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# FROM THE EXPANSION OF THE SPACE OF THE DISPLACEMENTS TOWARDS THE TRANSFORMATIONS OF THE PLANE WITH THE INVOLVEMENT OF THE LEBESGUE'S THEOREM AND THE LIMIT

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► **To cite this version:**

Edern Ollivier. FROM THE EXPANSION OF THE SPACE OF THE DISPLACEMENTS TOWARDS THE TRANSFORMATIONS OF THE PLANE WITH THE INVOLVEMENT OF THE LEBESGUE'S THEOREM AND THE LIMIT. 2022. hal-03572778v2

**HAL Id: hal-03572778**

**<https://hal.science/hal-03572778v2>**

Preprint submitted on 17 Mar 2022

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37 THEOREM 2.2 (Terminal Point Theorem). *There is the integration of the insta-*  
 38 *bility terminal point of the computed odometric model at the value of integration  $\Pi/2$*   
 39 *:*

$$40 \quad \beta(\Pi/2) = \int_0^{\Pi/2} \cos(x)^I dx = \Gamma(1/2 + I/2) \cdot \sqrt{\Pi} / (2 \cdot \Gamma(1 + I/2))$$

41 *In other words, the terminal value is computed to a Gamma.*

$$42 \quad \beta(\Pi/2) = \Gamma(1/2 + I/2) \cdot \sqrt{\Pi} / (2 \cdot \Gamma(1 + I/2))$$

43 Observe that [Theorems 2.1](#) and [2.2](#) correctly mix definitions of the value of inte-  
 44 gration.

45 COROLLARY 2.3. *Let  $f(x)$  be continuous and differentiable everywhere. If  $f(x)$*   
 46 *has at least two terminal points, then  $\int_0^{\Pi/2} f(x)^I dx$  must have at least one value.*

47 *Proof.* Let  $a$  and  $b$  be two distinct terminal points of  $f$ . And also with  $g$  the  
 48 limited development of the  $f$  function on the interval  $[a, b]$ . By [Theorem 2.2](#), there  
 49 exists a number  $c$  such that the serie converges normally by Lebesgue :

$$50 \quad \int_a^b f(x)^I dx \leq \sum_{n=0}^{\infty} \int_a^c g^n(x)^I dx.$$

51 For example let  $f(x)$  to be the  $\cos(x)$  function to be the following function to be  
 52 evaluated :

$$53 \quad f(x) = \cos(x)$$

54 And let  $g(x)$ , the evaluation is to be done normally around zero but here we can  
 55 approximate the function in  $c$  with  $c \leq b$  by a polynom of order  $2n$  by Taylor :

$$56 \quad g^n(x) = (-x)^n \cdot x^2 / (2 \cdot n)! \quad \square$$

57 COROLLARY 2.4. *Let  $\theta(t)$  be defined most of everywhere. If  $\theta(t)$  has at*  
 58 *least a limit point, then  $\lim_{t \rightarrow \Pi/2} \theta(t) = \infty$ .*

59 *Proof.* Trivially, the limit is given by the expression  $\lim_{t \rightarrow \Pi/2} -\ln(\cos(t)) = \infty \quad \square$

60 *Proof.* On one hand the space of displacements is a subgroup of the space of the  
 61 transformations of the plane implies that each element of the space of displacements is  
 62 the limit of a series of elements of this space of displacements. And another hand for  
 63 each element of this space of displacements [Proof 3](#) there exists a bounded sequence  
 64 for any displacements, cf. The work of Guillaume Artus on the slavings to be given  
 65 to the computer to make a robot go from one position to another in the plane [1],  
 66 in the space of the displacements such that the limit belongs to this space of the  
 67 displacements of the plane implies that the space of the displacements is completed  
 68 in the body of the transformations of the plane.

69 Note about the explanations on the first page of calculus : The theorem has  
 70 been fully proven so that now I can access the datas from the first series with the  
 71 linearized functions in the matrix of state and measured, given by the chinese team  
 72 in a paper published by the IEEE editor, I have done the primary effort to the  
 73 state matrix of functions, nevertheless they did the last one to ensure the compat-  
 74 ibility of the odometric model with the mathematical background of mine and also

75 to get the values with the vehicle [2]. All the datas have been computed in a ta-  
 76 ble by a program under Ubuntu 20.04 with the GCC and also the GLIBC software,  
 77 it used to be accessed on the DataPort of the IEEE website with the odometric key  
 78 word : [https://iee-dataport.org/open-access/odometric-datas-simulation-odometric-](https://iee-dataport.org/open-access/odometric-datas-simulation-odometric-model)  
 79 model to give an example from an linear approximation of the odometric model  
 80 with GCC. For the simulation of the odometric model please see at : [https://iee-](https://iee-dataport.org/documents/new-figure-simulation-odometric-model)  
 81 dataport.org/documents/new-figure-simulation-odometric-model to give an example  
 82 from a simulation of the odometric model with Scilab.

83 Note about the example on the first page of calculus : The best effort example  
 84 is the half turn like maneuver. In the space of the displacements there are two arcs  
 85 of circle to do for instance, instead of a rotation of  $\Pi$  in the space of the transfor-  
 86 mations in the plane, and usually its can be done for more figures of maneuver with  
 87 the homothety, or even with the translation. It is usually appointed the transverse  
 88 function in the space of the displacements.  $\square$

89 **3. Algorithm.** Our analysis leads to the algorithm in [Algorithm 3.1](#).

---

**Algorithm 3.1** Build spiral

---

```

Define croquette := proc(n)
Define locali, R, x, z
Define R := [[0, 0]]
Define i := 1
while i < n/2 do
  Define x := i/n *  $\Pi$ 
  Define z :=  $\sin(x)$  * hypergeom([1/2, 1/2 - I/2], [3/2],  $\sin(x)^2$ )
  Update z := evalf(z)
  Update R := [op(R), [Re(z), Im(z)]
  Update i := i + 1
end while
return R
Define end

```

---

90 **4. Experimental results.** [Figure 1](#) shows an example result of the integration.  
 91 [Figure 2](#) shows again an example result of the integration.  
 92 [Figure 3](#) shows again an example result of the integration.

93 **5. Discussion of  $z = [Re(z), Im(z)]$ .** As one discussion, I could say that the  
 94 values of the integrated computed odometric model is computed in the given figures  
 95 with a procedure recording the complex values at every steps of the procedure given  
 96 by a volunteer from the Inria of Saclay in France.

97 **6. Conclusions.** As one conclusion, I could say that the odometric model is well  
 98 known now and that everything is done for the commands of a model of automation  
 99 to be validated with the odometric model [3]. I should also emphase that a thesis can  
 100 be done with the first issue of mine about the so called linearization of the equations  
 101 of the odometric model given back by the issue of the chinese team of researchers [1].

102 **Appendix A. An appendix for the modulo  $\Pi$ .**

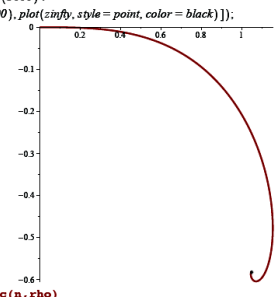
103 LEMMA A.1. *Idem with the  $3*\Pi/2$  value modulo  $\Pi$ .*

104 **Acknowledgments.** I would like to acknowledge the assistance of a volunteer  
 105 of the Inria of Saclay in France in solving together these figures.

```

> theta := -log(cos(t));
                                0 := -ln(cos(t))
=
> zinfly := int(cos(t)^t, t=0..Pi/2);
                                zinfly :=  $\frac{\Gamma\left(\frac{1}{2} + \frac{1}{2}\right)\sqrt{\pi}}{2\Gamma\left(1 + \frac{1}{2}\right)}$ 
=
> zinfly := [[Re(evalf(zinfly)), Im(evalf(zinfly))]];
                                zinfly := [[1.049478078, -0.5824562520]]
=
> int(cos(t)^t, t=0..7*Pi/100);
                                sin(7*Pi/100) hypergeom([1/2, 1/2 - 1/2], [3/2], sin(x)^2)
=
> croquette:=proc(n)
local i, R, x, z;
R:=[[0,0]];
i:=1;
while i<n/2 do
x:=i/n*Pi;
z:=sin(x)*hypergeom([1/2, 1/2-1/2], [3/2], sin(x)^2);
z:=evalf(z);
R:=[op(R), [Re(z), Im(z)]];
i:=i+1;
od;
R;
end;
> C3000 := croquette(3000);
> display([plot(C3000), plot(zinfly, style=point, color=black)]);
=
> kroquette:=proc(n, xho)

```



```

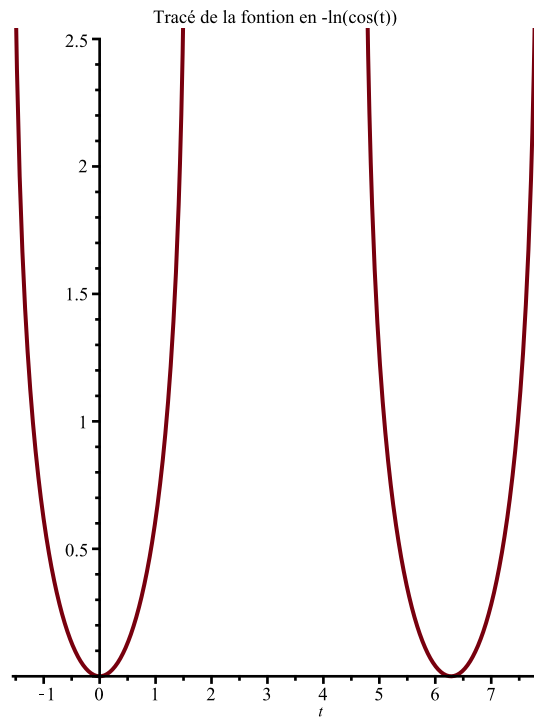
local i,R,x,z;
R:=[[0,0]]
i:=2;
while i<n do
  x:=pi/2-rho*i*pi/2;
  z:=sin(x)*hypergeom([1/2,1/2-I/2],[3/2],sin(x)^2);
  z:=evalf(z);
  R:=[op(R),[Re(z),Im(z)]];
  i:=i+1;
od;
R;
end:
> K1000 := kroquette(200,  $\frac{7}{8}$ );
> display([plot(K1000), plot(zinfy, style = point, color = black)]);

```

```

> with(plots):
[animate, animate3d, animatcurv, arrow, changecoords, complexplot, complexplot3d, conformal, (5)
conformal3d, contourplot, contourplot3d, coordplot, coordplot3d, densityplot, display,
dualaxisplot, fieldplot, fieldplot3d, gradplot, gradplot3d, implicitplot, implicitplot3d, inequal,
interactive, interactiveparams, intersectplot, listcontplot, listcontplot3d, listdensityplot, listplot,
listplot3d, loglogplot, logplot, matrixplot, multiple, odeplot, pareto, plotcompare, pointplot,
pointplot3d, polarplot, polygonplot, polygonplot3d, polyhedra_supported, polyhedraplot,
rootlocus, semilogplot, setcolors, setoptions, setoptions3d, shadebetween, spacecurve,
sparsematrixplot, surfdata, textplot, textplot3d, tubeplot]

```



106

107

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109 the transverse function approach : control design and simulation results. [Research Report]  
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112 jectory Planning and Tracking for Autonomous Vehicle Based On State Lattice And Model  
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