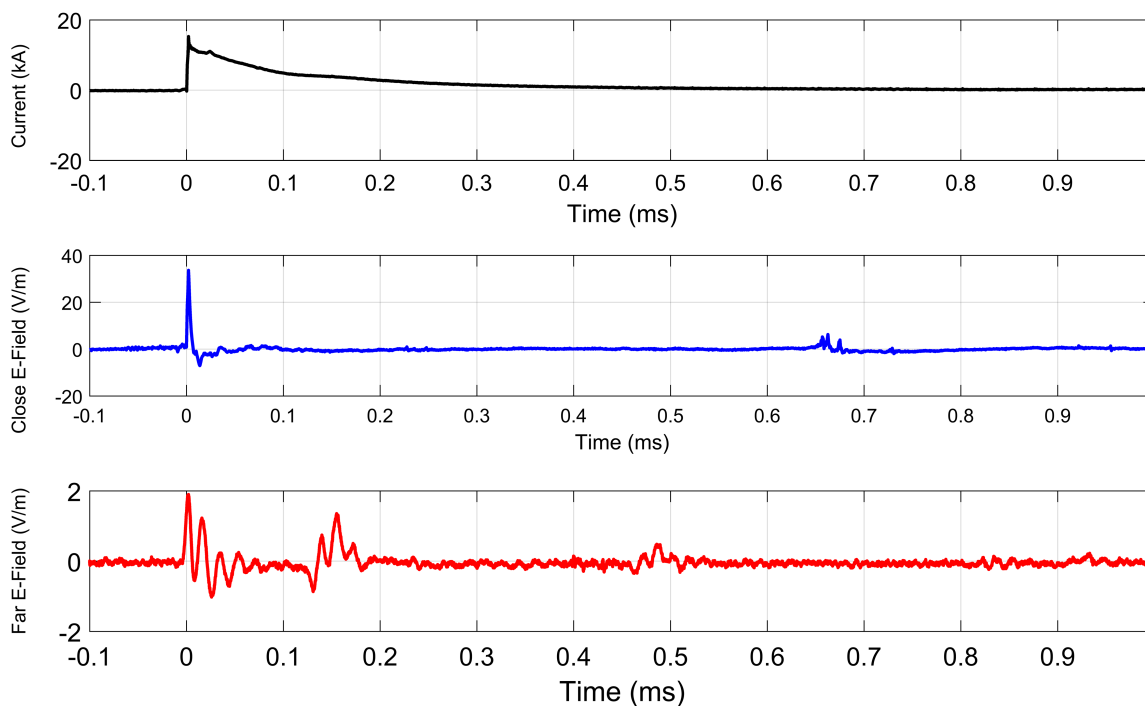


Fig. 8. Simultaneous channel-base current and electric field waveforms of an upward flash occurred on October 21, 2014 at 20:23:22.

V. CONCLUSION

We set up and utilized an EMTR algorithm to locate lightning discharges. We made use of a 2D-FDTD method to calculate the field in the forward and backward time phases of the EMTR technique.

The maximum field amplitude, maximum energy amplitude, and last local minimum of the entropy were tested as criteria to obtain the optimum time slice at which the waves refocus at the location of the source. We showed that, in the case of full-wave EM calculations, the last local minimum of the entropy can be used to identify the time at which the time-reversed back-propagated waves reach the source.

The results of our analysis revealed that the EMTR technique in the presence of scatterers can yield reasonable accuracy even using two sensors; however, using more sensors will increase the accuracy of the method.

Furthermore, we discussed the effect of a simplified medium in the backward propagation phase. Reasonable results were obtained using both, two and three sensors. However, using only two sensors in the case of a simplified medium might cause an ambiguity when all the scatterers are removed. Information on the medium, such as the number of scatterers and their positions affect the performance of the EMTR technique.

We used electric field data gathered at 14.7 and 380 km from the Sântis Tower to validate the proposed method experimentally. The recorded electric field waveforms from the

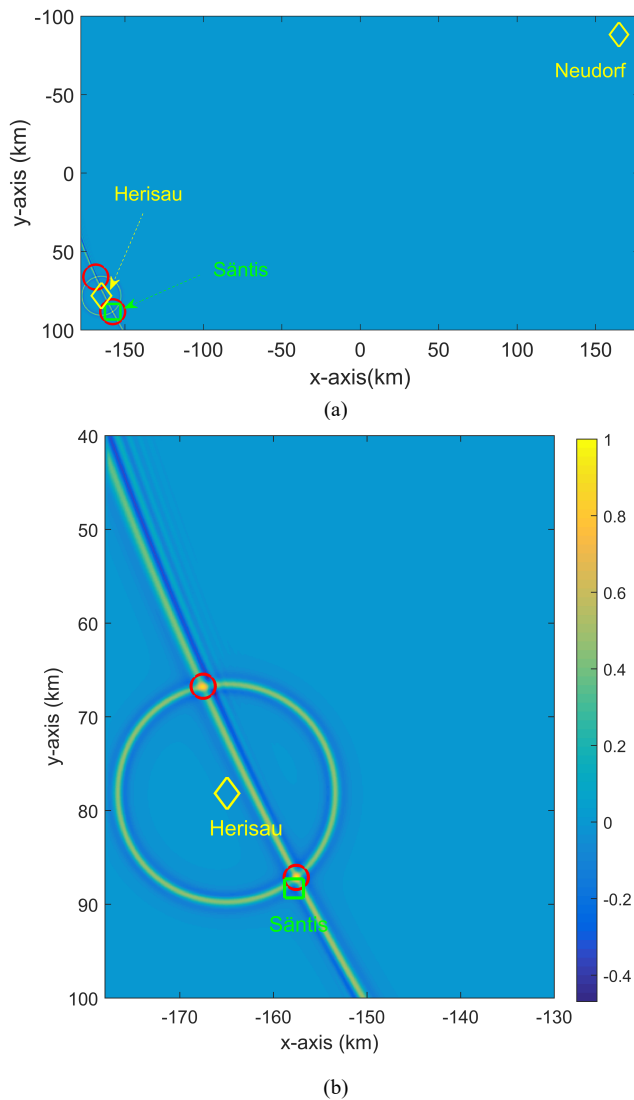


Fig. 9. Experimental validation of the EMTR technique. a) Overall view, b) expanded view in the vicinity of the Sântis Tower and the Herisau field station. The yellow diamonds mark the locations of the field sensors and the green rectangle is the location of the Sântis Tower. Red circles are the focusing points determined by the EMTR technique.

two sensors were time-reversed and back-injected into the simplified medium calculation domain (no scatterers, flat ground). It was found that, apart from the ambiguity, an excellent focusing accuracy can be achieved. Lightning location systems based on the described technique could use a lower number of sensors than current systems.

For the cases considered in this paper, a simple geometry for the presence of the mountains was used. We showed that even using such a simplified model, the proposed method can provide reasonable location accuracy. The proposed method can be extended to more complex and realistic 3D geometries. Further work is underway in this direction.

ACKNOWLEDGEMENT

Financial supports from the Swiss Federal Office of Energy (Grant Nr. SI/501706-01), the Swiss National Science Foundation (Project No. 200020_175594), the European

Union's Horizon 2020 research and innovation program (grant agreement. No 737033-LLR) are acknowledged.

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