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Pricing Decisions for an Omnichannel Retailing under Service Level Considerations

Minh Tam Tran^{1,2}[0000-0002-7496-9215], Yacine Rekik¹[0000-0002-9412-2310], and
Khaled Hadj-Hamou²[0000-0002-1223-3392]

¹ emlyon business school, 23 avenue Guy de Collongue, Ecully, Lyon 69134, France

² Univ Lyon, INSA Lyon, Université Claude Bernard Lyon 1, Univ Lumière Lyon 2,
DISP, EA4570, 69621 Villeurbanne, France

Abstract. An increasing number of retailers are presently moving to omnichannel configurations and embracing modern innovations to integrate the physical store and the online store to provide customers a comprehensive shopping experience. We develop a classical newsvendor model where a retailer buys items from a supplier and distributes them through two market segments, online vs. offline. We seek optimal prices for the product in the two channels under the newsvendor model with a single period, price-based stochastic demand, and cycle service level-based order quantity to maximize the retailer's profit. Motivated by market share models often used in marketing, we focus on a demand model involving multiplicative uncertainty and interaction between the two sales channels. The pricing problem arising is not to be well behaved because it is difficult to verify the joint concavity in prices of the objective function's deterministic version. However, we find that the objective function is still reasonably well behaved within the sense that there is a unique solution for our optimal problem. We observe such a situation through the visualization graphs in bounded conditions for prices and find the approximate optimal point.

Keywords: Inventory control · Newsvendor · Stochastic demand · Pricing · Omnichannel.

1 Introduction

E-commerce has shown impressive growth in the past decade in all the principal markets. In 2020, the global retail e-commerce sales amounted to 4.28 trillion US dollars and were projected to grow to 5.4 trillion US dollars in 2022. Online shopping is one of the most popular online activities worldwide [1].

This paper deals with demand modeling, stock management, and pricing under omnichannel configuration. Indeed, omnichannel also opens numerous modern challenges for price optimization. Classical retail pricing models augment channels' cost upon the presumption that there's no stock sharing and integration between channels. This inference does not hold in omnichannel where physical store's inventory is also utilized for matching the online client's orders. Another challenge is by cause of potential substitution of demand between the online and offline channels, which is influenced by the prices associated with them. Channel substitution is rejected in classical earnings management models, who admit that price only influences demand within the same channel. In

spite of these difficulties owing to the omnichannel environment, numerous retailers apply the classical price optimization frameworks that don't take into account channel interdependencies.

In literature, the interest in studying critical subjects relating to omnichannel retailing and pricing has progressively expanded, with a rapid increase in the contributions of academic experts covering this aspect ([2], [3]). Moreover, omnichannel retailing seems to be a promising stream for future research ([4]). Only a few literature reviews are already available that specifically analyze the distribution and optimal pricing problems faced by retailers selling items both online and through physical channels.

To formulate an omnichannel setting, a formal strategy is to extend the single channel pricing model. Concerning the optimal inventory levels and prices in a single channel, we refer them to the pricing newsvendor problem. Whittin [5], Mills [6], and Karlin [7] proposed the formulations of price-setting newsvendor problem for a single product and identified its optimal solutions. The unimodality of the objective function under certain assumptions on the statistical distribution was proved in [8], which was later extended by Petruzzi and Dada [9] on the price-dependent newsvendor problem. Aydin and Porteus [10] built on previous investigations by adding customer choice among multiple substitutable products through modeling demands for competing products by the multinomial logit demand model. Kocabiyikoglu and Popescu [11] studied the newsvendor problem with a more general demand function and derived general conditions for the unimodality of the objective function. The effect of demand uncertainty in a price-setting newsvendor model was studied in [12]. Kyparisis and Koulamas [13] investigated the price-setting newsvendor problem with nonnegative linear additive demand and showed that the problem still possesses an optimal solution and analyzed when the expected profit function is quasiconcave in price so that a unique optimal solution can be found. Given the nature of the subject, these reviews are somewhat ancient. They discuss neither multichannel systems, the correlation across channels in an omnichannel configuration, nor the impact of cycle service levels on the pricing problem. It leads to a research question on how to develop omnichannel modeling under these constraints.

Motivated by the prevailing omnichannel retailing practices and unexplored issues regarding customer service level, we utilize a newsvendor model to explore the impacts of customer service level on the retailer's pricing and ordering decisions considering each channel. Specifically, a retailer sells products to consumers via online and traditional stores, and the retailer maximizes the total profit of both channels. Our framework is inspired by the price-dependant demand model suggested by [2] in order to illustrate the integrated stock-pricing decision under an omnichannel configuration. Hence, we use attraction demand models to represent consumer choice across various channels when that the total market faces uncertainty. In contrast to [2], we consider a stochastic assumption for the attraction model where the global market facing the two channels is uncertain, and we adapt this model to the newsvendor configuration by trading-off underage and overage costs. [10] demonstrate the analytical complexity of the attraction model

through a single period inventory and pricing problem. The authors show that the objective function is not necessarily jointly quasi-concave in prices, even for deterministic demand. We show in this paper that the omnichannel attraction demand model suggested by [2] can lead to poor service levels for one or both channels. Such as poor service level resulting from the integrated pricing-stock decision is conflicting with the practitioners objective to offer a better service level to customers through the omnichannel experience. Our paper revisits the attraction model for the newsvendor by integrating the service level in the optimization process. We now summarize the main contributions of this paper:

- Omnichannel demand modeling and optimization framework: We introduce an omnichannel demand modeling configuration over which a series of advanced omnichannel retail analytical solutions can be built upon.
- Omnichannel price optimization: We study optimal pricing behavior within a given model and conditions on price and order quantity.

The rest of our study is structured as follows. The next section provides the problem formulation. The third section reports and discusses the results.

2 Problem formulation

Consider a supply chain consists of a supplier, a retailer, and clients. The retailer purchases items from the supplier and sells them to clients through two channels: online and physical stores. At the beginning of the selling season, the retailer is interested in determining the optimal order quantities and the optimal prices corresponding to two channels. Let I be the set of channels' indexes containing index 1 for online channel and 2 for physical channel. For each channel i , we let $r = [r_1, r_2]$ be the vector of the unit retail selling prices and s_i the unit salvage value ($s_i < r_i$). The demand for each channel is assumed to be stochastic characterized by a random variable $D_i(r)$ depending on the vector of prices. The order quantity Q_i for each channel is purchased by the retailer from the supplier on a fixed cost per unit, including operating expense c_i ($s_i < c_i < r_i$). The supplier is assumed to operate with no capacity restrictions, and an order placed by the buyer with the supplier is immediately filled. We denote Π_i the expected profit induced by the channel i .

2.1 Formulation of the demand distribution

We use the attraction demand functions to model the channel choice of consumers. We assume the demand $D_i(r)$ in each channel i has the following form

$$D_i(r) = \xi \frac{g_i(r_i)}{g_0 + \sum_i g_i(r_i)}, \quad (1)$$

where ξ is the total market size defined with the cumulative distribution function $F_\xi(x)$ and the probability density function $f_\xi(x)$, $g_i(r_i)$ is the attraction function of customers to channel i , g_0 is the attraction function of the non-purchase option usually assumed positive.

The market size represents the measure of consumers interested in the product and the market share, also generally referred to the purchase probability, shows how consumers prefer one through several channels. The attraction function depending on price is used to model the market share through channels. There are some well-known attraction models: linear attraction $g_i(r_i) = a_i - b_i r_i$ ($a_i, b_i, \min a_i - b_i r_i > 0$), multinomial logit (MNL) $g_i(r_i) = \exp(a_i - b_i r_i)$ ($a_i > 0$) and multiplicative competitive interaction (MCI) $g_i(r_i) = a_i r_i^{-b_i}$ ($a_i > 0, b_i > 1$). The constants a_i and b_i are assumed to satisfy the negative price elasticity of demand. This attraction demand is used by [2] to model the omnichannel setting. The authors consider the case when ξ is a positive deterministic scalar. In order to measure of market's uncertainty, we assume that ξ is modeled by a continuous random variable. In this scheme, we see that the channel i 's demand depends on both the prices r on two channels. Thus, the clients' demand in channel i is a continuous random variable $D_i(r) = \xi G_i(r)$, with

$$G_i(r) = \frac{g_i(r_i)}{g_0 + \sum_i g_i(r_i)}. \quad (2)$$

2.2 Formulation of the objective function

Before the sales period, the retailer favors determining order quantities Q_i , and then finding the related prices r_i corresponding to each channel i to maximize the expected total profit. Sales of the item occur at the end of selling season and: (i) if $Q_i \geq D_i(r)$, then $Q_i - D_i(r)$ units which are left over at the end of the period are salvaged by the retailer for a per unit revenue of s_i ; and (ii) if $Q_i < D_i(r)$, then $D_i(r) - Q_i$ units which represent lost sales cost the buyer zero per unit. Then, the actual end of period profit for the retailer is:

$$\Pi_i(D_i(r), Q_i, r_i) = (r_i - s_i)D_i(r) - (c_i - s_i)Q_i - (r_i - s_i)[D_i(r) - Q_i]^+, \quad (3)$$

$$\Pi_i(\xi, Q_i, r_i) = (r_i - s_i)G_i(r)\xi - (c_i - s_i)Q_i - (r_i - s_i)[G_i(r)\xi - Q_i]^+. \quad (4)$$

Thus, assuming $G_i(r) > 0$, the profit within the channel i is:

$$\Pi_i(\xi, Q_i, r_i) = (r_i - s_i)G_i(r)\xi - (c_i - s_i)Q_i - (r_i - s_i)G_i(r) \left[\xi - \frac{Q_i}{G_i(r)} \right]^+. \quad (5)$$

Since the demand has not been realized at the beginning of the selling season, the retailer cannot observe the actual profit. Hence, the traditional approach to analyze the problem is based on assuming a risk-neutral retailer who makes the optimal pricing decision at the beginning of the sales season to maximize total expected profit.

The expected profit within the channel i is:

$$\Pi_i(Q_i, r_i) = (r_i - s_i)G_i(r)E[\xi] - (c_i - s_i)Q_i - (r_i - s_i)G_i(r) \int_{\frac{Q_i}{G_i(r)}}^{\infty} \left(x - \frac{Q_i}{G_i(r)} \right) f_\xi(x) dx. \quad (6)$$

The expected profit for all channels is $\Pi(Q, r) = \sum_i \Pi_i(Q_i, r_i)$.

Induced profit function Π is a function of Q_i and r_i , it is separable and concave in the Q_i . Then, for any price vector r , the optimal order quantity in channel i , denoted $Q_i^*(r)$, is given by (see appendix I for the proof)

$$Q_i^*(r) = G_i(r) F_\xi^{-1} \left(\frac{r_i - c_i}{r_i - s_i} \right). \quad (7)$$

Let Q^* be the vector of all Q_i^* , the problem can be rewritten as maximizing the induced profit function below (see appendix II for the proof):

$$\Pi(Q^*, r) = \sum_i \left((r_i - s_i) G_i(r) \int_{-\infty}^{\frac{Q_i^*(r)}{G_i(r)}} x f_\xi(x) dx \right). \quad (8)$$

Let CSL_i^* be the induced cycle service level to channel $i \in I$ when the order quantity is set to Q_i^* , defined as

$$CSL_i^* = P(D_i(r) \leq Q_i^*(r)) = P \left(\xi \leq \frac{Q_i^*(r)}{G_i(r)} \right) = F_\xi \left(\frac{Q_i^*(r)}{G_i(r)} \right) = \frac{r_i - c_i}{r_i - s_i}. \quad (9)$$

Numerical Application: Let us consider the numerical setting and values considered in [10]: a pricing problem with stochastic logit demand in which the online (resp. physical) channel demand is characterised by $a_1 = 10$ and $b_1 = 1$ (resp. $a_2 = 25$ and $b_2 = 1$). We assume the non-purchased option normalised with the parameter $g_0 = 1$. With the higher parameter a_2 , the physical channel has more selling potential. We set the unit purchase costs $c_1 = 6$ and $c_2 = 20$. Here, the smaller value of c_1 comparing to c_2 tells that the total purchase cost and operation cost for the online channel is lower than the physical's one. The unit salvage costs are $s_1 = 4$ and $s_2 = 15$. We consider a uniform distribution for the global market demand with two settings: highly variable demand $\xi \sim U(100, 900)$ and low variable demand $\xi \sim U(400, 600)$.

The Table 1 represents the expectation and variation coefficient of total market (μ, σ) , online channel (μ_1^*, σ_1^*) , and physical channel (μ_2^*, σ_2^*) given the optimal prices r_1^*, r_2^* . CV is the coefficient of variation for both of them. $G_i^*, Q_i^*, CSL_i^*, PR_i$, and Π_i^* are the market sharing ratio, order quantity, service level, penalty ratio (the ratio between shortage penalty ($k_i = r_i - c_i$) and the overstock penalty ($h_i = c_i - s_i$), and profit induced from channel i under the optimal pricing. Π^* is the maximum total profit.

When the shortage cost is close to the overstock cost, the optimal order quantity is close to the mean of the demand. The safety stock is close to 0. This corresponds to channel 2 with two penalty ratios 0.95 and 0.85. If $k_i > h_i$, then the order quantity is higher than the demand expectation; thus the safety stock $Q_i^* - \mu_i$ is positive. This corresponds to channel 1 with two penalty ratios equal 1.98 and 2.05. Comparing two given cases, the optimal prices are close. In the higher deviation setting, the order quantities, the market sharing ratios, and the induced profits in the two channels are also close, which is false in the other case with a dominant channel. The more the market risk there is, the lower the maximum profit. In both cases, we have not very high service levels for both two channels. Moreover, since the shortage penalty doubles the overstock penalty of channel 1, the service level tends to be higher in order to avoid shortages.

Table 1. Characteristics of optimal pricing solution for induced profit function

ξ	μ	σ	μ_1^*	σ_1^*	μ_2^*	σ_2^*	CV	r_1^*	r_2^*
$U(100, 900)$	500	230.94	155.25	71.71	195.27	90.19	0.46	9.96	24.73
$U(400, 600)$	500	57.74	112.73	13.02	261.84	30.23	0.12	10.11	24.26

ξ	G_1^*	G_2^*	Q_1^*	Q_2^*	CSL_1^*	CSL_2^*	PR_1	PR_2	Π_1^*	Π_2^*	Π^*
$U(100, 900)$	0.31	0.39	196.13	190.98	0.66	0.48	1.98	0.95	450.05	544.38	994.43
$U(400, 600)$	0.23	0.52	120.51	257.68	0.67	0.46	2.05	0.85	432.64	996.31	1428.95

Cycle service level criteria for quantity ordering Instead of using the quantity order as the critical fractile solution, we can initially set the order quantity by the cycle service level. Let CSL_i be the target cycle service level to channel $i \in I$ when the order quantity is set to \hat{Q}_i defined as $P(D_i \leq \hat{Q}_i)$. By definition, the related order quantity \hat{Q}_i satisfies

$$CSL_i = P(D_i \leq \hat{Q}_i) = P\left(\xi \leq \frac{\hat{Q}_i}{G_i(r)}\right) = F_\xi\left(\frac{\hat{Q}_i}{G_i(r)}\right), \quad (10)$$

$$\text{thus } \hat{Q}_i = G_i(r)F_\xi^{-1}(CSL_i). \quad (11)$$

The profit function is now as below (see the appendix III for the proof):

$$\Pi(\hat{Q}, r) = \sum_i \left((r_i - c_i)\hat{Q}_i - (r_i - s_i)CSL_i\hat{Q}_i + (r_i - s_i)G_i(r) \int_{-\infty}^{\frac{\hat{Q}_i}{G_i(r)}} x f_\xi(x) dx \right). \quad (12)$$

3 Characterization of objective function: a case of study

For comparison purposes, let us consider the same setting for parameters as the numerical application for the induced profit function in the previous section. The Fig. 1 shows the shape of profit function varying with prices within high and low variation market. The characteristics of optimal pricing solutions are given in Table 2. Table 3 shows the dynamic of approximate optimal prices and profit with a set of different targeted $CSLs$. Comparing with induced optimal profit in the first numerical application, our new formulation improves the $CSLs$ from (0.66, 0.48), (0.67, 0.46) to the targeted values (0.90, 0.90) on both low and high variation market. However, the profit is degraded from 994.43 to 777.73 and from 1428.95 to 1334.32, respectively. Table 3 illustrates the results for other different targeted values of the two $CSLs$. In Table 2, within two schemes, the optimal prices are not very much different, the same situation as Table 1.

The $CSLs$ setting and boundary conditions on prices can cause the negativity on the total profit. The global market uncertainty also has a negative impact on this value. However, it is not necessary to balance the two channels in the high demand variability case. One channel can dominate the other in order to secure the service level and maximise profit. It can be seen that if the global market is more uncertain, then the maximum total profit is lower, but each channel' profit

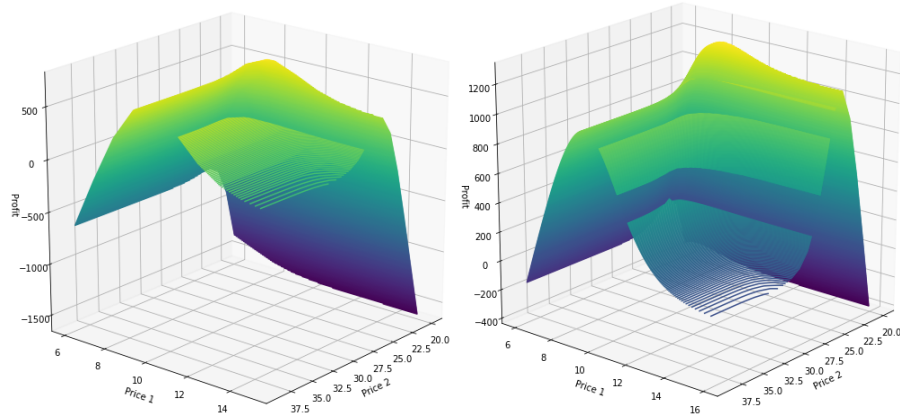
Table 2. Characteristics of optimal pricing solution under service level constraints

ξ	μ	σ	$\hat{\mu}_1^*$	$\hat{\sigma}_1^*$	$\hat{\mu}_2^*$	$\hat{\sigma}_2^*$	CV	\hat{r}_1^*	\hat{r}_2^*
$U(100, 900)$	500	230.94	220.80	101.98	84.54	39.05	0.46	9.87	25.83
$U(400, 600)$	500	57.74	136.27	15.73	227.60	26.28	0.12	10.00	24.49

ξ	\hat{G}_1^*	\hat{G}_2^*	\hat{Q}_1^*	\hat{Q}_2^*	CSL_1	CSL_2	PR_1	PR_2	$\hat{\Pi}_1^*$	$\hat{\Pi}_2^*$	$\hat{\Pi}^*$
$U(100, 900)$	0.44	0.17	362.11	138.65	0.9	0.9	1.94	1.17	562.38	215.35	777.73
$U(400, 600)$	0.27	0.46	158.07	264.02	0.9	0.9	2.00	0.90	499.69	834.63	1334.32

is not necessarily decreasing. Although the shortage penalty related to the first channel still doubles the overstock penalty, the service level is high as 0.9.

Fig. 1. The expected profit as a function of the prices, where the quantity order satisfies a constraint in service with the market’s high and low variation, resp.



4 Conclusion

In this paper, we proposed and investigated a pricing problem in an omnichannel scheme using the newsvendor model with a price-dependent stochastic demand and cycle service level-based order quantity. Under a demand model coming from an attractive interpretation of clients’ preference over the sales channels, our model adapts well within the nature of management. Within a set of boundary conditions, there exists a unique optimal price vector, which can be calculated numerically (by gradient descent algorithm). Being aware of the high level of customer expectations in omnichannel settings, we firstly showed the limitation of only considering profit maximization, and we then proposed a new formulation of the attraction model by considering service level considerations. Our model is applicable in various types of product: private goods, consumer goods, etc. For future research, one line of analysis is to extend the results to other demand models. In the case study, we consider the uniform market, one selling season, and one stage supply chain structure. It would therefore be interesting to improve our model under multiple ordering opportunities and dynamic pricing.

Table 3. Optimal prices and profit for given service levels

CSL		$\xi \sim U(100, 900)$			$\xi \sim U(400, 600)$		
CSL_1	CSL_2	\hat{r}_1^*	\hat{r}_2^*	$\Pi(\hat{Q}^*, \hat{r}^*)$	\hat{r}_1^*	\hat{r}_2^*	$\Pi(\hat{Q}^*, \hat{r}^*)$
0.7	0.5	9.98	24.73	993.22	10.11	24.27	1427.73
0.7	0.7	9.86	25.13	937.44	10.05	24.35	1401.78
0.7	0.9	9.66	25.96	841.80	9.94	24.50	1343.35
0.8	0.5	10.09	24.70	981.70	10.13	24.26	1425.63
0.8	0.7	9.96	25.10	922.19	10.08	24.35	1399.43
0.8	0.9	9.75	25.92	819.25	9.96	24.50	1340.43
0.9	0.5	10.24	24.65	959.10	10.17	24.26	1420.96
0.9	0.7	10.11	25.04	893.01	10.12	24.34	1394.30
0.9	0.9	9.87	25.83	777.73	10.00	24.49	1334.32

The decentralized supply chain case would be an interesting research direction to investigate the supply chain (un)coordination and its impact on the pricing strategies for the different sales channels.

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Appendix

Appendix I

Proof. The expected profit within the channel i is:

$$\Pi_i(Q_i, r_i) = (r_i - s_i)G_i(r)E[\xi] - (c_i - s_i)Q_i - (r_i - s_i)G_i(r) \int_{\frac{Q_i}{G_i(r)}}^{\infty} \left(x - \frac{Q_i}{G_i(r)}\right) f_\xi(x) dx.$$

To find the order quantity Q_i^* that maximizes the expected profit associated to the channel i within a given prices r_i , we compute the derivative of $\Pi_i(Q_i, r_i)$:

$$\begin{aligned} \frac{\partial \Pi_i(Q_i, r_i)}{\partial Q_i} &= -(c_i - s_i) + (r_i - s_i)G_i(r) \frac{1}{G_i(r)} \int_{\frac{Q_i}{G_i(r)}}^{\infty} f_\xi(x) dx \\ &= -(c_i - s_i) + (r_i - s_i) \left(1 - F_\xi\left(\frac{Q_i}{G_i(r)}\right)\right) \\ &= (r_i - c_i) - (r_i - s_i)F_\xi\left(\frac{Q_i}{G_i(r)}\right). \end{aligned}$$

In addition, the second derivative is negative:

$$\begin{aligned} \frac{\partial^2 \Pi_i(Q_i, r_i)}{\partial Q_i^2} &= -(r_i - s_i) \frac{\partial F_\xi\left(\frac{Q_i}{G_i(r)}\right)}{\partial Q_i} \\ &= -(r_i - s_i) \frac{1}{G_i(r)} f_\xi\left(\frac{Q_i}{G_i(r)}\right) < 0. \end{aligned}$$

The function $\Pi_i(Q_i, r_i)$ is therefore concave and is minimal if and only if:

$$\frac{\partial \Pi_i(Q_i, r_i)}{\partial Q_i} = 0 \Leftrightarrow (r_i - c_i) - (r_i - s_i)F_\xi\left(\frac{Q_i}{G_i(r)}\right) = 0 \Leftrightarrow F_\xi\left(\frac{Q_i^*(r)}{G_i(r)}\right) = \frac{r_i - c_i}{r_i - s_i}.$$

This proves the equation 7.

Appendix II

Proof. The induced profit function for the channel i :

$$\begin{aligned} \Pi_i(Q_i^*, r) &= (r_i - s_i)G_i(r)E[\xi] - (c_i - s_i)Q_i^*(r) - (r_i - s_i)G_i(r) \int_{\frac{Q_i^*(r)}{G_i(r)}}^{\infty} \left(x - \frac{Q_i^*(r)}{G_i(r)}\right) f_\xi(x) dx \\ &= (r_i - s_i)G_i(r)E[\xi] - (c_i - s_i)Q_i^*(r) \\ &\quad - (r_i - s_i)G_i(r) \left(E[\xi] - \int_{-\infty}^{\frac{Q_i^*(r)}{G_i(r)}} x f_\xi(x) dx - \frac{Q_i^*(r)}{G_i(r)} \left(1 - F_\xi\left(\frac{Q_i^*(r)}{G_i(r)}\right)\right) \right) \\ &= -(c_i - s_i)Q_i^*(r) + (r_i - s_i)G_i(r) \int_{-\infty}^{\frac{Q_i^*(r)}{G_i(r)}} x f_\xi(x) dx + (r_i - s_i)Q_i^*(r) \\ &\quad - (r_i - s_i)Q_i^*(r)F_\xi\left(\frac{Q_i^*(r)}{G_i(r)}\right) \end{aligned}$$

$$\begin{aligned}
&= (r_i - c_i)Q_i^*(r) + (r_i - s_i)G_i(r) \int_{-\infty}^{\frac{Q_i^*(r)}{G_i(r)}} x f_\xi(x) dx - (r_i - s_i)Q_i^*(r) \frac{r_i - c_i}{r_i - s_i} \\
&= (r_i - s_i)G_i(r) \int_{-\infty}^{\frac{Q_i^*(r)}{G_i(r)}} x f_\xi(x) dx.
\end{aligned}$$

Since $\Pi(Q^*, r) = \sum_i \Pi_i(Q_i^*, r)$, the equation 8 holds.

Appendix III

Proof. For each index i , we have

$$\begin{aligned}
\int_{\frac{\hat{Q}_i}{G_i(r)}}^{\infty} \left(x - \frac{\hat{Q}_i}{G_i(r)} \right) f_\xi(x) dx &= \int_{\frac{\hat{Q}_i}{G_i(r)}}^{\infty} x f_\xi(x) dx - \frac{\hat{Q}_i}{G_i(r)} \int_{\frac{\hat{Q}_i}{G_i(r)}}^{\infty} f_\xi(x) dx \\
&= E[\xi] - \int_{-\infty}^{\frac{\hat{Q}_i}{G_i(r)}} x f_\xi(x) dx - \frac{\hat{Q}_i}{G_i(r)} \left(1 - \int_{-\infty}^{\frac{\hat{Q}_i}{G_i(r)}} f_\xi(x) dx \right) \\
&= E[\xi] - \int_{-\infty}^{\frac{\hat{Q}_i}{G_i(r)}} x f_\xi(x) dx - \frac{\hat{Q}_i}{G_i(r)} \left(1 - F_\xi \left(\frac{\hat{Q}_i}{G_i(r)} \right) \right).
\end{aligned}$$

Thus

$$\begin{aligned}
\Pi_i(\hat{Q}_i, r) &= (r_i - s_i)G_i(r)E[\xi] - (c_i - s_i)\hat{Q}_i - (r_i - s_i)G_i(r) \int_{\frac{\hat{Q}_i}{G_i(r)}}^{\infty} \left(x - \frac{\hat{Q}_i}{G_i(r)} \right) f_\xi(x) dx \\
&= (r_i - s_i)G_i(r)E[\xi] - (c_i - s_i)\hat{Q}_i \\
&\quad - (r_i - s_i)G_i(r) \left(E[\xi] - \int_{-\infty}^{\frac{\hat{Q}_i}{G_i(r)}} x f_\xi(x) dx - \frac{\hat{Q}_i}{G_i(r)} \left(1 - F_\xi \left(\frac{\hat{Q}_i}{G_i(r)} \right) \right) \right) \\
&= -(c_i - s_i)\hat{Q}_i + (r_i - s_i)G_i(r) \int_{-\infty}^{\frac{\hat{Q}_i}{G_i(r)}} x f_\xi(x) dx + (r_i - s_i)\hat{Q}_i - (r_i - s_i)\hat{Q}_i F_\xi \left(\frac{\hat{Q}_i}{G_i(r)} \right) \\
&= (r_i - c_i)\hat{Q}_i - (r_i - s_i)CSL_i \hat{Q}_i + (r_i - s_i)G_i(r) \int_{-\infty}^{\frac{\hat{Q}_i}{G_i(r)}} x f_\xi(x) dx.
\end{aligned}$$

Since $\Pi(\hat{Q}, r) = \sum_i \Pi_i(\hat{Q}_i, r)$, the equation 12 is proved.