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Characterization and suppression of the hydrodynamic instability in the time domain for acoustic propagation in a lined flow duct

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Abstract

The gradient term suppression (GTS) method for removing the hydrodynamic instability appearing in the time-domain solutions of the linearized Euler equations (LEE) along a lined flow duct is assessed. For this, the characterization of a convective instability in the time domain, with the aid of a complementary modal analysis, is first presented. The effect of the mesh size and spatial filtering on the instability is investigated. In particular, a convergence of the instability in the time domain is achieved for a small enough grid size. The consequence of suppressing the mean flow gradient term on the modes is then investigated. It is shown that the unstable modes are indeed removed, but also that acoustic modes are significantly modified, especially for low Helmholtz numbers. The GTS method is finally applied to the NASA grazing impedance tube benchmark. It is found that tuning the weight of the mean flow gradient term within the LEE can be effective for suppressing the instability while conserving a reasonable accuracy of the acoustic component.

Keywords: Lined flow duct, hydrodynamic instability, removing of instability, time domain

1. Introduction

Acoustic liners are widely used in the intake or bypass of the aero-engines to attenuate the noise generated by airplanes. To study their interaction with acoustic waves in the presence of a mean flow, time-domain approaches, based on the linearized Euler equations (LEE), are well-suited, as broadband results can be obtained with a single simulation [1, 2, 3]. In addition, extension to account for nonlinear effects due to large sound pressure levels can be considered. Time-domain approaches are however especially sensitive to instabilities, which can contaminate the pressure field and complicate the extraction of the acoustic component.

The presence of an instability over acoustic liners was first detected experimentally through the transmission coefficient [4, 5, 6, 7]. The transmission coefficient became larger than 1 around the liner resonance at sufficiently high Mach numbers, implying a source of acoustic energy. The source was rightfully attributed to an instability. Optical flow measurements [8, 9] led to a full spatial characterization of the instability, and revealed its convective nature.

The use of the Ingard-Myers boundary condition [10, 11], which describes how acoustic perturbations interact with an impedance wall for a vanishingly-thin inviscid boundary layer, was the

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16 usual approach from the early numerical and theoretical works on the subject. By doing so, in-
17 stabilities were observed in many studies [12, 13, 1, 14, 15, 16]. Later, Brambley [17] showed that
18 the Ingard-Myers boundary condition is mathematically ill-posed in the time domain because the
19 model supports unstable modes of unlimited exponential growth at arbitrarily short wavelengths.
20 He [18] has shown later theoretically that accounting for a boundary layer thickness in the bound-
21 ary condition (modified Ingard-Myers condition) regularizes the problem. Since many studies have
22 questioned the validity of the Ingard-Myers boundary condition [19, 20], the use of a finite boundary
23 layer thickness has been explored to improve the modeling of the instabilities [21, 22, 23, 24, 25, 26].
24 Detailed numerical analysis with a mass-spring-damper impedance model showed that the use of a
25 mean velocity profile instead of the Ingard-Myers condition transformed the unstable mode from
26 absolute to convective [27]. However, even with a full velocity profile, absolute instabilities seems
27 to be present in time-domain simulations for liners with a more realistic broadband impedance [28].

28 The thermoviscous effects [29] or turbulent dissipation through an eddy viscosity profile [30,
29 31, 32] are capable of attenuating the instability. It can even lead the nature of instability to
30 change from absolute to convective [30, 29]. Including turbulent dissipation, the properties of the
31 instability appear to be close to the experimental results. More recently, numerical simulations of
32 a channel flow with an acoustic liner using implicit large eddy simulations [33] were performed.
33 Surface waves with characteristics similar to the instability predicted by linear stability analysis
34 were exhibited. However, very fine grids, and even tremendous computational resources for direct
35 numerical simulations, are needed in those studies to properly capture the instability, which is a
36 strong disadvantage if the goal of the simulations is the computation of the acoustic field.

37 Various strategies to attenuate the instability numerically in time-domain simulations have been
38 explored in the past. Since it is observed that the instability often appears in fine grids, the use of
39 coarse grids and artificial damping has been recommended to attain a stable simulation by some
40 authors. In particular, the studies of Gabard and Brambley [27, 34] suggest that the origin of the
41 absolute instability lies in the dispersion properties of the finite-difference schemes, and they proved
42 the effectiveness of numerical filtering at the boundary to avoid spurious numerical instabilities.
43 Marx [25] examined the case of a convective instability in the time-domain, and assessed the impact
44 of selective filtering and grid size. He considers that if the instability is physical, increasing the
45 grid size or strengthening the selective filters do not necessarily attenuate the instability.

46 Another approach, widely used in flows reminiscent of shear instabilities, is the substitution of
47 the underlying LEE by a new set of equations which are inherently stable. Among them we can
48 mention the gradient term suppression method (GTS) [35, 36, 37, 38, 39], the acoustic perturbation
49 equations [40], the linearized perturbed compressible equations [41, 42], or the gradient term filtering
50 method [43, 44]. These methods have been applied to deal with Kelvin-Helmholtz (KH) instabilities
51 in various cases with shear flows, such as sound radiation through two-dimensional shear layers and
52 sound radiation from a bypass duct [44, 36]. It has been shown that most of them are similarly
53 effective for these particular flows [44].

54 To the authors' knowledge, such methods have not been employed yet for the suppression of
55 instabilities generated in a lined flow duct. It is worth mentioning that the mechanism for these
56 instabilities, while related to KH instabilities, is different. Specifically, the instabilities in a lined
57 flow duct are related to the presence of acoustic liners and not only due to the shear flow. Among
58 the methods existing in the literature for KH instabilities, we have opted to restrict ourselves to
59 GTS in this study, as a first approach to the case of acoustic liners. However, other methods might
60 perform similarly or even better in certain cases, so a future comparative study would certainly be

61 useful.

62 The objective of the paper is to analyze the effectiveness of the GTS method in suppressing
 63 the hydrodynamic instability that appears in the time-domain simulations of acoustic propagation
 64 along a lined duct. For this, it is beforehand verified that the instability is not of numerical origin
 65 and is a part of the LEE solution and that its characteristics are in good agreement with the
 66 predictions of a reference modal analysis, following Marx [25]. In particular, the impact of the grid
 67 resolution and the amount of selective filtering on instabilities are analyzed.

68 The paper is organized as follows. The configuration of the model, the governing equations and
 69 the numerical schemes are presented in Section 2. An example of an instability in the time-domain
 70 approach is shown and is characterized in Section 3. In Section 4, the effect of the selective filter
 71 and mesh size on the instability is assessed. Finally, in Section 5, the performance of the partial
 72 GTS method to suppress the instability is analyzed, and is applied on the NASA GIT benchmark
 73 problem.

74 2. Numerical models

75 A two-dimensional partially lined duct, of size $H \times L$, is considered, as shown in Fig. 1. The
 76 acoustic liner is installed on the upper wall, while the lower wall is rigid. A point source is fixed
 77 in the center of the duct. The source is impulsive, unless stated otherwise. A parallel shear flow of
 78 velocity profile u_0 goes from left to right, and vanishes on the walls. ρ_0 is the mean density, and c_0
 79 is the sound speed. The physical quantities with dimensions of length, time, velocity, and pressure
 80 are nondimensionalized using H , H/c_0 , c_0 and $\rho_0 c_0^2$, respectively.

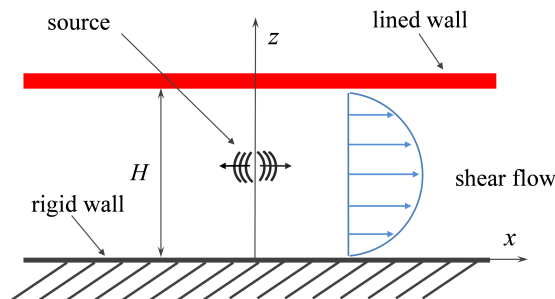


Fig. 1. Numerical configuration.

81 2.1. Governing equations

Acoustic propagation in a lined flow duct is governed by the LEE. Assuming the mean flow is homentropic and neglecting the gradient of mean pressure, the LEE in dimensionless form are written as:

$$\frac{\partial p}{\partial t} + u_0 \frac{\partial p}{\partial x} + \frac{\partial u}{\partial x} + \frac{\partial v}{\partial z} = Q \quad (1a)$$

$$\frac{\partial u}{\partial t} + u_0 \frac{\partial u}{\partial x} + v \frac{du_0}{dz} + \frac{\partial p}{\partial x} = 0 \quad (1b)$$

$$\frac{\partial v}{\partial t} + u_0 \frac{\partial v}{\partial x} + \frac{\partial p}{\partial z} = 0 \quad (1c)$$

82 where p , u and v are the acoustic pressure and components of the acoustic velocity along the x and
 83 z -direction, respectively.

84 The mean flow velocity profile is given by:

$$u_0(z) = M \frac{n_t + 1}{n_t} (1 - |1 - 2z|^{n_t}), \quad (2)$$

85 where M is the Mach number corresponding to the mean velocity. The parameter n_t specifies the
 86 flow profile and can be related to the boundary layer thickness. The displacement thickness, defined
 87 by:

$$\delta = \int_0^{1/2} \left(1 - \frac{u_0(z)}{u_0(1/2)} \right) dz \quad (3)$$

88 is equal to $\delta = 1/[2(n_t + 1)]$. Unless stated otherwise, the flow profile parameter is set to $n_t = 9$,
 89 corresponding to $\delta = 5\%$. Note the 2D velocity profile in Eq. (2) does not have an inflection point
 90 and should not support any KH instability. While not shown for conciseness, it has been verified
 91 that the time-domain simulations in a rigid duct with this sheared flow profile do not present any
 92 instability.

93 2.2. Time-domain solver

94 The LEE are solved using high order finite-difference time-domain methods. The spatial deriva-
 95 tives are calculated by optimized 4th-order finite-difference schemes over 11 points. Namely, the
 96 centered scheme of Bogey & Bailly [45] and the non-centered schemes of Berland *et al.* [46] are
 97 employed for the interior and boundary points, respectively. The optimized, 4th-order 6-stage
 98 Runge-Kutta algorithm of Berland *et al.* [47] is employed for time integration. The time step is
 99 $\Delta t = 0.0014$.

100 Selective filters are used to remove grid-to-grid oscillations. For the interior points, the centered
 101 11-point 6th-order selective filter of Bogey *et al.* [48] is adopted. For the boundary points, the
 102 selective filters of Berland *et al.* [46] are used. A parameter, denoted by σ and referred to as the
 103 filtering strength, allows one to adjust the effect of the selective filters. It ranges from 0 to 1, 0
 104 meaning that no filtering is applied and 1 meaning that grid-to-grid oscillations are totally removed.
 105 As the filtering is applied at every iteration, a filtering strength of 1 is usually not necessary and
 106 can even deteriorate longer, well-resolved wavelengths. The default value is $\sigma = 0.5$.

107 The mesh is uniform in the x -direction with a size of $\Delta x = 0.02$. This ensures that the number
 108 of points per acoustic wavelength $\lambda = 2\pi/\omega$ is larger than 10 up to $\omega = 10\pi$. It can be noticed
 109 that the duct cut-off frequency in the no-flow case corresponds to $\omega = \pi$. Damping zones with a
 110 length of 20, are implemented upstream and downstream to prevent reflections. In these zones, the
 111 mesh spacing gradually increases with a stretching factor of 3% and a Laplacian filter is applied to
 112 add artificial dissipation [49]. Along the z -direction, the mesh size decreases gradually towards the
 113 walls with a shrinking factor of 1%. The number of grid points along the duct height is n_z , with
 114 default value 175.

115 For the rigid wall, the boundary condition $v_z(x, z = 0) = 0$ is imposed. The lined wall is
 116 modelled through the impedance boundary condition:

$$p(x, z = 1, t) = [\tilde{z} * v_n](x, t), \quad (4)$$

117 where $*$ denotes the convolution operator, $v_n(x, t) = v(x, z = 1, t)$ and $\tilde{z}(t)$ is the impedance model

118 in the time domain. To avoid computing the convolution integral, the time-domain boundary
 119 condition proposed in Troian *et al.* [50] is employed. For this, the admittance in the frequency
 120 domain $\beta(\omega)$, [related to the time-domain impedance model by](#)

$$\beta(\omega)^{-1} = \int_{-\infty}^{+\infty} \tilde{z}(t)e^{-i\omega t} dt, \quad (5)$$

121 is approximated by a rational function:

$$\beta(\omega) = Y_{\infty} + \sum_{i=1}^P \frac{A_i}{\lambda_i + i\omega} + \sum_{i=1}^S \left(\frac{B_i - iC_i}{\alpha_i - i\beta_i + i\omega} + \frac{B_i + iC_i}{\alpha_i + i\beta_i + i\omega} \right), \quad (6)$$

122 where λ_i and $\alpha_i \pm i\beta_i$ are respectively the real poles and complex-conjugate pole pairs of $\beta(\omega)$, P
 123 and S denote their number and Y_{∞} , A_i , B_i and C_i are real coefficients. For a given broadband
 124 impedance model, the admittance parameters in Eq. (6) are determined using the vector fitting
 125 algorithm [51] in the frequency band of interest. The implementation of the time-domain impedance
 126 boundary condition is not further described here but interested readers can refer to Troian *et al.* [50]
 127 for details.

128 The source term Q in Eq. (1a) is chosen as

$$Q(x, z, t) = \lambda(t) \exp\left(-\frac{x^2 + (z - 0.5)^2}{B_s^2} \ln 2\right), \quad (7)$$

129 where B_s is the Gaussian half-width of the source, with a value of 0.104. Both impulsive and
 130 harmonic sources are used in this study. For impulsive sources, $\lambda(t)$ is defined as:

$$\lambda(t) = \frac{t - t_s}{t_c} \exp\left(-\frac{(t - t_s)^2}{t_c^2} \ln 2\right) H(t) \quad (8)$$

131 where $t_s = 5.42$ is a time shift, $t_c = 0.95$ specifies the frequency content of the source signal and
 132 $H(t)$ is the Heaviside function. In the case of a harmonic source $\lambda(t)$ is defined as:

$$\lambda(t) = \exp(i\omega t) H(t) \quad (9)$$

133 with ω the angular frequency. [A complex-valued harmonic source is chosen for an easier extraction
 134 of instability characteristics. It can be noticed that acoustic pressure and velocity fields in the time-
 135 domain numerical simulations are also complex-valued when this harmonic source is employed.](#)

136 2.3. Modal analysis approach

137 A modal analysis is also performed to determine the stability properties of the lined section, from
 138 which the wavenumbers and the mode shapes of all modes can be determined, including the unstable
 139 modes. Monochromatic waves propagating in an infinite, homogeneous duct are considered:

$$\begin{aligned} u &= U(z) e^{i(\omega t - kx)}, \\ v &= V(z) e^{i(\omega t - kx)}, \\ p &= P(z) e^{i(\omega t - kx)}, \end{aligned} \quad (10)$$

140 where U , V and P are the mode shapes, which depend only on z , and k is the mode wavenumber.
 141 Note that ω and k can be complex-valued.

142 In order to obtain this information, the next step is to introduce Eqs. (10) into Eqs. (1) to
 143 obtain an eigenvalue problem. The eigenvalue problem is not directly formulated using the physical
 144 variables; instead, the characteristic variables, i.e., U , $P - V$ and $P + V$, are used. Specifically,
 145 $P - V$ and $P + V$ are the characteristic waves travelling along the $-z$ and $+z$ -direction, respectively.
 146 The motivation for using the characteristic variables is described below. The discretization of the
 147 LEE on a grid with N points leads to a system of $3N$ equations for $3N$ unknowns. The boundary
 148 conditions however bring two additional equations. In order to avoid having an overdetermined
 149 system, it is thus necessary to choose two equations to eliminate, which is not trivial and somewhat
 150 arbitrary. The characteristic variables allow for a suitable choice (see Appendix A).

151 Doing so, the LEE in the frequency domain are written as:

$$\begin{pmatrix} 1 & \frac{-1}{2i\omega} \frac{du_0}{dz} & \frac{1}{2i\omega} \frac{du_0}{dz} \\ 0 & 1 - \frac{1}{i\omega} \frac{d}{dz} & 0 \\ 0 & 0 & 1 + \frac{1}{i\omega} \frac{d}{dz} \end{pmatrix} \begin{pmatrix} U \\ P - V \\ P + V \end{pmatrix} = \frac{k}{\omega} \begin{pmatrix} u_0 & \frac{1}{2} & \frac{1}{2} \\ 1 & u_0 & 0 \\ 1 & 0 & u_0 \end{pmatrix} \begin{pmatrix} U \\ P - V \\ P + V \end{pmatrix}. \quad (11)$$

152 This matrix system has to be completed with appropriate boundary conditions. On the rigid
 153 wall, the boundary condition becomes $[P + V](z = 0) = [P - V](z = 0)$. On the lined wall, the
 154 impedance boundary condition is written as:

$$[P - V](z = 1) = \frac{1 - \beta}{1 + \beta} [P + V](z = 1) \quad (12)$$

155 Note that the admittance in Eq. (12) depends on the frequency. In the stability analysis performed
 156 in Section 3, the frequency can be complex-valued and the admittance has to be calculated accord-
 157 ingly. To be noticed, admittance models given by a rational function in Eq. (6) are used in the
 158 paper.

159 The eigenvalue problem in Eq. (11) is solved numerically. The spatial derivatives are discretized
 160 using the Chebyshev spectral method [52], which is able to generate smaller numerical errors than
 161 finite-difference schemes for a given number of grid points. The gradient of the mean flow du_0/dz is
 162 also calculated numerically using the Chebyshev spectral method. The default value of the number
 163 of grid points N used in the modal analysis is 150. The discretized LEE are then recast into a
 164 generalized eigenvalue problem, which is solved by the `eig` function of *MATLAB*. Details on the
 165 discretization and on the implementation of boundary conditions are provided in Appendix A.

166 3. Stability analysis

167 As described in the previous section, both methods will be applied to study hydrodynamic
 168 instabilities. The occurrence of a hydrodynamic instability depends on the characteristics of the
 169 liner, the mean flow profile and the source frequency. Two cases with Mach numbers of $M = 0.1$
 170 and $M = 0.3$ are considered. The corresponding flow profiles are shown in Fig. 2. For simplicity, a

171 mass-spring-damper (MSD) liner is used, with impedance

$$Z(\omega) = R + i\omega m + \frac{K}{i\omega}, \quad (13)$$

172 where R , m and K are respectively the damping, mass and spring stiffness. The impedance
 173 spectrum of the liner used in our case is shown in Fig. 3, with $R = 0.2$, $m = 5.4 \times 10^{-3}$ and $K = 0$.

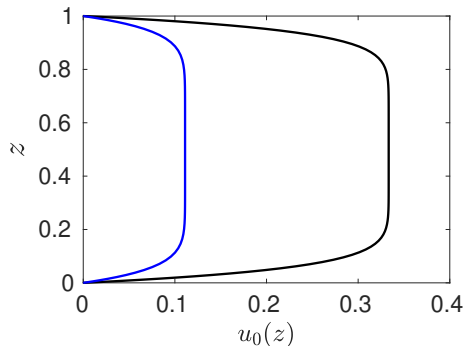


Fig. 2. Mean flow profile for: — $M = 0.1$ and — $M = 0.3$.

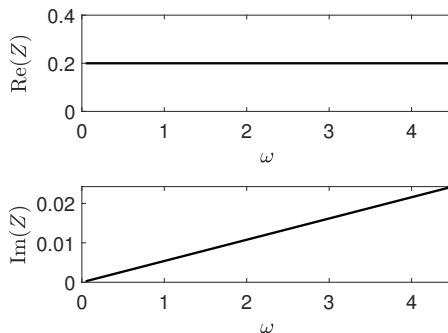


Fig. 3. Impedance of the MSD liner.

174 *3.1. Example of time-domain simulation*

175 Time-domain simulations are first performed using an impulsive source to illustrate the acous-
 176 tic propagation along the lined duct. Fig. 4 shows the pressure response varying with time and
 177 position along the x -axis on the lined wall. For $M = 0.1$, two branches originating from $t = t_s$ are
 178 observed, corresponding to upstream and downstream propagating waves. These acoustic waves
 179 are attenuated along the lined wall. After $t \approx 18$, they have left the computational domain and the
 180 remaining fluctuating pressure is almost null. For $M = 0.3$, these two branches are retrieved. Their
 181 orientation is modified due to the difference in the Mach number. Another contribution is however
 182 observed in the downstream direction. Its magnitude and its spatial extent increase as it propa-
 183 gates away from the source, indicating it is an instability. Moreover, since the pressure response is
 184 given as a function of the distance and time, the velocity of acoustic waves and instabilities can be

185 deduced from the slopes of the different branches. It can then be estimated that the propagation
 186 velocity of the instability is smaller than that of the acoustic waves.

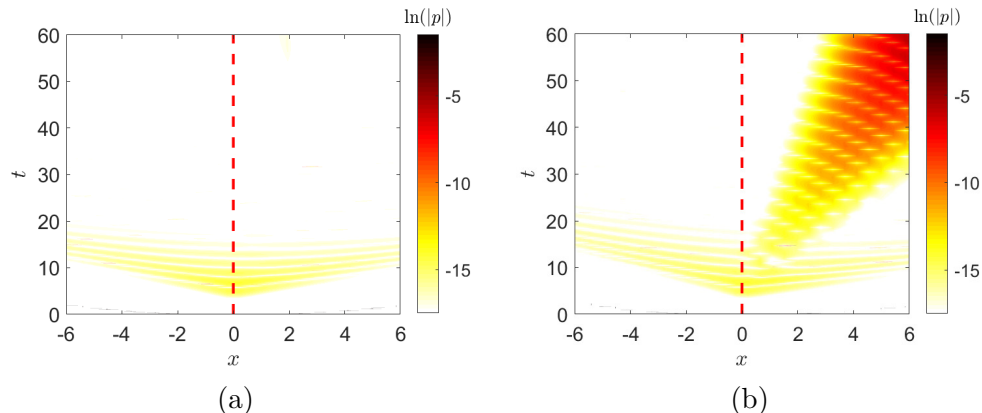


Fig. 4. Pressure response $\ln |p(x, z = 1, t)|$ to an impulsive source on the lined wall for (a) $M = 0.1$ and (b) $M = 0.3$. The source position is indicated by the vertical dashed line.

187 3.2. Characterization of the instability from modal analysis

188 This section aims at correlating the instability observed in the time-domain simulations with
 189 the modal analysis. Therefore, a harmonic source is imposed on both approaches. For predicting
 190 the response to a harmonic excitation, a spatial stability analysis is performed: the frequency is
 191 real-valued and the mode wavenumbers can be complex. Fig. 5 displays the wavenumbers of all
 192 modes at $\omega = 0.9271$ for both $M = 0.1$ and $M = 0.3$. This particular frequency is chosen because
 193 it corresponds to a dimensional frequency $f = 1000$ Hz for the benchmark data of the NASA GIT
 194 duct (sound speed $c_0 = 344.28$ m s $^{-1}$ and duct height $H = 0.0508$ m), which is considered in
 195 Section 5.3. It is also close to the frequency $\omega = 0.8344$ where the maximal spatial growth rate
 196 is observed (not shown). For $M = 0.1$, there is no mode in the upper-right complex k -plane and
 197 therefore no possible instability. For $M = 0.3$, the wavenumber of one mode ($k = 6.78 + 1.97i$)
 198 has both positive real and imaginary parts. This is possibly an unstable mode, and its stability
 199 depends on the propagation direction. If the mode is an upstream decaying mode, the propagation
 200 is stable. On the other hand, if it propagates in the downstream direction, it corresponds to an
 201 instability. The propagation direction of this mode therefore needs to be determined.

202 In Fig. 5, noticeable modes other than the acoustic modes and the instability are convected
 203 modes which satisfy the dispersion relation $ku_0(z) = \omega$. For a sheared mean flow, they come
 204 as a continuous spectrum, located in the k -plane along the horizontal half-line starting at $k =$
 205 $\omega/\max(u_0)$. In solving numerically the eigenvalue problem, this continuous spectrum turns into in-
 206 dividual modes, which are greatly dependent on the discretization. In particular, even the wavenum-
 207 bers of these modes may have small positive imaginary parts and thus be weakly unstable. As shown
 208 in the study of Brambley *et al.* [53], the contributions of the continuous spectrum are negligible
 209 when the point source is located at the duct centerline, where the mean flow is relatively unsh-
 210 eared. Note that the use of selective filters, as employed in the time-domain simulations, tends to move the
 211 convected modes towards the lower right quadrant of the k -plane [25] and thus to make all these
 212 modes stable. As we are primarily interested in the instability, no specific method is employed to

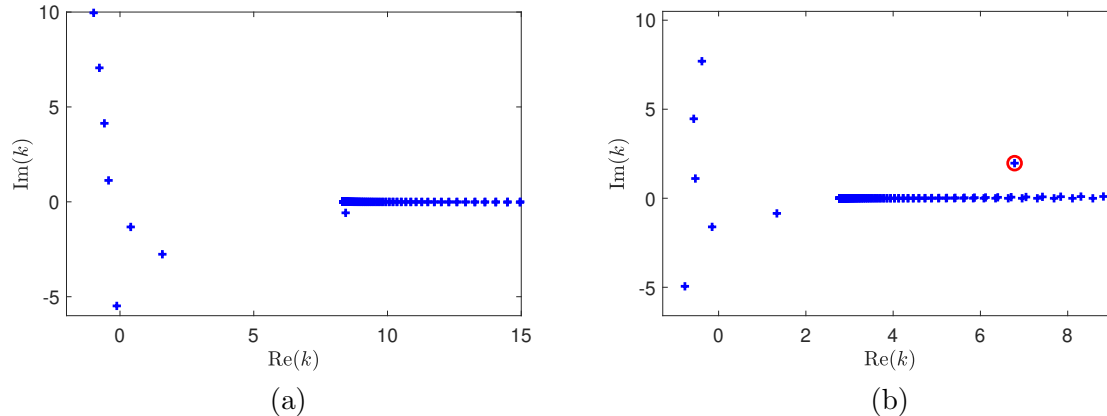


Fig. 5. Wavenumber obtained from modal analysis for $\omega = 0.9271$ and for (a) $M = 0.1$ and (b) $M = 0.3$. The encircled wavenumber indicates a possible unstable hydrodynamic mode.

213 have an accurate prediction of convective modes. Such a method has been for instance described
 214 in Vilenski and Rienstra [54].

215 The Briggs-Bers criterion [55, 56] is used to determine the propagation direction of the modes,
 216 and, if an unstable mode is present, whether it is convective or absolute. This criterion has been
 217 introduced to study the plasma instability and has been later considered for the stability analysis
 218 of acoustic propagation in a lined flow duct [17]. However, before using the Briggs-Bers criterion,
 219 it must be checked that it is applicable. For instance, as discussed by Brambley [17], the acous-
 220 tic propagation in a lined duct with a plug flow and the Myers impedance boundary condition is
 221 mathematically ill-posed, implying that Briggs-Bers criterion can not be used. In order to apply
 222 the Briggs-Bers criterion, one must first check that the growth rates of all modes, and in partic-
 223 ular the unstable modes, are bounded. To do so, a temporal stability analysis is conducted. The
 224 wavenumber here remains real, and the frequency ω , possibly complex-valued, satisfying the disper-
 225 sion relation is sought. If for a particular value of the wavenumber, one has $\text{Im}(\omega) < 0$, the system
 226 is then unstable. Fig. 6 shows the growth rate of the possible unstable mode for $M = 0.3$ as a
 227 function of $\text{Re}(\omega)$. It is observed that the growth rate is bounded at $\text{Im}(\omega) \approx -0.23$, meaning that
 228 the instability will not be amplified at an arbitrarily large growth rate for any given real frequency.
 229 The Briggs-Bers criterion can thus be applied in this case.

230 To determinate if a possible unstable mode is an instability, the trajectory of its wavenumber
 231 in the complex k -plane is recorded as $\omega = \text{Re}(\omega) + i \text{Im}(\omega)$ is varied. More specifically, $\text{Re}(\omega)$ is
 232 fixed while $\text{Im}(\omega)$ is varied from $-\infty$ to 0. Fig. 7 shows this process for $\text{Re}(\omega) = 0.9271$, while the
 233 imaginary part $\text{Im}(\omega)$ goes from -1.4 to 0. The wavenumber of the possible unstable mode crosses
 234 the real k -axis from the lower k -plane when $\text{Im}(\omega)$ is approaching 0. The crossing of the real k -axis
 235 from the lower half k -plane indicates that this mode is indeed a downstream propagating mode and
 236 is thus an instability.

237 In addition, in order to discard the existence of an absolute instability, it should be verified,
 238 that the trajectories of the wavenumbers in the k -plane for any $\text{Re}(\omega)$ do not show any saddle
 239 point, which corresponds to merging roots of k from different halves of the k -plane. This has been
 240 carefully checked on a large range of values of $\text{Re}(\omega)$. To sum up, for $\omega = 0.9271$, there is only one
 241 hydrodynamic instability, and it is a convective instability.

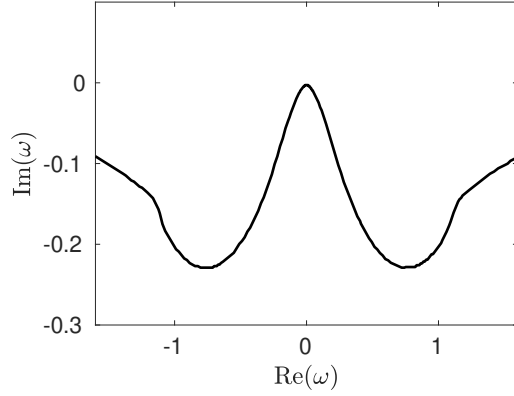


Fig. 6. Growth rate of the unstable mode for a real wavenumber and for $M = 0.3$.

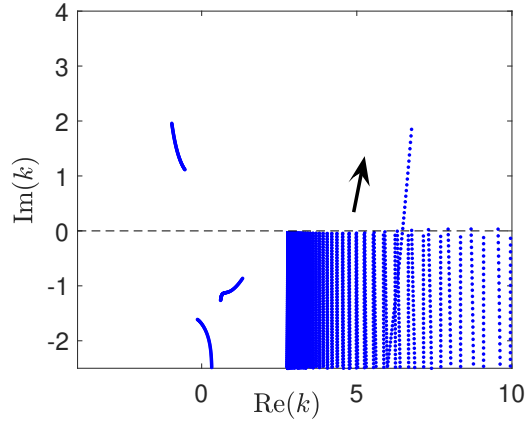


Fig. 7. Trajectories of the wavenumbers for $\text{Re}(\omega) = 0.9271$ as $\text{Im}(\omega)$ varies from -1.4 to 0 . The arrow indicates the direction of increasing $\text{Im}(\omega)$.

242 3.3. Characterization of the instability from time-domain simulation

243 3.3.1. Instability type

244 The type of the instability observed in the time-domain simulations can also be investigated by
 245 considering acoustic propagation driven by a harmonic source with $\omega = 0.9271$. The logarithm of
 246 $|p|$ on the lined wall $z = 1$ varying with t and x is shown in Fig. 8. With a harmonic source, the
 247 response of pressure has a constant value at $x = 0$. The attenuation by the liner can be noticed along
 248 the upstream direction, and the hydrodynamic instability is observed in the downstream direction.
 249 Once a steady-state is attained, the magnitude of the instability at a given location is not increasing
 250 over time. It thus appears that the instability observed in the time-domain simulation is also a
 251 convective one.

252 3.3.2. Wavenumber

253 If the instability is convective and is the main contribution to the pressure field, the fluctuating
 254 pressure will have the form:

$$p(x, z, t) = P_{\text{HI}}(z) e^{i\omega t} e^{-i\text{Re}(k_{\text{HI}})x} e^{\text{Im}(k_{\text{HI}})x}, \quad (14)$$

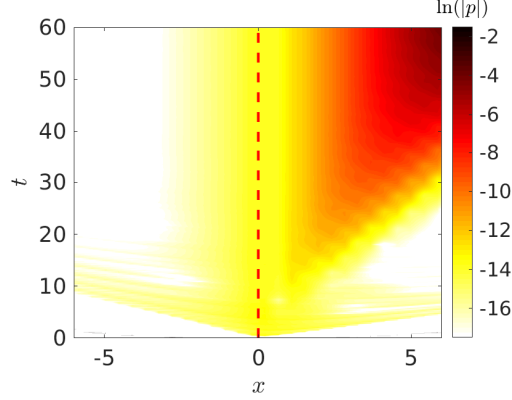


Fig. 8. Pressure response $\ln |p(x, z = 1, t)|$ to a harmonic source at $\omega = 0.9271$ obtained from the time-domain simulation for $M = 0.3$. The source position is indicated by the vertical dashed line.

255 where P_{HI} is the mode shape of the instability and k_{HI} is its wavenumber.

256 In this case, the phase of the complex pressure, denoted by \arg is given by:

$$\arg[p(x, z, t)] = -\text{Re}(k_{\text{HI}})x + \omega t + \arg[P_{\text{HI}}(z)]. \quad (15)$$

257 The real part of the instability wavenumber can then be estimated from the time-domain solution
 258 through the axial evolution of the phase of the complex pressure. Fig. 9 shows $\arg(p)$ along the
 259 liner ($z = 1$) at $t = 90$, for which it has been checked that the instability is well developed. The
 260 phase has been unwrapped to ensure that $\arg(p)$ is continuous. The estimated slope for $2 \leq x \leq 6$
 261 of the phase is -6.89 , which is very close to the prediction of modal analysis $\text{Re}(k_{\text{HI}}) = 6.78$.

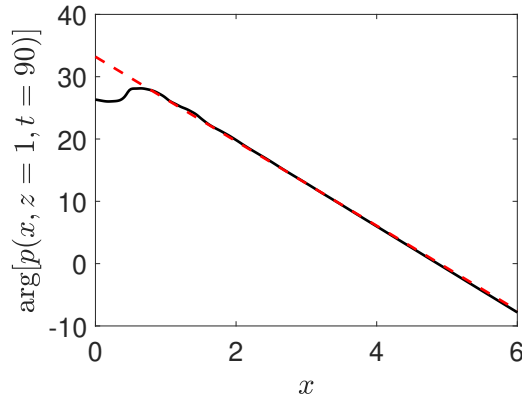


Fig. 9. Comparison of the phase of the pressure on the lined wall from the time-domain solution at $t = 90$ — and from the analytical solution in Eq. (15) ($\arg(p) \propto -6.78x$) - - - .

262 Similarly, the growth rate of the convective instability $\text{Im}(k_{\text{HI}})$ in Eq. (14) can be calculated
 263 from the logarithm of $|p(x, z, t)|$:

$$\ln |p(x, z, t)| = \text{Im}(k_{\text{HI}})x + \ln |P_{\text{HI}}(z)|. \quad (16)$$

264 A snapshot of $\ln |p|$ at $t = 90$ from the time-domain simulation is depicted in Fig. 10 (a). The

265 instability is seen growing from the source at the vicinity of the liner. Fig. 10 (b) shows the spatial
 266 variation of $\ln |p(x, z = 1, t = 90)|$. A linear growth is observed downstream, where the instability is
 267 the dominant contribution to the pressure field. The estimated slope for $2 \leq x \leq 6$ is 1.99, which is
 268 again in close agreement with the spatial growth rate predicted by modal analysis $\text{Im}(k_{\text{HI}}) = 1.97$.

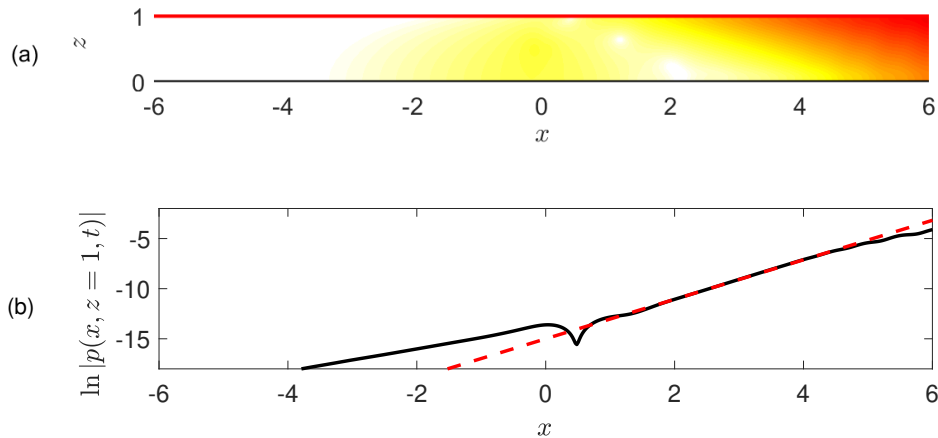


Fig. 10. (a) Snapshot of $\ln |p|$ at $t = 90$ and (b) comparison of the amplitude of the pressure on the lined wall from the time-domain solution — and from the analytical solution in Eq. (16) ($\ln |p| \propto 1.97x$) - - - .

269 Another example for a different frequency $\omega = 0.4636$ can be found in Appendix B. This shows
 270 the viability of the time-domain method in simulating the instability.

271 3.3.3. Mode shapes

272 The mode shapes P , U and V of the instability determined from modal analysis are compared
 273 with the vertical profiles of p , u and v calculated from the time-domain solution in Fig. 11. The
 274 results of the time-domain solution correspond to the section $x = 4$ at $t = 90$, for which it has been
 275 checked that the solution is not changing after additional time iterations. When the time-domain
 276 solution is in a steady-state, the vertical profiles of p , u and v should hold similar shapes regardless
 277 of x -position and time iteration. For comparison, the mode shapes of the instability and the vertical
 278 profiles of p , u and v are normalized by the maximum of the pressure modulus. The peaks of the
 279 amplitudes are observed near the lined wall. Then they gradually decrease to a small value on the
 280 rigid wall. The match between modal analysis and the time-domain solution is remarkable.

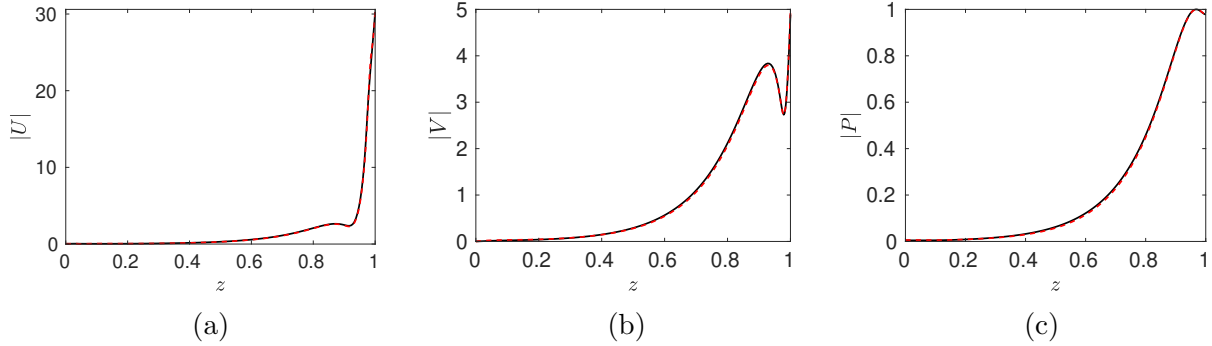


Fig. 11. Comparison of the mode shape of the instability for (a) $U(z)$, (b) $V(z)$ and (c) $P(z)$ — determined from the time-domain solution at $x = 4$ and $t = 90$ and - - - calculated by modal analysis.

281 4. Convergence of instability

282 Previous studies on the same topic, i.e., instabilities appearing during sound propagation in a
 283 lined flow duct, have investigated the impact of spatial filters and grid resolution on the instabilities.
 284 It was concluded that selective filtering is effective to remove or partly remove the instability [34, 3]
 285 and that the use of fine grids can reinforce the instability [27, 2], in particular when using the Myers
 286 impedance boundary condition instead of resolving the boundary layer as done here. Therefore,
 287 the effects of both factors on the hydrodynamic instabilities are studied.

288 4.1. Effect of the filtering strength

289 In the time-domain simulations, a selective filter of strength σ is applied on the whole domain
 290 to remove the grid-to-grid oscillations. In the previous sections, σ was set to 0.5. Simulations are
 291 now performed with different values of σ in order to study how the properties of the hydrodynamic
 292 instability are modified.

293 Fig. 12 shows the real and imaginary parts of the instability wavenumber obtained with the
 294 time-domain solutions as a function of σ . The results are compared with the reference wavenumber
 295 obtained by modal analysis. When the strength of selective filter is between 0.7 and 1, the strong
 296 dissipation of the selective filter results in a noticeable deviation between the reference result and
 297 the time-domain prediction. In particular, the growth rate of the instability is overestimated. As σ
 298 decreases, $\text{Re}(k_{\text{HI}})$ and $\text{Im}(k_{\text{HI}})$ start to approach the reference wavenumber. The value of σ does
 299 not make a significant difference to $\text{Re}(k_{\text{HI}})$ and $\text{Im}(k_{\text{HI}})$, when it is in the range between 0.2 and
 300 0.5. For σ below 0.2, spurious numerical waves with a short wavelength are rapidly growing near
 301 the duct walls and the simulations become unstable instantly. Therefore, for accurately predicting
 302 the hydrodynamic instability, the filtering strength should be chosen as small as possible while
 303 sufficiently large to stabilize the numerical calculation. A value of σ between 0.2 and 0.5 seems to
 304 be a good compromise considering the time step used in the time-domain simulations.

305 4.2. Effect of the mesh size along the duct height

306 Along the x -direction, centered high-order finite-difference schemes, with dispersion-relation
 307 preserving properties, are employed. It is assumed that the mesh size Δx is small enough so that
 308 the dispersion properties of the instability are well-reproduced. In particular, it has been checked

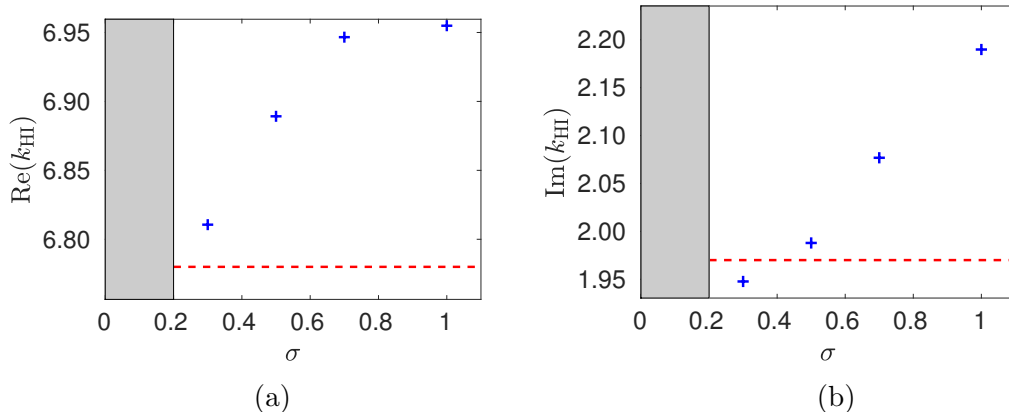


Fig. 12. (a) Real and (b) imaginary parts of the instability wavenumber $+$ estimated from the time-domain solution as a function of the filtering strength σ and $- - -$ predicted by modal analysis. The grey region indicates unstable simulations.

309 that $k^*(k_{\text{HI}})$ is very close to k_{HI} , where k^* is the effective wavenumber of the centered finite-
 310 difference scheme employed here. No particular effect in reducing Δx is thus expected. The grid
 311 along the z -direction is anticipated to have more impact on the instability. Indeed, non-centered
 312 schemes are used near the walls, which enhance dispersion. They can amplify or dissipate even
 313 constant-amplitude waves, contrary to centered schemes. In addition, the instability is directly
 314 related to the mean flow profile through u_0 and du_0/dz . The discretization of the mean flow
 315 profile, in particular within the boundary layer, is expected to play a role in the results.

316 Fig. 13 shows the impact of the number of grid points along the z -direction n_z on the real
 317 and imaginary parts of the instability wavenumber. For $n_z \leq 155$, the deviations of $\text{Re}(k_{\text{HI}})$ and
 318 $\text{Im}(k_{\text{HI}})$ estimated from the time-domain simulations and the modal analysis are noticeable. Note
 319 that the mesh is already fine enough for $n_z = 55$ to precisely calculate the acoustic propagation.
 320 As shown in [57], the pressure of instabilities should decay exponentially away from the lining
 321 as $\exp[-\text{Re}(\mu)(1-z)]$ with $\mu^2 = k^2 - (\omega - Mk)^2$. In the present case, $\text{Re}(\mu) = 6.69$ so that
 322 the number of points per wavelength is about $2\pi n_z / \text{Re}(\mu) \approx 52$, which is very large. However,
 323 that is still not sufficient to precisely calculate the hydrodynamic instability. As n_z increases, the
 324 features of hydrodynamic instability can be captured and the wavenumber gradually approaches
 325 the prediction of modal analysis. With $n_z \geq 175$ the time-domain solution is deemed converged.

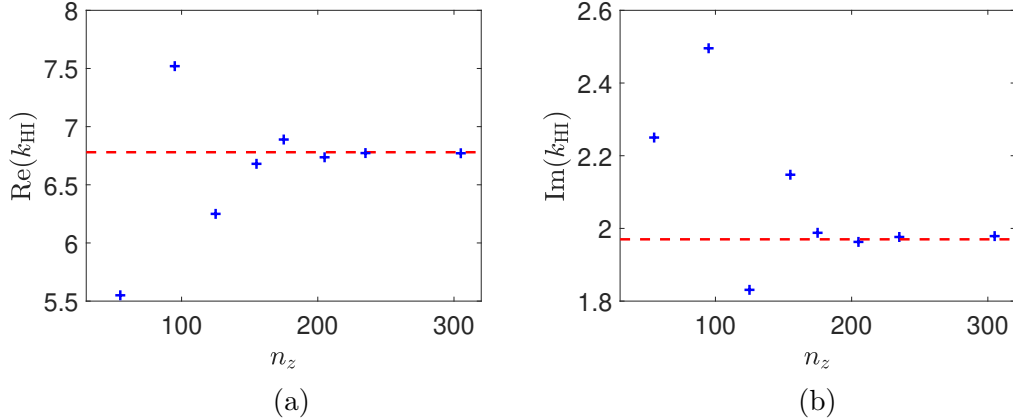


Fig. 13. (a) Real and (b) imaginary parts of the instability wavenumber $+$ estimated from the time-domain solution as a function of the number of grid points along the duct height n_z and $- - -$ predicted by modal analysis.

326 5. Suppression of the hydrodynamic instability

327 In all cases tested with realistic Mach numbers, it was not possible to prevent the appearance
 328 of the instability using coarse grids and increased selective filter strength. We turned therefore to
 329 the methods based on the stabilization via modification of the underlying equations. Given the
 330 similar performance of many of these methods in turbulent jets [44], we opted for the gradient term
 331 suppression (GTS) method due to its simplicity and performance.

332 In the case that the acoustic field is generated by an impulsive source, as in the present case,
 333 it may be possible to separate the acoustic and instability components in time through a temporal
 334 window. This would allow to account for the full LEE, but it is of use only when the instability
 335 is weak enough. In general, a robust suppression strategy is needed with a minimal impact on
 336 acoustic propagation.

337 5.1. Partial GTS

338 A natural generalization of the original GTS method is used here to suppress the instability,
 339 consisting of a partial suppression of the gradient term instead of a complete suppression. Eqs. (1)
 340 are modified by adding a coefficient ϵ in front of the term of du_0/dz . Therefore, Eq. (1b) turns to

$$\frac{\partial u}{\partial t} + u_0 \frac{\partial u}{\partial x} + \epsilon \frac{du_0}{dz} v + \frac{\partial p}{\partial x} = 0 \quad (17)$$

341 where ϵ adjusts the strength of the mean flow gradient term and ranges from 1 to 0.

342 Combining Eqs. (1a), (17) and (1c) leads to the wave equation:

$$\frac{D}{Dt} \left(\frac{D^2 p}{Dt^2} - \nabla^2 p \right) + (\epsilon + 1) \frac{du_0}{dz} \frac{\partial^2 p}{\partial x \partial z} = \frac{D^2 Q}{Dt^2} \quad (18)$$

343 where $D/Dt = \partial/\partial t + u_0 \partial/\partial x$ is the material derivative. In particular, the Lilley's equation
 344 is recovered for $\epsilon = 1$. The decrease of ϵ from 1 to 0 is thus equivalent to transforming the
 345 original Lilley's equation, which sustains unstable modes, to a simplified form of Lilley's equation
 346 which appears to be inherently stable [35]. In addition, decreasing ϵ from 1 to 0 diminishes the

347 refraction term proportional to du_0/dz in the original Lilley's equation by a factor of two. A gradual
 348 suppression of the instability is then expected when diminishing the value of ϵ .

349 Another remarkable value is $\epsilon = -1$ for which Eq. (18) corresponds to the material derivative of
 350 the convected wave equation. However, Eq. (18) is already stable for $\epsilon = 0$. In addition, it will be
 351 shown that the prediction of the acoustic propagation tends to be deteriorated with the decrease
 352 of ϵ . Therefore, the study is hereafter restricted to $\epsilon \in [0, 1]$.

The GTS method is expected to provide an accurate prediction of acoustic propagation at high frequencies [35]. This can be explained by conducting an order of magnitude analysis. The order of the magnitude of each term in Eq. (1b) is as follows:

$$\frac{Du}{Dt} \sim (\omega - Mk)\hat{u}, \quad \frac{du_0}{dz}v \sim \frac{M}{\delta}\hat{v}, \quad \frac{\partial p}{\partial x} \sim k\hat{p},$$

353 where for acoustic modes the wavenumber is in the order of the frequency $k \sim \omega$ and the acoustic
 354 variables are in the same order of magnitude $\hat{u} \sim \hat{v} \sim \hat{p}$. The mean flow gradient term in Eq. (1b)
 355 can be neglected if

$$\left|v \frac{du_0}{dz}\right| \ll \left|\frac{\partial p}{\partial x}\right|, \left|\frac{Du}{Dt}\right|. \quad (19)$$

356 which, considering the orders of magnitude above, shows that the mean flow gradient term is
 357 expected to have a negligible effect on the acoustic propagation for $\omega\delta \gg M$. This explains why
 358 the high frequency components are less impacted by the partial GTS method.

359 5.2. Effect on the wavenumber and the mode shape

360 The wavenumbers are studied when ϵ varies from 1 to 0. Fig. 14 shows the wavenumbers in
 361 the k -plane determined by modal analysis for different values of ϵ for the rigid and partially lined
 362 duct. It is first noticed that modifying ϵ tends to move all the modes in the complex k -plane. The
 363 instability for the lined duct, observed for $\epsilon = 1$, is not seen for the smaller values of ϵ , which shows
 364 that a small decrease of ϵ is sufficient to suppress the instability. Also, the wavenumbers for the
 365 rigid duct are less sensitive to ϵ than for the lined duct.

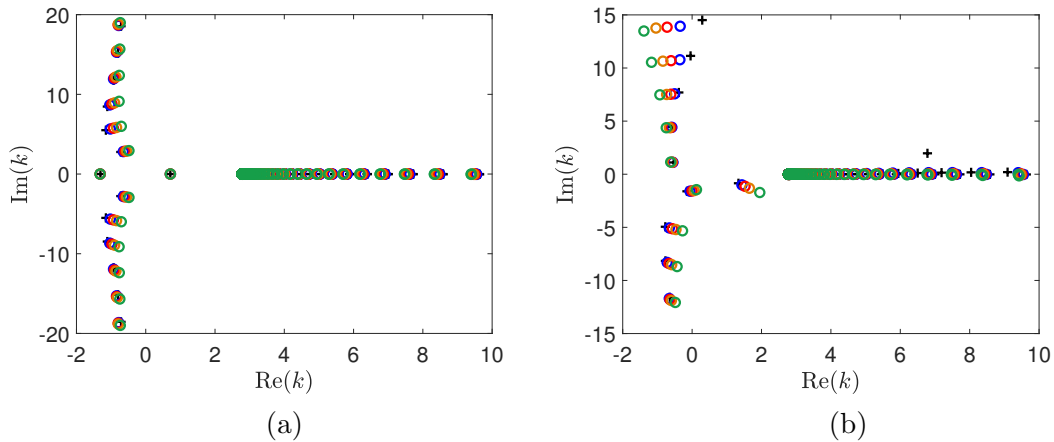


Fig. 14. Wavenumbers in the k -plane at $\omega = 0.9271$ for a duct (a) with two rigid walls, (b) with one rigid wall and for different strengths of gradient term: $+$ $\epsilon = 1$, \bullet $\epsilon = 0.7$, \circ $\epsilon = 0.5$, \circ $\epsilon = 0.3$ and \circ $\epsilon = 0$.

366 To analyze more finely the effect of ϵ on the instability, the variations of the instability wavenum-
 367 ber are plotted as a function of ϵ in Fig. 15. The results are shown for three different grid sizes
 368 ($N = 100, 200$ and 300). The growth rate of the unstable mode $\text{Im}(k_{\text{HI}})$ decreases gradually with
 369 ϵ which proves the stabilizing effect of decreasing ϵ . It is interesting to note that for $\epsilon \leq 0.94$, the
 370 wavenumber of the unstable mode starts depending on the grid size. There, the unstable mode
 371 merges into the continuous spectrum of convected modes near the real k -axis.

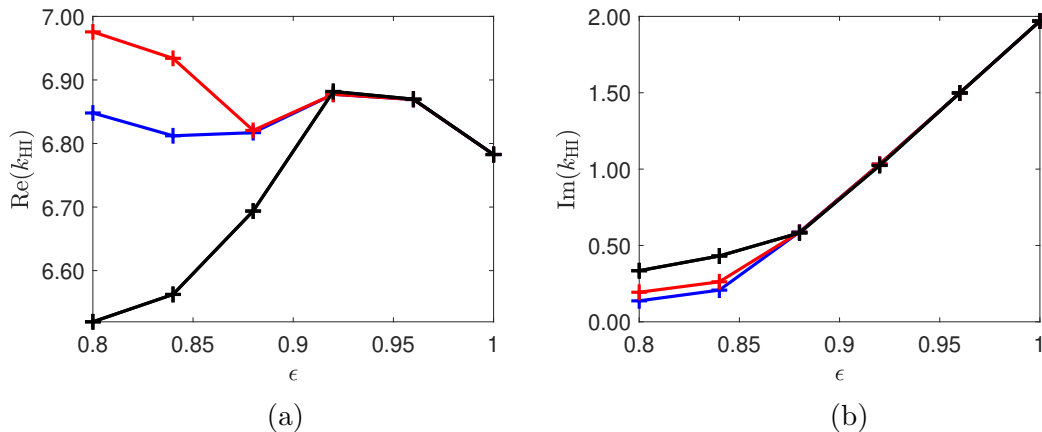


Fig. 15. Variations of the (a) real and (b) imaginary parts of the instability wavenumber k_{HI} as a function of ϵ for different numbers of grid points: \blacksquare $N = 100$, $\color{red}+$ $N = 200$ and $\color{blue}+$ $N = 300$.

372 Acoustic modes are also affected by reducing the parameter ϵ . To quantify the impact of
 373 the partial GTS method on the wavenumbers of the acoustic modes, the attenuation rate of the
 374 propagative acoustic mode is provided in Fig. 16 (a). The attenuation rate α , defined as $\alpha =$
 375 $20 \log_{10} e^{-\text{Im}(k)}$, expresses the decrease per unit distance (H) of the signal amplitude during the
 376 propagation along the axial direction. For $\omega = 0.9271$, the attenuation rate of the plane wave mode
 377 increases from 7.39 to 14.95 dB/ H when ϵ decreases from 1 to 0.

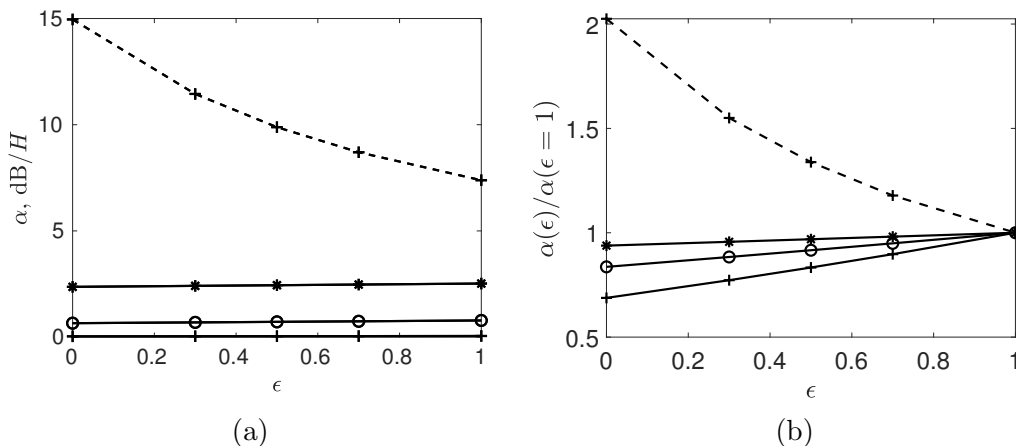


Fig. 16. (a) Attenuation rate of acoustic modes and (b) ratio of the attenuation rate determined for the partial GTS method to that for the original LEE for (dashed) $\omega = 0.9271$ and (full) $\omega = 9.271$: $+$ plane wave mode, o second downstream propagative mode and $*$ third downstream propagative mode.

378 In addition, Fig. 17 depicts the mode shapes of the acoustic velocity components and pressure for
 379 the plane wave mode and for the leading upstream and downstream evanescent modes for different
 380 values of ϵ . The modulus of all three variables are normalized by the maximum of $|P|$. Modifying
 381 ϵ has an evident impact on $|U|$ and $|V|$, noticeably near the lined wall. This effect is particularly
 382 dramatic for $|U|$, whose peak on the lined wall decreases by a factor of 10 from $\epsilon = 1$ to $\epsilon = 0$.
 383 Also, a noticeable increase of $|V|$ near the lined wall is observed as ϵ decreases. Concerning the
 384 pressure, the mode shape does not vary too much with ϵ . For the plane wave mode, a decrease of
 385 the amplitude is noticed near the rigid wall, while for the evanescent modes, the amplitude grows
 386 gradually near the lined wall as ϵ decreases.

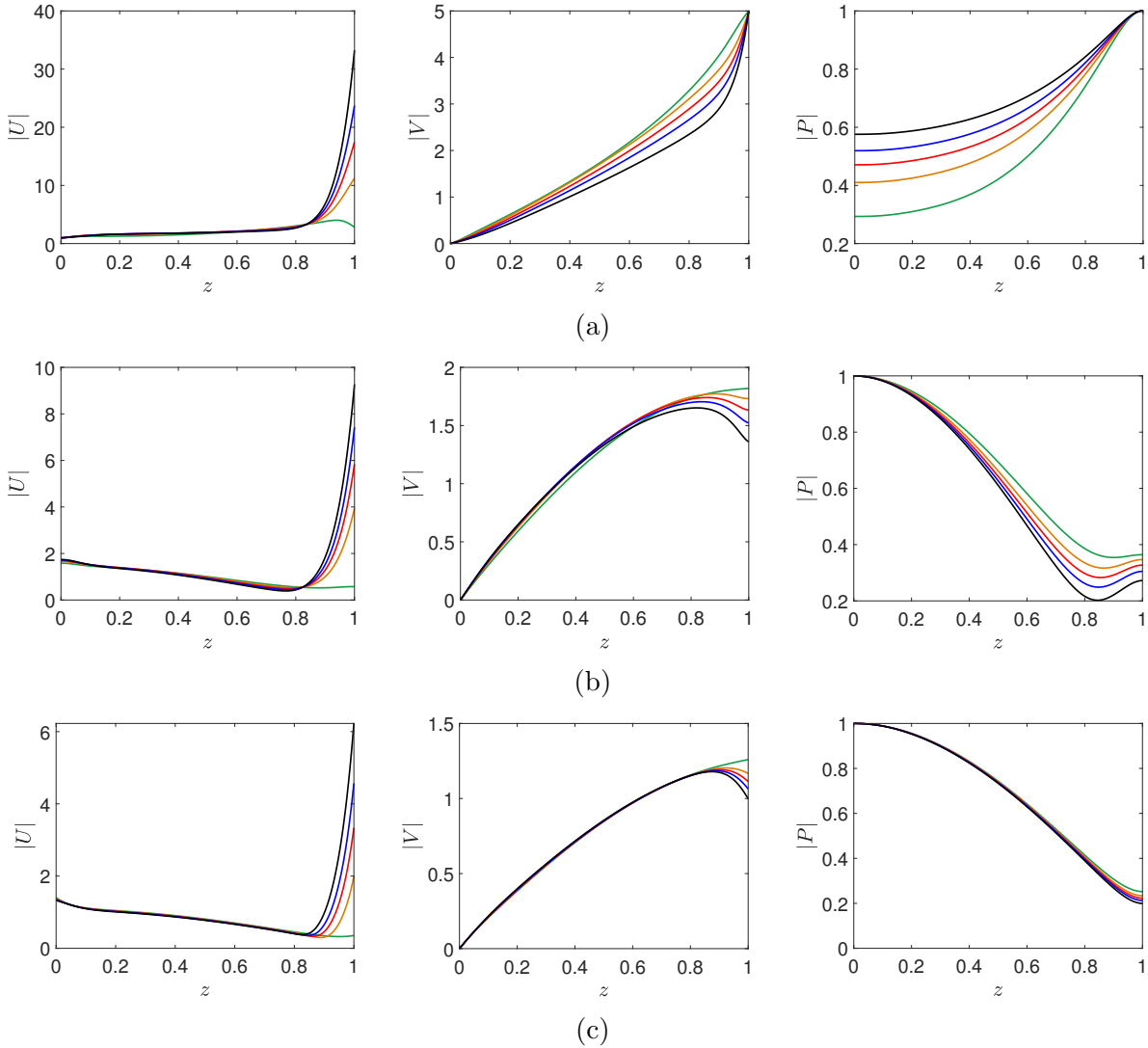


Fig. 17. Mode shapes for the MSD liner at $\omega = 0.9271$ and for several values of ϵ : (a) plane wave mode and leading (b) downstream and (c) upstream evanescent modes for, from left to right, velocity components along x and z -direction and pressure: — $\epsilon = 1$, — $\epsilon = 0.7$, — $\epsilon = 0.5$, — $\epsilon = 0.3$ and — $\epsilon = 0$.

387 The impact of the partial GTS method on the acoustic propagation at high Helmholtz numbers

388 is here studied. Fig. 18 provides the mode wavenumbers for $\omega = 9.271$, while keeping the same
389 impedance and boundary layer thickness as for $\omega = 0.9271$. Three downstream propagative modes
390 are noticed and no unstable mode is observed. The mode wavenumbers are less changed due to
391 the partial GTS method compared with Fig. 14. The attenuation rates for the three downstream
392 propagative modes are also plotted in Fig. 16 (a). The deviation caused by the gradient term
393 suppression are respectively 0.017, 0.13 and 0.16 dB/H for the three modes. The ratio of the
394 attenuation rate obtained with the partial GTS method to that determined with the original LEE
395 is also shown in Fig. 16 (b). It is seen that the largest changes of attenuation rate occur for low
396 Helmholtz numbers.

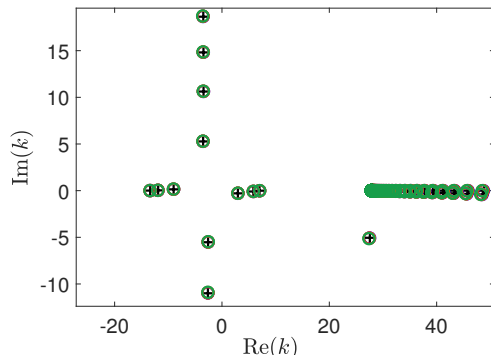


Fig. 18. Wavenumbers in the k -plane at $\omega = 9.271$ for a lined duct and for several values of ϵ : $+$ $\epsilon = 1$, \circ $\epsilon = 0.7$, \circ $\epsilon = 0.5$, \circ $\epsilon = 0.3$ and \circ $\epsilon = 0$.

397 5.3. NASA GIT benchmark

398 The GTS method is now applied to the GIT benchmark problem [58], which is widely used
399 for validation of aeroacoustic solvers of lined ducts under plane wave propagation. A number of
400 numerical studies have reported the existence of an instability at 1 kHz for this problem [16, 59, 28].
401 The tube has a dimension of $0.8128 \times 0.0508 \times 0.0508$ m. The upper wall is lined with a ceramic
402 tubular liner (CT57) with a length of 0.406 m. The sound pressure level (SPL) and phase of acoustic
403 waves have been measured at various Mach numbers and for frequencies from 500 Hz to 3 kHz.

404 The source is installed ahead of the section $x = 0$ at a distance of 0.05 m. The mean velocity
405 profile is given by Eq. (2), with a Mach number of 0.335 and a boundary layer thickness equal
406 to 2 % of the duct height. A grid of 55 points is used along the z -direction, while the grid size
407 along the x -direction is $\Delta x = 0.0011$ m. The time step has been set to 2×10^{-6} s. The broadband
408 admittance of the liner (see. Eq. (6)) is obtained by a fit of the educed values provided by Jones *et*
409 *al.* [58] for $M = 0.335$. In this case, two pairs of complex-conjugate poles are sufficient to have
410 a good match from 500 Hz up to 3 kHz, as shown in Fig. 19. The coefficients of the obtained
411 admittance model are given in Table 1.

412 Fig. 20 shows a series of successive snapshots of the pressure. The left and right columns
413 correspond to the results without GTS ($\epsilon = 1$) and full GTS ($\epsilon = 0$), respectively. We focus first
414 on the case without GTS. Once the incoming pulse encounters the liner leading edge, it splits into
415 two wave fronts, a first one that is transmitted downstream and is progressively attenuated by the
416 liner, and a second one that is reflected upstream. At the same time, the instability is generated

Table 1: Coefficients of the rational function for the normalized broadband admittance model of CT57

	α_i	β_i	B_i	C_i	Y_∞
$i = 1$	6135	-14886	-4584	3729	1.16
$i = 2$	507	5577	1788	-290	

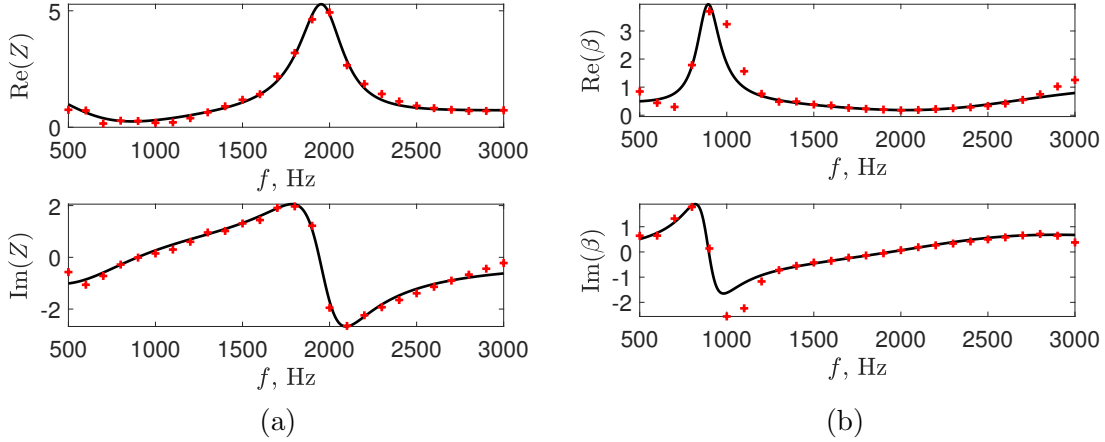


Fig. 19. (a) Impedance and (b) admittance of the CT57 liner: + NASA educed values and — fit obtained with the vector fitting algorithm.

417 at the leading edge of the liner. Briefly afterwards, when most of the transmitted and reflected
 418 acoustic pulses have left the computational domain, the instability takes over in the entire lined
 419 section, leading to a divergent pressure field. In the case of full GTS, the same overall behavior is
 420 observed for the acoustic component. However, no instability is generated and the acoustic energy
 421 falls to zero at increasing times.

422 Even though the full GTS method shows encouraging results for suppressing instabilities, it
 423 should be applied carefully. Indeed, it may not be always physically valid to ignore the instability.
 424 Alomar *et al.* [9] showed experimentally a feedback mechanism between the instability and the
 425 acoustic waves for a liner of finite length. The instabilities generated at the leading edge of the liner
 426 are scattered into acoustic waves at the trailing edge; these acoustic waves propagate upstream and
 427 trigger another instabilities once they are scattered at the leading edge. This feedback mechanism
 428 can not be reproduced when the full GTS method is employed.

429 The time series of the pressure at three virtual microphones located on the rigid wall, at different
 430 axial locations, are shown in Fig. 21 for several values of ϵ . The successive appearance of the initial
 431 acoustic pulse and the instability is observed at all three locations. The impact of ϵ is mainly on
 432 the instability component: decreasing values of ϵ induces a delay in the emergence of the instability.
 433 This is the expected behavior, as decreasing ϵ causes a decrease of the amplification of the unstable
 434 mode, and thus it takes more time for the instability to leverage the acoustic pressure levels. For
 435 $\epsilon = 0.3$ and 0, no instability appears within the simulation time.

436 In order to obtain the frequency spectrum of the pressure field, a tapered cosine window is
 437 applied to the pressure signals to discard the instability component, as shown in Fig. 22. The
 438 Fourier transform of the filtered signals is then computed. Note that for the cases for which the
 439 partial GTS method induces a delay in the appearance of the instability, the window could have

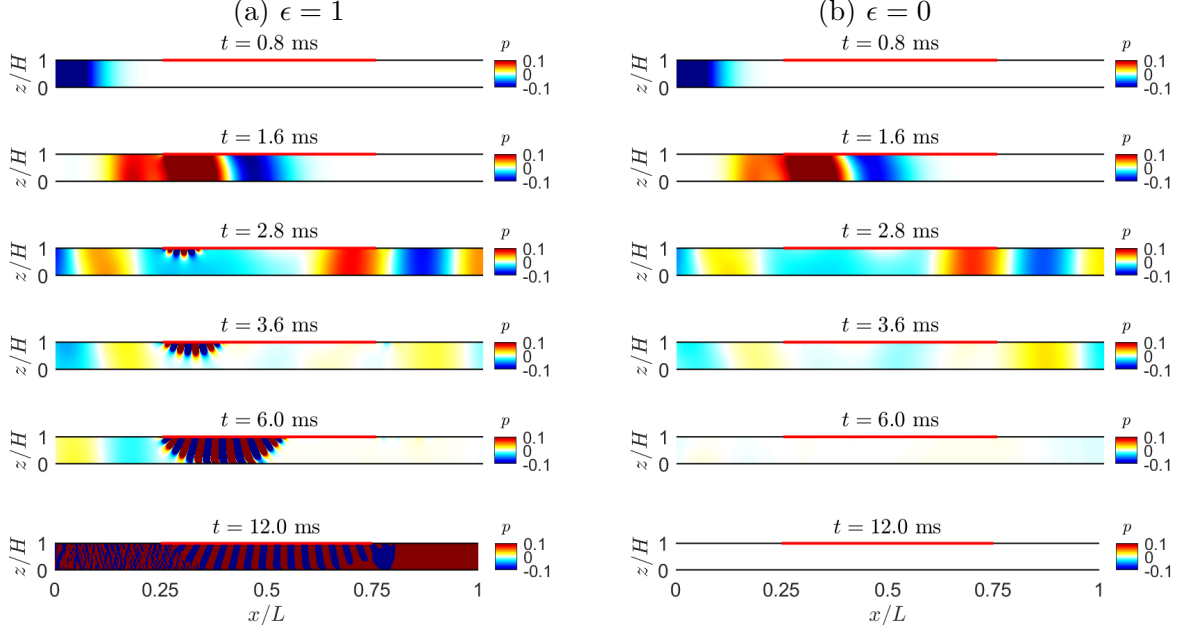


Fig. 20. Pressure map (Pa) along the duct at different times for (a) $\epsilon = 1$ and (b) $\epsilon = 0$.

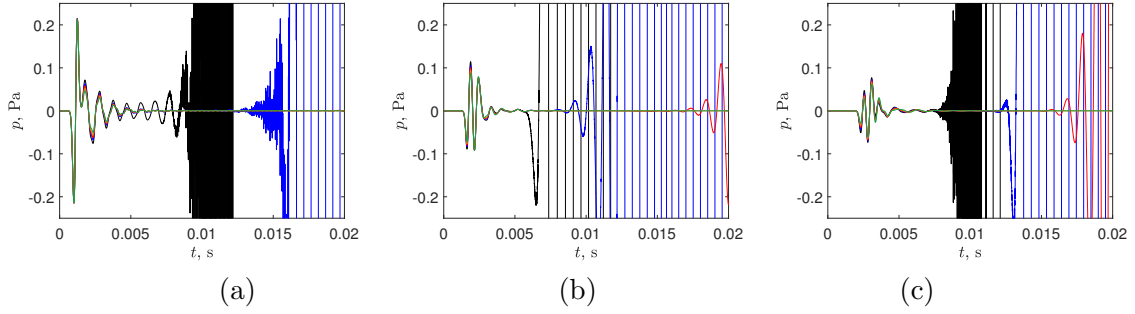


Fig. 21. Time series of pressure on the rigid wall at (a) $x = 0.12L$, (b) $x = 0.5L$ and (c) $x = 0.86L$ for: \blacksquare $\epsilon = 1$, \blacksquare $\epsilon = 0.7$, \blacksquare $\epsilon = 0.5$, \blacksquare $\epsilon = 0.3$ and \blacksquare $\epsilon = 0$.

440 be chosen longer; moreover, for the cases for which the partial GTS allows a suppression of the
 441 instability, the window is not necessary. However, for consistency, the same window is applied on
 442 all the pressure signals and for all the cases considered.

443 The resulting SPL and phase of the acoustic pressure along the duct wall opposite to the liner
 444 are shown in Fig. 23, at different frequencies and for different values of ϵ . For comparison with the
 445 NASA measurements, the curves are adjusted so that the SPL and the phase are 130 dB and 0° ,
 446 respectively, at $x = 0$. Without GTS ($\epsilon = 1$), some oscillations on the SPL and on the phase are
 447 observed at 500, 1000 and 1500 Hz in the lined section. These are due to the difficulty in isolating the
 448 acoustic contribution by simple windowing. Indeed, as seen in Fig. 21 (a) and (b), there is an overlap
 449 near $t = 0.005$ s between the acoustic pulse and the instability. An obvious deviation between the
 450 simulations and the experimental results is noticed at 3 kHz. One of the reasons is that the window
 451 function applied for calculating the Fourier transform does not account for the full acoustic signal.
 452 It can also be partly explained by the proximity of the cut-off frequency of the rigid duct, which

453 induced a significant error in the impedance eduction of the liner in the GIT experiment. Overall,
 454 the trends of both the SPL and the phase are insensitive to ϵ . Decreasing ϵ tends to underestimate
 455 the pressure levels, especially downstream of the liner. The largest discrepancies are observed at
 456 $f = 1$ and 3 kHz. For the other frequencies, the SPL predictions are within a few dBs from the
 457 measured values.

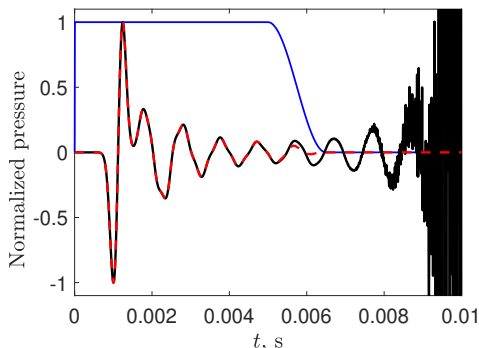


Fig. 22. Window function used for signal processing — blue — and example of (normalized) pressure signal at $x = 0.12L$ before — black — and after windowing — red dashed —.

458 To assess the effectiveness of the GTS, another quantity of interest, namely the insertion loss
 459 (IL), is considered. It is defined as the difference in acoustic power at the exit section $x = L$ with
 460 and without liner. Details on the calculation of the acoustic power from the time-domain numerical
 461 solution can be found in Troian *et al.* [50]. Fig. 24 shows the frequency variations of IL calculated
 462 for different values of ϵ , as well as estimated from the NASA measurements. A peak is observed
 463 close to the liner resonance (1.1 kHz) for $\epsilon = 1$ and 0.7, in good agreement with the experimental
 464 values. The impact of ϵ on the IL is weak except in the immediate vicinity of the resonance and
 465 also for $f > 2.5$ kHz. Note that even if a significant increase of IL is observed for $f > 2.5$ kHz when
 466 decreasing ϵ , a good match is achieved for $\epsilon = 0.5$. Figs. 21 and 23 show that, for this particular
 467 case, $\epsilon = 0.5$ seems to be the optimal value ensuring a good compromise between stable simulation
 468 and accurate prediction. However, this is not a general conclusion, as the optimal value of ϵ may
 469 depend on the flow profile, the Mach number and the liner, among others, and may thus be different
 470 from case to case.

471 The NASA GIT benchmark deals with small Helmholtz numbers ($\omega \leq 2.8$). To examine the
 472 performance of the partial GTS method for high Helmholtz numbers, an additional set of time-
 473 domain simulations are conducted for a duct whose height $H = 0.508$ m is 10 times larger than
 474 the one of the NASA GIT duct. The lengths of the duct and of the liner are also increased, with
 475 respective values of 4.06 m and 2.03 m. The boundary layer thickness of the mean flow remains
 476 equal to 2 % of the duct height. The numerical parameters are the same as before except the
 477 number of grid points along the z -direction, set to $n_z = 175$.

478 The time series of the pressure at $x = 0.5L$ on the rigid wall of the large duct are presented for
 479 several values of ϵ in Fig. 25. Compared with the waveforms for the smaller duct in Fig. 21, the
 480 signals are more complex because of the multimodal propagation. Instabilities are only observed
 481 for $\epsilon = 1$ and do not appear when the partial GTS method is applied. A view on the pressure
 482 response for $0.004 \text{ s} < t < 0.01 \text{ s}$ is shown in Fig. 25 (b). The signals for the several values of ϵ are
 483 almost superimposed even if the amplitude of the waveform is slightly reduced as ϵ decreases.

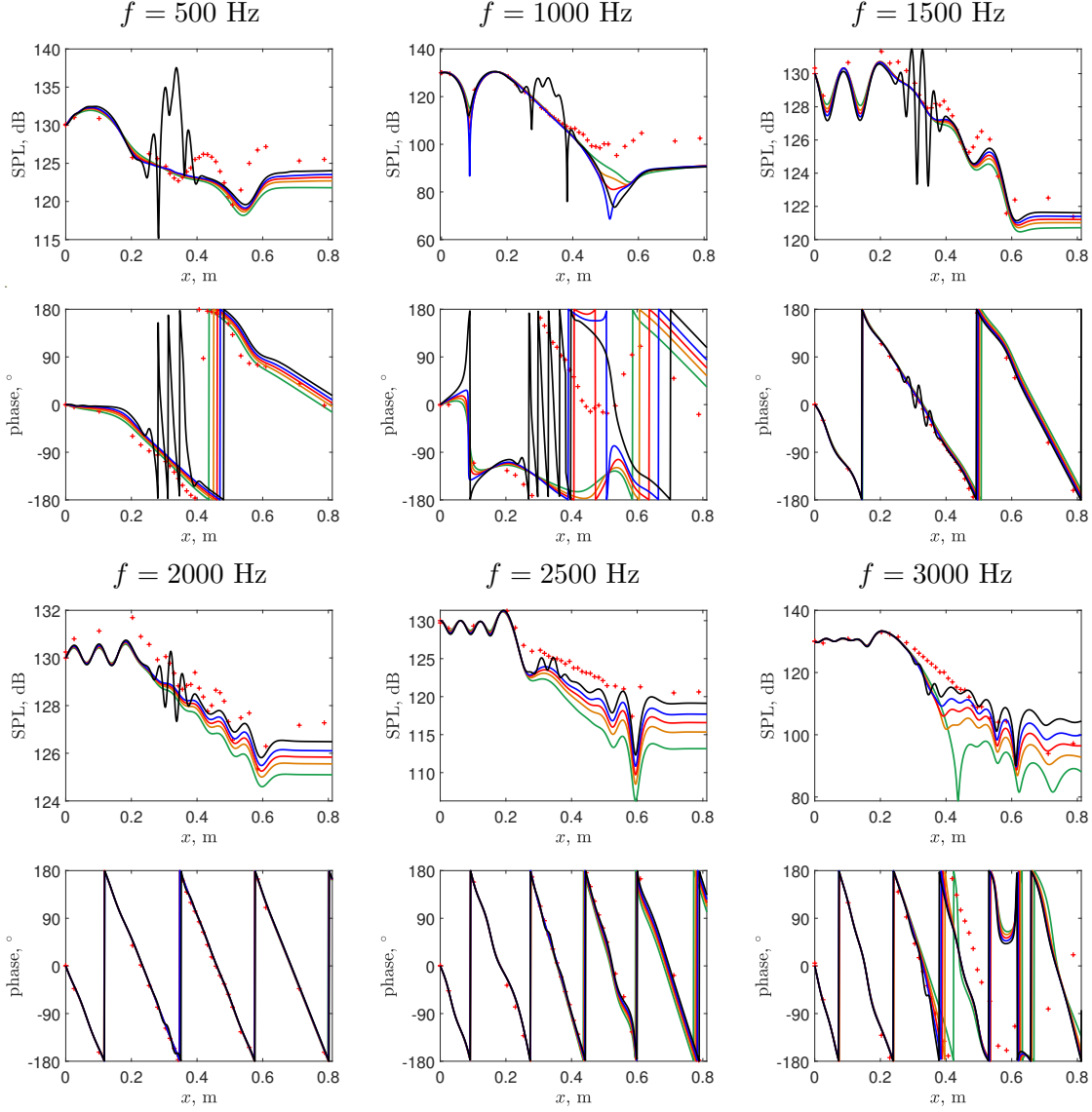


Fig. 23. SPL and phase of the acoustic pressure along the duct for $M = 0.335$ and for different values of ϵ : $\epsilon = 1$, $\epsilon = 0.7$, $\epsilon = 0.5$, $\epsilon = 0.3$, $\epsilon = 0$, + NASA experiments.

484 The SPL and phase along the large duct on the wall opposite to the liner are provided for three
485 frequencies and for several values of ϵ in Fig. 26. A window function, shown in Fig. 25 (a) is also
486 applied to avoid the influence of instabilities and is wider than the one shown in Fig. 22. Compared
487 with the results for the NASA GIT duct shown in Fig. 23, the differences due to the suppression
488 of the gradient term are relatively small.

489 Finally, the powers W at the exit plane $x = L$ are compared for several values of ϵ and for both
490 ducts. Fig. 27 shows the power difference ΔW , which is defined as $\Delta W = 10 \log_{10}[W(\epsilon)/W(\epsilon = 1)]$,
491 where $W(\epsilon)$ is the power when the partial GTS method is applied and $W(\epsilon = 1)$ is the power
492 calculated from the original LEE solution. For the NASA GIT case, ΔW has a large peak near
493 $f = 1.1$ kHz due to the suppression of the instability and has largest variations near $f = 3$ kHz.

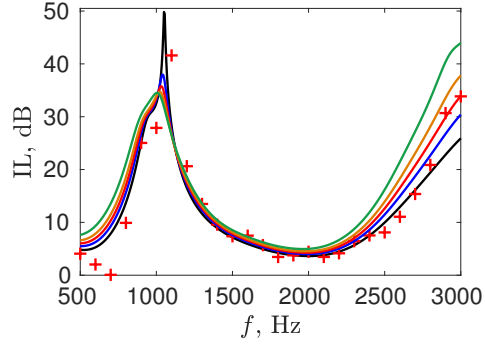


Fig. 24. Insertion loss for $M = 0.335$ and for: — $\epsilon = 1$, — $\epsilon = 0.7$, — $\epsilon = 0.5$, — $\epsilon = 0.3$, — $\epsilon = 0$, + NASA experiments.

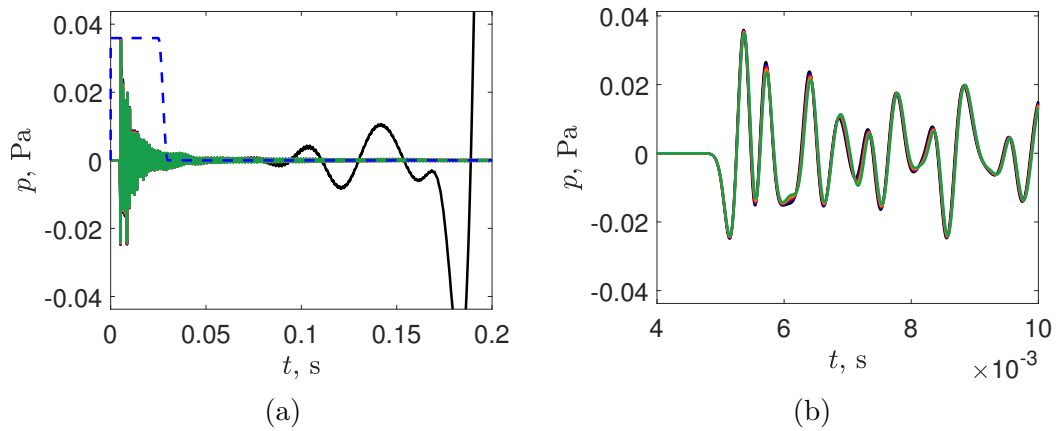


Fig. 25. (a) Time series of pressure on the rigid wall of the large duct at $x = 0.5 L$ and (b) view for $0.004 \text{ s} \leq t \leq 0.01 \text{ s}$ for — $\epsilon = 1$, — $\epsilon = 0.7$, — $\epsilon = 0.5$, — $\epsilon = 0.3$ and — $\epsilon = 0$. The dashed line shows the shape of the window function used in signal processing.

494 For the large duct, ΔW takes much smaller values. Over the entire frequency band of interest, it
 495 is smaller than 3 dB for the full GTS method and than 1 dB for the partial GTS method and for
 496 $\epsilon = 0.7$. The results shown in this section confirm that the partial GTS method noticeably impacts
 497 the acoustic propagation for low Helmholtz numbers but should be acceptable for large Helmholtz
 498 numbers.

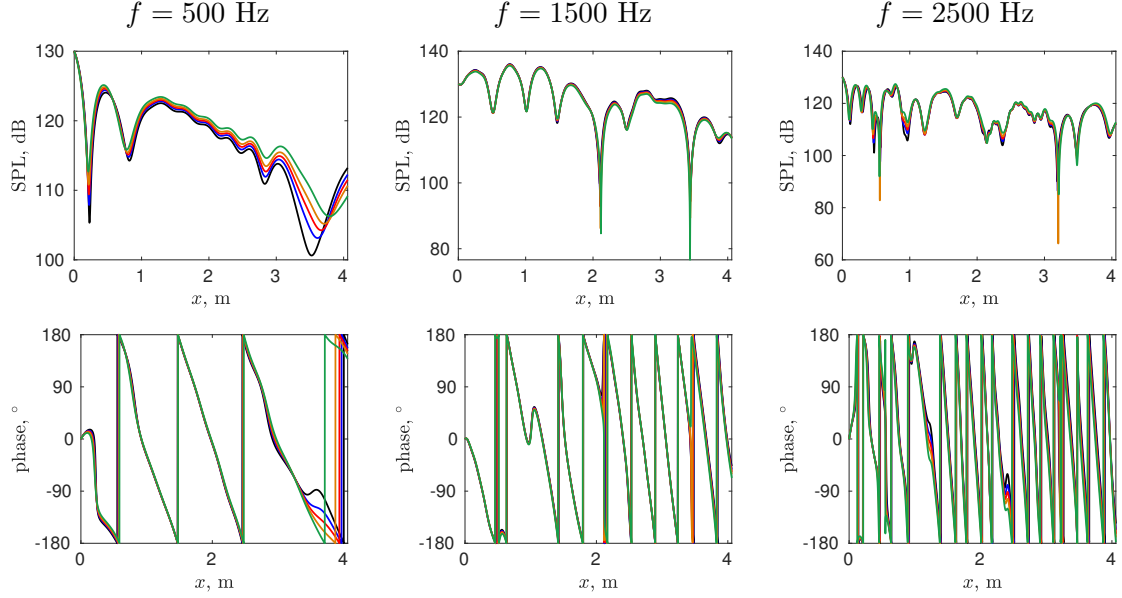


Fig. 26. SPL and phase of the acoustic pressure along the large cross-section duct for $M = 0.335$ and for different values of ϵ : — $\epsilon = 1$, — $\epsilon = 0.7$, — $\epsilon = 0.5$, — $\epsilon = 0.3$, — $\epsilon = 0$.

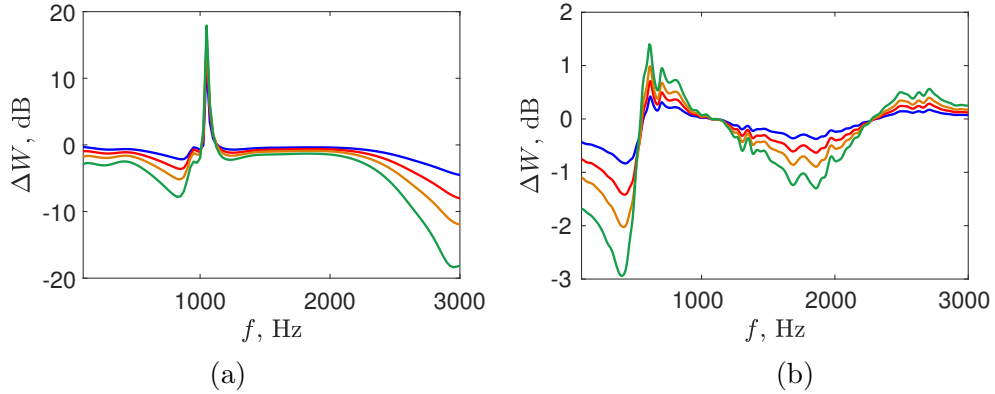


Fig. 27. Power difference $W(\epsilon)/W(\epsilon = 1)$ as a function of the frequency (a) for the NASA GIT duct and (b) for the large duct and for different values of ϵ : — $\epsilon = 0.7$, — $\epsilon = 0.5$, — $\epsilon = 0.3$, — $\epsilon = 0$.

499 6. Conclusion

500 An analysis of the partial GTS method for suppressing hydrodynamic instabilities in a lined
501 flow duct has been performed. The case of a convective instability generated in a lined flow duct
502 was first illustrated. The instability was calculated by solving the linearized Euler equations with a
503 finite-difference time-domain approach. The characteristics of the instability were compared with
504 the predictions of modal analysis. It was shown that both methods indicate that the instability is
505 convective. Perfect matches of wavenumber and mode shapes were also obtained. The effect of the
506 grid resolution and selective filter, which have been reported in the literature to have a significant
507 impact on the instabilities, was investigated. It turns out that very fine grids, compared with what
508 is required for acoustic waves, are needed for precisely calculating the instability.

509 The partial GTS method for suppressing the instability was then assessed. The impact of
510 diminishing the mean flow gradient term on the modes has been investigated. The unstable mode
511 turns to be highly sensitive to the mean flow gradient term. The acoustic modes are also greatly
512 altered for small Helmholtz numbers, but are less affected for high Helmholtz numbers. The partial
513 GTS method was finally applied on the NASA GIT benchmark. The effectiveness of this method
514 has been shown for removing the instabilities in time-domain simulations. The SPL was however
515 underestimated by several dBs for certain frequencies. In particular, a total suppression of the
516 mean flow gradient term seems too severe to accurately predict sound propagation in a lined flow
517 duct, while a partial suppression of this term seems to provide an acceptable prediction, especially
518 in the high frequency range.

519 In addition, even if removing instabilities allows for a stable numerical calculation, it may be,
520 in particular cases, not physically valid: as an example, the feedback mechanism between the
521 instability and acoustic waves for a liner of finite length discussed in Alomar and Aurégan [9] could
522 not be reproduced when the instability is suppressed.

523 Future works can be conducted to study an absolute instability in the time-domain approach.
524 Since the effectiveness of the partial GTS method has been proved, other techniques proposed for
525 dealing with shear instabilities [40, 41, 42, 44, 43] can also be considered.

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532 Appendix A. Matrices of the discretized eigenvalue problem

533 This appendix details the discretization of the eigenvalue problem to obtain the mode wavenum-
534 ber and mode shapes. The number of grid points is denoted by N . Eq. (11) could be rewritten as:
535 $\mathbf{A}\mathbf{X} = \frac{k}{\omega}\mathbf{B}\mathbf{X}$ with \mathbf{X} being the column vector of unknowns at the grid points:

$$\mathbf{X} = [U(z_1) \cdots U(z_N) (P - V)(z_1) \cdots (P - V)(z_N) (P + V)(z_1) \cdots (P + V)(z_N)]^T \quad (\text{A.1})$$

536 where $(z_i)_{1 \leq i \leq N}$, with $z_1 = 0$ and $z_N = 1$, are the grid points. At the boundary points, the
537 variables should satisfy both the LEE equations and the boundary conditions. To avoid having an
538 over-constrained system, some information must be discarded. The use of characteristic variables
539 allows one to make a suitable choice. Thus, at $z = 0$, the incident wave on the boundary is $P - V$
540 and the reflected wave is $P + V$. Therefore, $P + V$ is determined with the boundary condition
541 and not from the LEE. Similarly, at $z = 1$, the incident and reflected wave are $P + V$ and $P - V$,
542 respectively and, hence, $P - V$ is determined with the boundary condition.

Bringing the LEE equations and the boundary conditions together lead to the $3N \times 3N$ matrices:

$$\mathbf{A} = \left[\begin{array}{c|c|c} \mathbf{I}_{(1:N),(1:N)} & -\frac{\epsilon}{2i\omega}(\mathbf{D}\mathbf{U}_0)_{(1:N),(1:N)} & \frac{\epsilon}{2i\omega}(\mathbf{D}\mathbf{U}_0)_{(1:N),(1:N)} \\ \hline \mathbf{0}_{N,N} & \left(\mathbf{I} - \frac{1}{i\omega}\mathbf{D}\right)_{(1:N-1),(1:N)} & \mathbf{0}_{N-1,N} \\ \hline 0 & \dots & 0 & -1 - \beta & 0 & \dots & 0 & 1 - \beta \\ \hline \mathbf{0}_{N,N} & 1 & 0 & \dots & 0 & -1 & 0 & \dots & 0 \\ \hline & & \mathbf{0}_{N-1,N} & & \left(\mathbf{I} + \frac{1}{i\omega}\mathbf{D}\right)_{(2:N),(1:N)} & & & & \end{array} \right] \quad (\text{A.2})$$

$$\mathbf{B} = \left[\begin{array}{c|c|c} \mathbf{U}_0_{(1:N),(1:N)} & \frac{1}{2}\mathbf{I}_{(1:N),(1:N)} & \frac{1}{2}\mathbf{I}_{(1:N),(1:N)} \\ \hline \mathbf{I}_{(1:N-1,N)} & \mathbf{U}_0_{(1:N-1),(1:N)} & \mathbf{0}_{N,N} \\ \hline \mathbf{0}_{1,N} & \mathbf{0}_{1,N} & \\ \hline \mathbf{0}_{1,N} & & \mathbf{0}_{1,N} \\ \hline \mathbf{I}_{(2:N,N)} & \mathbf{0}_{N,N} & \mathbf{U}_0_{(2:N),(1:N)} \end{array} \right] \quad (\text{A.3})$$

544 where $\mathbf{0}_{I,J}$ is the zero matrix with I rows and J columns and \mathbf{D} and \mathbf{I} are the differentiation and
 545 identity matrix, respectively, both square matrices of size N . The matrix \mathbf{U}_0 is a diagonal matrix
 546 of size N with diagonal elements $\mathbf{U}_0_{i,i} = u_0(z_i)$. In addition, the notation $\mathbf{M}_{(I:J),(K:L)}$ indicates
 547 the submatrix of \mathbf{M} formed by rows I to J and columns K to L . Finally, the parameter ϵ in matrix
 548 \mathbf{A} controls the mean flow gradient term and is equal to $\epsilon = 1$ for the LEE equations and to $\epsilon = 0$
 549 for the full GTS method.

550 Appendix B. Comparison of the instability characteristics for another frequency

551 In this appendix, the comparison of the instability characteristics determined from the time-
 552 domain numerical solution with those predicted by the modal analysis approach, done in Section 3
 553 for $\omega = 0.9271$, is performed for another frequency, namely $\omega = 0.4636$. The objective is to confirm
 554 the viability of the time-domain method in simulating the instability.

555 Fig. B.28 displays the wavenumbers of the modes determined by modal analysis for $\omega = 0.4636$.
 556 The reference result $\omega = 0.9271$ is also indicated. Note that the MSD liner model is still used
 557 and the liner impedance at these two frequencies is different. The wavenumbers of the acoustic
 558 propagating mode ($k = 0.79 - 0.33i$) and the unstable mode ($k_{\text{HI}} = 2.57 + 1.31i$) show the most
 559 evident change. Compared with $\omega = 0.9271$, the spatial growth rate of the unstable mode decreases
 560 only slightly. On the other hand, the wavelength becomes three times longer. Fig. B.29 shows the
 561 evolution of $\arg(p)$ and $\ln|p|$ along the lined wall for $\omega = 0.4636$ obtained from the time-domain
 562 solution at $t = 90$ and predicted with the analytical solution in Eq. (14), where the value of k_{HI}
 563 obtained from modal analysis has been used. Here again, an excellent match is observed.

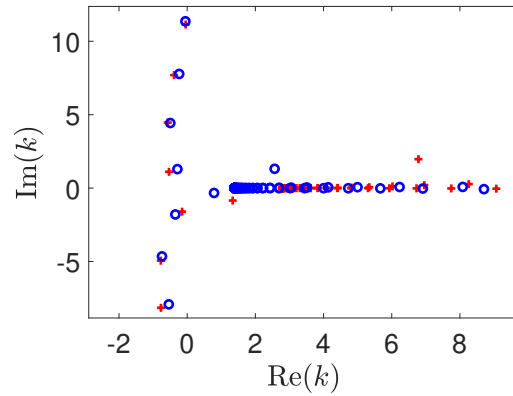


Fig. B.28. Wavenumber obtained by modal analysis for $\circ \omega = 0.4636$ and $+ \omega = 0.9271$.

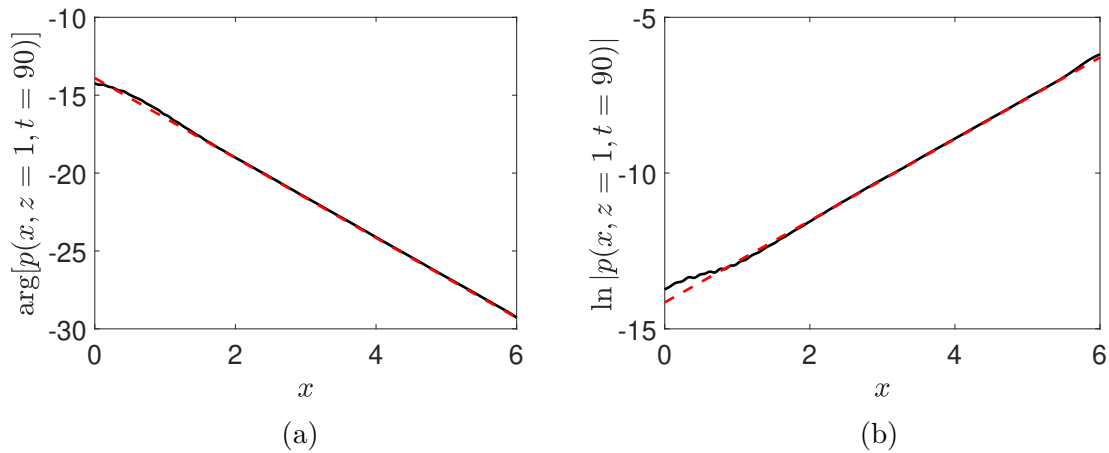


Fig. B.29. Comparison of (a) the phase and (b) amplitude of the pressure on the lined wall for $\omega = 0.4636$ from the time-domain solution at $t = 90$ — and from the analytical solution - - - .

564 **References**

565 [1] Y. Özyörük, L. N. Long, M. G. Jones, Time-domain numerical simulation of a flow-impedance
566 tube, *J. Comput. Phys.* 146 (1) (1998) 29–57.

567 [2] H. Ju, K.-Y. Fung, Time-domain impedance boundary conditions with mean flow effects, *AIAA*
568 *J.* 39 (9) (2001) 1683–1690.

569 [3] C. Richter, F. H. Thiele, X. D. Li, M. Zhuang, Comparison of time-domain impedance bound-
570 ary conditions for lined duct flows, *AIAA J.* 45 (6) (2007) 1333–1345.

571 [4] E. Meyer, F. Mechel, G. Kurtze, Experiments on the influence of flow on sound attenuation
572 in absorbing ducts, *J. Acoust. Soc. Am.* 30 (3) (1958) 165–174.

573 [5] M. Brandes, D. Ronneberger, Sound amplification in flow ducts lined with a periodic sequence
574 of resonators, Munich, Germany, 1995, pp. 893–901.

- 575 [6] Y. Aurégan, M. Leroux, Experimental evidence of an instability over an impedance wall in a
576 duct with flow, *J. Sound Vib.* 317 (3-5) (2008) 432–439.
- 577 [7] Y. Aurégan, D. Kumar Singh, Experimental observation of a hydrodynamic mode in a flow
578 duct with a porous material, *J. Acoust. Soc. Am.* 136 (2) (2014) 567–572.
- 579 [8] D. Marx, Y. Aurégan, H. Bailliet, J.-C. Valière, PIV and LDV evidence of hydrodynamic
580 instability over a liner in a duct with flow, *J. Sound Vib.* 329 (18) (2010) 3798–3812.
- 581 [9] A. Alomar, Y. Aurégan, Particle image velocimetry measurement of an instability wave over
582 a porous wall in a duct with flow, *J. Sound Vib.* 386 (2017) 208–224.
- 583 [10] U. Ingard, Influence of fluid motion past a plane boundary on sound reflection, absorption,
584 and transmission, *J. Acoust. Soc. Am.* 31 (7) (1959) 1035–1036.
- 585 [11] M. Myers, On the acoustic boundary condition in the presence of flow, *J. Sound Vib.* 71 (3)
586 (1980) 429–434.
- 587 [12] B. J. Tester, The propagation and attenuation of sound in lined ducts containing uniform or
588 “plug” flow, *J. Sound Vib.* 28 (2) (1973) 151–203.
- 589 [13] C. K. Tam, L. Auriault, Time-domain impedance boundary conditions for computational
590 aeroacoustics, *AIAA J.* 34 (5) (1996) 917–923.
- 591 [14] S. W. Rienstra, A classification of duct modes based on surface waves, *Wave Motion.* 37 (2)
592 (2003) 119–135.
- 593 [15] S. W. Rienstra, B. J. Tester, An analytic Green’s function for a lined circular duct containing
594 uniform mean flow, *J. Sound Vib.* 317 (3-5) (2008) 994–1016.
- 595 [16] X. D. Li, C. Richter, F. Thiele, Time-domain impedance boundary conditions for surfaces with
596 subsonic mean flows, *J. Acoust. Soc. Am.* 119 (5) (2006) 2665–2676.
- 597 [17] E. J. Brambley, Fundamental problems with the model of uniform flow over acoustic linings,
598 *J. Sound Vib.* 322 (4-5) (2009) 1026–1037.
- 599 [18] E. J. Brambley, Well posed boundary condition for acoustic liners in straight ducts with flow,
600 *AIAA J.* 49 (6) (2011) 1272–1282.
- 601 [19] S. Rienstra, G. Vilenski, Spatial instability of boundary layer along impedance wall, in: 14th
602 AIAA/CEAS Aeroacoustics Conference, AIAA Paper 2008–2932, Vancouver, BC, Canada,
603 2008, pp. 1–13.
- 604 [20] Y. Renou, Y. Aurégan, Failure of the Ingard–Myers boundary condition for a lined duct: An
605 experimental investigation, *J. Acoust. Soc. Am.* 130 (1) (2011) 52–60.
- 606 [21] S. Rienstra, Impedance models in time domain, including the extended Helmholtz resonator
607 model, in: 12th AIAA/CEAS Aeroacoustics Conference, AIAA Paper 2006-2686, Cambridge,
608 MA, USA, 2006.
- 609 [22] S. W. Rienstra, M. Darau, Boundary-layer thickness effects of the hydrodynamic instability
610 along an impedance wall, *J. Fluid Mech.* 671 (2011) 559–573.

- 611 [23] G. Boyer, E. Piot, J.-P. Brazier, Theoretical investigation of hydrodynamic surface mode in a
612 lined duct with sheared flow and comparison with experiment, *J. Sound Vib.* 330 (8) (2011)
613 1793–1809.
- 614 [24] D. Marx, A piecewise linear mean flow model for studying stability in a lined channel, *J. Sound*
615 *Vib.* 331 (16) (2012) 3809–3823.
- 616 [25] D. Marx, Numerical computation of a lined duct instability using the linearized Euler equa-
617 tions, *AIAA J.* 53 (8) (2015) 2379–2388.
- 618 [26] L. Pascal, E. Piot, G. Casalis, Global linear stability analysis of flow in a lined duct, *J. Sound*
619 *Vib.* 410 (2017) 19–34.
- 620 [27] G. Gabard, E. Brambley, A full discrete dispersion analysis of time-domain simulations of
621 acoustic liners with flow, *J. Comput. Phys.* 273 (2014) 310–326.
- 622 [28] Y. Deng, D. Dragna, M.-A. Galland, A. Alomar, Comparison of three numerical methods for
623 acoustic propagation in a lined duct with flow, in: 25th AIAA/CEAS Aeroacoustics Confer-
624 ence, AIAA Paper 2019-2658, Delft, The Netherlands, 2019, pp. 1–15.
- 625 [29] D. Khamis, E. J. Brambley, Viscous effects on the acoustics and stability of a shear layer over
626 an impedance wall, *J. Fluid Mech.* 810 (2017) 489–534.
- 627 [30] D. Marx, Y. Aurégan, Effect of turbulent eddy viscosity on the unstable surface mode above
628 an acoustic liner, *J. Sound Vib.* 332 (15) (2013) 3803–3820.
- 629 [31] J. Rodríguez Sánchez, Étude théorique et numérique des modes propres acoustiques dans un
630 conduit avec écoulement et parois absorbantes, PhD thesis, Toulouse, ISAE (2016).
- 631 [32] B. Xin, D. Sun, X. Jing, X. Sun, Numerical study of acoustic instability in a partly lined flow
632 duct using the full linearized Navier–Stokes equations, *J. Sound Vib.* 373 (2016) 132–146.
- 633 [33] R. Sebastian, D. Marx, V. Fortuné, Numerical simulation of a turbulent channel flow with an
634 acoustic liner, *J. Sound Vib.* 456 (2019) 306–330.
- 635 [34] E. Brambley, G. Gabard, Time-domain implementation of an impedance boundary condition
636 with boundary layer correction, *J. Comput. Phys.* 321 (2016) 755–775.
- 637 [35] C. Bogey, C. Bailly, D. Juvé, Computation of flow noise using source terms in linearized Euler’s
638 equations, *AIAA J.* 40 (2) (2002) 235–243.
- 639 [36] C. Richter, H. Lueck, L. Panek, F. Thiele, Methods for suppressing shear layer instabilities for
640 CAA, *J. Comput. Acoustics* 19 (02) (2011) 181–203.
- 641 [37] Y. Özyörük, B. J. Tester, Application of frequency-domain linearized Euler solutions to the
642 prediction of aft fan tones and comparison with experimental measurements on model scale
643 turbofan exhaust nozzles, *J. Sound Vib.* 330 (16) (2011) 3846–3858.
- 644 [38] A. Iob, R. Arina, C. Schipani, Frequency-domain linearized Euler model for turbomachinery
645 noise radiation through engine exhaust, *AIAA J.* 48 (4) (2010) 848–858.

- 646 [39] X. Li, C. Schemel, U. Michel, F. Thiele, Azimuthal sound mode propagation in axisymmetric
647 flow ducts, *AIAA J.* 42 (10) (2004) 2019–2027.
- 648 [40] R. Ewert, W. Schröder, Acoustic perturbation equations based on flow decomposition via
649 source filtering, *J. Comput. Phys.* 188 (2) (2003) 365–398.
- 650 [41] J.-H. Seo, Y. J. Moon, Perturbed compressible equations for aeroacoustic noise prediction at
651 low Mach numbers, *AIAA J.* 43 (8) (2005) 1716–1724.
- 652 [42] J. H. Seo, Y. J. Moon, Linearized perturbed compressible equations for low Mach number
653 aeroacoustics, *J. Comput. Phys.* 218 (2) (2006) 702–719.
- 654 [43] X. Zhang, X. Chen, J. Gill, Gradient term filtering for stable sound propagation with linearized
655 Euler equations, in: 20th AIAA/CEAS Aeroacoustics Conference, AIAA Paper 2014-3306,
656 Atlanta, GA, 2014, pp. 1–14.
- 657 [44] Y. Sun, R. Fattah, S. Zhong, X. Zhang, Stable time-domain CAA simulations with linearised
658 governing equations, *Comput. Fluids.* 167 (2018) 187–195.
- 659 [45] C. Bogey, C. Bailly, A family of low dispersive and low dissipative explicit schemes for flow
660 and noise computations, *J. Comput. Phys.* 194 (1) (2004) 194–214.
- 661 [46] J. Berland, C. Bogey, O. Marsden, C. Bailly, High-order, low dispersive and low dissipative
662 explicit schemes for multiple-scale and boundary problems, *J. Comput. Phys.* 224 (2) (2007)
663 637–662.
- 664 [47] J. Berland, C. Bogey, C. Bailly, Low-dissipation and low-dispersion fourth-order Runge–Kutta
665 algorithm, *Comput. Fluids.* 35 (10) (2006) 1459–1463.
- 666 [48] C. Bogey, N. de Cacqueray, C. Bailly, A shock-capturing methodology based on adaptative
667 spatial filtering for high-order non-linear computations, *J. Comput. Phys.* 228 (5) (2009) 1447–
668 1465.
- 669 [49] C. Bogey, C. Bailly, Three-dimensional non-reflective boundary conditions for acoustic sim-
670 ulations: far field formulation and validation test cases, *Acta Acust. united Ac.* 88 (2002)
671 463–471.
- 672 [50] R. Troian, D. Dragna, C. Bailly, M.-A. Galland, Broadband liner impedance eduction for
673 multimodal acoustic propagation in the presence of a mean flow, *J. Sound Vib.* 392 (2017)
674 200–216.
- 675 [51] B. Gustavsen, A. Semlyen, Rational approximation of frequency domain responses by vector
676 fitting, *IEEE Trans. Power Deliv.* 14 (3) (1999) 1052–1061.
- 677 [52] L. N. Trefethen, *Spectral methods in MATLAB*, Vol. 10, Siam, 2000.
- 678 [53] E. J. Brambley, M. Darau, S. W. Rienstra, The critical layer in linear-shear boundary layers
679 over acoustic linings, *J. Fluid Mech.* 710 (2012) 545–568.
- 680 [54] G. G. Vilenski, S. W. Rienstra, Numerical study of acoustic modes in ducted shear flow, *J.*
681 *Sound Vib.* 307 (3-5) (2007) 610–626.

- 682 [55] R. Briggs, *Electron–stream interaction with plasmas*, MIT Press, Cambridge, 1964.
- 683 [56] P. Huerre, P. A. Monkewitz, Local and global instabilities in spatially developing flows, *Annual*
684 *review of fluid mechanics* 22 (1) (1990) 473–537.
- 685 [57] E. J. Brambley, N. Peake, Classification of aeroacoustically relevant surface modes in cylindrical
686 lined ducts, *Wave Motion*. 43 (4) (2006) 301–310.
- 687 [58] M. Jones, W. Watson, T. Parrott, Benchmark data for evaluation of aeroacoustic propagation
688 codes with grazing flow, in: *11th AIAA/CEAS Aeroacoustics Conference*, AIAA Paper 2005-
689 2853, Monterey, CA, USA, 2005, pp. 1–18.
- 690 [59] M. O. Burak, M. Billson, L.-E. Eriksson, S. Baralon, Validation of a time- and frequency-
691 domain grazing flow acoustic liner model, *AIAA J.* 47 (8) (2009) 1841–1848.