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Optimizing the ecological connectivity of landscapes with generalized flow models and preprocessing [☆]

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Abstract

In this paper we consider the problem of optimizing the ecological connectivity of a landscape under a budget constraint by improving habitat areas and ecological corridors between them. We consider a formulation of this problem in terms of graphs in which vertices represent the habitat areas and arcs represent a probability of connection between two areas that depend on the quality of the respective corridor. We propose a new generalized flow model that improves existing models for this problem and an efficient preprocessing algorithm that reduces the size of the graphs on which generalized flows is computed. Reported numerical experiments highlight the benefice of these two contributions on computation times and show that larger problems can be solved using them. Our experiments also show that several variants of greedy algorithms perform relatively well on practical instances while they return arbitrary bad solutions in the worst case.

Keywords: Environment and climate change, landscape connectivity, generalized network flow, robust shortest path, linear programming, distances in graphs.

1. Introduction

Habitat loss is a major cause of the rapid decline of biodiversity [5]. Further than reducing the available resources, it also increases the discontinuities among small habitat areas called *patches*, this is a phenomenon known under the name *habitat fragmentation* [9]. While habitat loss tends to reduce the size of populations of animals and plants, which may be deleterious to their survival in the long term, habitat fragmentation also makes it harder for organisms to move around in landscapes. This constrains their access to resources and reduces gene flow among populations, which can also be deleterious for their persistence

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in the long term. *Landscape connectivity*, defined as the degree to which the landscape facilitates the movement of organisms between habitat patches [19], then becomes of major importance for biodiversity and its conservation. Accounting for landscape connectivity in restoration or conservation plans thus appears as a key solution to maximize the return on investment of the scarce financial support devoted to biodiversity conservation. For this purpose, graph-theoretical approaches are useful in modelling habitat connectivity [20]. In this paper, we consider the *landscape connectivity* that is distinct from the graph theoretical notion of *connectivity* defined in terms of minimum cut.

A landscape can be viewed as a directed graph in which vertices represent the habitat patches and each arc represents a way for individuals to travel from one patch to another. Each vertex has a weight that represents the quality of the patch – patch area is often used as a surrogate for quality – and each arc has a weight that represents the difficulty for an individual to make the corresponding travel – often approximated by the border to border distance between the patches. Interestingly, this approach can be used for a variety of ecological systems like terrestrial (patches of forests in an agricultural area, networks of lakes or wetlands), riverine (patches are segments of river than can be separated by human constructions like dams that prevent fishes’ movement) or marine (patches can be reefs that are connected by flows of larvae transported by currents). With this formalism, ecologists have developed many connectivity indicators [11, 12, 17] that aim to quantify the quality of a landscape with respect to the connections between its habitat patches. Among the proposed indicators, the probability of connection – PC – [17] and its derivative the equivalent connected area – ECA – [16] seem to perform well; they have received encouraging empirical support [3, 14, 18]. Definitions and relationship between these two indicators are described in Section 2. In this paper, our purpose is not to discuss the merit of ECA with respect to other landscape connectivity indicators but to study the corresponding combinatorial optimization problem from an algorithmic point of view. One key question for which landscape connectivity indicators have been used in the last years is to identify which elements of the landscape (habitat patches, corridors) should be preserved from destruction or restored in order to maintain a well-connected network of habitat under a given budget constraint [2]. This means, identifying the set of vertices or arcs that optimally maintain a good level of connectivity. Many mathematical programming models have been introduced in the literature to help decision-makers to protect biodiversity, for a deep and wide review of these models, we refer to the survey paper [4] and the references

therein.

Here, we will focus on this problem by measuring ecological connectivity with ECA. The budget-constrained ECA optimization problem consists in finding the best combination of graph elements (arcs or vertices) among a set Φ of elements that could be restored in order to maximize the ECA in the landscape within a limited budget. Interestingly, this problem addresses both restoration and conservation cases. The restoration case starts from the current landscape and aims to restore a set of elements among the different feasible options. The conservation case starts with a landscape in which we search for the elements to protect among the threatened ones, and before the unprotected ones get degraded. The conservation problem and the restoration problem are equivalent from an algorithmic point of view. Indeed, an instance of the conservation problem can be viewed as an instance of the restoration problem in which the initial landscape is the landscape in which all threatened elements are degraded and the set Φ of elements that can be restored in the instance of the restoration problem is the set of elements that can be protected in the instance of the conservation problem. Until now, ecologists have mostly tackled this problem by ranking each conservation or restoration option by its independent contribution to ECA, i.e. the amount by which ECA varies if the option is purchased alone. Such an approach overlooks the cumulative effects of the decisions made like unnecessary redundancies or potential synergistic effects, e.g. consecutive arcs. This could lead to solutions that are more expensive or less beneficial to ECA than an optimal solution. Some studies have tried to overcome this limitation by considering tuples of options [13, 15]. In [15], the authors show that the brute force approach rapidly becomes impractical for landscape with more than 20 patches of habitat. However, few studies explore the search for an optimal solution, most of which being developed for river dendritic networks. For instance, in [21], a polynomial time approximation scheme is described for the problem where the landscape is represented by a tree graph. Recently a mixed integer formulation that is applicable to our problem has been introduced [22]. This formulation is however expensive to solve. It uses $O(|V|^2|A|)$ flow variables and $O(|V|^3)$ constraints while $O(|\Phi| + |V|^2)$ integer variables are required to approximate the non-linear objective function. The authors of [22] show that solving exactly this formulation does not scale to landscapes of few hundreds of patches. Their main contribution was to propose a XOR sampling method for approximating it. Since it is possible to use the XOR sampling method with our exact and more compact MIP model, it is better to view our contributions as a complement

to those of [22]. Therefore, we did not perform numerical experiments to compare our methodology against those developed in [22].

Our model is based on a generalized flow formulation (see for instance [1]) instead of a standard network flow formulation, which has two main advantages. On one side, our formulation has a linear objective function. We do not need to use a piecewise constant approximation nor to introduce additional binary variables to handle non-linearity. On the other side, the new formulation is much more compact because it aggregates into a single generalized flow the contribution to the connectivity of several source/sink pairs having the same source. This reduces significantly the number of variables and constraints of the model. It uses $O(|V||A|)$ flow variables with $O(|V|^2)$ constraints, and $|\Phi|$ integer variables instead of $O(|V|^2|A|)$ flow variables with $O(|V|^3)$ constraints, and $O(|\Phi| + |V|^2)$ integer variables in the model of [22]. Recall that Φ is the set of elements that are threatened or could be restored. Another contribution of this paper is to propose a preprocessing step that speeds-up the resolution of the problem by reducing the size of the directed graph on which a generalized flow to a particular sink t have to be computed. The idea is to identify, for each sink t , the set of arcs (u, v) that are always on a shortest path from u to t for all the $2^{|\Phi|}$ possible restoration plans. Such arcs, called t -strong arcs, are always carrying flow and can be contracted in the generalized flow formulation to t . Conversely, an arc (u, v) that is never on a shortest path from u to t , called t -useless, can be removed from the generalized flow formulation to t because it never carries any flow. Once t -strong arcs have been contracted and t -useless arcs have been removed in each generalized flow formulation to a particular sink t , it remains to solve the problem on a much smaller instance containing only the arcs that are not t -strong nor t -useless. Obviously it would be prohibitive in terms of computation times to solve a shortest path problem for each of the $2^{|\Phi|}$ possible restoration plans in order to determine whether a given arc is t -strong (resp. t -useless) or not. In Section 4, we design and analyze a very efficient algorithm, that computes directly the set of all sinks t such that a given arc is t -strong. This algorithm takes implicitly into consideration all $2^{|\Phi|}$ possible restoration plans but has the same complexity as the Dijkstra shortest path algorithm. It improves previous result for the problem of computing t -strong or t -useless arcs [6]. We anticipate that this algorithmic result is of independent interest and could be useful in many other contexts, in particular when shortest path computations are embedded in robust optimization models.

In Section 2 we give more details about the landscape modelization and describe for-

mally the problem. Section 3 is devoted to the description of our mixed integer formulation of the problem. In Section 4 we first explain why the removal of t -useless arcs and the contraction of t -strong arcs does not modify the objective function of any restoration/conservation plan. Then, we design and analyze a $O(|A| + |V| \log |V|)$ algorithm that computes the set of all vertices t for which a given arc (u, v) is t -strong or the set of all vertices t for which (u, v) is t -useless. Section 5 presents experimental cases for which we compare our optimization approach with simple greedy algorithms in terms of running times and quality of the solutions found. These experiments also shows that the preprocessing is very effective on some real instances.

2. Budget Constrained ECA Optimization problem

Our optimization problem takes as input a landscape represented by a directed graph $G = (V, A)$. Each patch $s \in V$ has a weight w_s that represents its size. Each ordered pair of patches (s, t) such that an individual can move from patch s to patch t without crossing another patch is joined by an arc of G . For each arc (u, v) , we suppose that an estimate of the probability $\pi_{(u,v)}$ that an individual successfully moves from u to v . This value will be determined by the landscape and by the species studied. The input of our optimization problem also includes a list of arcs that can be improved to make them easier or less dangerous to cross and the costs of these improvements. As objective function and measure of the quality of the landscape, we use the Equivalent Connected Area defined by the following formula [16]:

$$ECA(G, w, l) = \sqrt{\sum_{s \in V} \sum_{t \in V} (w_s w_t \Pi_{st})}$$

where Π_{st} is the *probability of connection* from the patch s to the patch t . This probability is defined as the probability of the most probable (reliable) path from s to t where the probability of a path is the product of the probabilities of its arcs. The probability of an arc a is often defined as $\pi_a := \exp(-\alpha l_a)$ where α is a parameter that depends on the dispersal ability of the species and l_a is the euclidean distance between the centers of the two patches joined by a . This dispersal model has been used in many applications. Its ecological justification, discussed for instance in [7, 8], is out of the scope of this paper. By monotonicity of the exponential function, a path P is a most reliable path between s and t if and only if it is a shortest path with respect to arc lengths l and $\Pi_{st} = \exp(-\alpha d(s, t))$ where $d(s, t)$ is the length of a shortest st -path with respect to l . The equivalent connected

area of a landscape is the area of a single patch equivalent to it. This equivalence comes from the indicator PC on which ECA is based [16]. Indeed, suppose that the landscape is a rectangle of area \mathcal{W} containing all patches. We consider a stochastic process consisting of choosing two points p and q uniformly at random in the rectangle. The indicator PC is the expected value of a random variable equal to 0 if either p or q does not belong to a patch and Π_{st} if p belongs to s and q belongs to t (recall that $\Pi_{st} = 1$ if $s = t$). Since the probability that p belongs to a patch u is w_u/\mathcal{W} , and the events $p \in s$ and $q \in t$ are independent, by linearity of expectation, PC can be expressed as follows:

$$PC(G, w, l) = \frac{\sum_{s,t \in V} w_s w_t \Pi_{st}}{\mathcal{W}^2} = \frac{ECA(G, w, l)^2}{\mathcal{W}^2}$$

where \mathcal{W} , w_s and w_t are respectively the area of the rectangle, patch s and patch t . The equivalent connected area of a landscape is the area of a single patch whose PC value is equal the PC value of the original landscape. If the area of the patches and the landscape are normalized to make \mathcal{W} equal to 1 then PC is the square of ECA. Therefore, optimizing PC and optimizing ECA are equivalent problems. ECA is often considered by researchers interested in landscape connectivity because it represents an area, a more concrete quantity than the expected value of a random variable.

An instance $I = (V, A, \Phi, w, l, l^-, c, B)$ of the *Budget Constrained ECA Optimization Problems – BC-ECA-Opt* – is defined by a graph $G = (V, A)$ with a weight w_u for each vertex $u \in V$ and a length l_a for each arc $a \in A$, a subset $\Phi \subseteq A$ of arcs that can be improved, for each arc $a \in \Phi$, the cost c_a of reducing its length to l_a^- and the total budget B for improving the arcs. An optimal solution of the instance I is a subset of arcs $S \subseteq \Phi$ of total cost $c(S) := \sum_{a \in S} c_a$ at most B that maximizes $ECA(G, l', w)$ where $l'_a = l_a^-$ if $a \in S$ and l_a otherwise. For the sake of clarity, we first consider only arcs improvements, but, as explained in Section 3.1, our model can be extended to the case where it is also possible to improve the landscape by increasing the weight of the vertices. In this case, a subset of vertices W is given, as well as the cost c_u of increasing the weight of u to w_u^+ for each vertex $u \in W$, and we want to find a subset of arcs $S \subseteq \Phi$ and a subset of vertices $T \subseteq W$ of total cost $c(S) + c(T) \leq B$ that maximizes $ECA(G, l', w')$ where $l'_a = l_a^-$ if $a \in S$ and l_a otherwise and $w'_u = w_u^+$ if $u \in T$ and w_u otherwise.

Remark 1. Our optimization problem is stated in terms of arc length reduction but it would be equivalent to state it in terms of arc probability augmentation.

Remark 2. In our model, it is possible to add a new connection between two existing patches u and v . It suffices to place (u, v) in the set of improvable arcs Φ with an initial length $l_{(u,v)} = \infty$ (i.e. $\pi_{(u,v)} = 0$) and a finite improved length $l_{(u,v)}^- < \infty$.

3. An improved MIP formulation for BC-ECA-Opt

We first show how to compute $f^t = \sum_{s \in V} w_s \cdot \Pi_{st}$ as the maximum quantity of generalized flow that can be sent to t across the network if, for every patch $s \in V$, w_s units of flow are available at s and appropriate multipliers are chosen for each arc of the network. Recall that a generalized flow differs from a standard flow by the fact that each arc a has a multiplier π_a such that the quantity of flow leaving arc a is equal to the quantity of flow ϕ_a entering in a multiplied by π_a . In our case $\pi_a = \exp(-\alpha l_a)$. For each vertex $u \in V$, let δ_u^{out} be the set of arcs leaving u and δ_u^{in} the set of arcs entering u . As explained below, the linear program (\mathcal{P}) has f^t as optimum value.

$$(\mathcal{P}) \left\{ \begin{array}{ll} \max & z \\ \text{s.t.} & \sum_{a \in \delta_u^{\text{out}}} \phi_a - \sum_{b \in \Gamma_u^-} \pi_b \cdot \phi_b \leq w_u \quad u \in V \setminus \{t\} \quad (\text{A1}) \\ & \sum_{a \in \delta_t^{\text{out}}} \phi_a - \sum_{b \in \delta_t^{\text{in}}} \pi_b \cdot \phi_b = w_t - z \quad (\text{A2}) \\ & \phi_a \geq 0 \quad a \in A \quad (\text{A3}) \end{array} \right.$$

Constraints (A1) require that the total quantity of flow leaving u is at most w_u plus the total quantity of flow entering u , i.e. the quantity of flow available in vertex u . Constraint (A2) requires that z is equal to the total quantity of flow entering t plus w_t minus the total quantity of flow leaving t . Finally, constraints (A3) state that each arc carries a positive quantity of flow from its source to its sink.

Lemma 1. *Any optimal solution of (\mathcal{P}) is obtained by sending, for each every vertex $s \in V - \{t\}$, w_s units of flow from s to t along most reliable st -paths.*

Proof. First notice that the quantity of flow arriving at t when w_s units flow are sent from s to t along an st -path P is w_s times the probability of path P . Let ϕ' be an optimal solution of (\mathcal{P}) maximizing the quantity of flow routed along a path which is not a shortest

path. Suppose by contradiction that ϕ' send $\epsilon > 0$ units of flow along an st -path P' whose probability is smaller than the probability of a most reliable st -path P . Let ϕ be the flow obtained from ϕ' by decreasing the flow sent on path P' by ϵ and increasing the flow sent on path P by ϵ . Since the probability of P is larger than the probability of P' , ϕ send more flow to t than ϕ' , a contradiction. \square

Corollary 1. *The optimal value of (\mathcal{P}) is $f^t = \sum_{s \in V} w_s \cdot \Pi_{st}$.*

Proof. Since the objective is to maximize z , no flow leaves t in any optimal solution of (\mathcal{P}) , i.e. $\sum_{a \in \delta_t^{\text{out}}} \phi_a = 0$. Constraint (A2) ensures that z is the quantity of flow received by t plus $w_t = w_t \cdot \Pi_{tt}$. By Lemma 1, there exists an optimal solution ϕ such that every vertex s distinct from t send w_s units of flow on a most reliable st -path. Hence, for every vertex s distinct from t , the flow received by t from s is $w_s \cdot \Pi_{st}$ and thus the value of z is $\sum_{s \in V} w_s \cdot \Pi_{st}$. \square

With this linear programming definition of f^t , we can now present our MIP program of BC-ECA-Opt. For each arc $a = (u, v) \in \Phi$, we add another arc $a' = (u, v)$ of length $l_{a'} = l_a^-$ that can be viewed as an improved copy of arc a . This improved copy can be used only if the improvement of arc a is purchased. For each arc $a \in \Phi$, that could be improved, the variable x_a is equal to one if the improvement of arc a is purchased and zero otherwise. We denote by Ψ the set of improved copies of arcs in Φ . In the following MIP program, δ_u^{out} and δ_u^{in} are defined with respect to the set of arcs $A' = A \cup \Psi$.

$$\text{BC-ECA-Opt} \left\{ \begin{array}{ll} \max & \sum_{t \in V} w_t \cdot f^t \\ \text{s.t.} & \sum_{a \in \delta_u^{\text{out}}} \phi_a^t - \sum_{b \in \delta_u^{\text{in}}} \pi_b \cdot \phi_b^t \leq w_u \quad t \in V, u \in V \setminus \{t\} \quad (\text{B1}) \\ & \sum_{a \in \delta_t^{\text{out}}} \phi_a^t - \sum_{b \in \delta_t^{\text{in}}} \pi_b \cdot \phi_b^t = w_t - f^t \quad t \in V \quad (\text{B2}) \\ & \phi_{a'}^t \leq x_a \cdot M_a \quad t \in V, a \in \Phi \quad (\text{B3}) \\ & \sum_{a \in F} c_a \cdot x_a \leq B \quad (\text{B4}) \\ & x_a \in \{0, 1\} \quad a \in \Phi \quad (\text{B5}) \\ & \phi_a^t \geq 0 \quad t \in V, a \in \Phi \quad (\text{B6}) \end{array} \right.$$

Constraints of (B1) and (B2) are simply constraints of (A1) and (A2) for all possible target vertex t . For each arc $a \in \Phi$, the big-M constraints (B3) ensure that if the im-

provement of arc a is not purchased, i.e. $x_a = 0$, then the flow on arc a' is null. M_a is an upper bound of the flow value on arc a . We could simply take $M_a = \sum_{u \in V} w_u$ for all $a \in \Phi$ but smaller estimations are important to guaranty a good linear relaxation and thus a faster resolution of the MIP program. Constraint (B4) ensures that the total cost of the improvements is at most B . This MIP program has $O(|V|(|A| + |\Phi|))$ flow variables, $|\Phi|$ binary variables and $O(|V|^2 + |V||\Phi|)$ constraints.

3.1. Extension to patch improvements

To extend our model to the version with patch improvements where it is possible to increase the weight of a vertex u from w_u to w_u^+ at cost c_u , we process all the occurrences of w_u as follows. Let y_u be a binary variable equal to 1 if the vertex u is improved and 0 otherwise. We add a term $y_u c_u$ in the budget constraint for every vertex u in the set W of vertices that can be improved. When w_u appears as an additive constant, we simply replace it by $w_u + y_u(w_u^+ - w_u)$. Note that w_u only appears as a coefficient in the objective function in the form $w_u f_t$. In this case, to avoid a quadratic term, we use a standard McCormick linearization [10]. We replace the product $w_u f_t$ by $w_u f_t + (w_u^+ - w_u) f_t'$ where f_t' is a new variable that is equal to f_t if $y_u = 1$ and 0 otherwise. To achieve this values of f_t' we add the constraints $f_t' \leq f_t$ and $f_t' \leq y_u M$ where M is larger than any values of f_t . As it is a maximization program and f_t' appears with a positive coefficient in the objective function, the first constraint guaranties that $f_t' = f_t$ if $y_u = 1$ in any optimal solution and the second constraint guaranties that $f_t' = 0$ if $y_u = 0$.

4. Preprocessing

The size of the mixed integer programming formulation of BC-ECA-Opt given in Section 3 grows quadratically with the size of the graph that represents the landscape. In this section, we describe a preprocessing step that reduces the size of this graph. For that, we introduce a notion of *strongness* of an arc with respect to a target vertex $t \in V$. We will call a solution $x \in \{0, 1\}^\Phi$ of BC-ECA-Opt a *scenario* and say that the distances are *computed under scenario* x when the length of every $a \in A$ is l_a^- if $x_a = 1$ and l_a otherwise. We denote $d_x(s, t)$ the distance between s and t when the arc lengths are set according to the scenario x . An arc (u, v) is said to be *t-strong* if, for every scenario $x \in \{0, 1\}^\Phi$, (u, v) belongs to a shortest path from u to t , i.e. $d_x(u, t) = d_x(u, v) + d_x(v, t)$ for every $x \in \{0, 1\}^\Phi$. An arc (u, v) is said to be *t-useless* if, for every scenario $x \in \{0, 1\}^\Phi$, arc

(u, v) does not belong to any shortest ut -path when arc lengths are set according to the scenario x . We denote by $S(t)$ the set of arcs (u, v) such that (u, v) is t -strong and by $W(t)$ the set of arcs (u, v) such that (u, v) is t -useless.

Of course, the naive algorithm that tests whether (u, v) belongs to a shortest ut -path for every scenario $x \in \{0, 1\}^\Phi$ is very inefficient. In Section 4.2, we describe and analyse an algorithm that computes the set $S(t)$ and $W(t)$ for every sink t in a much more efficient way. But first, we explain how the knowledge of these sets can be used to define a smaller equivalent instance of the problem.

4.1. Operations to reduce the size of the graph

Removal of an arc from $W(t)$. Let $(u, v) \in W(t)$, since for all scenarios $x \in \{0, 1\}^\Phi$, (u, v) does not belong to any shortest path from u to t , its removal does not affect the distance from any vertex to t . It is clear, from the definition of ECA, that the removal of (u, v) does not affect the contribution of t to ECA. Let $f_t^x(G) := \sum_{s \in V} w_s w_t \Pi_{st}^x$ be the contribution to ECA of all the pairs having sink t in the graph G when the probability of connection Π_{st}^x is computed under the scenario x .

Lemma 2. *Let $(u, v) \in W(t)$ and let G' be a graph obtained from G by removing arc (u, v) then, for all scenario $x \in \{0, 1\}^\Phi$, $f_t^x(G) = f_t^x(G')$.*

Proof. For every scenario $x \in \{0, 1\}^\Phi$, (u, v) does not belong to any shortest path from u to t . Therefore, the removal of (u, v) cannot affect the probability of connection Π_{ut} for any vertex t and $f_t^x(G) = f_t^x(G')$. \square

Contraction of an arc in $S(t)$. Now, assume that $(u, v) \notin \Phi$, $(u, v) \in S(t)$. The contraction of (u, v) consists in replacing every arc $(w, u) \in \delta_u^{\text{in}}$ by an arc (w, v) of length $l'_{wv} = l_{wu} + l_{uv}$ and by removing the vertex u and all its outgoing arcs. The weight of u in G is moved to the weight of v in G' . Namely, the weight of v in the new graph is $w'_v = w_v + w_u \exp(-\alpha l_{uv})$. Let G' be the graph obtained from G by contracting (u, v) . The next lemma establishes that the contribution of t to ECA in G is equal to its contribution in G' .

Lemma 3. *Let $(u, v) \in S(t)$ and let G' be the graph obtained from G by contracting arc (u, v) and modifying accordingly the weight of w_v . For every scenario $x \in \{0, 1\}^\Phi$, $f_t^x(G) = f_t^x(G')$.*

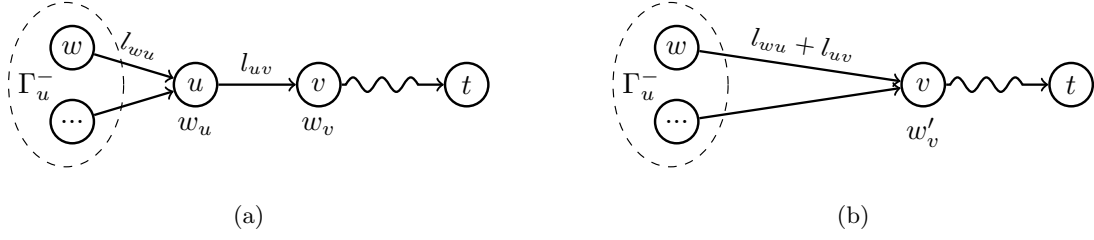


Figure 1: (a) represents a graph G before the contraction of an arc (u, v) . (b) represents the graph G' obtained from G by contracting arc (u, v) . The weight w'_v of v in G' is equal to $w_v + w_u \exp(-\alpha l_{uv})$.

Proof. Let s be a vertex of G and x be a scenario in $\{0, 1\}^\Phi$. We denote $f_{st}^x(G)$ the contribution to ECA of the pair st with scenario $x \in \{0, 1\}^\Phi$ in G . If $s \notin \{u, v\}$, it is easy to check that, for any $x \in \{0, 1\}^\Phi$ the length of the shortest path from s to t in G is equal to the length of the shortest path from s to t in G' . Moreover, the weight of s is the same in G and in G' . Therefore, by the definition of ECA, $f_{st}^x(G) = f_{st}^x(G')$, for any $x \in \{0, 1\}^\Phi$. On the other hand, the contributions of the pairs ut and vt in G sum to the contribution of v in G' . Indeed,

$$\begin{aligned}
f_{ut}^x(G) + f_{vt}^x(G) &= w_u \exp(-\alpha d_x(u, t)) + w_v \exp(-\alpha d_x(v, t)) \\
&= w_u \exp(-\alpha(l_{uv} + d_x(v, t))) + w_v \exp(-\alpha d_x(v, t)) \\
&= (w_v + w_u \exp(-\alpha l_{uv})) \exp(-\alpha d_x(v, t)) \\
&= w'_v \exp(-\alpha d_x(v, t)) \\
&= f_{vt}^x(G')
\end{aligned}$$

We conclude that, for any $x \in \{0, 1\}^\Phi$, $f_t^x(G) = \sum_{s \in V} f_{st}^x(G) = f_{ut}^x(G) + f_{vt}^x(G) + \sum_{s \in V - \{u, v\}} f_{st}^x(G) = f_{vt}^x(G') + \sum_{s \in V - \{u, v\}} f_{st}^x(G') = f_t^x(G')$. \square

Graph reduction. We obtain G_t by deleting every arcs of $W(t)$ and contracting every arcs of $S(t)$. By induction the contribution of t is preserved. The experiments of Section 5 show that, when the number of arcs that can be protected is much smaller than the total number of arcs, replacing G by G_t in the construction of the mixed integer program reduces significantly the size of the MIP formulation and the running times.

The problem of identifying $S(t)$ and $W(t)$ has been studied as a preprocessing step for other combinatorial optimization problems having as input a graph whose arc lengths may change [6]. In [6], given an arc (u, v) and a vertex t , the authors identify a sufficient (but not necessary) condition for $(u, v) \in S(t)$ and use it to design a $O(|A| + |V| \log |V|)$

algorithm that can, in most cases, identify if $(u, v) \in S(t)$. The authors also propose a $O(|A| + |V| \log |V|)$ algorithm to test whether a given arc (u, v) belongs to $W(s)$ or not.

Given an arc (u, v) of G , we will show how to adapt the Dijkstra's algorithm to compute in $O(|A| + |V| \log |V|)$ the set of all vertices t such that $(u, v) \in S(t)$ or the set of all vertices t such that $(u, v) \in W(t)$. This improves the results of [6] by providing a necessary and sufficient condition for (u, v) to be t -strong, i.e. we compute the entire set $S(t)$ while the algorithm of [6] computes a subset of $S(t)$. It also reduces the time complexity for computing $S(t)$ and $W(t)$ for all t by a factor $|V|$ as we only need to run the algorithms on each arc instead of each pair of arc and vertex.

4.2. Computing $S(t)$ for all t

Given a scenario $x \in \{0, 1\}^\Phi$, the fiber $F_x(u, v)$ of arc (u, v) is the set of vertex t such that (u, v) belongs to a shortest path from u to t when arc lengths are set according to x , i.e.

$$F_x(u, v) = \{t \in V : d_x(u, t) = l_x(u, v) + d_x(v, t)\}$$

Let $F(u, v)$ be the intersection of the fibers of (u, v) over all possible scenarios $x \in \{0, 1\}^\Phi$, i.e. $F(u, v) := \bigcap_x F_x(u, v)$. By definition, an arc (u, v) is t -strong if t belongs to $F_x(u, v)$ for every scenario x i.e.

$$S(t) = \{(u, v) : t \in F(u, v)\}$$

In order to compute every $S(t)$, we first compute $F(u, v)$ for every arc (u, v) and then transpose the representation to get $S(t)$ for every vertex $t \in V$. Let $y \in \{0, 1\}^\Phi$ be the following scenario :

$$y_{wt} = \begin{cases} 0 & w \in F(u, v) \text{ or } (w, t) = (u, v) \\ 1 & w \notin F(u, v) \end{cases} \quad (1)$$

Lemma 4. *The intersection $F(u, v)$ of the fibers of (u, v) over all possible scenarios is the fiber of (u, v) under the scenario y , i.e. $F(u, v) = F_y(u, v)$.*

Proof. Since the inclusion $F(u, v) \subseteq F_y(u, v)$ is obvious, it suffices to prove that $F_y(u, v) \subseteq F_x(u, v)$ for any scenario $x \in \{0, 1\}^\Phi$. By way of contradiction, suppose that x is a scenario such that $F_y(u, v) \setminus F_x(u, v)$ contains a vertex t at minimum distance from u in the scenario y . Let P be a shortest ut -path containing (u, v) under the scenario y . For every vertex

w of P , $d_y(u, w) \leq d_y(u, t)$ and $w \in F_y(u, v)$. Hence, by the choice of t , w belongs to $F_x(u, v)$ for every scenario x . Therefore w belongs to $F(u, v)$ and the length of every arc of P is set to its upper bound in the scenario y , i.e. $l_y(P) = l(P)$. Now, let Q be a shortest ut -path in the scenario x . If the path Q contains a vertex $z \in F(u, v)$ then $d_x(u, t) = d_x(u, z) + d_x(z, t) = l_x(u, v) + d_x(v, z) + d_x(z, t) = l_x(u, v) + d_x(v, t)$, a contradiction with $t \notin F_x(u, v)$. Therefore, the vertices of Q do not belong to $F(u, v)$ and thus their lengths are set to their lower bound in y , i.e. $l(Q) = l^-(Q)$. We deduce that $l_x(P) \leq l_y(P) \leq l_y(Q) \leq l_x(Q)$ a contradiction with $t \notin F_x(u, v)$. \square

We describe a $O(|A| + |V| \log |V|)$ time algorithm that, given an arc (u, v) , computes simultaneously the scenario y defined by (1) and the fiber $F_y(u, v)$ of arc (u, v) with respect to this scenario. The algorithm is an adaptation of the Dijkstra's shortest path algorithm that assigns colors to vertices. We prove that it colors a vertex w in blue if w belongs to $F_y(u, v)$ and in red otherwise. At each step, we consider a subset of vertices $S \subseteq V$ whose colors have been already computed. Before the first iteration, $S = \{u\}$ and the length of every arc leaving u is set to its lower bound except the length of (u, v) which is set to its upper bound according to scenario y . At each step, since the color of every vertex of S is known, the length, under scenario y , of every arc (w, t) with $w \in S$ is also known. Therefore, it is possible to find a vertex $t \in V - S$ at minimum distance from u , under scenario y , in the subgraph $G[S \cup \{t\}]$ induced by $S \cup \{t\}$. Following Dijkstra's algorithm analysis, we know that the distance under scenario y from u to t in $G[S \cup \{t\}]$ is in fact the distance between u and t in G . This allows us to determine if there exists a shortest path from u to t in the scenario y passing via (u, v) and to color the vertex t accordingly. We end-up the iteration with a new vertex t whose color is known and that can be added to S before starting the next iteration. The algorithm terminates when $S = V$.

For every vertex $w \in V - S$, the estimated distance $d(w)$ is the length of a uw -path under the scenario y in the subgraph $G[S \cup \{w\}]$. An arc a is blue if $a = uv$ or if its origin is blue, the other arcs are red, i.e. a is blue if $y_a = 0$ and red if $y_a = 1$. The estimated color $\gamma(w)$ of w is blue if there exists a blue uw -path of length $d(w)$ in $G[S \cup \{w\}]$ and red otherwise. The correctness of Algorithm 1 follows from the following Lemma.

Lemma 5. *For every vertex $t \in S$, $\gamma(t)$ is blue if and only if $t \in F_y(u, v)$.*

Proof. We proceed by induction on the number of vertices of S . When $S = \{u\}$, the property is verified. Now, suppose the property true before the insertion in S of the

Algorithm 1: Computes the set S of vertices t such that (u, v) is t -strong

Input : $G = (V, A, l, l^-)$, $(u, v) \in A$

Output: $\{t \in V : (u, v) \text{ is } t\text{-strong}\}$

foreach $(u, w) \in \delta_u^{out} \setminus \{(u, v)\}$ **do**

$d(w) \leftarrow l_{uw}^-$

$\gamma(w) \leftarrow red$

$d(v) \leftarrow l_{uv}$; $\gamma(v) \leftarrow blue$

$S \leftarrow \{u\}$; $\gamma(u) \leftarrow red$

while $S \neq V$ **do**

 Pick $t \in V - S$ with smallest $d(t)$ breaking tie by choosing a vertex t such that

$\gamma(t) = blue$ if it exists

if $\gamma(t)$ is blue **then**

foreach $(t, w) \in \delta_t^{out}$ such that $d(w) \geq d(t) + l_{tw}$ **do**

$d(w) \leftarrow d(t) + l_{tw}$

$\gamma(w) \leftarrow blue$

else

foreach $(t, w) \in \delta_t^{out}$ such that $d(w) > d(t) + l_{tw}^-$ **do**

$d(w) \leftarrow d(t) + l_{tw}^-$

$\gamma(w) \leftarrow red$

$S \leftarrow S \cup \{t\}$

return $\{t \in V : \gamma(t) = blue\}$

vertex t such that $d(t)$ is minimum. By induction hypothesis, the lengths of arcs having their source in S are set according to y . Therefore, following Dijkstra's algorithm analysis, we deduce that $d(t)$ is the length of the shortest path from u to t in the graph G under scenario y . If t has been colored blue then there exists a blue vertex $w \in S$ such that $d(t) = d(w) + l_{wt}$. By induction hypothesis $w \in F_y(u, v)$ and there exists a shortest ut -path under scenario y containing (u, v) , i.e. $t \in F_y(u, v)$. Now, suppose that t has been colored in red. By contradiction, assume there exists a shortest ut -path under scenario y that contains (u, v) . This path cannot contain a vertex outside S except t because otherwise, since arc lengths are non-negative, its length according to y would be greater than $d(t)$ by the choice of t . Therefore, the predecessor w of t in this path belongs to S . Since w belongs to a shortest ut -path passing via (u, v) , by induction, it was colored blue. But in this case, there exists a blue vertex w such that $d(w) + l(w, t) = d(t)$, and t was colored blue as well, a contradiction. \square

We are now ready to state the main result of this section.

Proposition 1. *Given a graph $G = (V, A)$, two arc-length functions l^- and l such that $0 \leq l_a^- \leq l_a$ for every arc $a \in A$, and an arc (u, v) , Algorithm 1 computes in $O(|A| + |V| \log |V|)$ the set of vertex t such that (u, v) is t -strong.*

Proof. The correctness of Algorithm 1 is given by Lemma 4 and Lemma 5. Analogously to the Dijkstra's algorithm, it can be implemented to run in $O(|A| + |V| \log |V|)$ using a Fibonacci heap as priority queue. \square

4.3. Computing $W(t)$ for all t

The algorithm that computes, for every arc (u, v) , the set of vertices t such that (u, v) is t -useless is obtained from Algorithm 1 by applying two small changes. Before describing these changes, we explain how the two problems are related. For that, we first introduced a strengthening of the notion of strongness. We will say that an arc (u, v) is strictly t -strong if it belongs to all shortest ut -paths for every scenario $x \in \{0, 1\}^\Phi$. Recall that t -strongness requires only the existence of a shortest ut -path passing via (u, v) for every scenario $x \in \{0, 1\}^\Phi$. Algorithm 1 can be easily adapted to compute for every arc (u, v) the set of vertex t such that (u, v) is strictly t -strong. It suffices to change the way, the algorithm breaks tie between a red and a blue path and the choice of t in case of tie. Namely, in the first internal loop, the condition for coloring w in blue becomes $d(w) > d(t) + l_{tw}$ while the condition for coloring w in red in the second internal loop becomes $d(w) \geq d(t) + l_{tw}^-$. Moreover, when we choose t such that $d(t)$ is minimal, we break tie by choosing a red vertex if it exists. Clearly, these small changes exclude the existence of a red path of length $d(w)$ between u and a blue vertex w . Therefore, (u, v) belongs to every path of length $d(w)$ and (u, v) is strictly w -strong whenever w is blue. We call the resulting algorithm the strict version of Algorithm 1. The next step is to extend the notion of strict strongness to a subset of arcs having the same source. For any vertex $u \in V$, a subset $\Gamma \subseteq \Gamma_u^+$ of arcs is strictly t -strong if, for every scenario $x \in \{0, 1\}^\Phi$, all shortest ut -paths intersect Γ . By definition, an arc (u, v) is t -useless if and only if $\Gamma_u^+ \setminus \{(u, v)\}$ is strictly t -strong. Indeed, every ut -path avoiding (u, v) intersects $\Gamma_u^+ \setminus \{(u, v)\}$ and, conversely, (u, v) belongs to every ut -path avoiding $\Gamma_u^+ \setminus \{(u, v)\}$. Hence, computing the set of vertex t such that (u, v) is t -useless amounts to compute the set of vertex t such that $\Gamma_u^+ \setminus \{(u, v)\}$ is strictly t -strong. An algorithm that computes this

set of vertices can be obtained from the strict version of Algorithm 1 by modifying only the initialization step: arc (u, v) is colored in red and its length is set to l_{uv}^- while arcs of $\delta_u^{\text{out}} \setminus \{(u, v)\}$ are colored in blue and their lengths are set to their upper bound. A correctness proof very similar to the one of Algorithm 1 (and that we will not repeat) shows that a vertex is colored blue by Algorithm 2 if and only if (u, v) is t -useless. Since the two algorithms have clearly the same time complexity, we conclude this section with the following result.

Algorithm 2: Computes the set of vertex t such that (u, v) is t -useless

Input : $G = (V, A, l, l^-), (u, v) \in A$

Output: $\{t \in V : (u, v) \text{ is } t\text{-useless}\}$

$d(v) \leftarrow l_{uv}^-; \gamma(v) \leftarrow \text{red}$

foreach $(u, w) \in \delta_u^{\text{out}} \setminus \{(u, v)\}$ **do**

| $d(w) \leftarrow l_{uw}$
 | $\gamma(w) \leftarrow \text{blue}$

$S \leftarrow \{u\}; \gamma(u) \leftarrow \text{blue}$

while $S \neq V$ **do**

| Pick $t \in V - S$ with smallest $d(t)$ breaking tie by choosing a vertex t such that
 | $\gamma(t) = \text{red}$ if it exists

| **if** $\gamma(t)$ is blue **then**

| **foreach** $(t, w) \in \delta_t^{\text{out}}$ such that $d(w) > d(t) + l_{tw}$ **do**

| | $d(w) \leftarrow d(t) + l_{tw}$
 | | $\gamma(w) \leftarrow \text{blue}$

| **else**

| **foreach** $(t, w) \in \delta_t^{\text{out}}$ such that $d(w) \geq d(t) + l_{tw}^-$ **do**

| | $d(w) \leftarrow d(t) + l_{tw}^-$
 | | $\gamma(w) \leftarrow \text{red}$

| $S \leftarrow S \cup \{t\}$

return $\{t \in V : \gamma(t) = \text{blue}\}$

Proposition 2. Given a graph $G = (V, A)$, two arc-length functions l^- and l such that $0 \leq l_a^- \leq l_a$ for every arc $a \in A$, and an arc (u, v) , Algorithm 2 computes in $O(|A| + |V| \log |V|)$ the set of vertex t such that (u, v) is t -useless.

5. Numerical experiments

In this section, we report our computational experiments in order to demonstrate the added benefit of our MIP formulation and preprocessing step. The numerical experiments were performed on a desktop computer equipped with an Intel(R) Core(TM) i7-8700k 4.8 gigahertz and a memory of 16 gigabytes. Our model was implemented in Gurobi 9.0.1 with default parameters and the other algorithms were coded in multithreaded C++ using the LEMON Graph Library.

5.1. Quality of the solutions

In the lack of an efficient method to compute an optimal solution of BC-ECA-Opt, ecologists often use greedy algorithms to compute sub-optimal solutions. In this subsection we will compare an optimal solution obtained with our MIP model to the solutions obtained with different *greedy algorithms*. The *incremental greedy* algorithm (IG) starts from the graph with no improved element, at each step i , the algorithm selects the element e with the greatest ratio δ_e^i/c_e until no more element fits in the budget. Here, δ_e^i denotes the difference between the value of ECA with and without the improvement of the element e at the step i and c_e is the cost of improving the element e (that can be either an arc or a vertex). The *decremental greedy* (DG) starts from the graph with all improvements performed and iteratively removes the improvement of the element e with the smallest ratio δ_e^i/c_e . DG finishes with incremental steps to ensure there is no free budget left. These algorithms perform at most $|\Phi|$ steps and at each step i need to compute δ_e^i for each element e . It is easy to implement the IG algorithm in $O(|V|^3 + |\Phi|^2 \cdot |V|^2)$ by using an all pair shortest path algorithm to compute in $O(|V|^3)$ the initial distance matrix and then by performing $|\Phi|$ steps in which the computation of δ_e^i for each arc $e \in \Phi$ takes $O(|V|^2)$. Indeed, when we decrease the length of an edge we can update the distance matrix in $O(|V|^2)$. For the implementation of DG, when we increase the length of an edge we cannot update the distance matrix as easily as for the IG algorithm. We can recompute in $|V|^3$ the whole distance matrix for computing each δ_e^i and the algorithm runs in $O(|\Phi|^2 \cdot |V|^3)$ (we did not try to improve the implementation). These complexities are already too large for the practical instances handled by ecologists which can have few thousands of patches. Most of the studies using the PC or ECA indicators use simpler algorithms that we call *static increasing* (SI) and *static decreasing* (SD). These algorithms are simpler variants of the greedy algorithms which do not recompute the ratio δ_e/c_e of

each element e at each step and thus are faster but do not account for cumulative effects nor redundancies.

In the next section, we provide instances on which IG and DG performs poorly compared to an optimal solution. On these instances, it is easy to check that the solutions returned by algorithms SI and SD are not better than the solutions returned by algorithms IG and DG.

5.1.1. Limits of the greedy algorithms

In the instances presented below, all arcs have a probability 1 if improved and 0 otherwise and have unitary costs. Recall that a spider is a tree (sorry for ecologists) consisting of several paths glued together on a central vertex.

Case A: The graph is a spider with $2k$ branches: k long branches with two edges, an intermediate node of weight 0 and a leaf node of weight 1, and k short branches consisting of a single edge with a leaf of very small weight $\epsilon > 0$, see Fig.1 (a). All branches are connected to a central node of weight 1. IG performs poorly on this instance. Indeed, IG is tricked into purchasing short branches with very small ECA improvement because purchasing an edge of a long branch alone does not increase ECA at all. An optimal solution results in larger value of ECA by improving pairs of arcs of long branches.

In this case, IG does not perform well while DG finds an optimal solution computed by the MIP solver except when the budget is 1. In this case, the reverse occurs: DG performs badly while IG is optimal. Indeed, DG realizes that the budget is not sufficient to improve two arcs of a long branch only after removing the improvements of all short branches.

Case B: The graph is obtained from a star with $k + 1$ branches by replacing one branch by a path of length k . The central node and all leaves except the leaf of the path have weight 1. The leaf of the path has weight $1 + \epsilon$. The internal nodes of the path have weight 0. DG performs poorly on this instance because it removes one by one the branches of the star for which $\delta_e/c_e = 1$ before removing an edge of the path for which $\delta_e/c_e = 1 + \epsilon$. When the budget is at least 2, an optimal solution removes all the edges of the path before removing an edge of another branch.

Case C: The graph is a spider with $k + 1$ branches. All branches except one are paths of length 2 with an internal node of weight 0 and a leaf of weight 1. The last branch is a path

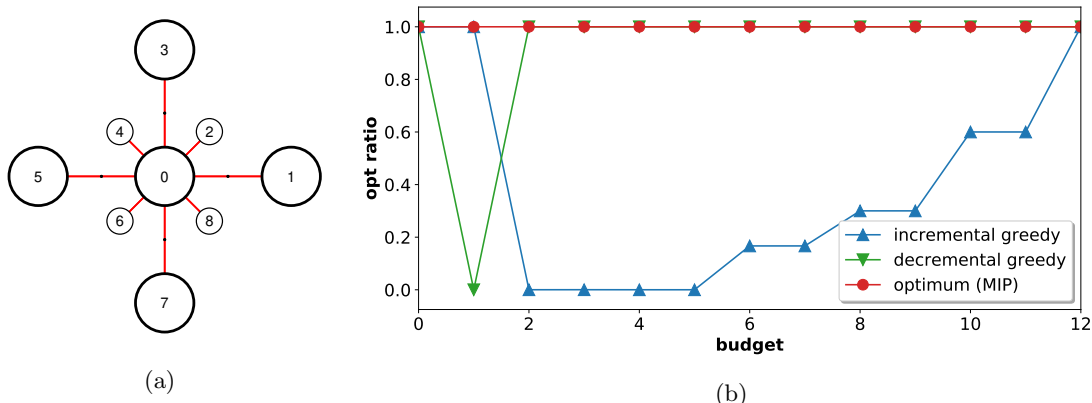


Figure 2: An instance on which Incremental Greedy fails. (a) the graph of case A with $k = 4$, (b) ratio of the increase in ECA between the solutions returned by IG and DG and an optimal solution for several budgets.

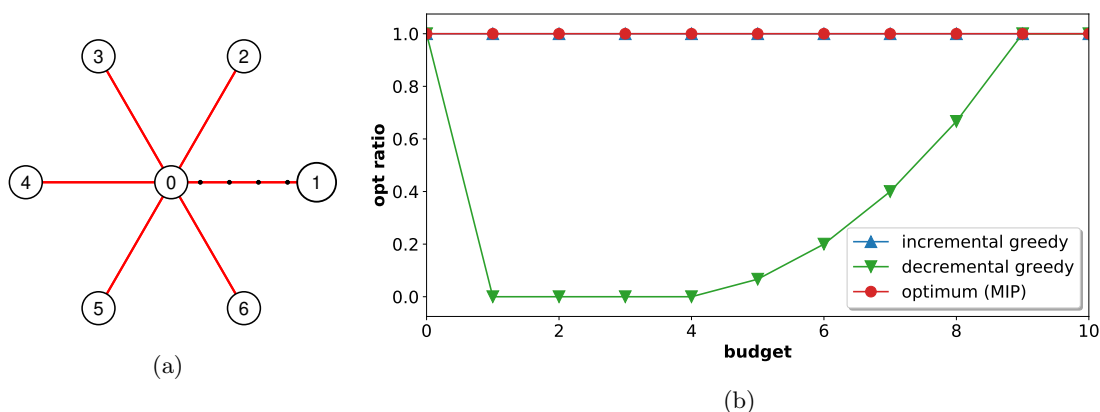
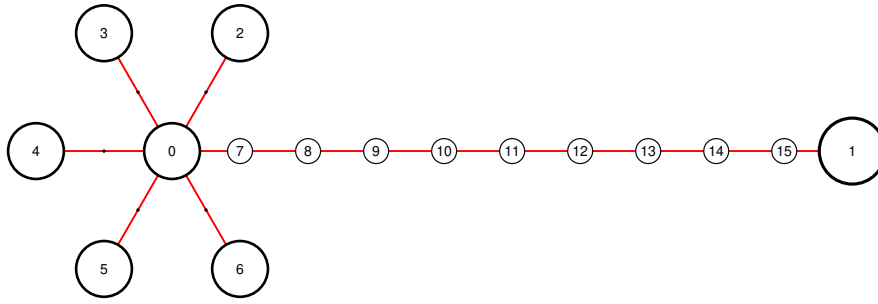
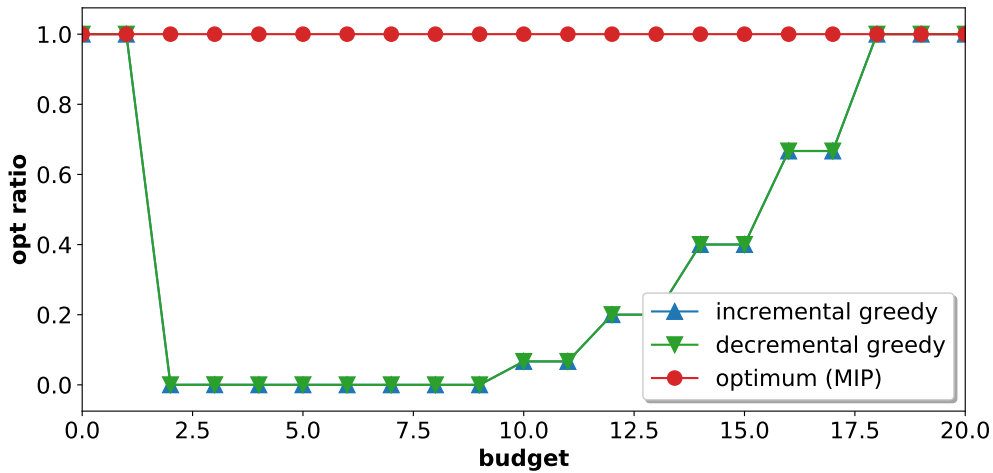


Figure 3: An instance on which Decremental Greedy fails. (a) the graph of case B with $k = 5$, (b) ratio of the increase in ECA between the solutions returned by IG and DG and an optimal solution for several budgets.

of length $2k$ with internal nodes of weight $\epsilon > 0$ and a leaf of weight $1 + \epsilon$. All branches intersect in a central node of weight 1. In this case, both Incremental and Decremental Greedy fail. On one hand, IG selects the edges of the path of length $2k$ one by one and does not realize that by taking two edges of a short branch it could improve much more ECA. On the other hand, DG removes first the edges of the short branch because the weight of leaf of a long branche is $1 + \epsilon$ while the weight of the leaf of a short branch is 1. Hence, DG and IG return the same low quality solution.



(a)



(b)

Figure 4: An instance on which both Incremental and Decremental Greedy algorithms fail. (a) the graph of case C with $k = 5$, (b) ratio of the increase in ECA between the solutions returned by IG and DG and an optimal solution for several budgets.

Case C illustrates the fact that the greedy algorithms IG and DG do not provide any constant approximation guarantee (even for trees), i.e. for any constant $0 < \alpha < 1$ there exists an instance of BC-ECA-Opt such that $ALG < \alpha OPT$ where ALG is the value of ECA for the best solution among those returned by IG and DG and OPT is the value of ECA for an optimal solution.

5.1.2. In Practice

In this subsection we investigate on the performances of the greedy algorithms we discuss in the previous subsection on a real instance. We perform these numerical experiments on a model of the landscape around Montréal from [2]. The landscape model of [2] is a Gabriel graph – a subset of the Delaunay triangulation – of 8733 patches. For each patch, we have an estimation of its area that could be lost by 2050. In order to obtain

several examples with a reasonable computing time, we extract 72 subgraphs of sizes between 150 and 400 nodes from this large instance. Then, we consider that protecting a patch corresponds to buying the portion of it that would otherwise be lost and has a cost proportional to the purchased area. To model the protection of a patch u , we set its default weight, w_u , to the area it would have in 2050 if unprotected and its improved weight, w_u^+ , to its current area. If a patch is threatened of total loss, i.e. all its area would be lost by 2050, we prevent it to contribute to shortest paths if it is not purchased as follows. We add a node u_{out} , replace each outgoing arc (u, v) by an arc (u_{out}, v) and connect u and u_{out} with an arc (u, u_{out}) of default probability 0 and enhanced probability 1. In this way, if a patch threatened of total loss is not protected then it does not contribute to ECA by its quality nor by its use as stepping stone for other pairs of nodes.

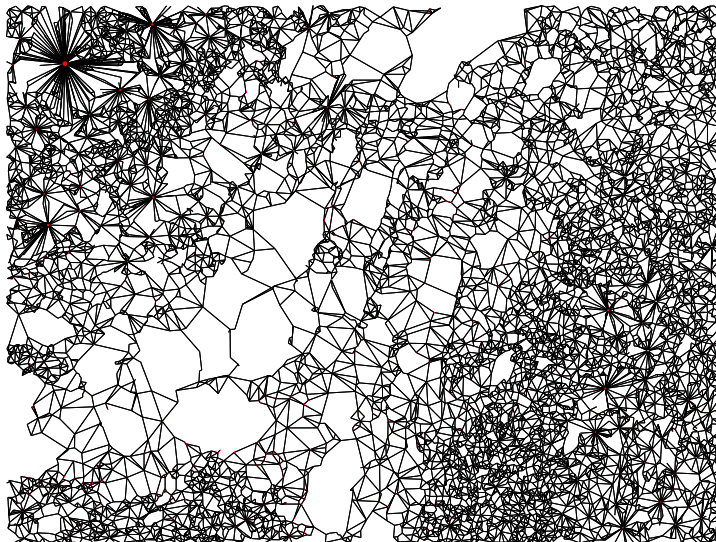


Figure 5: Quebec landscape modelization from [2]

Contrastingly to what we have seen in the previous section, greedy algorithms perform surprisingly well in the experiments we perform on this real case study. As shown in Fig 6, greedy algorithms lead on average to gains that reach 95% of the one obtained with optimal resolution with respect to the gain expected with a random solution.

5.2. Scalability and benefits of the preprocessing

In this subsection we address the scalability of our approach and the added benefits of our preprocessing step. For this purpose we build a plausible instance from which we can easily vary the number of changing elements and integer variables. We take the case of the city of Marseille which is surrounded by 3 big forest massifs and contains 54 parks of

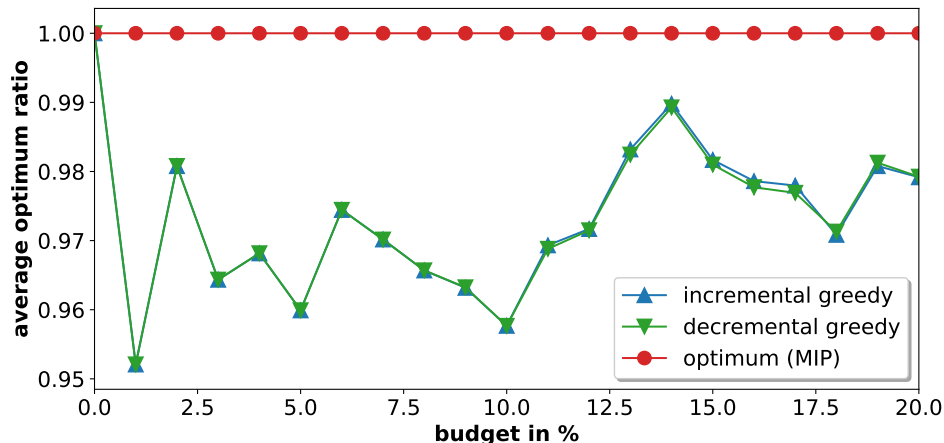


Figure 6: Average value of returned solutions normalized between the average random solution and an optimal one

#wasteland	MIP			preprocessed MIP			greedy
	#var	#const	time	#var	#const	time	time
20	345775	18410	12246	10716	7197	316	766
50	717901	36410	142306	34613	21485	1964	7217
80	1306597	59810	1914478	76281	42039	13141	29479
110	-	-	-	132225	66759	62268	87948
140	-	-	-	207999	96600	200558	208179
170	-	-	-	308355	132701	1701684	416751

Table 1: Comparison of the MIP, the preprocessed MIP and the greedy (algorithm DG) according to the number of variables (#var), the number of constraints (#const) and the time (in ms) it takes to execute

variable sizes. The idea is to consider a certain number of wastelands throughout the city that can be protected to preserve the stepping stones between massifs and parks. The base graph is composed of 96 nodes of which 42 models the frontier of massifs, 43 represents small parks and 11 big parks. The graph is complete and each arc is weighted with the border to border distance between the two patches it connects. We model each wasteland as a patch which is totally threatened (see Subsection 5.1.2). A non purchased wasteland has then a null quality and does not contribute to any shortest path. We perform our experiments by adding various number of wastelands, randomly generated across the city, and looking for variations of the MIP formulation size and computation time.

We see in Table 1 that the computation time of the MIP without preprocessing in-

creases very quickly. It takes more than 30 minutes on average for instances with 80+ wastelands whereas the preprocessed one can be solved in less than 30 minutes with up to 170 wastelands. This is due to the preprocessing step that significantly reduces the time required to solve the linear relaxation by reducing the number of variables, constraints and non-zero entries of the mixed integer program. The preprocessed MIP is faster than the greedy algorithm for instances with at most 140 wastelands (the preprocessing step was not used for greedy algorithms).

Finally, we run our preprocessing on 400 instances randomly generated from the Quebec’s graph (Fig. 5) in order to study the impact of the preprocessing on the MIP formulation size. For building these instances, we take 20 connected subgraphs of 500 nodes and for each graph we create 20 instances by randomly picking a percentage of arcs whose length could be improved by half. Figure 7 shows the percentage of reduction of the number of constraints, variables and non-zero entries of the MIP formulation with respect to the percentage of arcs that could be restored. The preprocessing removes almost all the elements of the MIP formulation when the number of restorable arcs arrives close to zero. When 20% of the arcs could be restored, the preprocessing removes more than 75% of the model’s variables and non-zero entries and around 60% of the constraints.

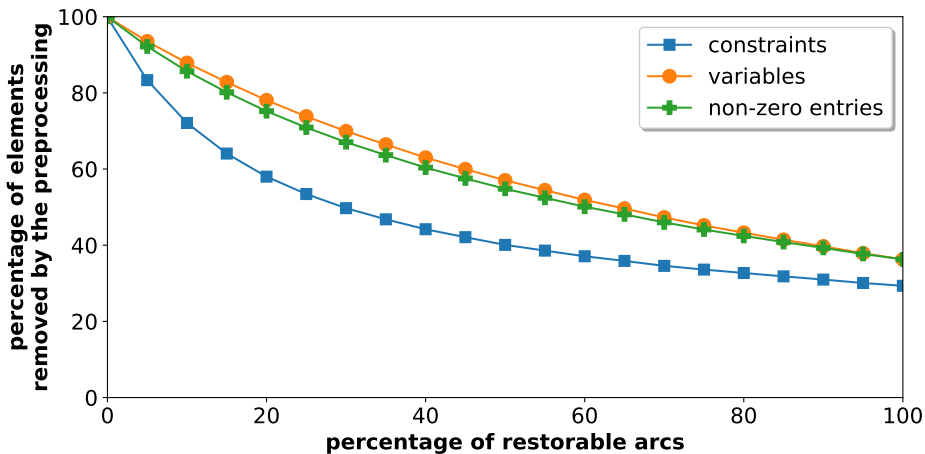


Figure 7: Average percentage of constraints, variables and non-zero entries that the preprocessing removes with respect to the percentage of arcs that could be restored

6. Conclusion

In this paper we introduce a new MIP formulation for BC-ECA-Opt and we show that this formulation allows solving optimally instances with around 150 habitat patches while

previous formulations [22] are limited to instances of about 30 patches. In order to make our formulation scale to even larger instances, we present a preprocessing step that reduces significantly the size of the graphs on which a generalized flow has to be computed. We show that this preprocessing step allows to greatly reduce the MIP formulation size and that its benefits increase when the proportion of arcs whose lengths can change decrease. Thus allowing us to treat instances of up to 300 patches.

This new method allows us to solve optimally larger instances of BC-ECA-Opt and to compare optimum solutions with the solutions returned by greedy algorithms. Interestingly, we found that greedy algorithms perform well in practice despite the arbitrary bad cases we spotted. Our next goals will be to experiment our approach on other practical instances of the problem arising from different ecological contexts. On the theoretical side, we would like to investigate the problem from an approximation algorithm point of view. For instance, is it possible to find reasonable assumptions under which greedy algorithms are guaranteed to return a solution whose ECA value is at least a constant fraction of the optimal ECA ? If these assumptions are fulfilled by the real instances that we considered, this would explain our experimental observations. Moreover, since a polynomial time approximation scheme has been given in the case of trees [21], it could also be interesting to know for which larger classes of graphs constant factor approximation algorithms for this problem exist. Another interesting question is to determine whether good solutions could be obtained by decomposing geographically the problem, by solving independently a subproblem for each region and then by reassembling the solutions. In this case, a notion of fairness could help to allocate the budget among the regions so that each region can enhance its own *internal connectivity* keeping a part of the budget to enhance the connectivity between the regions. Since ECA is based on the equivalence between a landscape and a patch, such a multilevel optimization approach looks promising.

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