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Chemical hyperstructures for Astatine, Tellurium and for Bismuth

Madeline Al Tahan¹, Bijan Davvaz^{2,3}

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Abstract

The concept of weak algebraic hyperstructures constitutes a generalization of the well-known algebraic hyperstructures. The aim of this paper is to present some of the results, methods and ideas about chemical examples of weak algebraic hyperstructures. More precisely, we give chemical hyperstructures of chemical reactions for: a set of Astatine (At), a set of Tellurium (Te) and a set of Bismuth (Bi).

Keywords: Hypergroup, H_v -semigroup, oxidation state, redox reaction.

Resumen

El concepto de hiperestructuras algebraicas débiles constituye una generalización de las hiperestructuras algebraicas conocidas. El objetivo de este trabajo es presentar algunos de los resultados, métodos e ideas sobre ejemplos químicos de hiperestructuras algebraicas débiles. Más precisamente, ofrecemos hiperestructuras químicas de reacciones químicas para: un conjunto de Ástato (At), un conjunto de Telurio (Te) y un conjunto de Bismuto (Bi).

Palabras claves: Hipergrupos, H_v -semigrupo, estado de oxidación, reaccion redox.

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1 Introduction

In 1935, F. Marty [1] introduced algebraic hyperstructures which are generalized concepts of algebraic structures. A generalization of algebraic hyperstructures was introduced in 1990 by T. Vougiouklis [2] where he defined weak hyperstructures. Many researchers such as Corsini [3], Corsini and Leoreanu [4], Davvaz [5,6], Davvaz and Leoreanu-Fotea [7], and Vougiouklis [8] wrote books related to hyperstructure theory and their applications. One of motivations for the study of hyperstructures comes from chemical reactions. In [9], Davvaz and Dehghan-Nezhad provided examples of hyperstructures associated with chain reactions. In [10], Davvaz et al. introduced examples of weak hyperstructures associated with dismutation reactions. In [11], Davvaz et al. investigated the examples of hyperstructures and weak hyperstructures associated with redox reactions. Also, see [12–18]. In [15], Chung et al. investigated mathematical structures of chemical reactions for three consecutive oxidation states of elements. In [12], the authors presented three different examples of weak hyperstructures associated to electrochemical cells. Moreover, Chun in [13] presented chemical hyperstructures of chemical reactions for a set of Titanium.

In this paper, we consider the redox reactions: Astatine and Tellurium and Bismuth as algebraic hyperstructures and it is organized as follows: After an Introduction, Section 2 presents some definitions and concepts related to (weak) hyperstructures that are used throughout the paper. Section 3 presents a new example of chemical hyperstructures (and non-isomorphic to previous examples). More precisely, it considers redox reactions of Astatine as algebraic hyperstructures. Finally, Sections 3 and 4 consider redox reactions of Tellurium and Bismuth as algebraic hyperstructures.

2 Preliminaries

In this section, we present some definitions and concepts related to (weak) hyperstructures that are used throughout the paper.

Definition 1. *Let H be a non-empty set. Then, a mapping $\circ : H \times H \rightarrow \mathcal{P}^*(H)$ is called a binary hyperoperation on H , where $\mathcal{P}^*(H)$ is the family of all non-empty subsets of H . The couple (H, \circ) is called a hypergroupoid.*

In the above definition, if A and B are two non-empty subsets of H and $x \in H$, then we define:

$$A \circ B = \bigcup_{\substack{a \in A \\ b \in B}} a \circ b, \quad x \circ A = \{x\} \circ A \text{ and } A \circ x = A \circ \{x\}.$$

H_v -structures were introduced by T. Vougiouklis as a generalization of the well-known algebraic hyperstructures [2, 19], also see [8, 20]. Some axioms of classical algebraic hyperstructures are replaced by their corresponding weak axioms in H_v -structures. Most of H_v -structures are used in representation theory.

A hypergroupoid (H, \circ) is called H_v -semigroup if $(x \circ (y \circ z)) \cap ((x \circ y) \circ z) \neq \emptyset$ for all $x, y, z \in H$. A subset K of an H_v -semigroup is an H_v -subsemigroup if K is an H_v -semigroup. An element $x \in H$ is called *idempotent* if $x^2 = x \circ x = x$ and an element $e \in H$ is called an *identity* of (H, \circ) if $x \in x \circ e \cap e \circ x$, for all $x \in H$. The hypergroupoid (H, \circ) is said to be *commutative* if $x \circ y = y \circ x$, for all $x, y \in H$. A hypergroupoid (H, \circ) is called a *semihypergroup* if for every $x, y, z \in H$, we have $x \circ (y \circ z) = (x \circ y) \circ z$ and is called a *quasihypergroup* if for every $x \in H$, $x \circ H = H = H \circ x$. This condition is called the reproduction axiom. The couple (H, \circ) is called a *hypergroup* if it is a semihypergroup and a quasihypergroup. Two hypergroupoids (G, \circ) and (H, \star) are said to be *isomorphic* if there exists a bijective function $f : G \rightarrow H$ such that for all $x, y \in G$, we have $f(x \circ y) = f(x) \star f(y)$.

3 Redox reactions of Astatine as algebraic hyperstructures

Inspired by the work done by Chun and Davvaz et al. in [13, 16], we apply their methods to Astatine.

Astatine, denoted as At, is a highly radioactive element and it is the heaviest known halogen. The Latimer diagram of Astatine in basic solution is given as follows [21]:

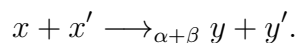


Astatine has four different oxidation states: +5, +3, 0 and -1. We denote AtO_3^- by At^{5+} and AtO^- by At^+ . We compute the potential differences of the following reduction:

- (1) $At^{5+} \longrightarrow At_2, E = \frac{0.5(4)+0(1)}{5} = 0.4,$
- (2) $At^{5+} \longrightarrow At^-, E = \frac{0.5(4)+0(1)+0.2(1)}{6} = 0.367,$

$$(3) \quad At^+ \longrightarrow At^-, E = \frac{0(1)+0.2(1)}{2} = 0.1.$$

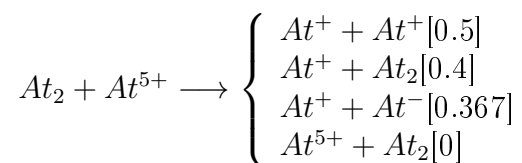
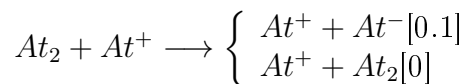
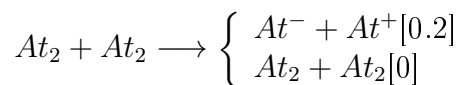
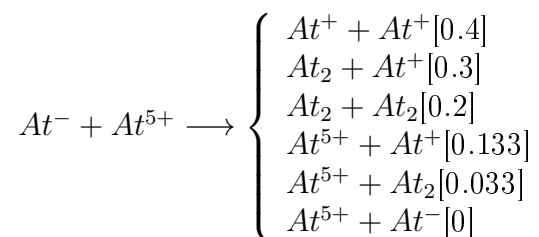
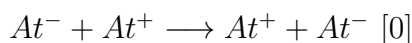
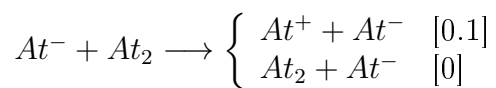
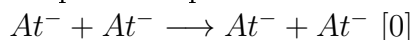
Let $\{x, x', y, y'\} \subseteq \{At^{5+}, At^+, At_2, At^-\}$ such that $x \longrightarrow_{\alpha} x'$ and $y \longrightarrow_{\beta} y'$ where α, β are potential differences. We get the following redox reaction:



If $E = \alpha + \beta > 0$ then our redox reaction is spontaneous. Otherwise, it is not spontaneous.

Remark 1. The redox reaction $At^+ + At^+ \longrightarrow At^{5+} + At_2$ is not spontaneous as $E = -0.5 < 0$.

We consider the set $G = \{At^{5+}, At^+, At_2, At^-\}$ of Astatine. The following are all possible spontaneous redox combinations for G .



$$At^+ + At^+ \longrightarrow At^+ + At^+ [0]$$

$$At^+ + At^{5+} \longrightarrow At^{5+} + At^+ [0]$$

$$At^{5+} + At^{5+} \longrightarrow At^{5+} + At^{5+} [0].$$

Let $G = \{At^-, At_2, At^+, At^{5+}\}$ and define " \oplus " on G as follows: For all $x, y \in G$, $x \oplus y = z$, where z is in the product of the spontaneous redox reaction with greatest potential difference that occurs between x and y . Then we obtain the following table for (G, \oplus) .

\oplus	At^-	At_2	At^+	At^{5+}
At^-	At^-	$\{At^-, At^+\}$	$\{At^-, At^+\}$	At^+
At_2		$\{At^-, At^+\}$	$\{At^-, At^+\}$	At^+
At^+			At^+	$\{At^+, At^{5+}\}$
At^{5+}				At^{5+}

By setting $a = At^-$, $b = At_2$, $c = At^+$, $d = At^{5+}$, we get the following table.

\oplus	a	b	c	d
a	a	$\{a, c\}$	$\{a, c\}$	c
b	$\{a, c\}$	$\{a, c\}$	$\{a, c\}$	c
c	$\{a, c\}$	$\{a, c\}$	c	$\{c, d\}$
d	c	c	$\{c, d\}$	d

Remark 2. (G, \oplus) is not a quasi-hypergroup.

Theorem 1. (G, \oplus) is a commutative H_v -semigroup.

Proof. Let $x, y, z \in G$. We consider the following cases:

- Case $(x, y, z) = (a, a, a)$. We have $x \oplus (y \oplus z) = (x \oplus y) \oplus z = \{a\}$.
- Case $(x, y, z) = (d, d, d)$. $x \oplus (y \oplus z) = (x \oplus y) \oplus z = \{d\}$.
- Case $(x, y, z) \neq (a, a, a)$ and $(x, y, z) \neq (d, d, d)$. It is easy to see that $c \in (x \oplus (y \oplus z)) \cap ((x \oplus y) \oplus z)$. \square

Remark 3. (G, \oplus) is not a semihypergroup. This is clear as $\{c\} = b \oplus (d \oplus d) \neq (b \oplus d) \oplus d = \{c, d\}$.

Proposition 1. (G, \oplus) has two H_v -subsemigroups up to isomorphisms.

Proof. One can easily see that $(\{a, c\}, \oplus)$, $(\{c, d\}, \oplus)$ and $(\{a, c, d\}, \oplus)$ are the only H_v -subsemigroups of G . Moreover, $(\{a, c\}, \oplus)$ and $(\{c, d\}, \oplus)$ are isomorphic hypergroups. \square

Remark 4. From the table of (G, \oplus) , we deduce that At^+ is most likely to occur in nature.

Let $G = \{At^-, At_2, At^+, At^{5+}\}$ and define " \otimes " on G as follows:

For all $x, y \in G$, $x \otimes y = z$, where z is in the product of any spontaneous redox reaction that occurs between x and y . Then we obtain the following table for (G, \otimes) .

\otimes	a	b	c	d
a	a	$\{a, b, c\}$	$\{a, c\}$	G
b	$\{a, b, c\}$	$\{a, b, c\}$	$\{a, b, c\}$	G
c	$\{a, c\}$	$\{a, b, c\}$	c	$\{c, d\}$
d	G	G	$\{c, d\}$	d

Proposition 2. (G, \otimes) is a commutative quasi-hypergroup with identity.

Proof. Since all elements of G are present in every row and column of the table corresponding to (G, \otimes) , it follows that (G, \otimes) is a quasi-hypergroup. Moreover, every element in G is an identity. \square

Proposition 3. $(\{a, c\}, \otimes)$ and $(\{c, d\}, \otimes)$ are hypergroups.

Proof. The proof results from having $(\{a, c\}, \otimes)$ and $(\{c, d\}, \otimes)$ the form of the Biset hypergroup. \square

Proposition 4. (G, \otimes) is a semihypergroup.

Proof. Since $(\{a, c\}, \otimes)$ and $(\{c, d\}, \otimes)$ are hypergroups, it suffices to consider the following cases to prove associativity:

- $x \otimes (y \otimes z) = (x \otimes y) \otimes z$ for $x \in \{a, c\}$ and $\{y, z\} \not\subseteq \{a, c\}$;
- $b \otimes (y \otimes z) = (b \otimes y) \otimes z$ for $\{y, z\} \not\subseteq G$;
- $d \otimes (y \otimes z) = (d \otimes y) \otimes z$ for $\{y, z\} \not\subseteq \{c, d\}$.
- $a \otimes (b \otimes z) = \begin{cases} G, & z = d; \\ \{a, b, c\}, & \text{otherwise.} \end{cases} = (a \otimes b) \otimes z.$
- $a \otimes (d \otimes z) = (a \otimes d) \otimes z = G.$
- $a \otimes (y \otimes b) = \begin{cases} G, & y = d; \\ \{a, b, c\}, & \text{otherwise.} \end{cases} = (a \otimes y) \otimes b.$
- $c \otimes (b \otimes z) = \begin{cases} G, & z = d; \\ \{a, b, c\}, & \text{otherwise.} \end{cases} = (c \otimes b) \otimes z.$
- $c \otimes (y \otimes b) = \begin{cases} G, & y = d; \\ \{a, b, c\}, & \text{otherwise.} \end{cases} = (c \otimes y) \otimes b.$
- $c \otimes (a \otimes d) = (c \otimes a) \otimes d = G$ and $c \otimes (d \otimes a) = (c \otimes d) \otimes a = G.$
- $b \otimes (y \otimes z) = \begin{cases} G, & \text{if } y = d \text{ or } z = d; \\ \{a, b, c\}, & \text{otherwise.} \end{cases} = (b \otimes y) \otimes z.$
- $d \otimes (y \otimes z) = \begin{cases} d, & y = z = d; \\ \{c, d\}, & y = z = c \text{ or } y = c, z = d \text{ or } y = d, z = c; \\ G, & \text{otherwise.} \end{cases} = (d \otimes y) \otimes z.$

\square

Theorem 2. (G, \otimes) is a commutative hypergroup.

Proof. The proof results from Propositions 2 and 4. \square

We present now some chemical interpretation for our results.

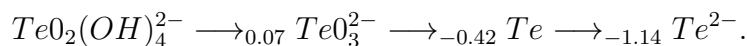
Remark 5. From the table of (G, \otimes) , we deduce that At^+ is most likely to occur in nature and At^{5+} is least likely to occur in nature.

Remark 6. (G, \otimes) is not isomorphic to that presented by Chun and Davvaz in [13, 16] as our hypergroup contains three idempotents only whereas their hypergroups contain four idempotents. This may be due to the differences in the Latimer diagrams of each where in the Latimer diagram of Titanium, the potential difference decreases as the oxidation number decreases whereas in the Latimer diagram of Astatine, the potential difference decreases and then increases.

Remark 7. Having each element x in (G, \otimes) an identity implies that $y \in G$ is present in the product of at least one of spontaneous reactions that that occurs between x and y .

4 Redox reactions of Tellurium as algebraic hyperstructures

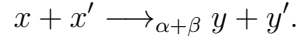
Tellurium, denoted as Te, is a brittle, mildly toxic, rare, silver-white metalloid and it is chemically related to selenium and sulfur. It is occasionally found in native form as elemental crystals. Tellurium is far more common in the universe as a whole than on Earth. The Latimer diagram of Tellurium in basic solution is given as follows [21]:



Tellurium has four different oxidation states: +6, +4, 0 and -2. We denote $TeO_2(OH)_4^{2-}$ by Te^{6+} and TeO_3^{2-} by Te^{4+} . We compute the potential differences of the following reduction:

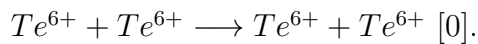
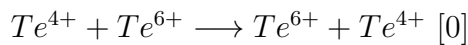
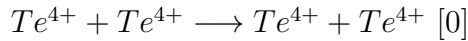
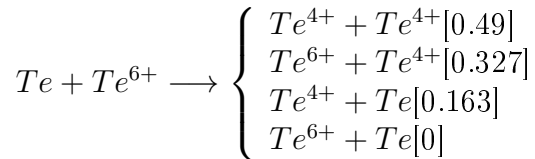
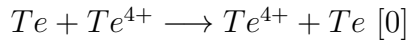
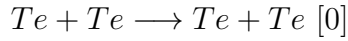
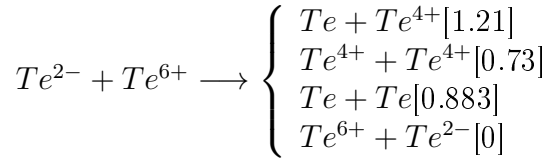
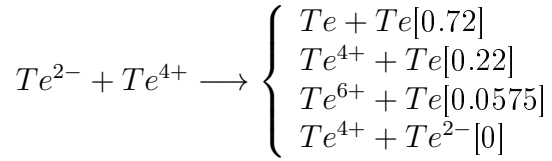
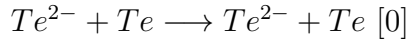
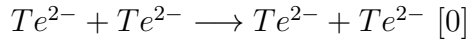
- (1) $Te^{6+} \longrightarrow Te, E = \frac{0.07(2) - 0.42(4)}{6} = -0.257,$
- (2) $Te^{6+} \longrightarrow Te^{2-}, E = \frac{0.07(2) - 0.42(4) - 1.14(2)}{8} = -0.4775,$
- (3) $Te^{4+} \longrightarrow Te^{2-}, E = \frac{-0.42(4) - 1.14(2)}{6} = -0.66.$

Let $\{x, x', y, y'\} \subseteq \{Te^{6+}, Te^{4+}, Te, Te^{2-}\}$ such that $x \xrightarrow{\alpha} x'$ and $y \xrightarrow{\beta} y'$ where α, β are potential differences. We get the following redox reaction:



If $E = \alpha + \beta > 0$ then our redox reaction is spontaneous. Otherwise, it is not spontaneous.

We consider the set $H = \{Te^{6+}, Te^{4+}, Te, Te^{2-}\}$ of Tellurium. The following are all possible spontaneous redox combinations for H .



For all $x, y \in H$, $x \oplus_1 y = z$, where z is in the product of the spontaneous redox reaction with greatest potential difference that occurs between x and y .

Then we obtain the following table for (H, \oplus_1) .

\oplus_1	Te^{2-}	Te	Te^{4+}	Te^{6+}
Te^{2-}	Te^{2-}	$\{Te^{2-}, Te\}$	Te	$\{Te, Te^{4+}\}$
Te		Te	$\{Te, Te^{4+}\}$	Te^{4+}
Te^{4+}			Te^{4+}	$\{Te^{4+}, Te^{6+}\}$
Te^{6+}				Te^{6+}

By setting $a = Te^{2-}$, $b = Te$, $c = Te^{4+}$, $d = Te^{6+}$, we get the following table.

\oplus_1	a	b	c	d
a	a	$\{a, b\}$	b	$\{b, c\}$
b	$\{a, b\}$	b	$\{b, c\}$	c
c	b	$\{b, c\}$	c	$\{c, d\}$
d	$\{b, c\}$	c	$\{c, d\}$	d

Theorem 3. (H, \oplus_1) is a commutative H_v -semigroup.

Proof. (H, \oplus_1) is isomorphic to the H_v -semigroup of Titanium presented in Theorem 3.2, [13]. \square

Let $H = \{Te^{6+}, Te^{4+}, Te, Te^{2-}\}$ and define " \otimes_1 " on H as follows:

For all $x, y \in H$, $x \otimes_1 y = z$, where z is in the product of the any spontaneous redox reaction that occurs between x and y .

Then we obtain the following table for (H, \otimes_1) .

\otimes_1	a	b	c	d
a	a	$\{a, b\}$	H	H
b	$\{a, b\}$	b	$\{b, c\}$	$\{b, c, d\}$
c	H	$\{b, c\}$	c	$\{c, d\}$
d	H	$\{b, c, d\}$	$\{c, d\}$	d

Theorem 4. (H, \otimes_1) is a commutative hypergroup.

Proof. (H, \otimes_1) is isomorphic to the hypergroup of Titanium presented in Theorem 3.4, [13]. \square

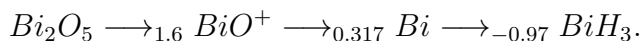
Next, we present some chemical interpretation for our results.

Remark 8. From the table of (H, \otimes_1) , we deduce that Te and Te^{4+} are most likely to occur in nature and Te^{2-} is least likely to occur in nature.

Remark 9. The hyperstructures of Astatine are not isomorphic to that of Tellurium. This may be due the differences in the Latimer diagram of each. In the Latimer diagram of Tellurium, the potential difference decreases as the oxidation number decreases whereas in the Latimer diagram of Astatine, the potential difference decreases and the increases.

5 Redox reactions of Bismuth as algebraic hyperstructures

Bismuth, denoted by Bi, is a pentavalent post-transition metal and one of the pnictogens, chemically resembles its lighter homologs arsenic and antimony. Elemental bismuth may occur naturally, although its sulfide and oxide form important commercial ores. It is a brittle metal with a silvery white color when freshly produced, but surface oxidation can give it a pink tinge. Bismuth is slightly radioactive and the most naturally diamagnetic element, and has one of the lowest values of thermal conductivity among metals. The Latimer diagram of Bismuth in acidic solution is given as follows [21]:



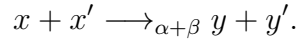
Bismuth has four different oxidation states: $+5, +3, 0$ and -3 . We denote Bi_2O_5 by Bi^{5+} , BiO^+ by Bi^{3+} and BiH_3 by Bi^{3-} . We compute the potential differences of the following:

$$(1) Bi^{5+} \longrightarrow Bi, E = \frac{1.6(2)+0.317(3)}{5} = 0.8302,$$

$$(2) Bi^{5+} \longrightarrow Bi^{3-}, E = \frac{1.6(2)+0.317(3)-0.97(3)}{8} = 0.155125,$$

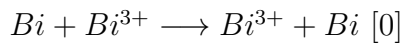
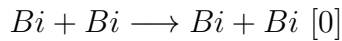
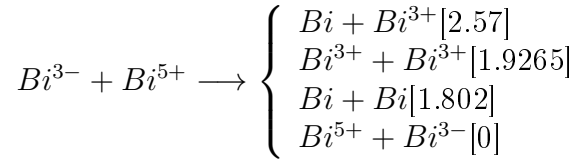
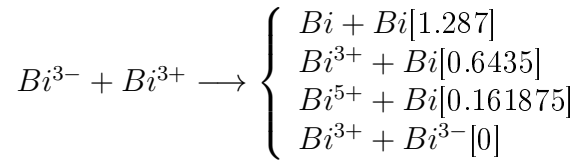
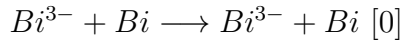
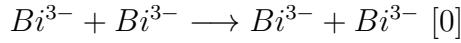
$$(3) Bi^{3+} \longrightarrow Bi^{3-}, E = \frac{0.317(3)-0.97(3)}{6} = -0.3265.$$

Let $\{x, x', y, y'\} \subseteq \{Bi^{5+}, Bi^{3+}, Bi, Bi^{2-}\}$ such that $x \xrightarrow{\alpha} x'$ and $y \xrightarrow{\beta} y'$ where α, β are potential differences. We get the following redox reaction:



If $E = \alpha + \beta > 0$ then our redox reaction is spontaneous. Otherwise, it is not spontaneous.

We consider the set $K = \{Bi^{5+}, Bi^{3+}, Bi, Bi^{2-}\}$ of Bismuth. The following are all possible spontaneous redox combinations for K .



$$Bi + Bi^{5+} \longrightarrow \begin{cases} Bi^{3+} + Bi^{3+}[1.283] \\ Bi^{5+} + Bi^{3+}[0.768] \\ Bi^{3+} + Bi[0.515] \\ Bi^{5+} + Bi[0] \end{cases}$$

$$Bi^{3+} + Bi^{3+} \longrightarrow Bi^{3+} + Bi^{3+} [0]$$

$$Bi^{3+} + Bi^{5+} \longrightarrow Bi^{5+} + Bi^{3+} [0]$$

$$Bi^{5+} + Bi^{5+} \longrightarrow Bi^{5+} + Bi^{5+} [0]$$

For all $x, y \in K$, $x \oplus_2 y = z$, where z is in the product of the spontaneous redox reaction with greatest potential difference that occurs between x and y .

Then we obtain the following table for (K, \oplus_2) .

\oplus_2	Bi^{3-}	Bi	Bi^{3+}	Bi^{5+}
Bi^{3-}	Bi^{3-}	$\{Bi^{3-}, Bi\}$	Bi	$\{Bi, Bi^{3+}\}$
Bi		Bi	$\{Bi, Bi^{3+}\}$	Bi^{3+}
Bi^{3+}			Bi^{3+}	$\{Bi^{3+}, Bi^{5+}\}$
Bi^{5+}				Bi^{5+}

By setting $a = Bi^{3-}$, $b = Bi$, $c = Bi^{3+}$, $d = Bi^{5+}$, we get the following table.

\oplus_2	a	b	c	d
a	a	$\{a, b\}$	b	$\{b, c\}$
b	$\{a, b\}$	b	$\{b, c\}$	c
c	b	$\{b, c\}$	c	$\{c, d\}$
d	$\{b, c\}$	c	$\{c, d\}$	d

Theorem 5. (K, \oplus_2) is a commutative H_v -semigroup.

Proof. The proof follows from Theorem 3 and having (K, \oplus_2) isomorphic to (H, \oplus_1) . \square

Let $K = \{Bi^{5+}, Bi^{3+}, Bi, Bi^{3-}\}$ and define " \otimes_2 " on K as follows:
For all $x, y \in H$, $x \otimes_2 y = z$, where z is in the product of the any spontaneous redox reaction that occurs between x and y .
Then we obtain the following table for (K, \otimes_2) .

\otimes_2	a	b	c	d
a	a	$\{a, b\}$	K	K
b	$\{a, b\}$	b	$\{b, c\}$	$\{b, c, d\}$
c	K	$\{b, c\}$	c	$\{c, d\}$
d	K	$\{b, c, d\}$	$\{c, d\}$	d

Theorem 6. (K, \otimes_2) is a commutative hypergroup.

Proof. The proof follows from Theorem 4 and having (K, \otimes_2) isomorphic to (H, \otimes_1) . \square

Remark 10. The hyperstructures of Astatine are not isomorphic to that of Bismuth.

We present now some chemical interpretation for our results.

Remark 11. From the table of (K, \otimes_2) , we deduce that Bi and Bi^{3+} are most likely to occur in nature and Bi^{3-} is least likely to occur in nature.

Remark 12. The hyperstructures of Bismuth are isomorphic to that of Tellurium and Titanium. This may be due the similarity in the Latimer diagram of each where the potential difference in the Latimer diagrams of each decreases as the oxidation number decreases.

6 Conclusion

This paper dealt with two non-isomorphic hyperstructures of three different elements with four different oxidation states. For future work, it will be interesting to generalize our work and investigate all algebraic hyperstructures for chemical reactions for four consecutive oxidation states of an arbitrary element.

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