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ON THE TRADEOFF BETWEEN RESOLUTION AND AMBIGUITIES FOR NON-UNIFORM LINEAR ARRAYS

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ABSTRACT

The performance of most direction of arrival (DoA) estimation algorithms is driven by two different kinds of errors: small, local errors in the main lobe and possibly large errors due to sidelobe peaks ambiguities. Reducing these two kinds of errors simultaneously is not possible and therefore a compromise has to be made. Motivated by automotive applications with small arrays, we consider solving this tradeoff by modifying the positions of the sensors of the array. In this letter, two new criteria are proposed to solve the above mentioned tradeoff. Optimal solutions are derived and illustrations are provided with a 3-element non-uniform linear array.

***Index Terms*—** Non Uniform Linear Array, Ambiguities, Radar

1. INTRODUCTION

DoA estimation is a crucial issue for many radar applications. This is especially the case for automotive systems designed to early detect and avoid pedestrians, cyclists or other cars. The system needs the best DoA estimation accuracy in order to precisely estimate the targets trajectories and decide whether there is a collision risk or not. Most often, uniform linear arrays (ULA) are employed, which consists of equidistant sensors, usually spaced a half-wavelength apart in order to avoid wavefront under sampling. In order to improve the antenna resolution while keeping a low system cost and hardware complexity, non-uniform linear arrays (NULA) have been investigated, see [1] and references therein. Indeed, increasing the antenna size while keeping the number of sensors constant results in a sharpener main beam and a possible better DoA precision, but also a sidelobe raising and potential ambiguities. Most of the time, the so-called minimum redundancy linear array (MRLA) geometry is considered when dealing with NULA because this kind of array allows to measure all correlation lags as the same size ULA, yet with a minimum of sensors. However, the MRLA geometry does not guarantee any optimality with respect to resolution or sidelobe level.

The best achievable DoA estimation variance is given by the Cramér-Rao Bound (CRB). In [2] Chambers *et al.* showed that the Cramér-Rao Bound (CRB) is inversely proportional to the spatial variance of the sensors. This dependence is also confirmed in [3] where the positions of the sensors are optimized with respect to array resolution using genetic algorithms. Briefly speaking, the larger the spatial variance of the sensors of the linear array, the better the resolution. Unfortunately the CRB only considers errors around the true DoA while considering only the main beam shape independently of any sidelobe level. Possible larger errors due to ambiguities are not taken into account. Improving the antenna resolution is incompatible with reducing the ambiguities and a compromise has to be found when one wants to optimize the antenna geometry to achieve the best DoA precision. Observe that such a compromise depends on signal to noise ratio (SNR). Indeed, at high SNR, it is unlikely that a sidelobe peak be mistaken for a true DoA and therefore focus will be on optimizing the main lobe width. In contrast, with low SNR, the probability to make a large error due to high sidelobe levels increases, and optimization should be carried out with this potential problem in mind. In practice though, it is impossible to adapt in real time the antenna geometry to the source SNR. Moreover, SNR is a-priori unknown in many array antenna applications. A compromise independent from any SNR has to be found, this is the main objective of this letter.

More precisely we focus on the case of collision avoidance radars. In the most advanced automotive applications, arrays with a small number of elements are used, typically 3 or 4. Moreover, these systems usually use a large bandwidth, resulting in a very high range resolution and hence the number of targets belonging to the same range-Doppler bin is typically one. In this letter, we focus on the simplest case of a three-element NULA where the two first elements are placed

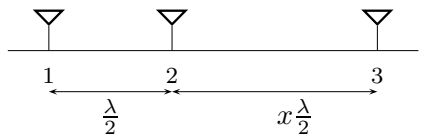

Fig. 1. Three-sensors non uniform linear array

a half-wavelength apart to prevent DoA aliasing, as illustrated on figure 1. Spatial resolution is improved when the distance $x\frac{\lambda}{2}$ between the last two sensors increases, i.e. the spatial variance maximized, while keeping the array unambiguous. Obviously as x increases, resolution improves but the increase of sidelobes level could result in large DoA estimation errors. In this paper, we try to define the best antenna size $((x+1)\frac{\lambda}{2})$ that satisfies the compromise between high resolution and low sidelobes level. Towards this end, two different criteria are proposed and optimized with respect to x. Accordingly, we propose to find the value of x that maximizes the directivity of the array or, equivalently, that minimizes the integral of the beam pattern over all DoA. Indeed, this integral is proportional to the main lobe width and inversely proportional to the sidelobes levels. Hence, maximizing the directivity amounts to minimize both the main beam and the sidelobes levels. Additionally, we also consider a variance-type criterion of the array beampattern and compare it to the first criterion.

This paper is organized as follows. We first introduce the framework at hand. Then, we present the criterion to be minimized in order to satisfy the resolution/sidelobe levels compromise and solve it with respect to x. We also introduce another possible optimization criterion which leads to a size close to the standard ULA. To asses the validity of the analysis, we compare the RMSE for different antenna sizes and for a classical DoA estimation algorithm, namely MUSIC.

2. DATA MODEL AND ANTENNA SIZE OPTIMIZATION

As stated in the introduction, we consider the three-sensors array represented in figure 1. Considering spatial frequencies, the steering vector model is as follows :

$$\boldsymbol{a}(f) = [1 \quad e^{2i\pi f} \quad e^{2i\pi f(1+x)}]^T \tag{1}$$

where $.^T$ stands for the vector transpose operator. The beam pattern for a given spatial frequency f_0 is defined as $g_{f_0}(f) = \left|\boldsymbol{a}(f_0)^H \boldsymbol{a}(f)\right|^2$, where $.^H$ stands for transpose and conjugate.

We do not consider here any specific DoA estimation algorithm and our analysis is based on the beam pattern characteristics. In fact, evaluating the maximum of the latter corresponds to maximum likelihood (ML) estimation under the single source hypothesis and, as stated in the introduction, this will be the case in the majority of the situations encounter with an automotive radar. Furthermore, $g_{f_0}(f)$ corresponds to the square modulus of the cross-correlation between any steering vector and the steering vector of interest. Hence, its shape will impact the performance of any DoA estimation method: ambiguities in case of high sidelobes and a good precision in case of sharp main lobe.

As displayed in figure 2, it is well known that increasing x results in a main lobe width reduction but a sidelobes level increase, two effects that have opposite consequences on the final DoA estimation performance.

The first criterion that comes in mind to manage this compromise is to maximize the directivity. The directivity represents the maximum radiation intensity divided by the average radiation intensity (averaged over the whole sphere) and can be interpreted as the array gain against isotropic noise. The directivity is defined as [1]

$$D = \left(\frac{1}{2}\int_{-\frac{1}{2}}^{\frac{1}{2}} g_{f_0}(f)df\right)^{-1}. \tag{2}$$

Hence maximizing the directivity consists in minimizing the area under the beam pattern diagram and consequently both minimizing the main lobe width and the sidelobes levels. The best directivity would be obtained for a dirac-like beam pattern but is impossible due to cross-correlations between DoA. Notice that this technique has been widely used to derive weights that minimize the sidelobe levels in case of NULA, see [4], [5], [6], [7] for example.

If we consider $f_0 = 0$ as the source position, we are simply looking for x_D that minimizes

$$C_1(x) = \int_{-\frac{1}{2}}^{\frac{1}{2}} \left|\boldsymbol{a}(0)^H \boldsymbol{a}(f)\right|^2 df = \boldsymbol{a}(0)^H \boldsymbol{\Gamma}_1 \boldsymbol{a}(0) \tag{3}$$

with

$$\boldsymbol{\Gamma}_1 = \begin{bmatrix} 1 & \int_{-\frac{1}{2}}^{\frac{1}{2}} e^{-2i\pi f}df & \int_{-\frac{1}{2}}^{\frac{1}{2}} e^{-2i\pi f(1+x)}df \\ \int_{-\frac{1}{2}}^{\frac{1}{2}} e^{2i\pi f}df & 1 & \int_{-\frac{1}{2}}^{\frac{1}{2}} e^{-2i\pi fx}df \\ \int_{-\frac{1}{2}}^{\frac{1}{2}} e^{2i\pi f(1+x)}df & \int_{-\frac{1}{2}}^{\frac{1}{2}} e^{2i\pi fx}df & 1 \end{bmatrix}$$
$$= \begin{bmatrix} 1 & 0 & \frac{\sin(\pi(1+x))}{\pi(1+x)} \\ 0 & 1 & \frac{\sin(\pi x)}{\pi x} \\ \frac{\sin(\pi(1+x))}{\pi(1+x)} & \frac{\sin(\pi x)}{\pi x} & 1 \end{bmatrix} \tag{4}$$

so that

$$x_D = \arg\min_x \left[\frac{\sin(\pi(1+x))}{\pi(1+x)} + \frac{\sin(\pi x)}{\pi x}\right]. \tag{5}$$

This function is plotted on figure 3 together with the directivity. The global minimum is obtained for $x_D \simeq 1.39$ so that the global NULA antenna size is $1.195.\lambda$ (to be compared with the λ width of the three-sensors ULA equivalent array).

A second approach can be considered in minimizing the following variance-like criterion:

$$C_2(x) = \int_{-\frac{1}{2}}^{\frac{1}{2}} \left|\boldsymbol{a}(0)^H \boldsymbol{a}(f)\right|^2 f^2 df = \boldsymbol{a}(0)^H \boldsymbol{\Gamma}_2 \boldsymbol{a}(0) \tag{6}$$

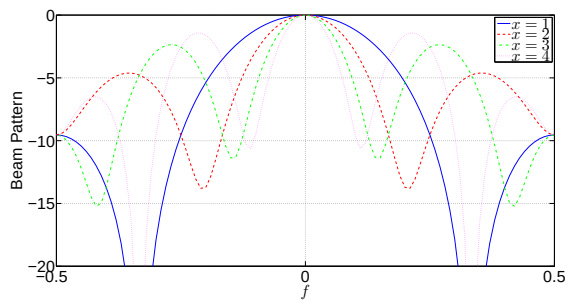

Fig. 2. Examples of beam pattern for $x = 1, x = 2, x = 3, x = 4$

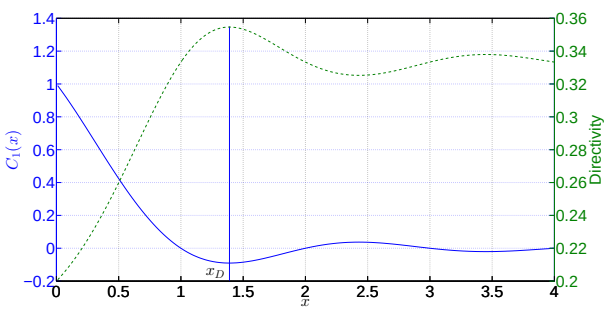

Fig. 3. $C_1(x)$ and the corresponding directivity vs. x

The difference compared with the previous criterion is a non homogeneous weighting on the frequencies. Indeed, considering that possible ambiguities errors due to sidelobes far from the main beam are more damageable because of a larger DoA errors, we choose to penalize the sidelobes levels with respect to their potential effects on the global DoA error. The above criterion can be viewed as the variance of the error $(f - f_0)$ considering a distribution for f proportional to the beampattern. In this case, we have

$$\mathbf{\Gamma}_2 = \begin{bmatrix} \int_{-\frac{1}{2}}^{\frac{1}{2}} f^2 df & \int_{-\frac{1}{2}}^{\frac{1}{2}} f^2 e^{-2i\pi f} df & h(1+x) \\ \int_{-\frac{1}{2}}^{\frac{1}{2}} f^2 e^{2i\pi f} df & \int_{-\frac{1}{2}}^{\frac{1}{2}} f^2 df & h(x) \\ h(1+x)^* & h(x)^* & \int_{-\frac{1}{2}}^{\frac{1}{2}} f^2 df \end{bmatrix} \tag{7}$$

and

$$\begin{aligned} h(x) &= \int_{-\frac{1}{2}}^{\frac{1}{2}} f^2 e^{-2i\pi f x} df \\ &= \frac{1}{4}\frac{\sin(\pi x)}{\pi x} + \frac{1}{2\pi^2 x^2}\left(\cos(\pi x) - \frac{\sin(\pi x)}{\pi x}\right) \end{aligned}$$

so that minimizing (6) is equivalent to

$$\min_x \left[h(x) + h(1+x)\right] \tag{8}$$

whose global minimum is obtained for $x_D \simeq 0.96$, a value very close to the standard ULA size composed of sensors

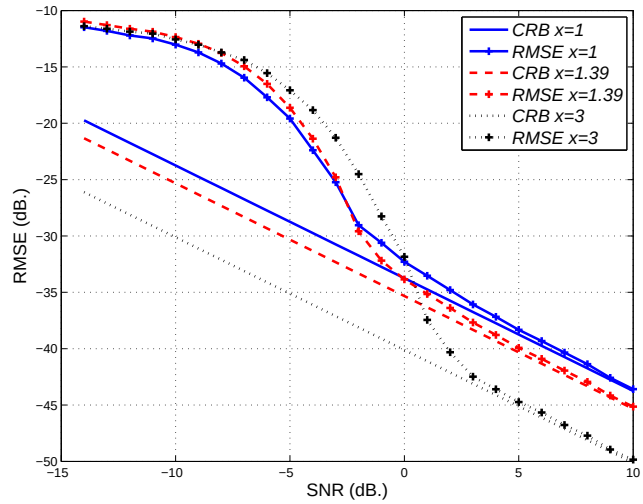

Fig. 4. Music and Capon performance analysis for 4 antenna geometries

placed a half-wavelength apart. The standard ULA configuration is close to optimal when considering this variance criterion, and hence, in the sequel, we compare the ULA configuration with the one obtained by maximizing directivity.

3. NUMERICAL ILLUSTRATIONS

In this section, we compare the performance of a very popular DoA estimation algorithm, namely spectral MUSIC, for 4 different antenna geometries. We consider the case of a single source impinging on the array with spatial frequency $f_0 = 0$, so that the snapshot received at time $t = 0, .., (N-1)$ can be written as

$$\boldsymbol{x}_t = \boldsymbol{a}(f_0)s_t + \boldsymbol{n}_t \tag{9}$$

where $\boldsymbol{n}_t$ is assumed to be a white Gaussian noise. The sample covariance matrix (SCM) ($\hat{\boldsymbol{R}} = \frac{1}{N}\sum_{t=0}^{N-1} \boldsymbol{x}_t \boldsymbol{x}_t^H$) is calculated from $N = 15$ snapshots. Its eigenvalue decomposition is as follows:

$$\hat{\boldsymbol{R}} = \lambda_1 \boldsymbol{u}_s \boldsymbol{u}_s^H + \boldsymbol{U}_n \mathbf{\Lambda}_n \boldsymbol{U}_n^H. \tag{10}$$

MUSIC estimates the DoA as

$$\hat{f}_{MUSIC} = \arg\max_f \left[\frac{1}{\boldsymbol{a}(f)^H \boldsymbol{U}_n \boldsymbol{U}_n^H \boldsymbol{a}(f)}\right]. \tag{11}$$

We only consider here MUSIC algorithm as the majority of the other procedures result approximately in the same performance for this single source case. Figure 4 represents the corresponding RMSE as well as the Cramér-Rao Bound (CRB) for each array configuration. As expected, the larger x, i.e., the larger the array aperture, the smaller the CRB. Also, as it is well known, MUSIC is efficient at high SNR where its RMSE is equal to the CRB. At very high SNR, it is thus

preferable to increase the antenna size in order to favor the main lobe width reduction against the sidelobes level. However, the SNR required for MUSIC to achieve the CRB is also increased. In fact, in most applications where a moderate SNR is encountered, the main objective is to decrease the SNR threshold at which most methods depart from the CRB. With this respect, the solution which optimizes the antenna directivity $(x_D = 1.39)$ offers a very good compromise as it allows a 2dB gain in the asymptotic zone compared with the non-ambiguous ULA and departs from the CRB approximately at the same SNR. Hence, the RMSE will always be better with this antenna size excepted in the no-information zone, where unfortunately none of the solutions is valid.

4. CONCLUSIONS

In this letter, we proposed an analysis of the geometry of a 3-sensors NULA, the two first sensors position being fixed so as to fulfill the non-aliasing constraint. This kind of antenna is extremely interesting for automotive collision avoidance radars or for any low-cost application. We propose to maximize the directivity of such an antenna to achieve the compromise between precision (thin mainlobe) and ambiguities (low sidelobes). This size optimization led to a distance between the last 2 sensors of $1.39\frac{\lambda}{2}$. Numerical simulations show that this configuration could be a good choice compared to the standard ULA.

Acknowledgment

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5. REFERENCES

[1] H. L. V. Trees, Ed., *Optimum Array Processing*. New York: Wiley, 2002.

[2] C. Chambers, T. C. Tozer, K. C. Sharman, and T. S. Durrani, "Temporal and spatial sampling influence on the estimated of superimposed narrowband signals: When less can mean more," *IEEE Transactions Signal Processing*, vol. 44, no. 12, pp. 3085–3098, December 1996.

[3] A. B. Gershman and J. F. Bohme, "A note on most favorable array geometries for doa estimation and array interpolation," *IEEE Signal Processing Letters*, vol. 4, no. 8, pp. 232–235, 1997.

[4] C. A. Olen and R. T. Compton, "A numerical pattern synthesis algorithm for arrays," *IEEE Transactions Antennas Propagation*, vol. 38, pp. 1666–1676, October 1990.

[5] M. H. Er, "Array pattern synthesis with a controlled mean-square sidelobe level," *IEEE Transactions Signal Processing*, vol. 40, pp. 977–981, April 1992.

[6] C. Y. Tseng and L. J. Griffiths, "A simple algorithm to achieve desired patterns for arbitrary arrays," *IEEE Transactions Signal Processing*, vol. 40, pp. 2737–2746, November 1992.

[7] K. L. Bell, H. L. V. Trees, and L. J. Griffiths, "Adaptive beampattern control using quadratic constraints for circular array stap," in *Proceedings ASAP*, MIT, Lincoln Lab., Lexington MA, March 2000, pp. 43–48.