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ABSTRACT

In this manuscript, we study the influence of various parameters on the dynamics of a semiconductor laser with phase-conjugate feedback. We report the chaos bandwidth of its chaotic states, and simultaneously the frequency of its self-pulsing modes. We demonstrate that both indicators are unaffected by the delay length at any given feedback strength. A higher pumping current enhances both indicators, as does an increase of the carrier lifetime. We further investigate the influence of the linewidth enhancement factor (α-*factor*): restabilization occurs for higher values of feedback strength, thereby enhancing the system's limit of attaining higher values of the chaos bandwidth. The effective bandwidth is observed to follow the general trend of the chaos bandwidth with an exception of transition dynamical solutions. We show that the PCF system is able to generate high-frequency and broadband chaos without any additional active optical element. Because of the high quality of chaos induced by the PCF system, delayed laser systems are strong candidate to generate high-performance chaotic signals.

Keywords: Phase-conjugate feedback, Chaotic laser diodes, High-frequency dynamics of laser diodes, Complex dynamics of laser diodes

1. INTRODUCTION

A semiconductor laser subjected to optical feedback can easily become unstable.[1,2] In particular, it can exhibit regular self-pulsing dynamics, steady-states and chaos, depending on the feedback strength.[3] To observe a wider range of dynamics, a relatively simple way is to introduce the phase-conjugate feedback. For low feedback strength values, chaos appears from the undamping of the relaxation oscillations,[4–7] as in conventional feedback.[8] However, the phase-conjugate feedback chaotic states are more broadband than the chaos obtained with conventional feedback.[9] If we increase the feedback strength, the chaos bandwidth increases, as the chaos centered around the laser relaxation oscillations bifurcates into unique broadband chaos.[10]

Then, the system locks-on completely new dynamics. The output power oscillates at a frequency equal to a multiple of the inverse of the output roundtrip time and this frequency is often higher than the relaxation oscillation frequency.[6,7] Analogous to the external cavity modes of the conventional feedback, these latter states are named external cavity modes (ECMs).[11] For even higher values of feedback, the laser restabilizes into a steady-state solution, ending the chaotic and self-pulsing states.[5,7,12]

Our focus in this study is the influence of the various parameters of the laser and of the external cavity on the frequencies associated with the laser dynamics.

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2. EXPERIMENTAL AND THEORETICAL BACKGROUND

2.1 Typical experimental setup

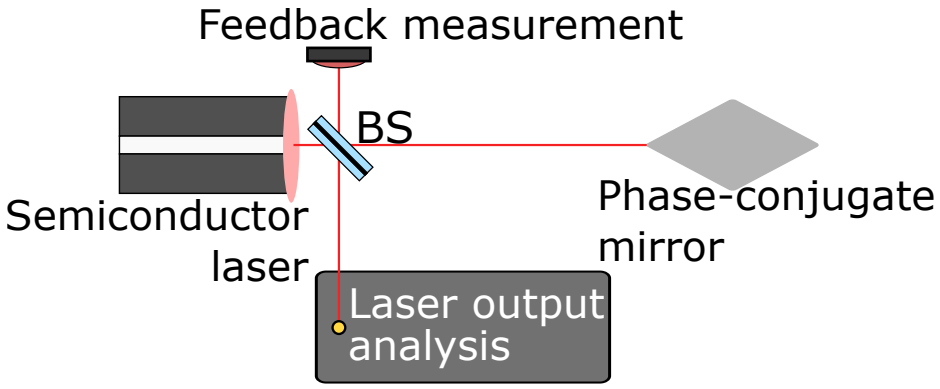

Figure 1. Typical setup of a phase-conjugate feedback experiment.

A typical setup is shown Fig. 1 . The output beam from a semiconductor laser is sent to a phase-conjugate mirror. The phase-conjugated beam is reflected back into the laser, while part of this feedback beam is deviated by a beamsplitter (BS) to monitor the feedback strength. The same beamsplitter sends part of the laser output into a analysis system, to enable the study of the laser dynamical states.

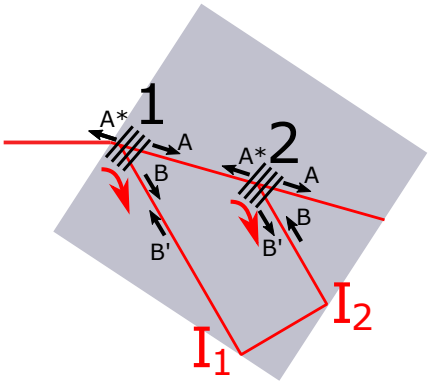

Figure 2. Typical setup of a *cat* self-pumped phase-conjugate mirror. The light enters from the left, part of it is fanned to the bottom and reflected on the edges of the crystal. The phase-conjugate wave is obtained by four-wave mixing in gratings 1 and 2.

Phase-conjugation can be achieved by several means, generally through four-wave mixing in non-linear medias, such as photorefractive crystals.[13] In particular, we have experimentally studied phase-conjugate feedback with rhodium-doped barium titanate crystals, in *cat* self-pumped phase-conjugate mirror configuration (*cat*-SPPCM).[14] In these mirrors, as detailed in Fig. 2, part of the input light is deviated by the photorefractive effect (beam fanning). The deviated light from 1 (B) is then reflected on the sides of the crystal (at I_1 and I_2). If the geometry enables it, four-wave mixing occurs in 2 between the light that travels in a straight direction A, the deviated light B and the light being deviated there B'. The fourth wave, $A*$, is the backward propagative phase-conjugate of the input beam. Similarly, wave mixing also occurs in grating 1.

2.2 Theoretical model

To simulate a laser with phase-conjugate feedback, we are using normalized equations derived from the Lang-Kobayashi equations:[15]

$$\dot{E}(t) = (1 + i\alpha)E(t)N(t) + \gamma F(t) \tag{1}$$

$$T\dot{N}(t) = P - N(t) - (1 + 2N(t))|E(t)|^2 \tag{2}$$

Where, E(t) is the normalized complex output electric field, N(t) the normalized carrier density above threshold and $\gamma F(t)$ the feedback electric field; γ being the feedback ratio. α is the standard linewidth enhancement factor, P the pump parameter above threshold and T the carrier lifetime. All the times are normalized by the photon lifetime.

In an initial approach, $F(t)$ was equal to the phase-conjugate of the delayed laser output field $E^*(t-\theta)$, θ being the roundtrip time delay.[16] This model has been able to reproduce some of the dynamics of the phase-conjugate feedback lasers, such as chaos[17] and ECMs.[11] However, we are not able to obtain steady-state solutions, that have been observed experimentally at high values of feedback. In 1998, a new model was introduced, namely, the filtered phase-conjugate feedback field, to account for the nonlinear interactions in the phase-conjugate feedback.[18] In the case of a *cat*-SPPCM, this filter can be linked to the length of interaction of light between 1 and 2 on Fig. 2. Such a model is able to numerically reproduce the restabilization of the laser via an Hopf bifurcation.[19] Therefore, we introduce the following equations for the feedback output field :

$$\tau_r \dot{F}(t) = E^*(t - \theta) - F(t) \tag{3}$$

τ_r is named finite penetration depth, and is proportional to the length of the phase-conjugate feedback.[18]

We set the parameters as in table 1, chosen on the basis of a previous study of the position of the external cavity modes.[20] These values have been chosen to simulate a laser with relaxation oscillations of 3.6 GHz, a pumping current equal to 1.6016 times the threshold current, an external cavity single-trip length of 52.5 cm and a cubic phase-conjugate mirror with edges of 5 mm.[7]

Table 1. Typical values of laser and feedback parameters used in this study as default values

parameter	name	value
τ_p	photon lifetime	1.4 ps
α	linewidth enhancement factor	2
P	pump parameter above threshold	0.6016
θ	normalized delay	2500
T	normalized carrier lifetime	1200
τ_r	normalized finite penetration depth	50
γ	feedback ratio	variable

2.3 Studied parameters

Our focus in this manuscript are two key indicators, one for each main dynamical state of the chaos bandwidth. To study the ECMs, we perform an analysis of the frequency of the regular self-pulsing states. The ECMs can be confused with the undamped relaxation oscillations, but these latter solutions can be obtained only at low feedback values.[19] Then, to study the chaotic states, we compute their chaos bandwidth. The chaos bandwidth is an indicator of the performances of a chaotic signal and is defined as the span of frequencies that contains 80% of the total power, starting from the DC frequency. A high chaos bandwidth means that the chaotic signal spans onto high frequencies.[21] We also study the effective chaos bandwidth, defined as the sum of the discrete spectral components that contain 80% of the total power. A high effective chaos bandwidth indicates that the spectrum is broadly distributed on many frequencies.[22]

3. NUMERICAL RESULTS

We want to study the influence of the feedback strength γ, of the α-factor, of the pump P, of the carrier lifetime T, of the delay θ and of the finite depth penetration θ. We use the fourth order Runge-Kutta algorithm with an integration step equal to the carrier lifetime to obtain 4000000 points-long time series, and we perform frequency analysis on time series with regular oscillations and chaos bandwidth analysis on chaotic ones, after the first 50000 points have been removed.

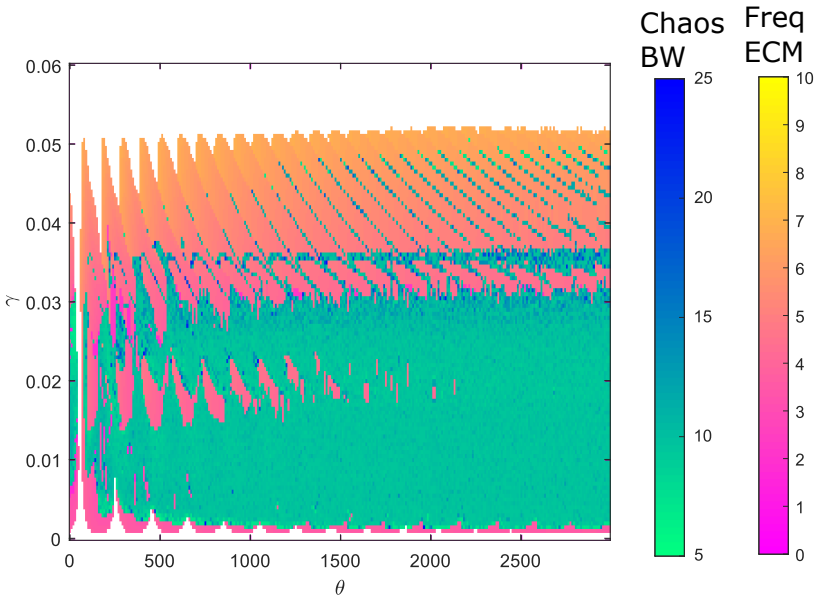

Figure 3. Influence of the delay θ (horizontal axis) and of the feedback strength γ (vertical axis) on the frequency of the external cavity modes for the self-pulsing states (pink to yellow color-scales) and on the chaos bandwidth (green to blue color-scales). The steady-states are displayed in white.

We present in Fig. 3 the evolution of the chaos bandwidth and of the frequency of the ECMs versus the feedback strength γ and the normalized delay length θ. Simulations of the frequency of the ECMs has already demonstrated it to be globally independent of the delay at a fixed feedback strength, the laser oscillating at the nearest multiple of the external cavity frequency.[23] Our simulation extends this result further to the chaos bandwidth. The chaos bandwidth seems to be independent of the delay at a fixed value of feedback. Between successive external cavity modes, small regions exhibiting chaos bandwidth of up to 25 GHz are observed.

Figure 4 shows the evolution of the chaos bandwidth and of the frequency of the ECMs versus the feedback strength γ and the finite penetration depth in the phase-conjugate mirror, τ_r. Previous simulations had shown its influence on the restabilization: increasing τ_r decreases the value of the restabilization for the feedback strength.[19] On the other hand, a smaller value of τ_r also enables more broadband chaos. Indeed, for $\tau_r < 10$, broadband chaos (up to 25 GHz) can be obtained for $\gamma \approx 0.05$. However, reducing τ_r requires a shorter phase-conjugate mirror, and this may complicate the attainment of high feedback values.

In Fig. 5, we trace the chaos bandwidth and the frequency of ECMs when tuning the α-factor and the feedback strength γ. For low values of α ($\alpha < 0.9$), the laser remains stable, which is consistent with the definition of α as an amplitude-phase coupling responsible for the destabilization of lasers. At a given value of γ, the chaos bandwidth seems to be rather independent from α. However, decreasing α forces the transition to ECMs and the restabilization to occur at lower values of γ, and hence the region of high values of chaos bandwidth at high values of γ cannot be reached.

We display on Fig. 6 (respectively 7) the influence of the pump above threshold P (respectively the carrier lifetime T) and the feedback strength γ on the chaos bandwidth and of the frequency of the ECMs. Increasing P has the same effect as increasing $^1/_T$ (and reversely). The relaxation oscillation frequency is proportional to

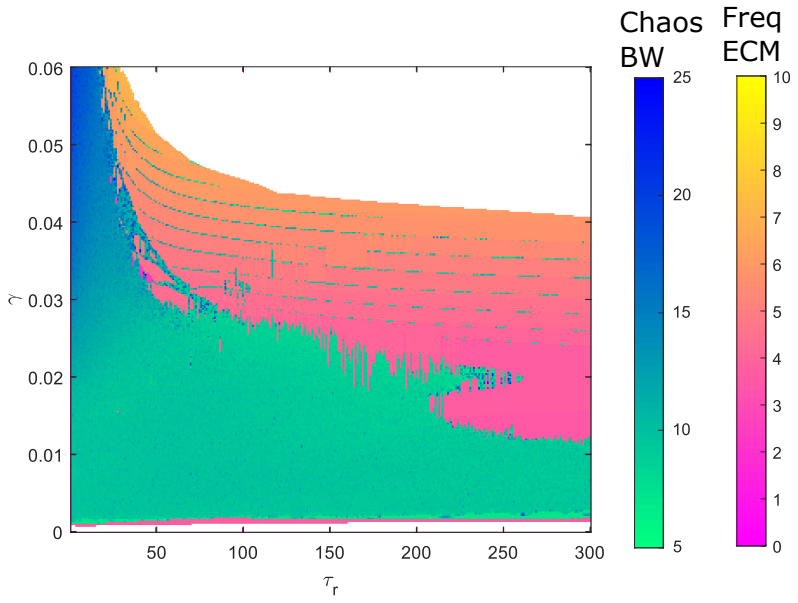

Figure 4. Influence of the finite penetration depth τ_r (horizontal axis) and of the feedback strength γ (vertical axis) on the frequency of the external cavity modes for the self-pulsing states (pink to yellow color-scales) and on the chaos bandwidth (green to blue color-scales). The steady-states are displayed in white.

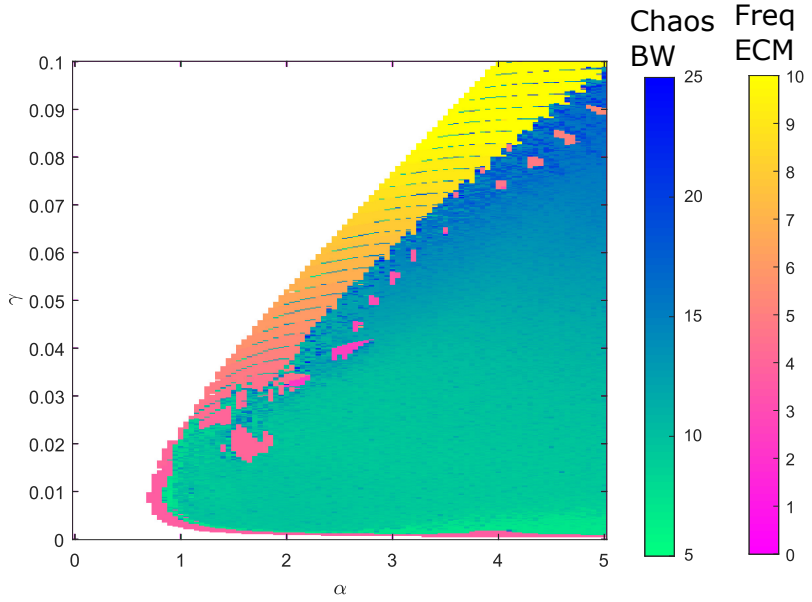

Figure 5. Influence of the α-factor (horizontal axis) and of the feedback strength γ (vertical axis) on the frequency of the external cavity modes for the self-pulsing states (pink to yellow color-scales) and on the chaos bandwidth (green to blue color-scales). The steady-states are displayed in white.

$\sqrt{P/T}$[24] and a similar square-root dependency seems to occur for the restabilization value. A strong decrease of T (i.e. $T \approx 1$) not only enables the apparition of broadband chaos and high-frequency ECMs but also increases the range of low feedback values where the laser remains stable. This condition is often achieved in quantum cascade lasers and this latter result has been confirmed with a more accurate 3-levels model of a QCL subjected to phase-conjugate feedback.[25]

In all these parametric studies, we also observe a direct link between the presence of high-frequency ECMs

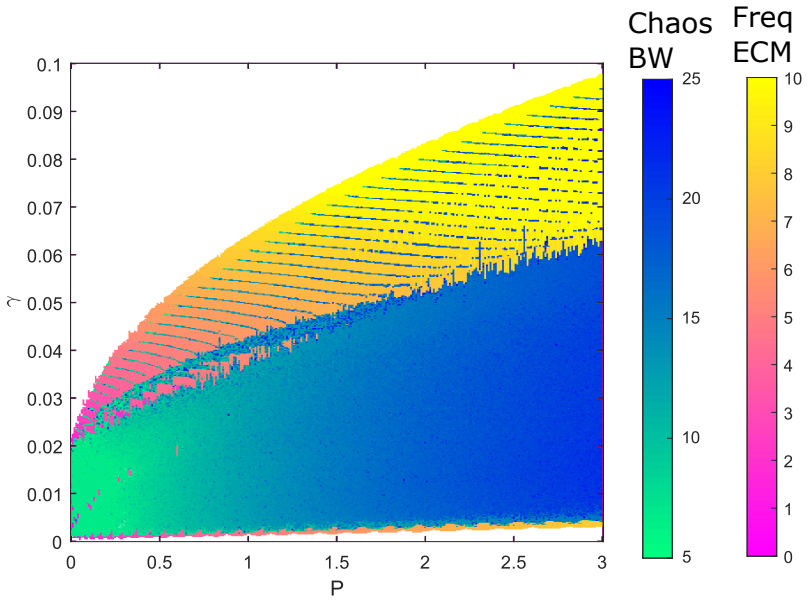

Figure 6. Influence of the pump parameter P (horizontal axis) and of the feedback strength γ (vertical axis) on the frequency of the external cavity modes for the self-pulsing states (pink to yellow color-scales) and on the chaos bandwidth (green to blue color-scales). The steady-states are displayed in white.

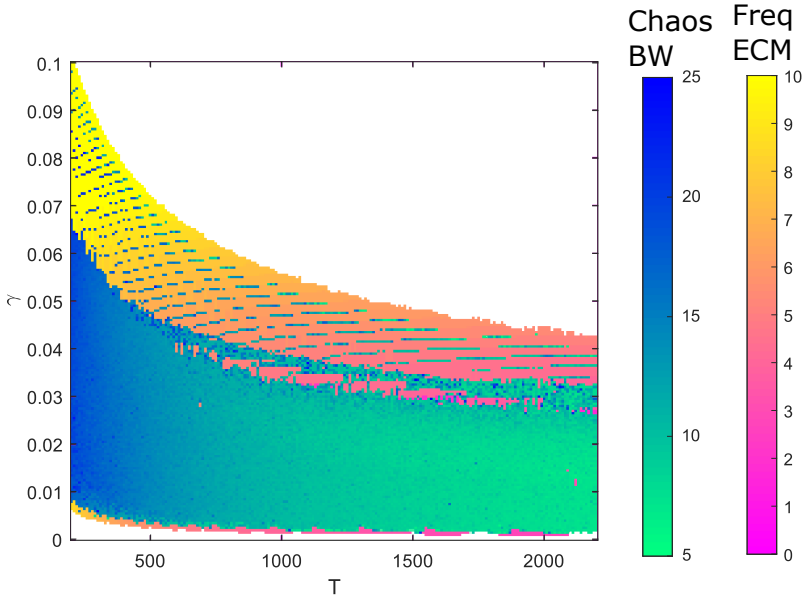

Figure 7. Influence of the carrier lifetime T (horizontal axis) and of the feedback strength γ (vertical axis) on the frequency of the external cavity modes for the self-pulsing states (pink to yellow color-scales) and on the chaos bandwidth (green to blue color-scales). The steady-states are displayed in white.

and broadband chaos. Both chaos bandwidth and ECM frequency are related to a general "speed" of the system. Furthermore, high values of chaos bandwidth are obtained between two succesive ECMs. This result has been also observed experimentally, with a strong enhancement of the chaos bandwidth in the region of the ECMs.[10]

Finally, we study the effective chaos bandwidth. For the default parameters indicated Tab. 1, we plot on Fig. 8 (a) the steady-states, the frequency of the external cavity modes and, for the chaotic states, their effective (chaos) bandwidth and their chaos bandwidth, versus the feedback strength γ. As in a bifurcation diagram, each

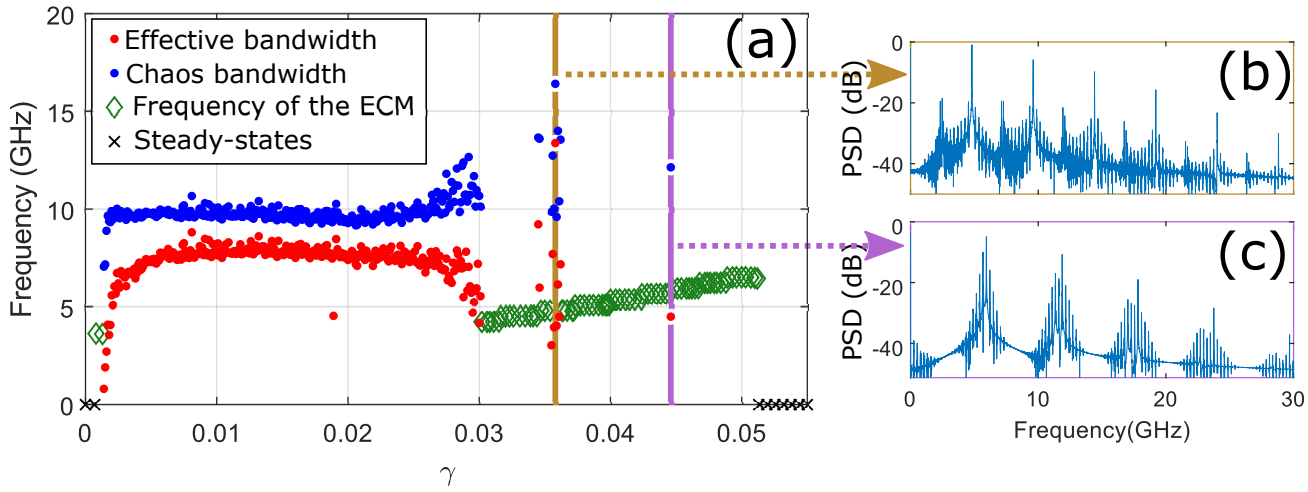

Figure 8. (a) Evolution of chaos bandwidth and effective (chaos) bandwidth (for chaotic states) and oscillation frequency (for ECMs) for a typical laser with phase-conjugate feedback. (b) and (c) present the power spectral density (PSD) for $\gamma = 0.0358$ (brown line on (a)) and $\gamma = 0.0446$ (violet line on (a)).

simulation is initialized with the output of the simulation for the previous value of γ. The chaos bandwidth and the effective bandwidth follow the same trend. Moreover, as previously reported the chaos bandwidth rises as the ECMs appears (for $\gamma > 0.03$). At $\gamma = 0.0358$ (brown vertical bar), the chaos bandwidth equals 16.42 GHz and the effective chaos bandwidth 13.39 GHz. A study of the spectrum of this time series, displayed Fig. 8 (b), shows many frequency components, with a maxima at 4.8 GHz and $\approx 2.4\ GHz$. 4.8 GHz is the frequency of the nearest ECM, while the other is half this value. However, in this region of mixed chaos and ECMs, the effective chaos bandwidth can decrease to low values while the chaos bandwidth remains high. For instance, at $\gamma = 0.0446$ (violet vertical bar), the effective chaos bandwidth is equal to 4.5 GHz while the chaos bandwidth is as high as 12.15 GHz. A look at the spectrum, see Fig. 8 (c), shows the energy is centered around 5.91 GHz, which is also the frequency of the nearest ECM. This high value of frequency (and the harmonics of this value) enhance the chaos bandwidth but not the effective chaos bandwidth since most of the energy is located on a few values. Fortunately, experimental fluctuations seem to prevent this semi-destabilized ECM to be obtained in reality.[12]

In conclusion, we have studied all the parameters to manipulate the chaos bandwidth and the ECM frequency of a laser diode with phase-conjugate feedback. Both chaos bandwidth and ECM frequency can be enhanced by changing laser properties (pump P, carrier lifetime T, α-factor) or mirror size (finite penetration depth τ_r). On the other hand, the length of the external cavity has no influence on the chaos bandwidth. This means that once the two phase-conjugate feedback components have been fixed, the chaos bandwidth keeps its value at a given feedback strength γ. Phase-conjugate feedback can therefore be a strong candidate to obtain a robust and broadband chaotic laser source with a semiconductor laser and only one passive component.

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