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Pierre Gaillard. Other patterns for the first and second order rational solutions to the KPI equation. 2022. hal-03576395

**HAL Id: hal-03576395**

**<https://hal.science/hal-03576395>**

Preprint submitted on 16 Feb 2022

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# Other patterns for the first and second order rational solutions to the KPI equation

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## Abstract

We present rational solutions to the Kadomtsev-Petviashvili equation (KPI) in terms of polynomials in  $x$ ,  $y$  and  $t$  depending on several real parameters. We get an infinite hierarchy of rational solutions written as a quotient of a polynomial of degree  $2N(N+1) - 2$  in  $x$ ,  $y$  and  $t$  by a polynomial of degree  $2N(N+1)$  in  $x$ ,  $y$  and  $t$ , depending on  $2N - 2$  real parameters for each positive integer  $N$ .

We construct explicit expressions of the solutions in the simplest cases  $N = 1$  and  $N = 2$  and we study the patterns of their modulus in the  $(x, y)$  plane for different values of time  $t$  and parameters. In particular, in the study of these solutions, we see the appearance not yet observed of three pairs of two peaks in the case of order 2.

PACS numbers :

33Q55, 37K10, 47.10A-, 47.35.Fg, 47.54.Bd

## 1 Introduction

We consider the Kadomtsev-Petviashvili (KPI) equation in the following normalization

$$(4u_t - 6uu_x + u_{xxx})_x = 3u_{yy}, \quad (1)$$

where subscripts  $x$ ,  $y$  and  $t$  denote partial derivatives.

This equation first appears in 1970, in a paper written by Kadomtsev and Petviashvili [56]. The equation (1) was derived as a model for surface and internal water waves [1], and in nonlinear optics [61].

A lot of methods have been used to solve this equation; these methods are reviewed in the book by Ablowitz and Clarkson published in 1991 [2]. Matveev used the Darboux transformation to construct solutions [60] in 1979. Dubrovin gave for the first time solutions in terms of Riemann theta functions in the framework of algebraic geometry in 1981 [3]. Freeman and Nimmo constructed in 1983 solutions in terms of wronskians [4, 5] by means of tau functions. The inverse scattering method has been applied to obtain solutions of the KPI equations in 1999 [64].

The first rational solutions were constructed in 1977 by Manakov, Zakharov, Bordag, Its and Matveev [59]. Among the various researches concerning rational solutions of the KPI equation, we can mention the works of the following authors : Krichever in 1978 [57, 58], Satsuma and Ablowitz in 1979 [65], Pelinovsky and Stepanyants in 1993 [62], Pelinovsky in 1994 [63].

We present multi parametric families of rational solutions to KPI equation as a quotient of two polynomials in  $x$ ,  $y$  and  $t$  depending on several real parameters. These solutions presented here belong to an infinite hierarchy of solutions of rational solutions written as a quotient of a polynomial of degrees  $2N(N + 1) - 2$  in  $x$ ,  $y$ , and  $t$  by a polynomial of degree  $N(N + 1)$  in  $x$ ,  $y$ , and  $t$ , depending on  $2N - 2$  real parameters for each positive integer  $N$ . We limit here the study to the case of the simplest  $N = 1$  and  $N = 2$ .

We study the patterns of the modulus of the solutions in the  $(x, y)$  plane for different values of time  $t$  and parameters.

This work is part of a project to find rational solutions to different classical partial differential equations. New types of rational solutions are proposed in this article.

We will give the ideas of the proof of the results in the case of order  $N$  in a future article. The proofs of the following results in the simple cases of the first orders can be carried out by hand or with the help of a formal calculation software.

## 2 Rational solutions of order 1

**Theorem 2.1** *The function  $v$  defined by*

$$v(x, y, t) = -2 \frac{n(x, y, t)}{d(x, y, t)^2} \quad (2)$$

*is a rational solution to the KPI equation (1)*

with

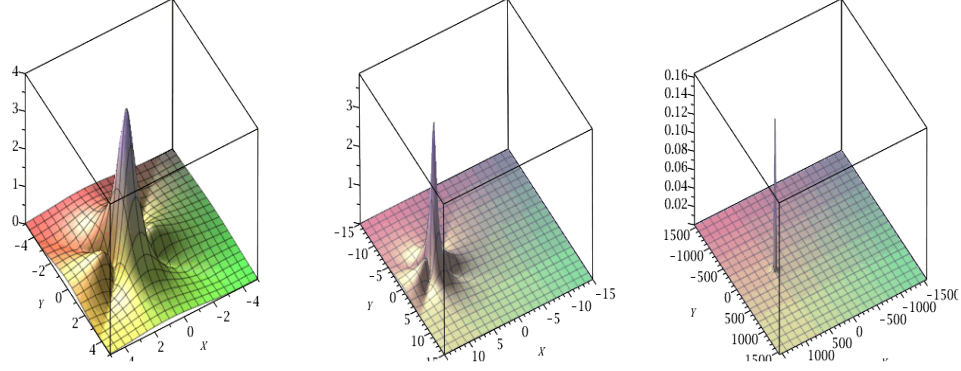
$$n(x, y, t) = -512x^2 + (768t + 1024)x + 512y^2 - 768t - 288t^2,$$

and

$$d(x, y, t) = 16x^2 + (-24t - 32)x + 9t^2 + 16y^2 + 24t + 32.$$

We represent the modulus of the solution for different values of  $t$  in the plane  $(x, y)$ . For  $t = 0$ , the maximum of the modulus is equal to 4. When the values of  $t$  increase, the maximum of the modulus of the solution decreases rapidly.

For example for  $t = 10^3$ , this value is equal to 0.16.



**Figure 1.** Solution of order 1 to KPI, on the left for  $t = 0$ ; in the center for  $t = 10$ ; on the right for  $t = 10^3$ .

### 3 Rational solutions of order 2 depending on 2 parameters

We always consider the KPI equation defined by (1).

**Theorem 3.1** *The function  $v$  defined by*

$$v(x, y, t) = -2 \frac{n(x, y, t)}{d(x, y, t)^2}, \quad (3)$$

*is a rational solution to the KPI equation (1), quotient of a polynomial  $n(x, y, t)$  of degree  $2N(N + 1) - 2 = 10$  in  $x, y, t$  by a polynomial  $d(x, y, t)^2$  of degree  $2N(N + 1) = 12$  in  $x, y, t$  depending on 2 real parameters  $a_1, b_1$*

with  $n$  and  $d$  defined by

$$\begin{aligned} n(x, y, t) = & -73383542784 x^{10} + (550376570880 t + 1712282664960)x^9 + (-18590497505280 - 1857520926720 t^2 - \\ & 11557907988480 t - 220150628352 y^2)x^8 + (1320903770112 t y^2 + 34673723965440 t^2 + 111542985031680 t + 3715041853440 t^3 + \\ & 123284351877120 + 4109478395904 y^2)x^7 + (-554779583447040 - 292800335708160 t^2 - 4875992432640 t^4 - 146767085568 y^4 - \\ & 647242847354880 t - 21574761578496 t y^2 - 33071516614656 y^2 - 60679016939520 t^3 - 3467372396544 t^2 y^2)x^6 + \\ & (660451885056 t y^4 - 782757789696 i\sqrt{3}y^2 a_1 + 68263894056960 t^4 + 4388393189376 t^5 + 782757789696 \sqrt{3}y^2 b_1 + \\ & 1775294667030528 + 1456296406548480 t^2 + 2054739197952 y^4 + 439200503562240 t^3 + 48543213551616 t^2 y^2 + 142853296619520 y^2 + \\ & 5201058594816 t^3 y^2 + 2496508125511680 t - 608811614208 i\sqrt{3}a_1 + 148821824765952 t y^2 + 608811614208 \sqrt{3}b_1)x^5 + \\ & (-4680952735334400 t^2 - 321909141012480 y^2 - 6657355001364480 t - 12719814082560 y^4 - 411750472089600 t^4 - \\ & 1820370508185600 t^3 - 2283043553280 \sqrt{3}t b_1 - 9132174213120 \sqrt{3}y^2 b_1 + 7871064440832 i\sqrt{3}a_1 - 260919263232 i a_1 b_1 - \\ & 4875992432640 t^4 y^2 - 1238347284480 t^2 y^4 - 279040921436160 t^2 y^2 - 60679016939520 t^3 y^2 - 7705271992320 t y^4 - \\ & 535699862323200 t y^2 + 2283043553280 i\sqrt{3}t a_1 + 9132174213120 i\sqrt{3}y^2 a_1 - 4049890959163392 + 146767085568 y^6 - \\ & 2742745743360 t^6 - 51197920542720 t^5 + 2935341711360 i\sqrt{3}t y^2 a_1 - 2935341711360 \sqrt{3}t y^2 b_1 - 7871064440832 \sqrt{3}b_1 + \\ & 130459631616 b_1^2 - 130459631616 a_1^2)x^4 + (9986032502046720 t^2 + 197254963003392 y^2 + 12149672877490176 t + \\ & 39529268379648 y^4 + 1365277881139200 t^4 + 4680952735334400 t^3 + 521838526464 \sqrt{3}y^4 b_1 + 23613193322496 \sqrt{3}t b_1 + \end{aligned}$$

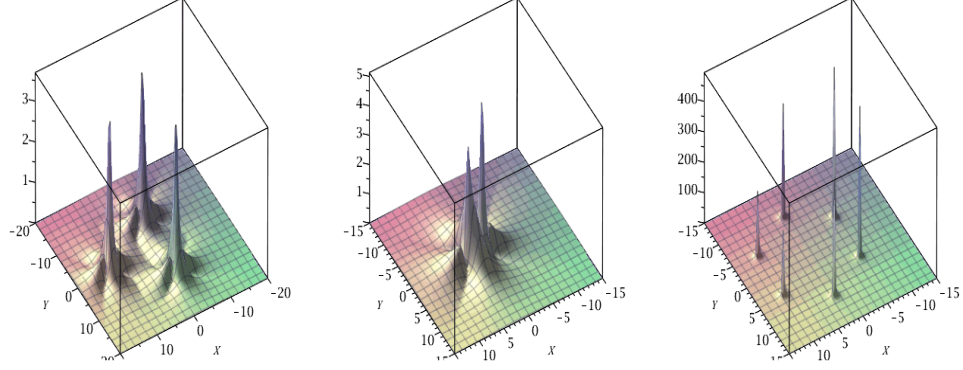
$$\begin{aligned}
& 3424565329920 \sqrt{3}t^2b_1 + 45052059451392 \sqrt{3}y^2b_1 - 39833674186752 i\sqrt{3}a_1 + 2435246456832 ia_1b_1 + 45509262704640 t^4y^2 + \\
& 11557907988480 t^2y^4 + 803549793484800 t^2y^2 + 279040921436160 t^3y^2 + 38159442247680 ty^4 + 965727423037440 ty^2 - \\
& 521838526464 iy^4\sqrt{3}a_1 - 23613193322496 i\sqrt{3}ta_1 - 3424565329920 i\sqrt{3}t^2a_1 - 45052059451392 i\sqrt{3}y^2a_1 + 782757789696 ita_1b_1 + \\
& 6308636406054912 - 1369826131968 y^6 + 25598960271360 t^6 + 247050283253760 t^5 - 4403012567040 i\sqrt{3}t^2y^2a_1 - \\
& 27396522639360 i\sqrt{3}ty^2a_1 + 4403012567040 \sqrt{3}t^2y^2b_1 + 27396522639360 \sqrt{3}ty^2b_1 + 39833674186752 \sqrt{3}b_1 - 391378894848 tb_1^2 - \\
& 1217623228416 b_1^2 + 1175462461440 t^7 - 440301256704 ty^6 + 391378894848 ta_1^2 + 1217623228416 a_1^2 + 2925595459584 t^5y^2 + \\
& 1238347284480 t^3y^4)x^3 + (-13668381987176448 t^2 + 840094797791232 y^2 - 14194431913623552 t - 46769777934336 y^4 - \\
& 2633035913625600 t^4 - 7489524376535040 t^3 - 3652869685248 \sqrt{3}y^4b_1 - 2568423997440 \sqrt{3}t^3b_1 - 89625766920192 \sqrt{3}tb_1 - \\
& 26564842487808 \sqrt{3}t^2b_1 - 118022480068608 \sqrt{3}y^2b_1 + 103324028239872 i\sqrt{3}a_1 - 9972914061312 ia_1b_1 - 156960518307840 t^4y^2 - \\
& 42929372528640 t^2y^4 - 1086443350917120 t^2y^2 - 602662345113600 t^3y^2 - 88940853854208 ty^4 - 443823666757632 ty^2 - \\
& 521838526464 iy^2a_1b_1 + 3652869685248 iy^4\sqrt{3}a_1 + 2568423997440 i\sqrt{3}t^3a_1 + 89625766920192 i\sqrt{3}ta_1 + 26564842487808 i\sqrt{3}t^2a_1 + \\
& 118022480068608 i\sqrt{3}y^2a_1 - 5479304527872 ita_1b_1 - 880602513408 it^2a_1b_1 - 5991619501228032 + 4109478395904 y^6 - \\
& 92643856220160 t^6 - 614375046512640 t^5 + 1174136684544 iy^4\sqrt{3}a_1 + 30821087969280 i\sqrt{3}t^2y^2a_1 + 3302259425280 it^3\sqrt{3}y^2a_1 + \\
& 101367133765632 i\sqrt{3}ty^2a_1 - 30821087969280 \sqrt{3}t^2y^2b_1 - 101367133765632 \sqrt{3}ty^2b_1 - 103324028239872 \sqrt{3}b_1 + \\
& 440301256704 t^2b_1^2 + 260919263232 y^2b_1^2 + 2739652263936 tb_1^2 - 3302259425280 \sqrt{3}t^3y^2b_1 - 1174136684544 \sqrt{3}ty^4b_1 + \\
& 4986457030656 b_1^2 - 8228237230080 t^7 - 330598817280 t^8 + 220150628352 y^8 + 3082108796928 ty^6 - 2739652263936 ta_1^2 - \\
& 1097098297344 t^6y^2 - 696570347520 t^4y^4 + 495338913792 t^2y^6 - 440301256704 t^2a_1^2 - 260919263232 y^2a_1^2 - \\
& 4986457030656 a_1^2 - 20479168217088 t^5y^2 - 8668430991360 t^3y^4)x^2 + (10645823935217664 t^2 - 2187416643305472 y^2 + \\
& 8987429251842048 t - 3913788948480 y^4 + 2808571641200640 t^4 + 6834190993588224 t^3 + 963158999040 \sqrt{3}t^4b_1 + \\
& 6175089229824 \sqrt{3}y^4b_1 + 13282421243904 \sqrt{3}t^3b_1 + 154986042359808 \sqrt{3}tb_1 + 67219325190144 \sqrt{3}t^2b_1 + 160204427624448 \sqrt{3}y^2b_1 + \\
& 115964116992 \sqrt{3}a_1^2b_1 - 260919263232 \sqrt{3}y^6b_1 + 2567836929097728 - 38654705664 i\sqrt{3}a_1^3 - 133851582038016 i\sqrt{3}a_1 + \\
& 17974438133760 ia_1b_1 + 2435246456832 iy^2a_1b_1 - 6175089229824 iy^4\sqrt{3}a_1 - 13282421243904 i\sqrt{3}t^3a_1 - 154986042359808 i\sqrt{3}ta_1 - \\
& 67219325190144 i\sqrt{3}t^2a_1 - 160204427624448 i\sqrt{3}y^2a_1 + 14959371091968 ita_1b_1 - 963158999040 i\sqrt{3}t^4a_1 + 4109478395904 it^2a_1b_1 + \\
& 440301256704 it^3a_1b_1 + 115964116992 i\sqrt{3}a_1b_1^2 + 260919263232 i\sqrt{3}y^6a_1 + 391378894848 ty^2a_1^2 - 391378894848 ty^2b_1^2 + \\
& 225998379417600 t^4y^2 + 66705640390656 t^2y^4 + 332887750068224 t^2y^2 + 543221675458560 t^3y^2 + 70154666901504 ty^4 - \\
& 1260142196686848 ty^2 - 1956894474240 y^6 + 153593761628160 t^6 + 789910774087680 t^5 - 5479304527872 iy^4\sqrt{3}a_1 - \\
& 76025350324224 i\sqrt{3}t^2y^2a_1 - 15410543984640 it^3\sqrt{3}y^2a_1 - 177033720102912 i\sqrt{3}ty^2a_1 + 782757789696 ity^2a_1b_1 - \\
& 1238347284480 i\sqrt{3}t^4y^2a_1 - 880602513408 i\sqrt{3}t^2y^4a_1 + 76025350324224 \sqrt{3}t^2y^2b_1 + 177033720102912 \sqrt{3}ty^2b_1 + \\
& 55099802880 t^9 + 133851582038016 \sqrt{3}b_1 - 38654705664 \sqrt{3}b_1^3 + 220150628352 t^3a_1^2 - 220150628352 t^3b_1^2 - 2054739197952 t^2b_1^2 - \\
& 1217623228416 y^2b_1^2 - 7479685545984 tb_1^2 + 235092492288 t^7y^2 + 208971104256 t^5y^4 - 247669456896 t^3y^6 - 330225942528 ty^8 + \\
& 1238347284480 \sqrt{3}t^4y^2b_1 + 880602513408 \sqrt{3}t^2y^4b_1 + 15410543984640 \sqrt{3}t^3y^2b_1 + 5479304527872 \sqrt{3}ty^4b_1 - 8987219066880 b_1^2 + \\
& 19852254904320 t^7 + 1542794480640 t^8 - 1027369598976 y^8 - 6164217593856 ty^6 + 7479685545984 ta_1^2 + 5119792054272 t^6y^2 + \\
& 3250661621760 t^4y^4 - 2311581597696 t^2y^6 + 2054739197952 t^2a_1^2 + 1217623228416 y^2a_1^2 + 8987219066880 a_1^2 + \\
& 47088155492352 t^5y^2 + 21464686264320 t^3y^4)x - 3370285969440768 t^2 + 1711891286065152 y^2 - 1925877696823296 t + \\
& 28991029248 \sqrt{3}tb_1^3 - 270582939648 \sqrt{3}a_1^2b_1 + 608811614208 \sqrt{3}y^6b_1 + 56456405581824 y^4 - 1281410811297792 t^4 - \\
& 2661455983804416 t^3 - 144473849856 \sqrt{3}t^5b_1 + 130459631616 \sqrt{3}y^4b_1 - 16804831297536 \sqrt{3}t^3b_1 - 100388686528512 \sqrt{3}tb_1 - \\
& 2490453983232 \sqrt{3}t^4b_1 - 89234388025344 \sqrt{3}y^2b_1 - 913217421312 ty^2a_1^2 + 913217421312 ty^2b_1^2 - 146767085568 t^2y^2a_1^2 + \\
& 146767085568 t^2y^2b_1^2 - 101854064148480 t^4y^2 - 26308000088064 t^2y^4 + 472553323757568 t^2y^2 - 83216937517056 t^3y^2 + \\
& 2935341711360 ty^4 + 1640562482479104 ty^2 - 3594887626752 iy^2a_1b_1 - 58119765884928 \sqrt{3}t^2b_1 + 16804831297536 i\sqrt{3}t^3a_1 + \\
& 100388686528512 i\sqrt{3}ta_1 + 58119765884928 i\sqrt{3}t^2a_1 + 89234388025344 i\sqrt{3}y^2a_1 - 13480828600320 ita_1b_1 + 2490453983232 i\sqrt{3}t^4a_1 - \\
& 5609764159488 it^2a_1b_1 + 144473849856 it^5\sqrt{3}a_1 - 82556485632 it^4a_1b_1 + 434865438720 iy^4a_1b_1 + 28991029248 i\sqrt{3}ta_1^3 - \\
& 1027369598976 it^3a_1b_1 - 270582939648 i\sqrt{3}a_1b_1^2 - 608811614208 i\sqrt{3}y^6a_1 + 90194313216 i\sqrt{3}a_1^3 + 63403380965376 i\sqrt{3}a_1 - \\
& 10436770529280 ia_1b_1 + 2935341711360 y^6 - 98738846760960 t^6 + 4631316922368 iy^4\sqrt{3}a_1 + 66387645038592 i\sqrt{3}t^2y^2a_1 + \\
& 19006337581056 it^3\sqrt{3}y^2a_1 + 120153320718336 i\sqrt{3}ty^2a_1 - 293534171136 it^2y^2a_1b_1 - 86973087744 i\sqrt{3}ta_1b_1^2 - \\
& 1826434842624 ity^2a_1b_1 + 185752092672 i\sqrt{3}t^5y^2a_1 + 220150628352 i\sqrt{3}t^3y^4a_1 - 195689447424 i\sqrt{3}ty^6a_1 + 2889476997120 i\sqrt{3}t^4y^2a_1 + \\
& 2054739197952 i\sqrt{3}t^2y^4a_1 - 421285746180096 t^5 - 66387645038592 \sqrt{3}t^2y^2b_1 - 86973087744 \sqrt{3}ta_1^2b_1 - 185752092672 \sqrt{3}t^5y^2b_1 -
\end{aligned}$$

$$\begin{aligned}
& 220150628352 \sqrt{3} t^3 y^4 b_1 + 195689447424 \sqrt{3} t y^6 b_1 - 120153320718336 \sqrt{3} t y^2 b_1 - 4132485216 t^{10} - 130459631616 i y^4 \sqrt{3} a_1 + \\
& 73383542784 y^{10} - 128566206720 t^9 - 63403380965376 \sqrt{3} b_1 + 90194313216 \sqrt{3} b_1^3 - 41278242816 t^4 a_1^2 + 41278242816 t^4 b_1^2 + \\
& 217432719360 y^4 a_1^2 - 217432719360 y^4 b_1^2 - 513684799488 t^3 a_1^2 + 513684799488 t^3 b_1^2 + 2804882079744 t^2 b_1^2 + \\
& 1797443813376 y^2 b_1^2 + 6740414300160 t b_1^2 - 22039921152 t^8 y^2 - 26121388032 t^6 y^4 + 46438023168 t^4 y^6 + 123834728448 t^2 y^8 - \\
& 548549148672 t^7 y^2 - 487599243264 t^5 y^4 + 577895399424 t^3 y^6 + 770527199232 t y^8 - 2889476997120 \sqrt{3} t^4 y^2 b_1 - \\
& 2054739197952 \sqrt{3} t^2 y^4 b_1 - 19006337581056 \sqrt{3} t^3 y^2 b_1 - 4631316922368 \sqrt{3} t y^4 b_1 + 5218385264640 b_1^2 - 16456474460160 t^7 - \\
& 1861148897280 t^8 + 2348273369088 y^8 + 1467670855680 t y^6 - 6740414300160 t a_1^2 - 5886019436544 t^6 y^2 - 4024628674560 t^4 y^4 + \\
& 2311581597696 t^2 y^6 - 2804882079744 t^2 a_1^2 - 1797443813376 y^2 a_1^2 - 5218385264640 a_1^2 - 33899756912640 t^5 y^2 - \\
& 16676410097664 t^3 y^4
\end{aligned}$$

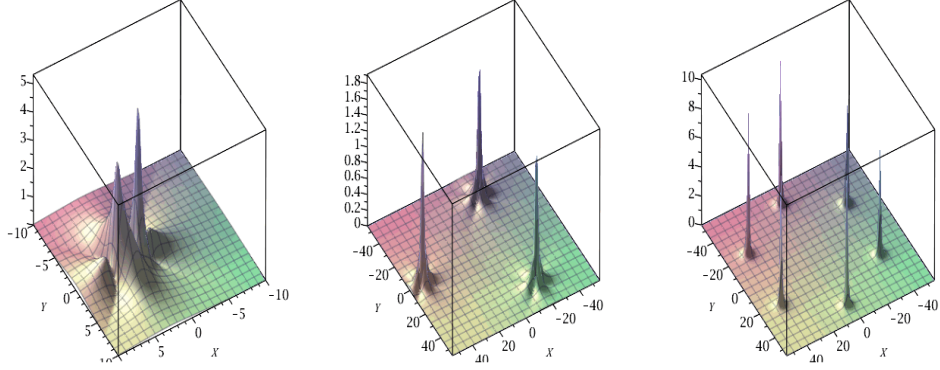
and

$$\begin{aligned}
d(x, y, t) = & -110592 x^6 + (497664 t + 1548288) x^5 + (-933120 t^2 - 331776 y^2 - 5806080 t - 9953280) x^4 + (933120 t^3 + \\
& 995328 t y^2 + 8709120 t^2 + 3096576 y^2 + 29859840 t - 98304 i \sqrt{3} a_1 + 98304 \sqrt{3} b_1 + 35831808) x^3 + (-524880 t^4 - \\
& 1119744 t^2 y^2 - 331776 y^4 - 6531840 t^3 - 6967296 t y^2 - 33592320 t^2 + 221184 i \sqrt{3} t a_1 - 221184 \sqrt{3} t b_1 - 14155776 y^2 - \\
& 80621568 t + 688128 i \sqrt{3} a_1 - 688128 \sqrt{3} b_1 - 71663616) x^2 + (157464 t^5 + 559872 t^3 y^2 + 497664 t y^4 + 2449440 t^4 + \\
& 5225472 t^2 y^2 + 1548288 y^4 + 16796160 t^3 - 165888 i \sqrt{3} t^2 a_1 + 165888 \sqrt{3} t^2 b_1 + 21233664 t y^2 + 294912 i \sqrt{3} y^2 a_1 - \\
& 294912 \sqrt{3} y^2 b_1 + 60466176 t^2 - 1032192 i \sqrt{3} t a_1 + 1032192 \sqrt{3} t b_1 + 34947072 y^2 + 107495424 t - 1572864 i \sqrt{3} a_1 + \\
& 1572864 \sqrt{3} b_1 + 71663616) x - 35831808 - 53747712 t - 2433024 y^4 - 40310784 t^2 - 39813120 y^2 - 104976 t^4 y^2 - \\
& 186624 t^2 y^4 - 7962624 t^2 y^2 - 1306368 t^3 y^2 - 1161216 t y^4 - 26210304 t y^2 - 110592 y^6 - 3149280 t^4 - 15116544 t^3 - \\
& 41472 \sqrt{3} t^3 b_1 - 1179648 \sqrt{3} t b_1 - 387072 \sqrt{3} t^2 b_1 + 688128 \sqrt{3} y^2 b_1 + 1179648 i \sqrt{3} t a_1 + 387072 i \sqrt{3} t^2 a_1 - 688128 i \sqrt{3} y^2 a_1 + \\
& 221184 \sqrt{3} t y^2 b_1 - 221184 i \sqrt{3} t y^2 a_1 - 786432 \sqrt{3} b_1 - 65536 b_1^2 + 65536 a_1^2 - 19683 t^6 - 367416 t^5 + 786432 i \sqrt{3} a_1 + \\
& 131072 i a_1 b_1 + 41472 i \sqrt{3} t^3 a_1.
\end{aligned}$$

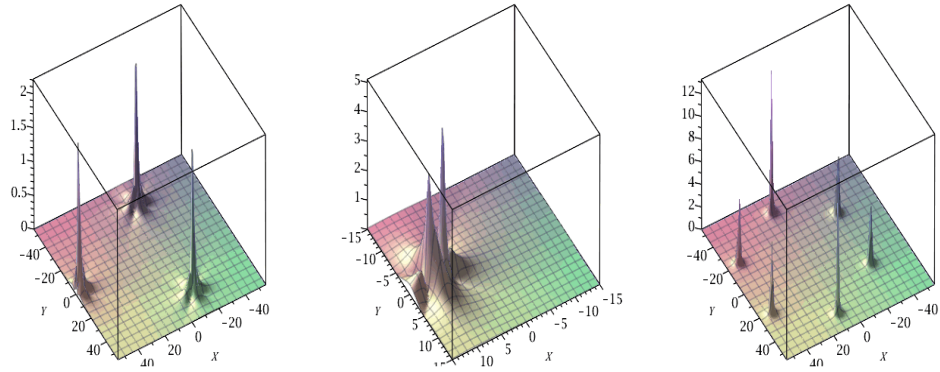
We represent the modulus of the solution for different values of  $t$  in the plane  $(x, y)$ .



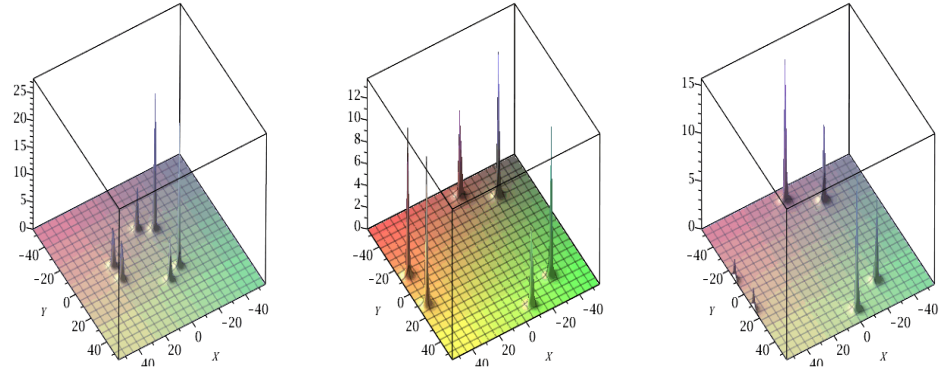
**Figure 2.** Solution of order 2 to KPI for  $t = 0$ , on the left  $a_1 = 0$ ,  $b_1 = 10^3$ ; in the center  $a_1 = 0$ ,  $b_1 = 0$ ; on the right  $a_1 = 10^3$ ,  $b_1 = 0$ .



**Figure 3.** Solution of order 2 to KPI for  $t = 1$ , on the left  $a_1 = 0, b_1 = 0$ ; in the center  $a_1 = 0, b_1 = 10^5$ ; on the right for  $a_1 = 0, b_1 = 10^5$ .



**Figure 4.** Solution of order 2 to KPI, on the left for  $t = 10, a_1 = 0, b_1 = 10^5$ ; in the center for  $a_1 = 0, b_1 = 0$ ; on the right for  $a_1 = 10^5, b_1 = 0$ .



**Figure 5.** Solution of order 2 to KPI, on the left for  $t = 0, a_1 = 10^4, b_1 = 10^4$ ; in the center for  $t = 1, a_1 = 10^5, b_1 = 10^5$ ; on the right for  $t = 10, a_1 = 10^5, b_1 = 0$ .

In this case of order 2, two types of configurations were highlighted.  
 For  $a_1 \neq 0$  and  $b_1 = 0$ , we get a figure with 3 peaks in which height decreases when  $a_1$  grows.  
 Conversely, for  $b_1 \neq 0$  and  $a_1 = 0$ , we see the appearance of 6 peaks on a ring in which height decreases as  $b_1$  grows.  
 For  $a_1 \neq 0$  and  $b_1 \neq 0$ , we see the appearance of 3 couples of peaks on a ring in which height decreases as  $a_1$  and  $b_1$  grow.

## 4 Conclusion

We have constructed rational solutions to KPI equation. To the best of our knowledge, all these solutions and configurations have never been presented before.

We propose ourselves in a future research to give the general method to construct rational solutions to KPI equation at order  $N$ .

We will show that these solutions can be written as a quotient of a polynomial of degree  $2N(N + 1) - 2$  in  $x$ ,  $y$  and  $t$  by a polynomial of degree  $2N(N + 1)$  in  $x$ ,  $y$  and  $t$ , depending on  $2N - 2$  parameters.

The method described in the present paper provides a powerful tool to get explicit solutions to the KPI equation and to understand the behavior of solutions. These solutions are different from those derived from the NLS equation [11, 12, 13, 29].

In both cases, peak modules are observed to decrease very rapidly with time  $t$ . What is new in the study of these solutions is the appearance not yet observed 3 pairs of two peaks in the case of order 2. One can hope to discover other new configurations in the study of solutions for higher orders.

## References

- [1] M.J. Ablowitz, H. Segur, On the evolution of packets of water waves, J. Fluid Mech., V. 92, 691-715, 1979.
- [2] M. Ablowitz, P.A. Clarkson, Solitons, nonlinear evolution equations and inverse scattering, Cambridge University Press, 1991.
- [3] B.A. Dubrovin, Theta functions and non-linear equations, Russian Math. Surveys, V.36, N.2, 11-92, 1981.
- [4] N. C Freeman, J.J.C. Nimmo, Rational solutions of the KdV equation in wronskian form, Phys. Letters, V. 96 A, N. 9, 443-446, 1983.
- [5] N. C Freeman, J.J.C. Nimmo, The use of Bäcklund transformations in obtaining N-soliton solutions in wronskian form, J. Phys. A : Math. Gen., V. 17 A, 1415-1424, 1984.



- [6] P. Gaillard, V.B. Matveev, Wronskian addition formula and its applications, Max-Planck-Institut für Mathematik, MPI 02-31, V. **161**, 2002
- [7] P. Gaillard, A new family of deformations of Darboux-Pöschl-Teller potentials, Lett. Math. Phys., V. **68**, 77-90, 2004
- [8] P. Gaillard, V.B. Matveev, New formulas for the eigenfunctions of the two-particle Calogero-Moser system, Lett. Math. Phys., V. **89**, 1-12, 2009
- [9] P. Gaillard, V.B. Matveev, Wronskian and Casorai determinant representations for Darboux-Pöschl-Teller potentials and their difference extensions, J. Phys A : Math. Theor., V. **42**, 404409-1-16, 2009
- [10] P. Dubard, P. Gaillard, C. Klein, V. B. Matveev, On multi-rogue wave solutions of the NLS equation and positon solutions of the KdV equation, Eur. Phys. J. Spe. Top., V. **185**, 247-258, 2010
- [11] P. Gaillard, Families of quasi-rational solutions of the NLS equation and multi-rogue waves, J. Phys. A : Meth. Theor., V. **44**, 435204-1-15, 2011
- [12] P. Gaillard, Wronskian representation of solutions of the NLS equation and higher Peregrine breathers, J. Math. Sciences : Adv. Appl., V. **13**, N. 2, 71-153, 2012
- [13] P. Gaillard, Degenerate determinant representation of solution of the NLS equation, higher Peregrine breathers and multi-rogue waves, J. Math. Phys., V. **54**, 013504-1-32, 2013
- [14] P. Gaillard, Wronskian representation of solutions of NLS equation and seventh order rogue waves, J. Mod. Phys., V. **4**, N. 4, 246-266, 2013
- [15] P. Gaillard, V.B. Matveev, Wronskian addition formula and Darboux-Pöschl-Teller potentials, J. Math., V. **2013**, ID 645752, 1-10, 2013
- [16] P. Gaillard, Two-parameters determinant representation of seventh order rogue waves solutions of the NLS equation, J. Theor. Appl. Phys., V. **7**, N. **45**, 1-6, 2013
- [17] P. Gaillard, Deformations of third order Peregrine breather solutions of the NLS equation with four parameters, Phys. Rev. E, V. **88**, 042903-1-9, 2013
- [18] P. Gaillard, Two parameters deformations of ninth Peregrine breather solution of the NLS equation and multi rogue waves, J. Math., V. **2013**, 1-111, 2013
- [19] P. Gaillard, Six-parameters deformations of fourth order Peregrine breather solutions of the NLS equation, J. Math. Phys., V. **54**, 073519-1-22, 2013

- [20] P. Gaillard, The fifth order Peregrine breather and its eight-parameters deformations solutions of the NLS equation, Commun. Theor. Phys., V. **61**, 365-369, 2014
- [21] P. Gaillard, Ten parameters deformations of the sixth order Peregrine breather solutions of the NLS equation, Phys. Scripta, V. **89**, 015004-1-7, 2014
- [22] P. Gaillard, Higher order Peregrine breathers, their deformations and multi-rogue waves, J. Of Phys. : Conf. Ser., V. **482**, 012016-1-7, 2014
- [23] P. Gaillard, M. Gastineau, Eighteen parameter deformations of the Peregrine breather of order ten solutions of the NLS equation, Int. J. Mod. Phys. C, V. **26**, N. 2, 1550016-1-14, 2014
- [24] P. Gaillard, Two parameters wronskian representation of solutions of nonlinear Schrödinger equation, eight Peregrine breather and multi-rogue waves, J. Math. Phys., V. **5**, 093506-1-12, 2014
- [25] P. Gaillard, Hierarchy of solutions to the NLS equation and multi-rogue waves, J. Phys. : Conf. Ser., V. **574**, 012031-1-5, 2015
- [26] P. Gaillard, Tenth Peregrine breather solution of the NLS, Ann. Phys., V. **355**, 293-298, 2015
- [27] P. Gaillard, M. Gastineau, The Peregrine breather of order nine and its deformations with sixteen parameters solutions of the NLS equation Phys. Lett. A, V. **379**, 1309-1313, 2015
- [28] P. Gaillard, Other  $2N-2$  parameters solutions to the NLS equation and  $2N+1$  highest amplitude of the modulus of the  $N$ -th order AP breather, J. Phys. A: Math. Theor., V. **48**, 145203-1-23, 2015
- [29] P. Gaillard, Multi-parametric deformations of the Peregrine breather of order  $N$  solutions to the NLS equation and multi-rogue waves, Adv. Res., V. **4**, N. **5**, 346-364, 2015
- [30] P. Gaillard, Higher order Peregrine breathers solutions to the NLS equation, Jour. Phys. : Conf. Ser., V. **633**, 012106-1-6, 2016
- [31] P. Gaillard, M. Gastineau Patterns of deformations of Peregrine breather of order 3 and 4, solutions to the NLS equation with multi-parameters, Journal of Theoretical and Applied Physics, V. **10**,1-7, 2016
- [32] P. Gaillard, M. Gastineau Twenty parameters families of solutions to the NLS equation and the eleventh Peregrine breather, Commun. Theor. Phys, V. **65**, N. **2**, 136-144, 2016
- [33] P. Gaillard, Rational solutions to the KPI equation and multi rogue waves, Annals Of Physics, V. **367**, 1-5, 2016

- [34] P. Gaillard, M. Gastineau Twenty two parameters deformations of the twelfth Peregrine breather solutions to the NLS equation, *Adv. Res.*, V. **10**, 83-89, 2016
- [35] P. Gaillard, Towards a classification of the quasi rational solutions to the NLS equation, *Theor. And Math. Phys.*, V. **189**, N. **1**, 1440-1449, 2016
- [36] P. Gaillard, Fredholm and Wronskian representations of solutions to the KPI equation and multi-rogue waves, *Jour. of Math. Phys.*, V. **57**, 063505-1-13, doi: 10.1063/1.4953383, 2016
- [37] P. Gaillard, M. Gastineau Families of deformations of the thirteenth Peregrine breather solutions to the NLS equation depending on twenty four parameters, *Jour. Of Bas. And Appl. Res. Int.*, V. **21**, N. 3, 130-139, 2017
- [38] P. Gaillard, From Fredholm and Wronskian representations to rational solutions to the KPI equation depending on  $2N^2$  parameters, *Int. Jour. of Appl. Sci. And Math.*, V. **4**, N. 3, 60-70, 2017
- [39] P. Gaillard, Families of rational solutions of order 5 to the KPI equation depending on 8 parameters, *New Hor. in Math. Phys.*, V. 1, N. 1, 26-31, 2017
- [40] P. Gaillard, 6-th order rational solutions to the KPI Equation depending on 10 parameters, *Jour. Of Bas. And Appl. Res. Int.*, V. **21**, N. 2, 92-98, 2017
- [41] P. Gaillard,  $N$ -Order rational solutions to the Johnson equation depending on  $2N - 2$  parameters, *Int. Jour. of Adv. Res. in Phys. Sci.*, V. **4**, N. 9, 19-37, 2017
- [42] P. Gaillard, Families of rational solutions to the KPI equation of order 7 depending on 12 parameters, *Int. Jour. of Adv. Res. in Phys. Sci.*, V. **4**, N. 11, 24-30, 2017
- [43] P. Gaillard, Rational solutions to the Johnson equation and rogue waves, *Int. Jour. of Inn. In Sci. and Math.*, V. **6**, N. 1, 14-19, 2018
- [44] P. Gaillard, Multiparametric families of solutions of the KPI equation, the structure of their rational representations and multi-rogue waves, *Theo. And Mat. Phys.*, V. **196**, N. 2, 1174-1199, 2018
- [45] P. Gaillard, The Johnson equation, Fredholm and wronskian representations of solutions and the case of order three, *Adv. In Math. Phys.*, V. **2018**, 1-18, 2018
- [46] P. Gaillard, Families of Solutions of Order 5 to the Johnson Equation Depending on 8 Parameters, *NHIMP*, V. **2**, N. 4, 53-61, 2018
- [47] P. Gaillard, Multiparametric families of solutions to the Johnson equation, *J. Phys. : Conf. Series*, V. **1141**, 012102-1-10, 2018

- [48] P. Gaillard, Rational solutions to the Boussinesq equation, *Fund. Jour. Of Math. And Appl.*, V. , 109-112, 2019
- [49] P. Gaillard, Differential relations for the solutions to the NLS equation and their different representations, *Comm. In Adv. Math. Sci.*, V. **2**, N. 4, 1-4, 2019
- [50] P. Gaillard, Multi-parametric families of solutions of order N to the Boussinesq and KP equations and the degenerate rational case, *UJMA*, V. **3**, N. 2, 44-52, 2020
- [51] P. Gaillard, The mKdV equation and multi-parameters rational solutions, *Wave Motion*, V. **100**, 102667-1-9, 2021
- [52] P. Gaillard, Degenerate Riemann theta functions, Fredholm and wronskian representations of the solutions to the KdV equation and the degenerate rational case, *Jour. Of Geom. And Phys.*, V. **161**, 104059-1-12, 2021
- [53] P. Gaillard, 2N parameter solutions to the Burgers equation, *Jour. Of Appl. Nonlin. Dyn.*, V. **1**, N. **1**, 1-6, 2021
- [54] P. Gaillard, Multi-Parametric rational Solutions to the KdV equation, *Asi. Jour. Of Res. and Rev. In Phys.*, V. **3**, N. 3, 14-21, 2021
- [55] P. Gaillard, Multiparametric solutions to the Gardner equation and the degenerate rational case, *Jour. Of Appl. Ana. And Appl.*, V. **11**, N. 4, 2102-2113, 2021
- [56] B.B. Kadomtsev, V.I. Petviashvili, On the stability of solitary waves in weakly dispersing media, *Sov. Phys. Dokl.*, V. 15, N. 6, 539-541, 1970.
- [57] I. Krichever, Rational solutions of the Kadomtcev-Petviashvili equation and integrable systems of n particules on a line, *Funct. Anal. and Appl.*, V. 12, N. 1 : 76-78, 1978.
- [58] I. Krichever, S.P. Novikov, Holomorphic bundles over Riemann surfaces and the KPI equation, *Funkt. Ana. E Pril.*, V. 12, 41-52, 1979.
- [59] S.V. Manakov, V.E. Zakharov, L.A. Bordag, V.B. Matveev, Two-dimensional solitons of the Kadomtsev-Petviashvili equation and their interaction, *Phys. Letters*, V. 63A, N. 3, 205-206, 1977.
- [60] V.B. Matveev, Darboux transformation and explicit solutions of the Kadomtcev-Petviaschvily equation depending on functional parameters, *Letters in Mathematical Physics*, V. 3, 213-216, 1979.
- [61] D.E. Pelinovsky, Y.A. Stepanyants, Y.A. Kivshar, Self-focusing of plane dark solitons in nonlinear defocusing media, *Phys. Rev. E*, V. 51, 5016-5026, 1995.

- [62] D.E. Pelinovsky, Y.A. Stepanyants, New multisolitons of the Kadomtsev-Petviashvili equation, *Phys. JETP Lett.*, V. 57, 24-28, 1993.
- [63] D.E. Pelinovsky, Rational solutions of the Kadomtsev-Petviashvili hierarchy and the dynamics of their poles. I. New form of a general rational solution, *J.Math.Phys.*, V. 35, 5820-5830, 1994.
- [64] B. Prinari, Inverse scattering theory for the KP equations, *Thesis*, 1999.
- [65] J. Satsuma, M.J. Ablowitz, Two-dimensional lumps in nonlinear dispersive systems, *J. Math. Phys.*, V. 20, 1496-1503, 1979.