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# Accurate comparison of tree sets using HMM-based descriptor vectors

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## Abstract

Trees are among the most studied data structures and several techniques have consequently been developed for comparing two trees belonging to the same category. Until the end of year 2020, there was a serious lack of suitable metrics for comparing two weighted trees or two trees from different categories. The problem of comparing two tree sets was not also specifically addressed. These limitations have been overcome in a paper published in 2021 where a customizable metric based on hidden Markov models have been proposed for comparing two tree sets, each containing a mixture of trees belonging to various categories. Unfortunately, that metric does not allow the use of non metric-dependent classifiers which take descriptor vectors as inputs. This paper addresses this drawback by deriving a descriptor vector for each tree set from the behavior of its associated models after a sufficiently long time. The comparison between two tree sets is then realized by comparing their associated descriptor vectors. Classification experiments carried out on the databases *FirstLast-L* (FL), *FirstLast-LW* (FLW) and *Stanford Sentiment Treebank* (SSTB) respectively showed best accuracies of 99.75%, 99.75% and 87.22%. These performances are respectively 40.75% and 20.52% better than the tree Edit distance respectively for FLW and SSTB. Additional clustering experiments exhibited 54.25%, 98.75% and 75.53% of correctly clustered instances respectively for FL, FLW and SSTB. No clustering was performed in existing work.

## Keywords

Trees; Comparison of tree sets; Distance between trees; Hidden Markov Models

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## I INTRODUCTION

Trees are among the most common and well-studied data structures in a great variety of applications, including compiler design, graph transformation, automatic theorem proving, information retrieval, structured text database, pattern recognition, and signal processing [1, 2]. Consequently, several techniques have been developed for comparing two labeled or unlabeled trees belonging to the same category (i.e: rooted or unrooted, ordered or unordered). Until the end of year 2020, these existing techniques were based on one of the 5 following concepts: *tree Edit* [3–14], *tree Alignment* [15, 16], *tree Inclusion* [17–21], *tree pattern matching* [22–27] and *Subtrees/supertrees similarity* [28–34]. These existing techniques unfortunately embedded the three following main drawbacks:

1. There was a serious lack of suitable metrics for comparing two weighted trees. However, the comparison of weighted trees is important for several applications such as the

match-making of agents in e-Business environments where product/service descriptions of seller/buyer agents are represented as arc-weighted trees [35]. An attempt for comparing such trees was proposed in [35], but that measure was limited because arc weights must always belong to the interval  $[0, 1]$ .

2. Existing techniques did not enable to compare two trees from different categories (e.g: comparing a rooted ordered tree and an unrooted unlabeled tree).
3. The problem of comparing two tree sets was not specifically addressed.
4. Existing techniques did not enable to explicitly specify the targeted nodes properties on which the tree comparison must be performed.

In 2021, these drawbacks have been overcome in [36] where a customizable metric based on Hidden Markov Models (HMMs) was proposed for comparing two tree sets, each containing a mixture of trees belonging to various categories. The metric proposed in that paper handles labeled and unlabeled trees, as well as weighted and unweighted trees where each node/arc can have several attributes (labels, weights). Unlike previous techniques, it allows the user to explicitly specify the targeted nodes properties on which the comparison should be performed and there is no boundary on the possible choices of these properties. The author of that paper relied on the *Depth-First Search (DFS)* traversal of a tree [37] to design the HMMs. The comparison between two tree sets was finally performed by comparing their associated HMMs. That metric showed perfect accuracies of 100% during flat classification experiments carried out on two online available synthetic databases <sup>1</sup> named *FirstLast-L (FL)* and *FirstLast-LW (FLW)* using the *Nearest Neighbor (NN)* classifier. This performance was 41% higher than the one exhibited when the *tree Edit* distance was selected for FL.

An important limitation of the metric proposed in that paper is related to the fact that it does not allow the use of non metric-dependent classifiers (i.e: *Support Vector Machines* [38], *Decision trees* [39], etc.) which absolutely require descriptor vectors as inputs. The current paper addresses this limitation by deriving a descriptor vector for each tree set from its associated models. Consider the set  $P = \{p_1, \dots, p_m\}$  of targeted nodes properties. Given a targeted nodes property  $p_i \in P$  and the model  $\lambda_i(T)$  associated with a tree set  $T$  according to  $p_i$ , we capture the overall proportion of time spent by  $\lambda_i(T)$  observing each symbol after a sufficiently long time, irrespective of the state from which this symbol is observed as a characteristic of  $T$ . We apply this for all the properties in  $P$  and the resulting values are sequentially gathered as the components of a descriptor vector  $\vec{T}$ . Finally, the comparison between two tree sets is realized by comparing their associated descriptor vectors. The accuracy of the proposed descriptor vectors is evaluated through classification and unsupervised clustering experiments carried out on two synthetic tree databases and one real world tree database.

The rest of this paper is organized as follows: the state of the art is presented in Section II, followed by a summarized presentation of HMMs in Section III. The description of the proposed approach is realized in Section IV, while experimental results are exhibited in Section V. The last section is dedicated to the conclusion.

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<sup>1</sup><http://perso-etis.ensea.fr/sylvain.iloga/FirstLast/index.html>

## II STATE OF THE ART

### 2.1 Related work

A tree can be defined as a connected acyclic graph. A detailed overview on graph theory and applications is available in [40]. Many distances have yet been proposed for comparing trees and they are based on one of the following concepts: *tree edit*, *tree alignment*, *tree inclusion*, *tree pattern matching*, *subtrees/supertrees similarity* and *HMMs*.

The *tree edit* distance [3–14] is based on the analysis of the number of edit operations required to transform a tree  $t_1$  into another tree  $t_2$ . The three following edit operations are considered for a given node: insertion, deletion and substitution. *Tree alignment* [15, 16] is a particular case of *tree edit* where insertions are always performed before deletions. Let  $t_1$  and  $t_2$  be two rooted trees with labeled nodes.  $t_1$  is *included* in  $t_2$  if there is a sequence  $S(t_1, t_2)$  of node deletions performed on  $t_2$  which makes  $t_2$  isomorphic to  $t_1$ . The tree inclusion problem is to decide if  $t_1$  can be included in  $t_2$  and the *tree inclusion* distance [17–21] is the sum of the costs of the delete operations found in  $S(t_1, t_2)$ . *Tree pattern matching* [22–27] consists in finding the instances of a given pattern tree in a specific target tree. *Subtrees/supertrees similarity* [28–34] are generally realized by finding the maximum agreement subtree, the largest common subtree, the smallest common supertree or the number of *pq-grams* that are not shared by 2 trees. A *pq-gram* of a tree  $t$  being a subtree of  $t$  with a specific shape determined by 2 user-defined integers  $p$  and  $q$  [34].

In 2021, a *HMM*-based technique have been proposed for comparing two finite tree sets [36]. The author of that paper remarked that *DFS* sequentially transits from one node depth to another and at each step, the algorithm observes the properties of the visited node. Consider the set  $P = \{p_1, \dots, p_m\}$  of targeted nodes properties. For every property  $p_i \in P$ , the former observation enabled to transform any tree  $t$  into a Markov chain  $\delta_i(t)$  where the hidden states were the nodes depths, while the symbols were the values of property  $p_i$  for every node of  $t$ . When this principle was applied on a tree set  $T = \{t_1, \dots, t_n\}$ , the set  $\Delta_i(T) = \{\delta_i(t_1), \dots, \delta_i(t_n)\}$  of Markov chains was obtained. The content of  $\Delta_i(T)$  was later used to initialize then to train the HMM  $\lambda_i(T)$  associated with  $T$  according to property  $p_i$ . Finally, the comparison between two tree sets was performed through the comparison of their associated HMMs for every property  $p_i \in P$ . The main assets of that technique are listed below:

1. It is designed for comparing finite tree sets.
2. It handles rooted/unrooted as well as ordered/unordered trees.
3. It compares weighted/unweighted as well as labeled/unlabeled trees.
4. It allows each node/arc to have many labels/weights.
5. It requires the specification of the targeted nodes properties.
6. It outperforms the *tree edit* distance with 41% of accuracy gain.

### 2.2 Problem statement

Until the end of year 2020, existing techniques for comparing two trees [3–34] embedded many drawbacks which were overcome in 2021 by the customizable HMM-based metric proposed in [36] for comparing two tree sets. An important limitation of the technique proposed in [36] is related to the fact that it does not allow the use of non metric-dependent classifiers which absolutely require descriptor vectors as inputs. The current paper tackles this limitation by deriving a descriptor vector for each tree set from its associated models. The comparison between two tree sets is then realized by comparing their respective associated descriptor vectors.

### III HIDDEN MARKOV MODELS

#### 3.1 HMM definition

A HMM  $\lambda = (A, B, \pi)$  is fully characterized by [41]:

1. The number  $N$  of states of the model. The set of states is  $S = \{s_1, s_2, \dots, s_N\}$ . The state of the model at time  $x$  is generally noted  $q_x \in S$ .
2. The number  $M$  of symbols. The set of symbols is  $\vartheta = \{v_1, v_2, \dots, v_M\}$ . The symbol observed at time  $x$  is generally noted  $o_x \in \vartheta$ .
3. The state transition probability distribution  $A = \{A[s_i, s_j]\}$  where  $A[s_i, s_j] = Prob(q_{x+1} = s_j | q_x = s_i)$  with  $1 \leq i, j \leq N$ .
4. The symbols probabilities distributions  $B = \{B[s_i, v_k]\}$  where  $B[s_i, v_k] = Prob(v_k \text{ at time } x | q_x = s_i)$  with  $1 \leq i \leq N$  and  $1 \leq k \leq M$ .
5. The initial state probability distribution  $\pi = \{\pi[s_i]\}$  where  $\pi[s_i] = Prob(q_1 = s_i)$  with  $1 \leq i \leq N$ .

#### 3.2 Manipulation of a HMMs

Consider a sequence of symbols  $O = o_1 o_2 \dots o_X$  and a HMM  $\lambda = (A, B, \pi)$ . The probability  $Prob(O|\lambda)$  to observe  $O$  given  $\lambda$  is efficiently calculated by the *Forward-Backward* algorithm [41] which runs in  $\theta(X.N^2)$ . Given a sequence of symbols  $O = o_1 o_2 \dots o_X$ , it is possible to iteratively re-estimate the parameters of a HMM  $\lambda = (A, B, \pi)$  in order to maximize the value of  $Prob(O|\bar{\lambda})$ , where  $\bar{\lambda} = (\bar{A}, \bar{B}, \bar{\pi})$  is the re-estimated model. The *Baum-Welch* algorithm [41] is generally used to perform this re-estimation. This algorithm runs in  $\theta(\beta.X.N^2)$  where  $\beta$  is the user-defined maximum number of iterations. The *Baum-Welch* algorithm can also train a HMM for multiple sequences. The algorithm maximizes the value of  $Prob(O|\bar{\lambda}) = \sum_{k=1}^K Prob(O^{(k)}|\bar{\lambda})$  where  $O = \{O^{(1)}, \dots, O^{(K)}\}$  is a set of  $K$  sequences of symbols and  $O^{(k)} = o_1^{(k)} \dots o_{X_k}^{(k)}$  is the  $k^{th}$  sequence of symbols of  $O$ . In the case of multiple sequences, this algorithm runs in  $\theta(\beta.(\sum_{k=1}^K X_k).N^2)$ . In this paper, the value  $\beta = 100$  is selected following [36].

#### 3.3 Stationary distribution of a HMM

A vector  $\varphi = (\varphi[s_1], \dots, \varphi[s_N])$  is a stationary distribution of a HMM  $\lambda = (A, B, \pi)$  if [42]<sup>2</sup>:

1.  $\forall j, (\varphi[s_j] \geq 0)$  and  $(\sum_j \varphi[s_j] = 1)$
2.  $\varphi = \varphi.A \Leftrightarrow (\varphi[s_j] = \sum_i \varphi[s_i] \times A[s_i, s_j], \forall j)$

$\varphi[s_j]$  estimates the overall proportion of time spent by  $\lambda$  in state  $s_j$  after a sufficiently long time.  $\varphi$  can be extracted from any line of the matrix  $A^w = A \times A \times \dots \times A$  ( $w$  times) when  $w \rightarrow +\infty$ . Therefore, the computation of  $\varphi$  requires  $\theta(w.N^3)$  arithmetic operations.

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<sup>2</sup>See Definition 9.1 of [42]

## IV THE PROPOSED APPROACH

### 4.1 Main idea

Consider the set  $P = \{p_1, \dots, p_m\}$  of targeted nodes properties and let us assume that the *DFS* traversal of a tree  $t$  is executed by a robot  $r$ . According to the observation made by the author of [36], for each property  $p_i \in P$ , the robot  $r$  sequentially transits from one node depth to another and at each step,  $r$  observes the value  $p_i(y)$  of the currently visited node  $y \in t$ . The MC  $\delta_i(t)$  resulting from this traversal of  $t$  executed by the robot  $r$  according to property  $p_i$  embeds relevant information related to the overall behavior (movements, observations) of  $r$ . Given a tree set  $T = \{t_1, \dots, t_n\}$  and its corresponding set  $\Delta_i(T) = \{\delta_i(t_1), \dots, \delta_i(t_n)\}$  of MCs, the HMM  $\lambda_i(T)$  was trained in [36] with the *Baum-Welch* algorithm to learn and to simulate the overall behavior of  $r$  during the traversal of the trees in  $T$ .

The main idea of the current paper is that, one can capture the behavior of  $r$  from  $\lambda_i(T)$  by evaluating the overall proportion of time spent by  $r$  observing each possible value of  $p_i$  after a sufficiently long time, irrespective of the node depth from which this value is observed. More precisely, we evaluate the probability  $\gamma(o|\lambda_i(T))$  of observing each symbol  $o$  after a sufficiently long time, given the model  $\lambda_i(T)$ , irrespective of the state from which  $o$  is observed. When this principle is applied for every property  $p_i \in P$  and for every possible symbol, the resulting probabilities are sequentially saved as the components of the descriptor vector  $\vec{T}$  associated with the tree set  $T$ .

### 4.2 Methodology

Consider a tree set  $T = \{t_1, \dots, t_n\}$  and the set  $P = \{p_1, \dots, p_m\}$  composed of  $m$  user-defined targeted nodes properties. The methodology applied in the current paper for deriving the descriptor vector  $\vec{T}$  associated with  $T$  can be summarized in the three following steps:

1. **Tree modeling:** For each property  $p_i \in P$ , the tree modeling principle proposed in [36]<sup>3</sup> is preserved here to obtain the model  $\lambda_i(T)$ . Therefore, this step is not described in the following sections.
2. **Probability computing:** Let  $\vartheta_i$  be the set of symbols of  $\lambda_i(T)$ . For every property  $p_i$  and for every symbol  $o \in \vartheta_i$ , we compute the probability  $\gamma(o|\lambda_i(T))$  of observing  $o$  after a sufficiently long time, given the model  $\lambda_i(T)$ , irrespective of the state from which  $o$  is observed.
3. **Construction of the descriptor vector:** All the probabilities computed at step 2 are sequentially saved as the components of the descriptor vector  $\vec{T}$  associated with  $T$ . Figure 1 summarizes the main steps of the proposed methodology.

### 4.3 Probability computing

Consider a tree set  $T$  and its associated model  $\lambda_i(T) = (A_i^T, B_i^T, \pi_i^T)$  according to property  $p_i \in P$ , where  $P = \{p_1, \dots, p_m\}$  is the user-defined set of targeted nodes properties. In order to compute  $\gamma(o|\lambda_i(T))$ , one must first evaluate the overall proportion of time spent by  $\lambda_i(T)$  observing  $o$  in state  $s_j$  after a sufficiently long time as follows:

1. Evaluate the overall proportion of time spent by  $\lambda_i(T)$  in state  $s_j$  after a sufficiently long time. This proportion is given by the  $j^{\text{th}}$  component  $\varphi_i^T[s_j]$  of the stationary distribution of  $\lambda_i(T)$ .

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<sup>3</sup>See Section 4.3 and Figure 4 of [36]

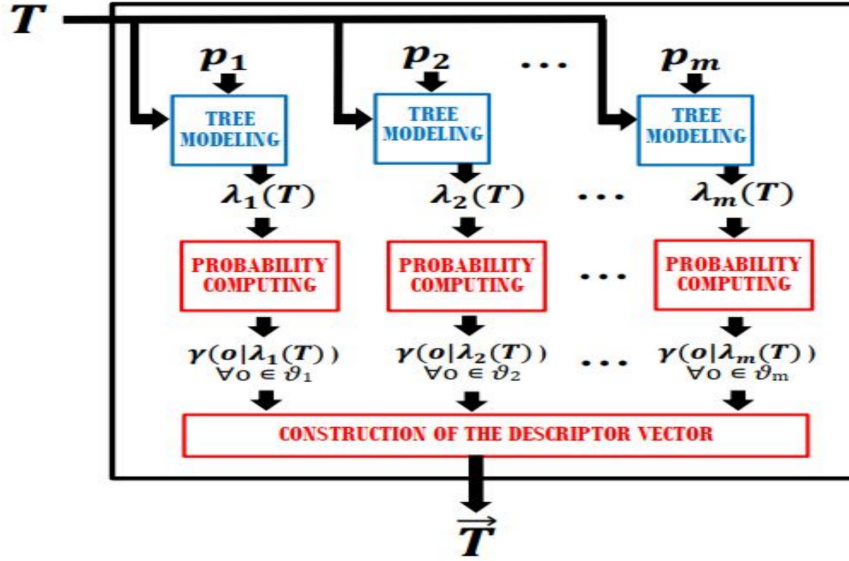


Figure 1: Methodology for deriving the descriptor vector  $\vec{T}$  associated with a tree set  $T$ .

2. Multiply the result of step 1 by the probability of observing  $o$  in state  $s_j$  which is  $B_i^T[s_j, o]$ . The value of  $\gamma(o|\lambda_i(T))$  is finally obtained by repeating this process for every state  $s_j$  and summing the resulting proportions as shown in Equation 1. An analogical probability computing scheme was performed in [43] where the authors used HMMs for realizing human activity recognition<sup>4</sup>.

$$\gamma(o|\lambda_i(T)) = \sum_{j=1}^N \varphi_i^T[s_j] \times B_i^T[s_j, o] \quad (1)$$

#### 4.4 Construction of the descriptor vector

The descriptor vector  $\vec{T}$  associated with  $T$  is constructed by sequentially saving the values of  $\gamma(o|\lambda_i(T))$  for every property  $p_i \in P$  and for every symbol  $o \in \vartheta_i$  as described in Algorithm 1. The first line of Algorithm 1 initializes the index of the components of the descriptor vector. Line 2 browses the properties, while line 3 browses the symbols for each property. The value of  $\gamma(o|\lambda_i(T))$  is computed at line 4 according to Equation 1 and saved as the current component of the descriptor vector at line 5. Line 6 moves to the index of the next component of the descriptor vector. The descriptor vector is finally returned at line 9.

A consequence of Algorithm 1 is that the dimension of the descriptor vector  $\vec{T}$  denoted in this paper as  $\alpha$  is the sum of the numbers of symbols of all its  $|P|$  associated models as precised in Equation 2. It can be easily demonstrated that Algorithm 1 requires  $\theta(\alpha.N)$  arithmetic operations.

$$\alpha = \sum_{i=1}^{|P|} |\vartheta_i| \quad (2)$$

<sup>4</sup>See Section IV-E of [43]

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**Algorithm 1**  $Vector(T, P, \{\lambda_i(T)\}, \{\vartheta_i\}, \{\varphi_i^T\})$ 

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```
1:  $k \leftarrow 1$ 
2: for each ( $p_i \in P$ ) do
3:   for each ( $o \in \vartheta_i$ ) do
4:      $\gamma(o|\lambda_i(T)) \leftarrow \sum_{j=1}^N \varphi_i^T[s_j] \times B_i^T[s_j, o]$ 
5:      $\vec{T}[k] \leftarrow \gamma(o|\lambda_i(T))$ 
6:      $k \leftarrow k + 1$ 
7:   end for
8: end for
9: return  $\vec{T}$ 
```

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The following conventional property is adopted here to associate a descriptor vector with the empty set: ‘The descriptor vector associated with  $\emptyset$  according to the set  $P = \{p_1, \dots, p_m\}$  of targeted nodes properties is ( $\vec{\emptyset} = \vec{0} \in \mathbb{R}^\alpha$ ).’

#### 4.5 Comparison of tree sets

The comparison between two tree sets  $T_1$  and  $T_2$  according to the set  $P = \{p_1, \dots, p_m\}$  of targeted nodes properties is performed here by calculating the value of any existing distance or similarity measure between their associated descriptor vectors  $\vec{T}_1$  and  $\vec{T}_2$ . Equation 3 shows how this can be done with the *Euclidean* distance and the *Manhattan* distance. Several other possible distance and similarity measures are online available <sup>5</sup>.

$$\begin{aligned} d_P(T_1, T_2) &= \sqrt{\sum_{k=1}^{\alpha} (\vec{T}_1[k] - \vec{T}_2[k])^2} && (Euclidean) \\ d_P(T_1, T_2) &= \sum_{k=1}^{\alpha} |\vec{T}_1[k] - \vec{T}_2[k]| && (Manhattan) \end{aligned} \tag{3}$$

## V EXPERIMENTAL RESULTS

### 5.1 Experimental tasks

The flat classification and clustering experiments realized in the current work are both realized with the *WEKA* soft [44]. Unlike [36] where only the *Nearest Neighbor* classifier was experimented, two additional non metric-dependent classifiers have also been experimented here. Thus, the three following classifiers have been experimented, their corresponding names in *WEKA* are in brackets: *Nearest Neighbor* (IBk), *Support Vector Machines* with polynomial kernels (SMO) and *Decision Trees* (J48). These classifiers have been used in *WEKA* with their default settings and a 10 folds cross-validation (90% – 10%) has been applied for each experiment.

The *Kmeans* [45] and the *Expectation-Maximization* (EM) [46] clustering algorithms have been selected here to evaluate how much the proposed descriptor vectors can enable to accurately

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<sup>5</sup><https://numerics.mathdotnet.com/Distance.html>



organize the experimental data (trees) into their corresponding classes (clusters) in an unsupervised process. The selected clustering algorithms have been used in *WEKA* with their default settings except the desired number of clusters which was initially fixed to the number of classes in each experimental database.

The *Euclidean* and the *Manhattan* distances have been both experimented as metrics for the *Nearest Neighbor* classifier and the *Kmeans* clustering algorithm.

## 5.2 Experimental databases

The two synthetic tree databases selected during the experiments of the current work are FL and FLW<sup>1</sup> which have both been constructed and experimented in [36]. Each database is composed of 4 classes, each containing 100 rooted ordered trees. FL contains node-labeled trees and the set of node labels is  $\{1, 2, \dots, 26\}$ . FLW contains the same trees found in FL, with weighted arcs. The trees in these two databases are characterized by non-trivial properties listed in [36]<sup>6</sup>.

The *Stanford Sentiment Treebank* (SSTB) tree database has also been experimented in the current work. This is a real world tree database originally proposed in [47] and containing 11.855 node-labeled binary trees associated with sentences from movie reviews organized into 5 classes of sentiments: *very negative* (1510 trees), *negative* (3140 trees), *neutral* (2242 trees), *positive* (3111 trees) and *very positive* (1852 trees). These classes are considered as node labels represented as integers belonging to  $\{0, 1, 2, 3, 4\}$ . In this database, each tree has the following recursive brackets representation:  $(label\ (left\ subtree)(right\ subtree))$  where the leftmost label is the overall sentiment of the sentence. Leave nodes have the following form:  $(label\ token)$  where token is a word of the sentence. During our experiments, the label of the root node of each tree was replaced by a new unique label (here 5) and tokens were removed from all the leave nodes following [34].

## 5.3 Experimental settings

We realized 2 main experiments for the databases FL and FLW. During the first one, the specific properties verified by each node in FL and FLW are intentionally ignored as if they were unknown. During the second main experience, these specific properties are considered. The sets  $\bar{P}_L = \{\bar{p}_1, \bar{p}_2\}$ ,  $\bar{P}_{LW} = \{\bar{p}_1, \bar{p}_2, \bar{p}_3\}$ ,  $P_L = \{p_1, \dots, p_8\}$  and  $P_{LW} = \{p_1, \dots, p_{12}\}$  of targeted nodes properties selected in [36]<sup>7</sup> for these experiences are preserved in the current work.

The 3 following sub-datasets of SSTB have been experimented and Table 1 presents the size of each class in these sub-datasets:

1. **SSTB5**: here, all the 5 original classes of sentiments are considered.
2. **SSTB3**: here, the content of the original classes *very negative* and *negative* (resp. *very positive* and *positive*) are merged to obtain one single class named *negative* (resp. *positive*). Thus, three classes are considered here.
3. **SSTB2**: This is SSTB3 where the class *neutral* is removed. Therefore, only two classes are considered here.

Only the default set  $\tilde{P} = \{\tilde{p}_1, \tilde{p}_2\}$  of targeted nodes properties has been experimented for the database SSTB, irrespective of its considered sub-dataset. For every node  $y$  of any tree in SSTB,  $\tilde{p}_1(y)$  is the degree of  $y$  and  $\tilde{p}_2(y)$  is the label of  $y$ . A detailed description of the experiments realized on the database SSTB and the corresponding data are available online<sup>8</sup>.

<sup>6</sup>See Table 5 of [36]

<sup>7</sup>See Section 5.2.2 of [36]

<sup>8</sup><http://perso-etis.ensea.fr/sylvain.iloga/index.html>

Table 1: Size of each class in the experimental sub-datasets of SSTB

	Very negative	Negative	Neutral	Positive	Very positive
SSTB5	1510	3140	2242	3111	1852
SSTB3	4650		2242	4963	
SSTB2	4650		0	4963	

In the context of each experiment and considering each tree  $t$  of the experimental databases as the singleton  $\{t\}$ , we have executed Algorithm 1 to construct the descriptor vector  $\vec{t}$  associated with  $t$ . The resulting descriptor vectors have then been saved into online available 'arff' files<sup>1,8</sup> which are taken as inputs by the soft *WEKA*.

#### 5.4 Classification results

Classification results are presented in Tables 2a, 2b and 2c respectively for FL, FLW and SSTB. Tables 2a and 2b reveal that best accuracy obtained for FL and FLW is 99.75%. This is quite equal to the 100% obtained in [36]. Table 2c reveals that the proposed approach also exhibits good performances for the database SSTB with accuracies always greater than 70% for the two non metric-dependent classifiers regarding the datasets SSTB3 and SSTB2, the best accuracy being 87.22%. This performance is 20.52% better than the 66.7% obtained in [14] for SSTB2<sup>9</sup>. These performances demonstrate the accuracy of the proposed approach in classification, even with a default set of targeted nodes properties.

Table 2: Classification results. Accuracies are in (%), best values are in Bold.

(a)- FL					(b)- FLW				
	IBk		SMO	J48		IBk		SMO	J48
	Eucli.	Manha.				Eucli.	Manha.		
$\bar{P}_L$	73.75	78	67.5	74.75	$\bar{P}_{LW}$	84	87.25	90.75	89.75
$P_L$	<b>99.75</b>	<b>99.75</b>	87.5	93.75	$P_{LW}$	<b>99.75</b>	<b>99.75</b>	<b>99.75</b>	99.5

(c)- SSTB using $\tilde{P}$				
	IBk		SMO	J48
	Eucli.	Manha.		
SSTB5	39.47	39.67	46.29	44.98
SSTB3	63.11	63.27	70.65	77.22
SSTB2	82.98	83.16	<b>87.22</b>	87.17

#### 5.5 Clustering results

Clustering results are presented in Tables 3a, 3b and 3c respectively for FL, FLW and SSTB. Tables 3a and 3b reveal that best clustering performance obtained for FL is 54.25%, while a quasi perfect performance of 98.75% is obtained for FLW. Given that the trees in FL are only characterized by properties related to the topology and the node-labels, experimental results demonstrate that this database does not embed enough information to distinguish the 4 classes during an unsupervised clustering process. But when the arc-weights are additionally considered in FLW, the 4 classes are accurately identified during the unsupervised clustering process with up to 98.75% correctly clustered instances when the 12 properties in  $P_{LW}$  are considered.

<sup>9</sup>See Table 1 of [14]

Table 3c reveals that the *Kmeans* clustering algorithm applied with the *Manhattan* distance always exhibits the best clustering results for SSTB5, SSTB3 and SSTB2 which are respectively 33.33%, 49.02% and 75.53% correctly clustered instances. These performances demonstrate the accuracy of the proposed approach in clustering, even with a default set of targeted nodes properties.

Table 3: Clustering results. Correctly clustered instances are in (%), best values are in Bold.

(a)- FL				(b)- FLW				(c)- SSTB using $\tilde{P}$			
	Kmeans		EM		Kmeans		EM		Kmeans		EM
	Eucli.	Manha.			Eucli.	Manha.			Eucli.	Manha.	
$\bar{P}_L$	47.5	47.25	<b>54.25</b>	$\bar{P}_{LW}$	54.25	55.25	42.25	SSTB5	32.63	33.33	29.17
$P_L$	45.5	44	42	$P_{LW}$	86.5	92	<b>98.75</b>	SSTB3	40.43	49.02	46.36
								SSTB2	50.97	<b>75.53</b>	61.73

## 5.6 Time cost

The time cost of the proposed technique for deriving the descriptor vector  $\vec{T}$  of a tree set  $T$  according to the set  $P$  of targeted nodes properties can be step-by-step estimated as follows:

1. Design the  $|P|$  HMMs associated with  $T$  which runs in  $\theta(|P| \cdot \beta \cdot (\sum_{t \in T} |t|) \cdot N^2)$  according to [36]<sup>10</sup>, where  $|t|$  is the number of nodes in the tree  $t$ .
2. Compute the stationary distributions of the  $|P|$  HMMs which runs in  $\theta(|P| \cdot w \cdot N^3)$  with  $w \rightarrow +\infty$ . In the current work, the value  $w = 100$  has been selected.
3. Construct the descriptor vector  $\vec{T}$  associated with  $T$  using Algorithm 1 which runs in  $\theta(\alpha \cdot N)$ .

It is according to these partial results that Equation 4 gives the expression of the theoretical time cost  $Time(\vec{T})$  of the proposed approach.

$$\begin{aligned}
 Time(\vec{T}) &= \theta(a \cdot N^3 + b \cdot N^2 + \alpha \cdot N) \text{ where} \\
 N &= \max\{\text{depth}(t) \mid t \in T\} + 1 \\
 a &= |P| \cdot w \\
 b &= |P| \cdot \beta \cdot \left( \sum_{t \in T} |t| \right)
 \end{aligned} \tag{4}$$

## 5.7 Comparisons with existing techniques

We have compared the proposed approach with:

1. [36] where the authors designed in 2021 the databases FL and FLW. They also proposed the original HMM-based technique for tree sets comparison which is improved in the current work.
2. [14] which is a work published in 2018 where the authors proposed a learning version of the *tree Edit* distance and experimented it on the dataset SSTB2.

It is important to mention that the dataset SSTB2 was more recently experimented in 2020 for tree classification in [34], but we performed no comparison with that work because the authors only randomly selected 100 trees for each class during their experiments. The technique proposed in the current paper:

<sup>10</sup>Cf. Section 5.3 of [36]

1. Is based on the HMMs designed in [36] for comparing two tree sets. It consequently inherits the main assets of that technique <sup>11</sup>.
2. Unlike [36], it associates a descriptor vector  $\vec{T} \in \mathbb{R}^\alpha$  whose components are interpretable with every tree set  $T$  such that all the operations that are applicable to vectors in  $\mathbb{R}^\alpha$  are now also applicable to tree sets.
3. Performs a finer characterization of a tree set than [36]. Indeed, a tree set was characterized in [36] by its associated HMMs themselves. In the current paper, a tree set is rather characterized by meta-information related to the overall behavior of these HMMs after a sufficiently long time.
4. Exhibits good classification performances (78% for FL and 90.75% for FLW) with the default sets  $\bar{P}_L$  and  $\bar{P}_{LW}$  of targeted nodes properties, unlike [36] where lower performances were obtained (44.25% for FL and 36% for FLW) in identical conditions.
5. Shows an excellent classification accuracy of 99.75% for FL and for FLW when the suitable sets  $P_L$  and  $P_{LW}$  of targeted nodes properties are selected. This performance is quite identical to the 100% obtained in [36].
6. Exhibits a good classification accuracy (87.22%) on the real world tree dataset SSTB2, this is 20.52% better than the 66.7% obtained in [14] for this same dataset.
7. Is capable to correctly cluster 54.25%, 98.75% and 75.53% of the instances of FL, FLW and SSTB2 respectively. No clustering was performed in [14, 36].

## VI CONCLUSION

The goal of this paper was to improve the HMM-based technique recently proposed in [36] for comparing two tree sets through the association of a descriptor vector with each tree set. Given a tree set  $T$  and a set  $P = \{p_1, \dots, p_m\}$  of targeted nodes properties, the author of [36] followed the principle of the *DFS* algorithm to associate  $|P|$  HMMs with  $T$ , each HMM  $\lambda_i(T)$  learned how much the nodes of the trees in  $T$  verify property  $p_i$ . The resulting models were finally compared to derive a distance and similarity between the two sets of trees. The technique proposed in [36] overcame the main limitations of the other existing techniques developed before its publication. Unfortunately, it did not allow the use of non metric-dependent classifiers which absolutely require descriptor vectors as inputs.

In order to derive the descriptor vector  $\vec{T}$  associated with a tree set  $T$  in the current paper, we capture the behavior of its associated HMMs by evaluating the overall proportion of time spent by every HMM at observing each symbol  $o$  after a sufficiently long time, irrespective of the state from which  $o$  is observed. The resulting proportions are then sequentially saved as the components of the descriptor vector. The comparison between two tree sets is finally realized by comparing their associated descriptor vectors. Classification experiments carried out on the databases FL, FLW and SSTB respectively showed best accuracies of 99.75%, 99.75% and 87.22%. These performances are respectively 40.75% and 20.52% better than the widespread *tree Edit* distance respectively for FLW and SSTB2. Additional clustering experiments exhibited 54.25%, 98.75% and 75.53% of correctly clustered instances respectively for FL, FLW and SSTB2. These performances demonstrate the dual accuracy of the proposed approach in classification and clustering. Most of the perspectives stated at the end of [36] remain valid here and must be explored by future work.

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<sup>11</sup>Cf. Section 5.4 of [36]

## REFERENCES

- [1] Gabriel Valiente. An efficient bottom-up distance between trees. In *spire*, pages 212–219, 2001.
- [2] Philip Bille. Tree edit distance, alignment distance and inclusion. Technical report, Cite-seer, 2003.
- [3] Kuo-Chung Tai. The tree-to-tree correction problem. *Journal of the ACM (JACM)*, 26(3):422–433, 1979.
- [4] Kaizhong Zhang and Dennis Shasha. Simple fast algorithms for the editing distance between trees and related problems. *SIAM journal on computing*, 18(6):1245–1262, 1989.
- [5] Kaizhong Zhang, Rick Statman, and Dennis Shasha. On the editing distance between unordered labeled trees. *Information processing letters*, 42(3):133–139, 1992.
- [6] Weimin Chen. New algorithm for ordered tree-to-tree correction problem. *Journal of Algorithms*, 40(2):135–158, 2001.
- [7] H el ene Touzet. Comparing similar ordered trees in linear-time. *Journal of Discrete Algorithms*, 5(4):696–705, 2007.
- [8] Erik D Demaine, Shay Mozes, Benjamin Rossman, and Oren Weimann. An optimal decomposition algorithm for tree edit distance. *ACM Transactions on Algorithms (TALG)*, 6(1):2, 2009.
- [9] Mateusz Pawlik and Nikolaus Augsten. Tree edit distance: Robust and memory-efficient. *Information Systems*, 56:157–173, 2016.
- [10] Stefan Schwarz, Mateusz Pawlik, and Nikolaus Augsten. A new perspective on the tree edit distance. In *International Conference on Similarity Search and Applications*, pages 156–170. Springer, 2017.
- [11] Thorsten Richter. *A new measure of the distance between ordered trees and its applications*. Inst. f ur Informatik, 1997.
- [12] Aida Ouangraoua, Pascal Ferraro, Laurent Tichit, and Serge Dulucq. Local similarity between quotiented ordered trees. *Journal of Discrete Algorithms*, 5(1):23–35, 2007.
- [13] Raghavendra Sridharamurthy, Bin Masood Talha, Kamakshidasan Adhitya, and Natarajan Vijay. Edit distance between merge trees. *IEEE TRANSACTIONS ON VISUALIZATION AND COMPUTER GRAPHICS*, pages 1–14, 2018.
- [14] Benjamin Paa en, Claudio Gallicchio, Alessio Micheli, and Barbara Hammer. Tree edit distance learning via adaptive symbol embeddings. In *International Conference on Machine Learning*, pages 3976–3985. PMLR, 2018.
- [15] Tao Jiang, Lusheng Wang, and Kaizhong Zhang. Alignment of trees—an alternative to tree edit. *Theoretical Computer Science*, 143(1):137–148, 1995.
- [16] Jesper Jansson and Andrzej Lingas. A fast algorithm for optimal alignment between similar ordered trees. In *Annual Symposium on Combinatorial Pattern Matching*, pages 232–240. Springer, 2001.

- [17] Pekka Kilpeläinen et al. Tree matching problems with applications to structured text databases. 1992.
- [18] Laurent Alonso and René Schott. On the tree inclusion problem. In *International Symposium on Mathematical Foundations of Computer Science*, pages 211–221. Springer, 1993.
- [19] Pekka Kilpeläinen and Heikki Mannila. Ordered and unordered tree inclusion. *SIAM Journal on Computing*, 24(2):340–356, 1995.
- [20] Thorsten Richter. A new algorithm for the ordered tree inclusion problem. In *Annual Symposium on Combinatorial Pattern Matching*, pages 150–166. Springer, 1997.
- [21] Weimin Chen. More efficient algorithm for ordered tree inclusion. *Journal of Algorithms*, 26(2):370–385, 1998.
- [22] Christoph M Hoffmann and Michael J O’Donnell. Pattern matching in trees. *Journal of the ACM*, 29(1):68–95, 1982.
- [23] S Rao Kosaraju. Efficient tree pattern matching. In *30th Annual Symposium on Foundations of Computer Science*, pages 178–183. IEEE, 1989.
- [24] Moshe Dubiner, Zvi Galil, and Edith Magen. Faster tree pattern matching. In *Proceedings [1990] 31st Annual Symposium on Foundations of Computer Science*, pages 145–150. IEEE, 1990.
- [25] RAMAKRISHNAN Ramesh and IV Ramakrishnan. Nonlinear pattern matching in trees. *Journal of the ACM (JACM)*, 39(2):295–316, 1992.
- [26] KZ Zhang, Dennis Shasha, and Jason Tsong-Li Wang. Approximate tree matching in the presence of variable length don’ t cares. *Journal of Algorithms*, 16(1):33–66, 1994.
- [27] Tyng-Luh Liu and Davi Geiger. Approximate tree matching and shape similarity. In *Proceedings of the Seventh IEEE International Conference on Computer Vision*, volume 1, pages 456–462. IEEE, 1999.
- [28] Martin Farach and Mikkel Thorup. Fast comparison of evolutionary trees. *Information and Computation*, 123(1):29–37, 1995.
- [29] Amihood Amir and Dmitry Keselman. Maximum agreement subtree in a set of evolutionary trees: Metrics and efficient algorithms. *SIAM Journal on Computing*, 26(6):1656–1669, 1997.
- [30] Sanjeev Khanna, Rajeev Motwani, and F Frances Yao. *Approximation algorithms for the largest common subtree problem*. Citeseer, 1995.
- [31] Tatsuya Akutsu and Magnús M Halldórsson. On the approximation of largest common subtrees and largest common point sets. *Theoretical Computer Science*, 233(1-2):33–50, 2000.
- [32] Arvind Gupta and Naomi Nishimura. Finding largest subtrees and smallest supertrees. *Algorithmica*, 21(2):183–210, 1998.
- [33] Naomi Nishimura, Prabhakar Ragde, and Dimitrios M Thilikos. Finding smallest supertrees under minor containment. *International Journal of Foundations of Computer Science*, 11(03):445–465, 2000.

- [34] Hikaru Shindo, Masaaki Nishino, Yasuaki Kobayashi, and Akihiro Yamamoto. Metric learning for ordered labeled trees with pq-grams. *arXiv preprint arXiv:2003.03960*, 2020.
- [35] Virendrakumar C Bhavsar, Harold Boley, and Lu Yang. A weighted-tree similarity algorithm for multi-agent systems in e-business environments. *Computational Intelligence*, 20(4):584–602, 2004.
- [36] Sylvain Iloga. Customizable hmm-based measures to accurately compare tree sets. *Pattern Analysis and Applications*, pages 1–23, 2021.
- [37] To-Yat Cheung. Graph traversal techniques and the maximum flow problem in distributed computation. *IEEE Transactions on Software Engineering*, (4):504–512, 1983.
- [38] Mariette Awad and Rahul Khanna. Support vector machines for classification. In *Efficient Learning Machines*, pages 39–66. Springer, 2015.
- [39] S Rasoul Safavian and David Landgrebe. A survey of decision tree classifier methodology. *IEEE transactions on systems, man, and cybernetics*, 21(3):660–674, 1991.
- [40] John Adrian Bondy, Uppaluri Siva Ramachandra Murty, et al. *Graph theory with applications*, volume 290. Macmillan London, 1976.
- [41] Lawrence R Rabiner. A tutorial on hidden markov models and selected applications in speech recognition. *Proceedings of the IEEE*, 77(2):257–286, 1989.
- [42] Jizhou Kang. Stat 243: Stochastic process. <https://bookdown.org/jkang37/stochastic-process-lecture-notes/lecture09.html>, 2021.
- [43] Sylvain Iloga, Alexandre Bordat, Julien Le Kernec, and Olivier Romain. Human activity recognition based on acceleration data from smartphones using hmms. *IEEE Access*, 9:139336–139351, 2021.
- [44] Witten Ian H. and Frank Eibe. Data mining: Practical machine learning tools and techniques. <http://weka.sourceforge.net/>, 2005.
- [45] Aristidis Likas, Nikos Vlassis, and Jakob J Verbeek. The global k-means clustering algorithm. *Pattern recognition*, 36(2):451–461, 2003.
- [46] Todd K Moon. The expectation-maximization algorithm. *IEEE Signal processing magazine*, 13(6):47–60, 1996.
- [47] Richard Socher, Alex Perelygin, Jean Wu, Jason Chuang, Christopher Manning, Andrew Ng, and Christopher Potts. Parsing With Compositional Vector Grammars. In *EMNLP*. 2013.