

# Analysis of hydrodynamic forces on non-spherical particles (Spherocylinder)

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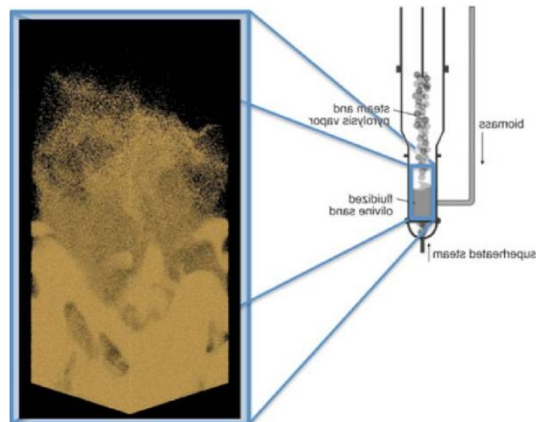
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Boris ARCEN (LRGP, CNRS)

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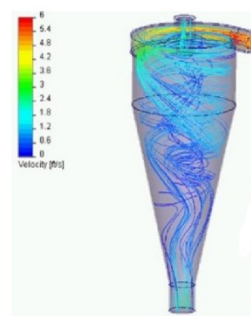
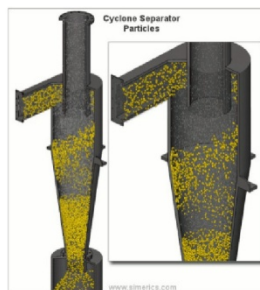
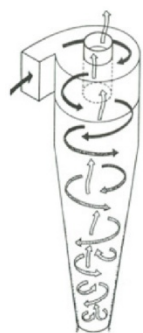
# Applications



Coal Combustion



Dispersion of Pollutants



Separation Process (cyclone)

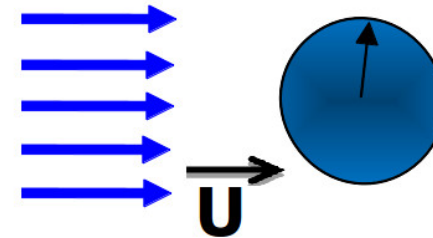
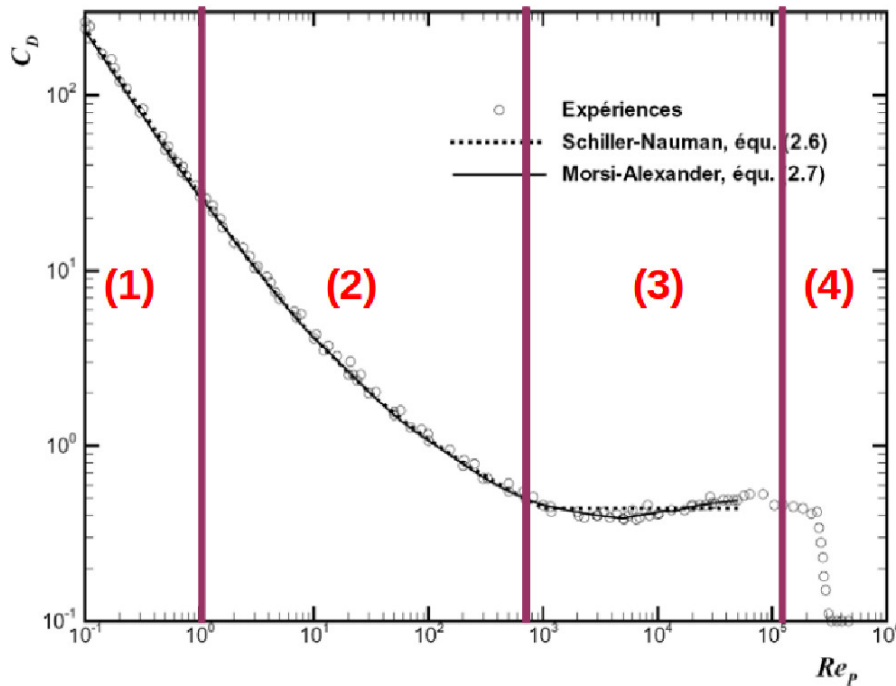
# Context and Objectives

- **This work is a part of program research about non-spherical particle dispersion. We are interested in simulating a dispersed two-phase flow using Direct Numerical Simulation under point force approximation.**
- **Before, we want to examine the force acting on each particle in order to choose or develop a model that will later be introduced in the DNS.**
- **To verify the ability of the CFD code to give the accurate results of hydrodynamic forces acting on non-spherical particles**

# Literature overview

# Spherical particle

Variation of the drag coefficient as a function of the Reynolds number for a spherical particle (Pr Ahmadi, Clarkson University)



- (1): Stokes regime
- (2): Intermediate regime
- (3): Newton regime
- (4): Trans-critical regime

$$Re_p = \frac{ud_p}{\nu}$$

$$C_D = \frac{24}{Re_p} \left[ 1 + 0.15Re_p^{0.687} \right]$$

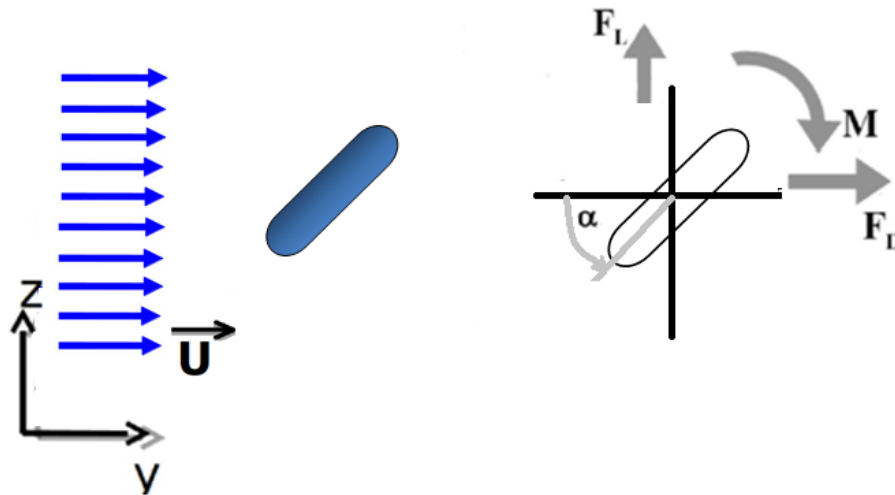
if  $Re_p \leq 1000$ ,

$$C_D = 0.44$$

if  $Re_p \geq 1000$ .

What's about non-spherical particles?

# Eulerian-Lagrangian methods for Two-Phase Flows



$F_D$ : Drag force  
 $F_L$ : Lift force  
 $T_p$ : Torque force  
 $A_p$ : Projected area  
 $u$ : Fluid velocity  
 $d_p$ : Particle diameter  
 $\rho$ : Fluid density

Drag coefficient

$$C_D = \frac{F_D}{\frac{1}{2} \rho u^2 A_p}$$

Lift coefficient

$$C_L = \frac{F_L}{\frac{1}{2} \rho u^2 \frac{\pi}{4} d_p^2}$$

Torque coefficient

$$C_T = \frac{T_p}{\frac{1}{2} \rho u^2 \frac{\pi}{16} d_p^3}$$

$$m_p \frac{d\vec{v}_p}{dt} = \vec{F}_{DRAG} + \vec{F}_{LIFT}$$

$$I_{x'} \frac{d\omega_{x'}}{dt} - \omega_{y'} \omega_{z'} (I_{y'} - I_{z'}) = T_{x'}$$

$$I_{y'} \frac{d\omega_{y'}}{dt} - \omega_{z'} \omega_{x'} (I_{z'} - I_{x'}) = T_{y'}$$

$$I_{z'} \frac{d\omega_{z'}}{dt} - \omega_{x'} \omega_{y'} (I_{x'} - I_{y'}) = T_{z'}$$

# Literature review (Drag coefficient)

## Before

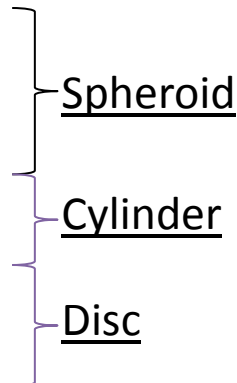
### *Correlation for arbitrary shaped particles*

1. Ganser [1993]
- 2. Haider and Levenspiel [1989]**
3. Hartman [1994]
4. Chien [1994] (
5. Swamme and Ojha [1991]



### *Correlation for specific shaped particles*

- Brenner [1963]
- Bowen and Masliyah [1973]
- Tripathi et al [1994]
- Militzer et al [1989]
- Huner and Hussey [1977]
- Ui et al [1984]
- Michael [1966]
- Shail and Norton [1969]
- Davis [1990]



C  
H  
A  
B  
R  
A  
1  
9  
9  
9

## After

### *Correlation for arbitrary shaped particles*

1. Holzer and Sommerfeld [2008]
2. Tran-Cong et al. [2004]

### *Correlation for specific shaped particles*

- Zastawny et al. [2012]
- Loth [2008]
- Yow [1994]
- Vakil and Green [2011]
- Mando and Rosendahl [2010]

### The correlations must take into account:

- Particle shape.
- Particle orientation.
- Particle rotation.



## Relevant parameters

Particle Reynolds number is calculated using the diameter of the equivalent sphere ( $d_{eq}$ )

$$Re_p = \frac{ud_{eq}}{\nu}$$

Sphericity ( $\phi$ ) is the ratio between the surface of sphere with the same volume as the particle and the surface area of the actual particle (S):

$$\phi = \frac{\pi d_{eq}^2}{S}$$

Aspect ratio ( $w$ ) is the ratio between the length (a) of the particle and its width(b):

$$w = \frac{a}{b}$$

# Recent correlations

Formula commonly admitted:

$$C_D = \frac{a}{\text{Re}} + \frac{b}{\sqrt{\text{Re}}} + c$$

Arbitrary shaped particles:

Holzer and Sommerfeld[2008]

$$C_D = \frac{8}{\text{Re}} \frac{1}{\sqrt{\phi_{\perp}}} + \frac{16}{\text{Re}} \frac{1}{\sqrt{\phi}} + \frac{3}{\sqrt{\text{Re}}} \frac{1}{\phi^{\frac{3}{4}}} + 0.42 * 10^{0.4(-\log \phi)^{0.2}} \frac{1}{\phi_{\perp}}$$

Specific shaped particles:

Rosendahl et al.[2010]

$$C_D = C_{D,\theta=90^\circ} + (C_{D,\theta=90^\circ} - C_{D,\theta=0^\circ}) \sin^3 \theta$$

Where :

$\begin{cases} C_{D,\theta=0^\circ} \\ C_{D,\theta=90^\circ} \end{cases}$  are determined from experimental or correlations results

Zastawny et al.[2012]

$$C_D = C_{D,\theta=90^\circ} + (C_{D,\theta=90^\circ} - C_{D,\theta=0^\circ}) \sin^{a_0} \theta$$

Where :

$$C_{D,\theta=0^\circ} = \frac{a_1}{\text{Re}^{a_2}} - \frac{a_3}{\text{Re}^{a_4}}$$

$$C_{D,\theta=90^\circ} = \frac{a_5}{\text{Re}^{a_6}} - \frac{a_7}{\text{Re}^{a_8}}$$

$$C_L = \left( \frac{b_1}{\text{Re}^{a_2}} - \frac{b_3}{\text{Re}^{a_4}} \right) \sin(\theta)^{b_5+b_6 \text{Re}^{b_7}} \cos(\theta)^{b_8+b_9 \text{Re}^{b_{10}}}$$

# Numerical simulation overview

# Governing equations and solutions parameters on ANSYS FLUENT

$$\nabla \cdot \vec{u} = 0$$

$$\frac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \nabla) \vec{u} = -\frac{1}{\rho} \nabla p + \nu \Delta \vec{u}$$

- Second-order solver for the three-dimensional Navier-Stokes equations.
- Laminar viscous model. Steady simulation.
- The **SIMPLE** algorithm (Semi Implicit Method for Pressure-Linked Equations) is used.

# Characteristics and parameters of the simulation

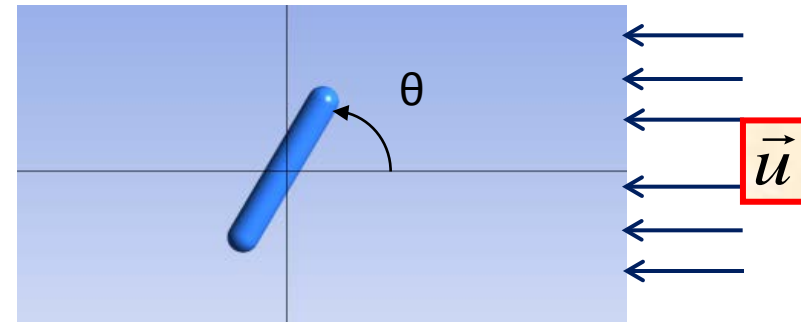
## Particle:

**Spherocylinder particle**

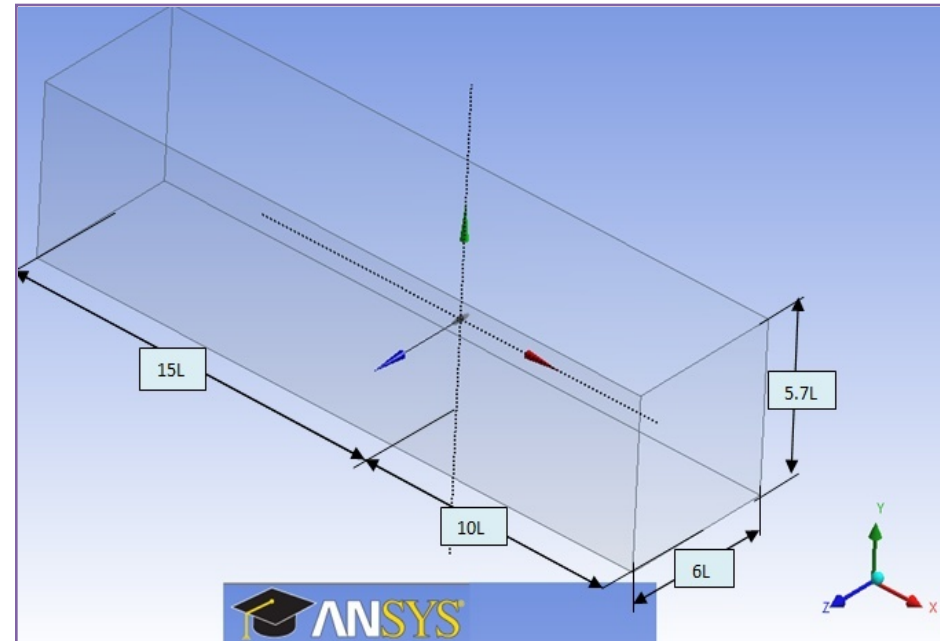
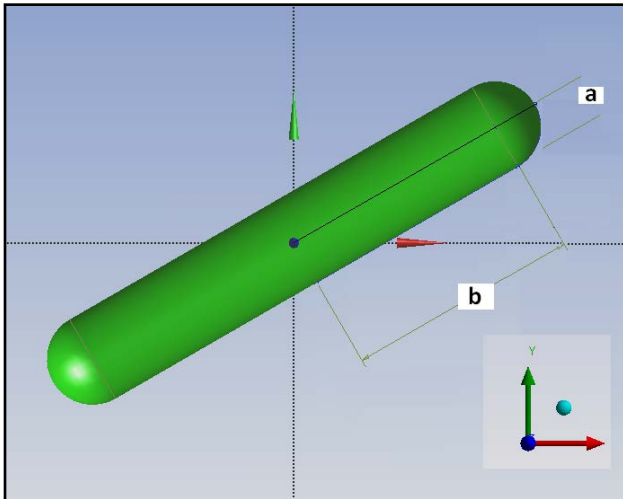
- $0^\circ \leq \theta \leq 90^\circ$
- Axis ratio  $(b/a)=5$
- Fixed and rigid particle

## Flow:

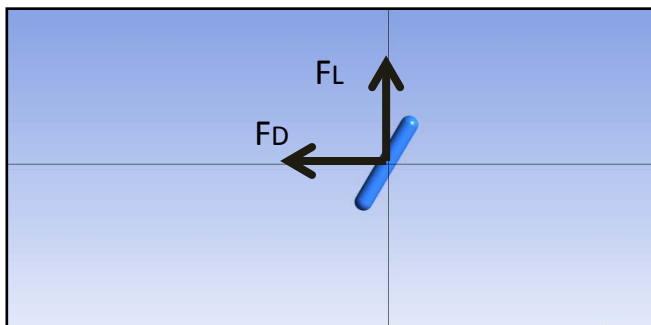
- Uniform flow
- $0.1 \leq Re_p \leq 300$



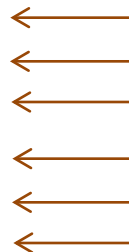
# The domain and boundary conditions



Moving no-slip wall boundary condition



Pressure outlet



Velocity-Inlet

Case : Uniform flow

**Results of numerical simulation  
ANSYS-FLUENT  
Uniform flow at  $Re_p = 10$  and  $Re_p = 300$**

# Drag coefficient

## Comparison with

### Correlations:

- Zastawny et al[2012];
- Holzer and Sommerfeld[2008];
- Rosendahl [2010].

### DNS (immersed boundary method):

Zastawny et al[2012].

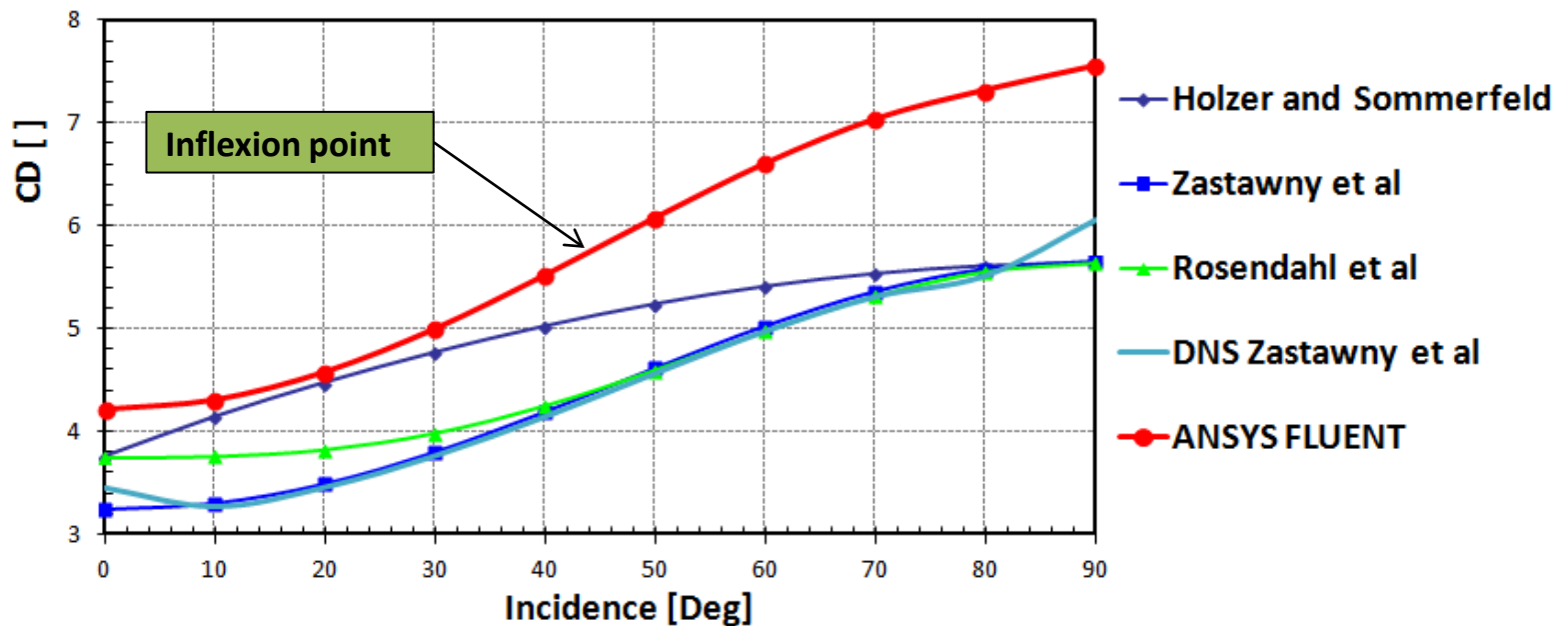
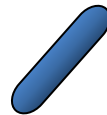


## Comparison at $Re_p = 10$

Spherocylinder

$w=5/1$

$\phi=0.69$



A similar tendency is noted for our results and those given by Zastawny et al. ( $CD_{max} / CD_{min} \approx 1.75$ ), a deviation of 30% is observed.

## Results of DNS of Zastawny et al.[2012]

### Ellipsoid 1

$$w=5/2$$

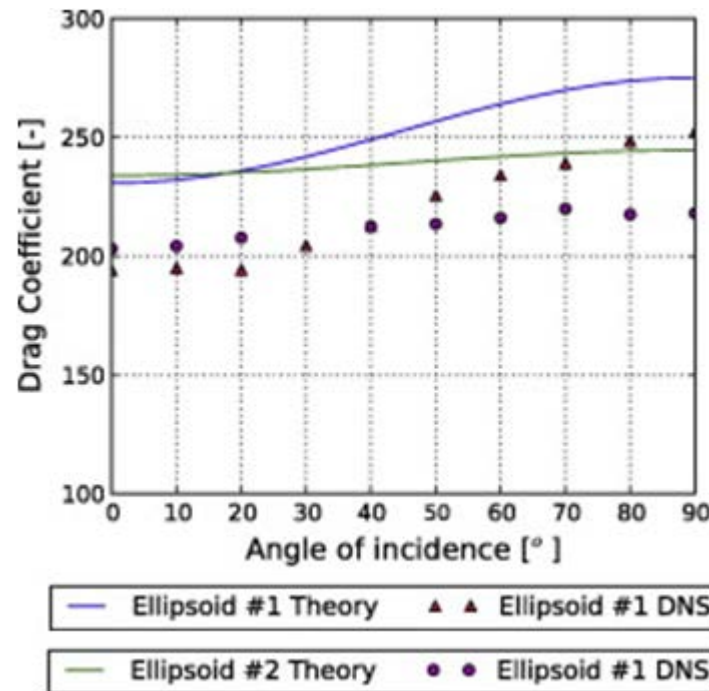
$$\phi=0.88$$



### Ellipsoid 2

$$w=5/4$$

$$\phi=0.99$$



Under-prediction of the CD at low Reynolds number by DNS Zastawny compared to theoretical results.

**Deviation 14%**

Comparison of the drag and lift coefficients obtained from *Brenner (1963)* with the results issued from **DNS of Zastawny for two ellipsoids at  $Re < 0.1$**

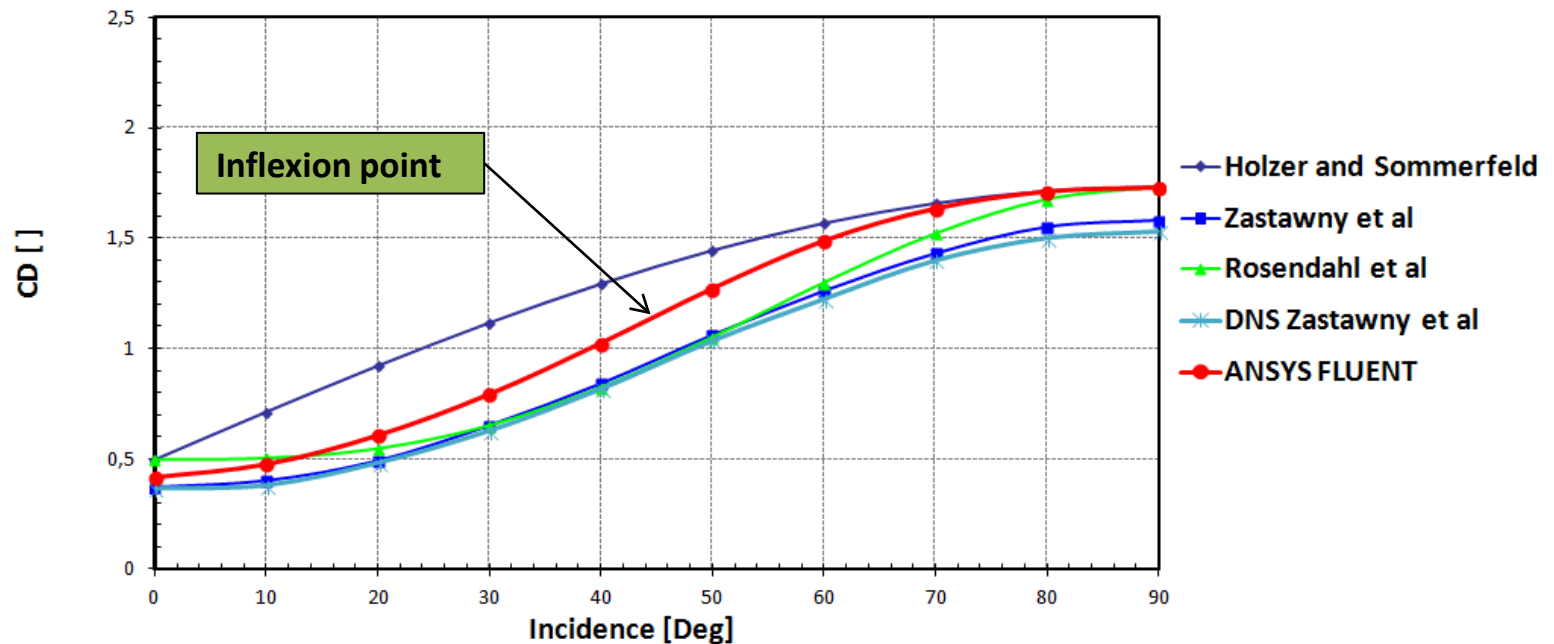
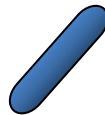
**At low Reynolds DNS of Zastawny under-predict CD compared to the theoretical results of Brenner.**

## Comparison at $Re_p = 300$

Spherocylinder

$w=5/1$

$\phi=0.69$



A similar trend is noted for our results and those given by Zastawny et al. ( $CD_{max} / CD_{min} \approx 4.2$ ). The difference is lower than 10%.

# Lift coefficient

## Comparison with

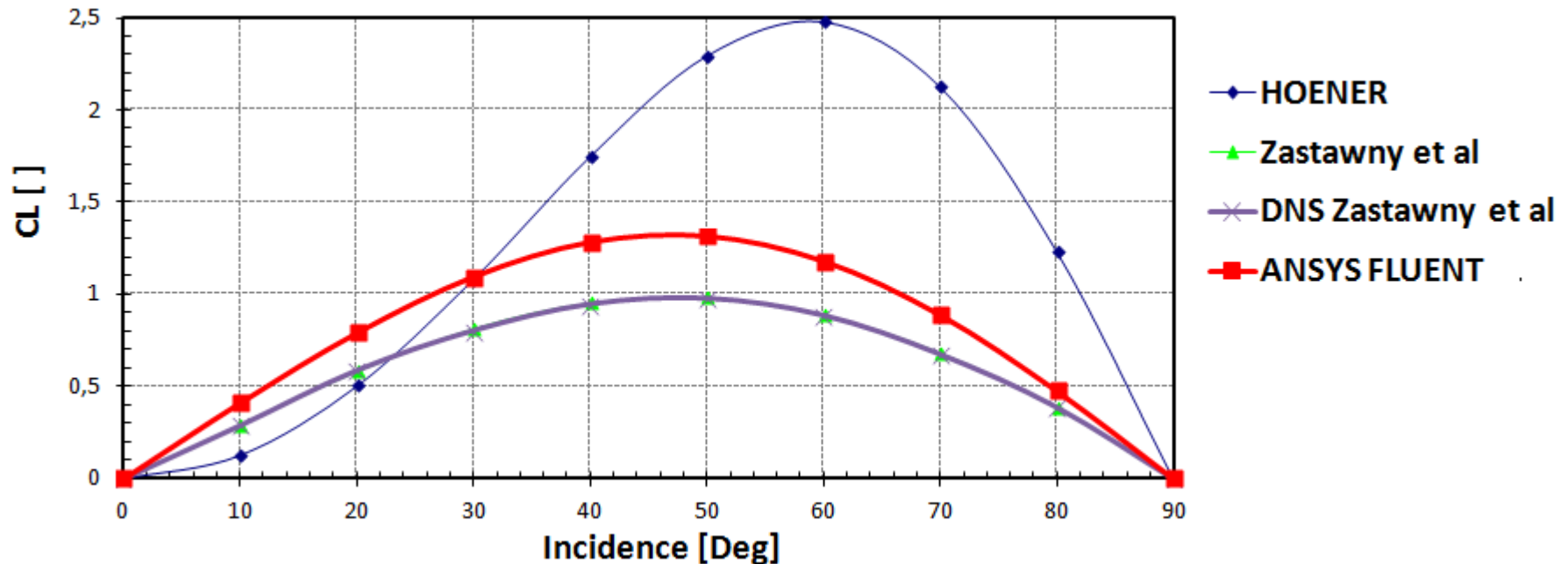
### Correlations:

- Zastawny et al[2012];
- Hoener[1963].

### DNS (immersed boundary method):

Zastawny et al[2012].

## Comparison at $Re_p = 10$

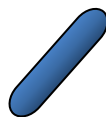


A similar tendency is noted for our results and those given by Zastawny et al. The deviation is the same to those of the drag coefficient 30%.

Spherocylinder

$w=5/1$

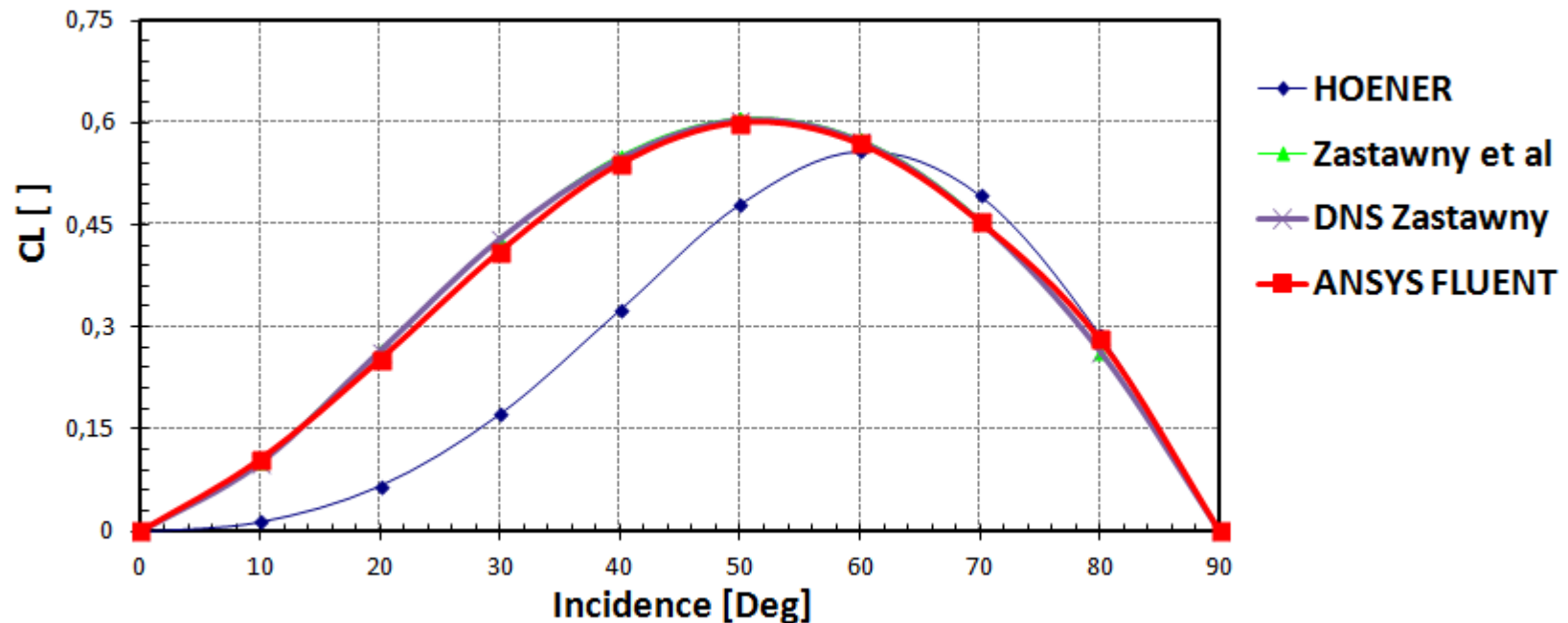
$\phi=0.69$



Hoener correlation:

$$\frac{C_L}{C_D} = \sin^2 \theta \cos \theta$$

## Comparison at $Re_p = 300$

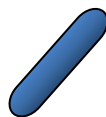


A similar trend is noted for our results and those given by Zastawny et al. The difference is less than 2%.

Spherocylinder

$w=5/1$

$\phi=0.69$



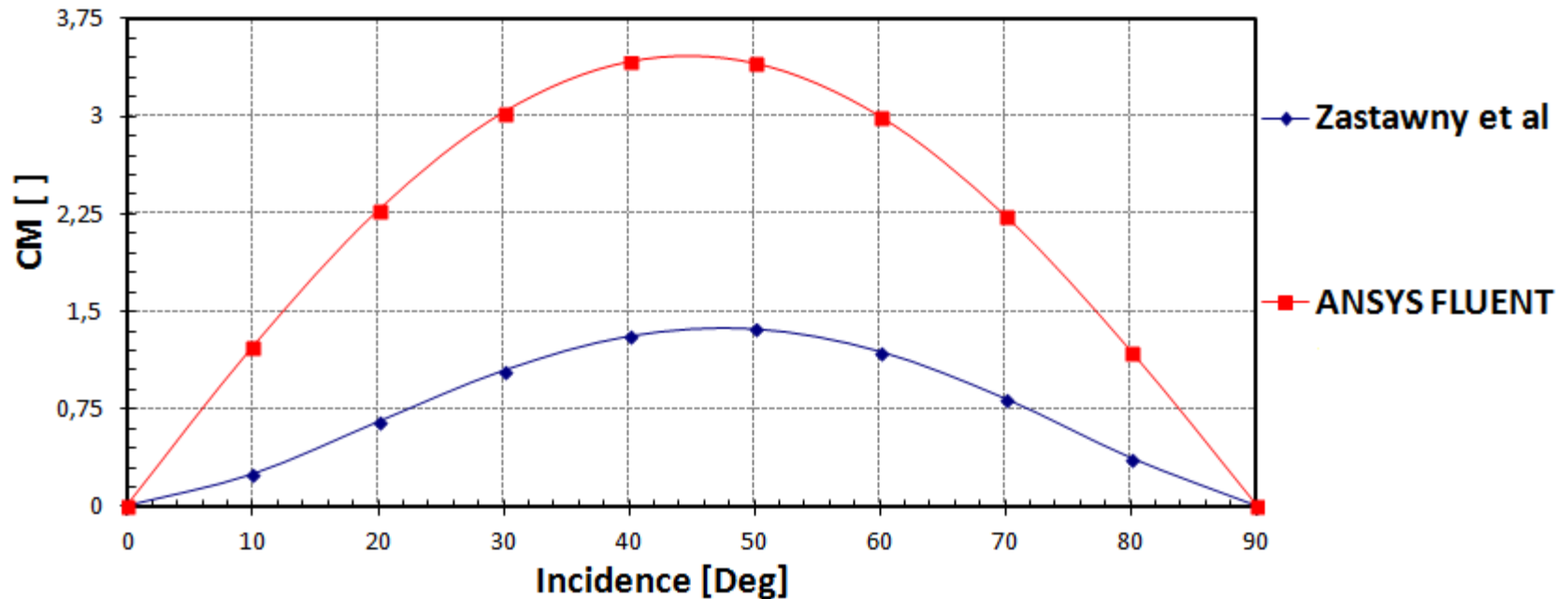
# Torque coefficient

**Comparison with**

**Correlation:**

**Zastawny et al[2012].**

## Comparison at $Re_p = 10$

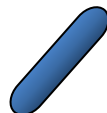


The torque in the Z direction given by Zastawny et al. is significantly lower than the one extracted from our simulations. Deviation  $\approx 179\%$

Spherocylinder

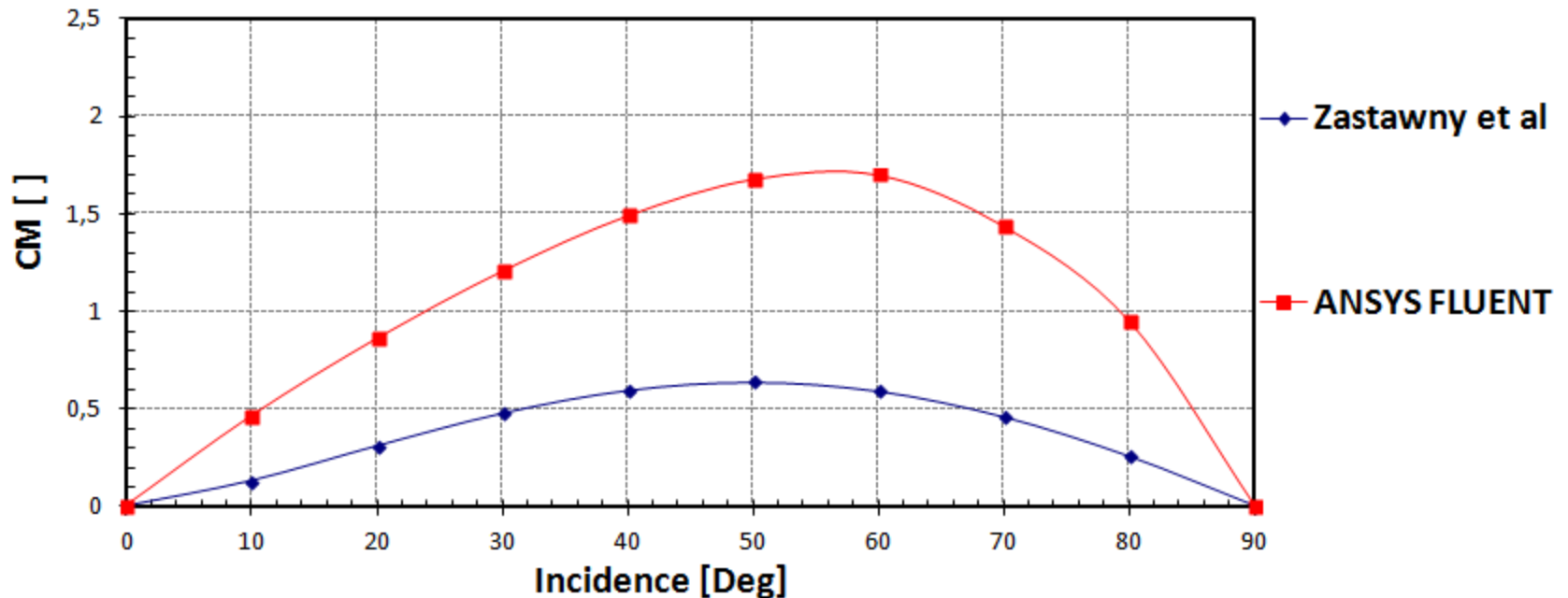
$w=5/1$

$\phi=0.69$





## Comparison at $Re_p = 300$

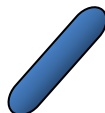


The torque in the Z direction given by Zastawny et al. is significantly lower than the one extracted from our simulations. Deviation  $\approx 174\%$ .

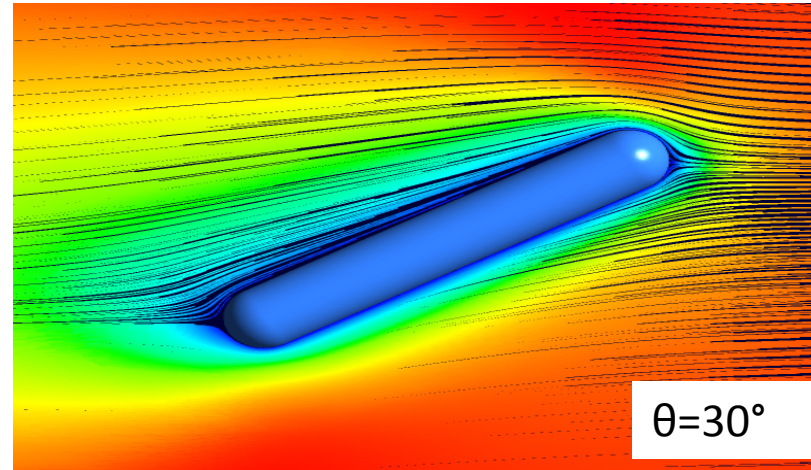
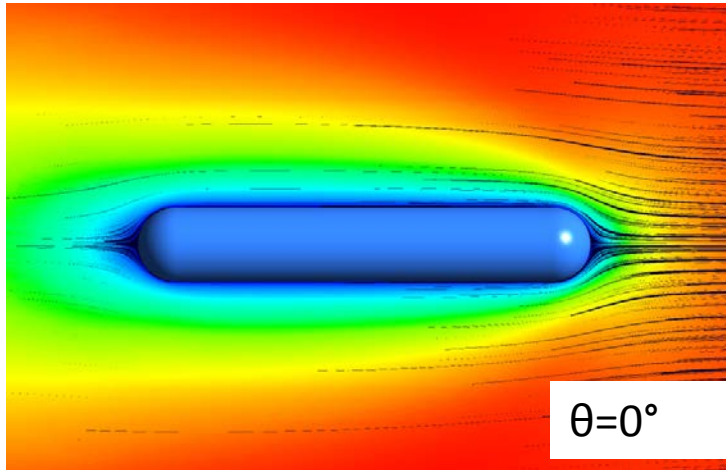
Spherocylinder

$w=5/1$

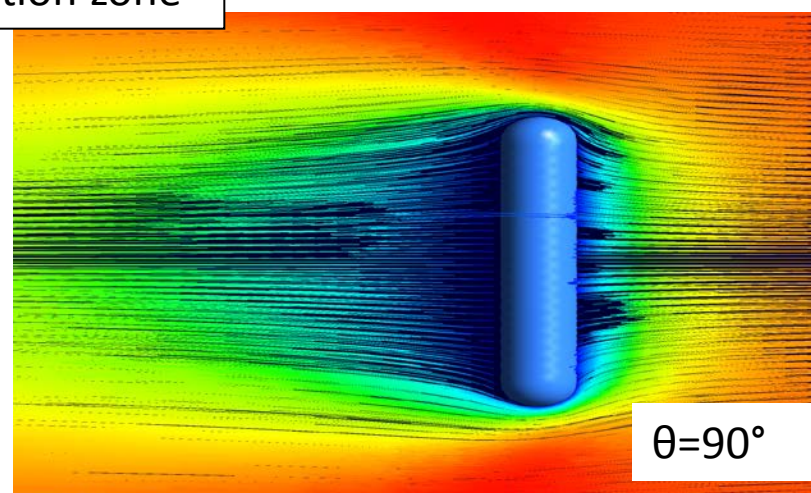
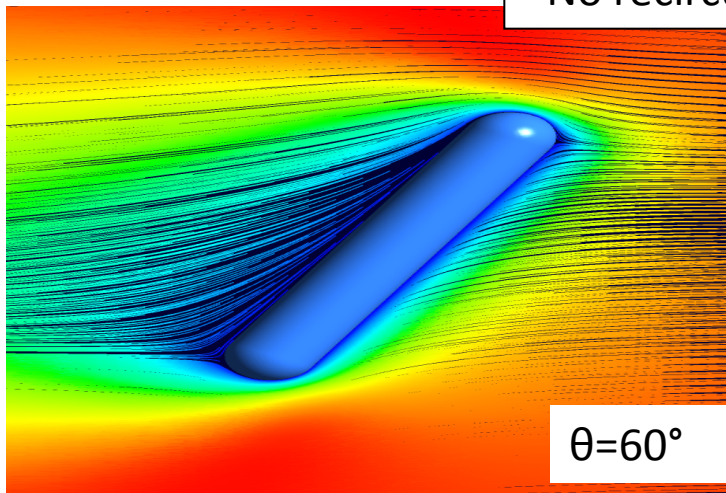
$\phi=0.69$



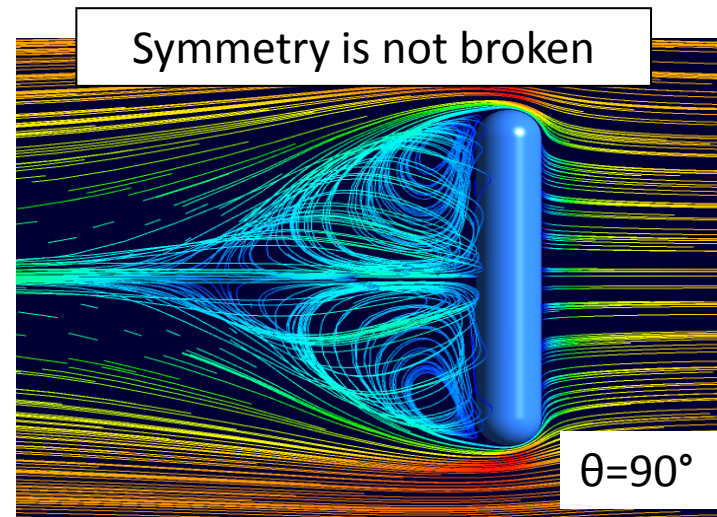
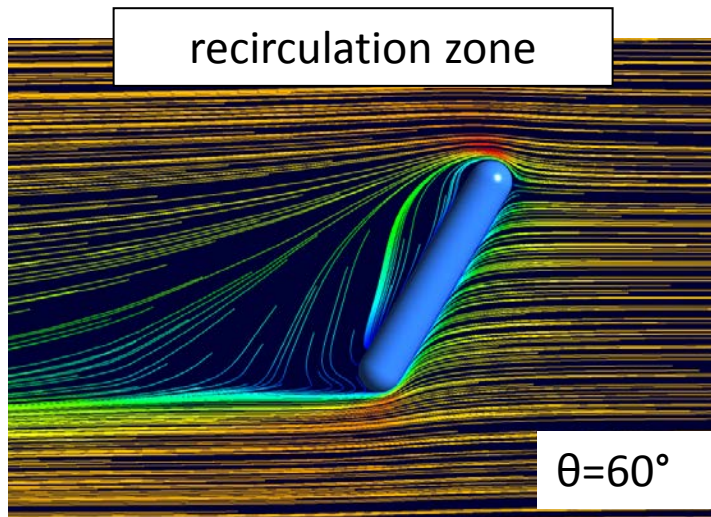
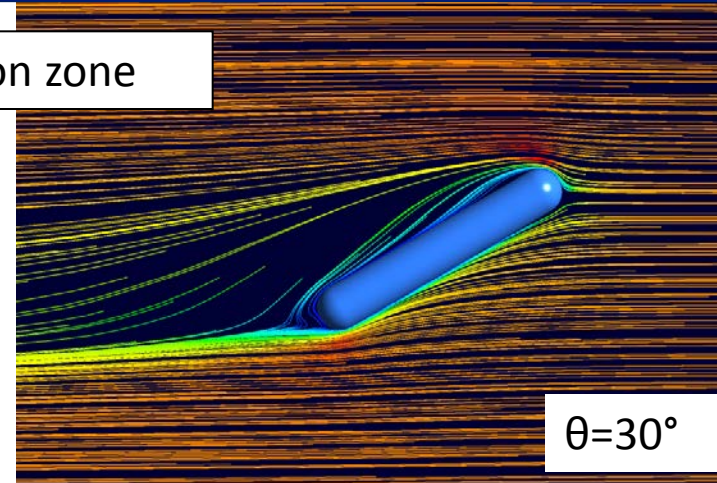
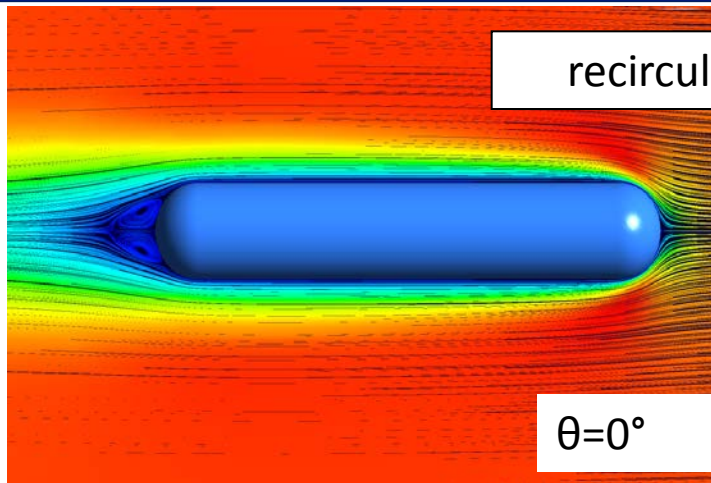
# Flow visualization at $Re_p=10$ ANSYS FLUENT



No recirculation zone



# Flow visualization at $Re_p=300$ ANSYS FLUENT





## Comparison between ellipsoid and spherocylinder at $Re_p=300$

- The same aspect ratio does not give the same behavior of the flow.

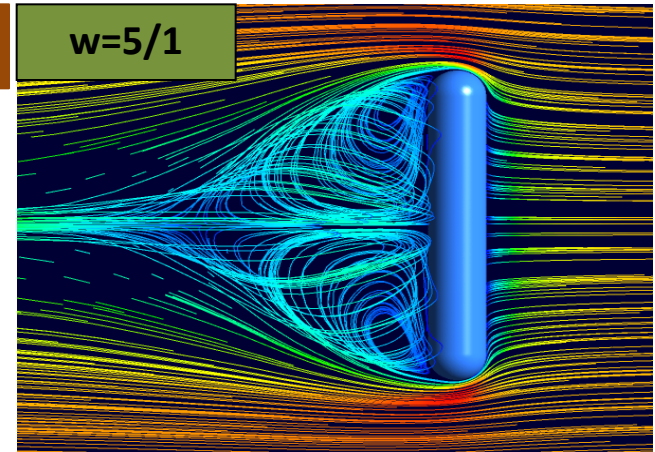
- The symmetry of the streamlines is not broken.

- The critical Reynolds number is not the same for both particles even if it has the same aspect ratio.

Spherocylinder

Symmetry

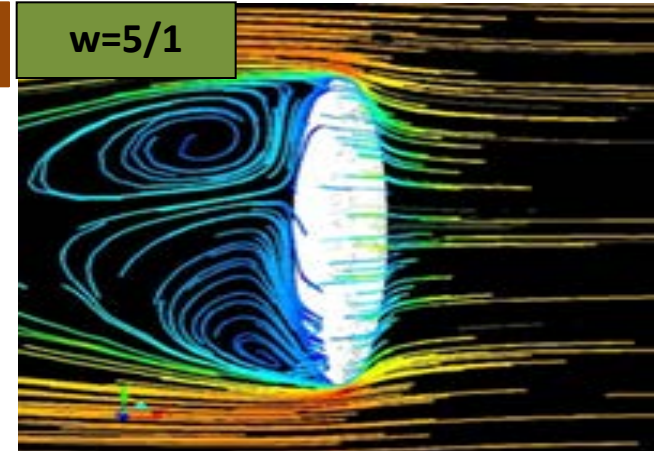
$w=5/1$



Ellipsoid

Beginning of the dissymmetry

$w=5/1$

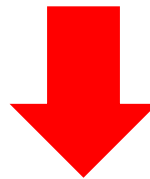


## Conclusion

- The present results show some significant differences at low Reynolds number for CD and CL with DNS of Zastawny.
- The present torque coefficients are not in good accordance with the results of Zastawny.



Not enough results in order to conclude about the pertinence of the existing estimations of the hydrodynamic forces (spherocylinder and ellipsoid).



The difficulty remains again in the choice of the correlation for spherocylinder particles in order to model the motion of non spherical particles.

## Future plans

- We will determine which is the critical Reynolds number for a spherocylinder.
- Simulation with an other type of the particles.

# References

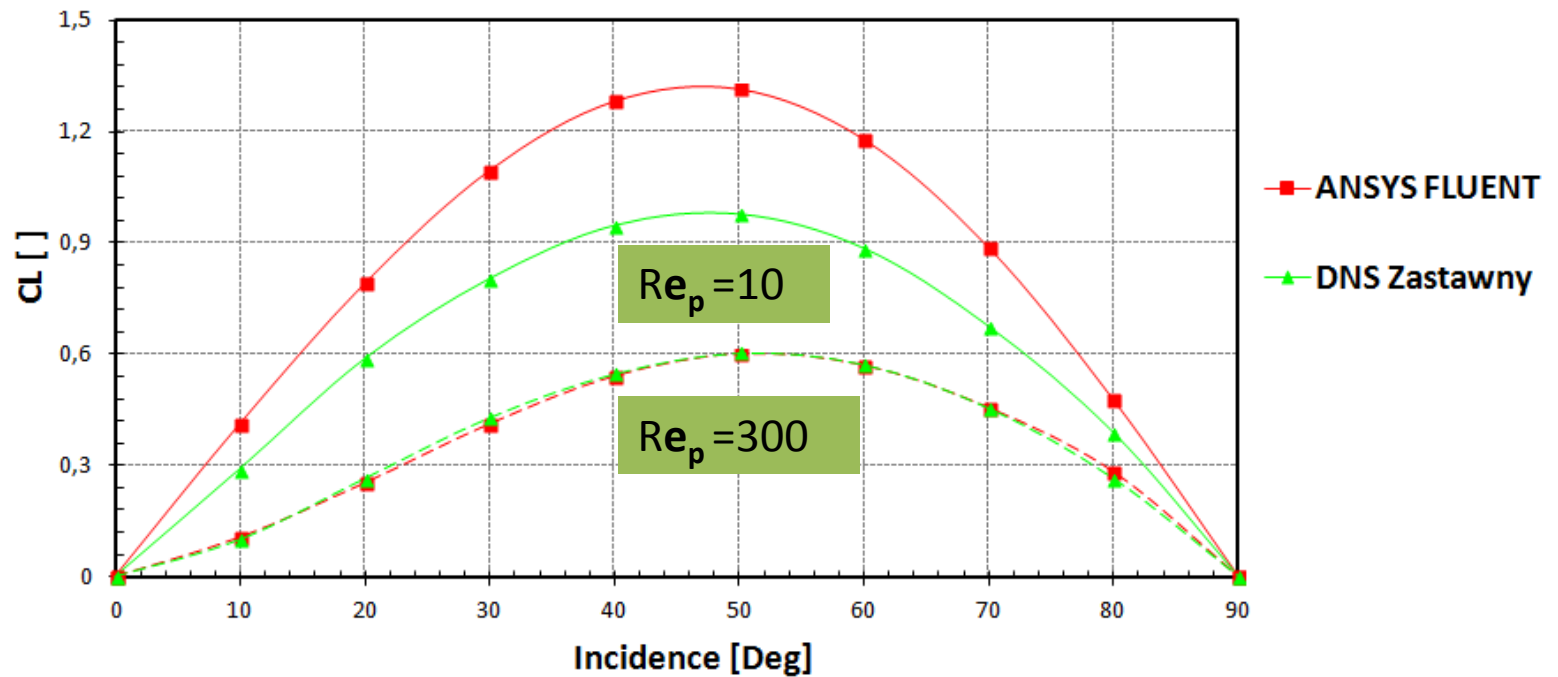
- [1] Hölzer, A and Sommerfeld, M. 2008. New simple correlation formula for the drag coefficient of non-spherical particles. *Powder Technol*, 184, 361-365
- [2] Zastawny, M., Mallouppas, G., Zhao, F., van Wachem, B. 2011. Derivation of drag and lift force and torque coefficients for non-spherical particles in flows. *International Journal of Multiphase Flow*. 39, 227-239
- [3] Mando, M and Rosendahl, L. 2010. On the motion of non-spherical particles at high Reynolds number. *Powder Technology*, 202, 1-13





# ANNEXE

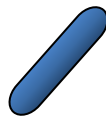
## Comparison between DNS of Zastawny and the ANSYS FLUENT simulation



Spherocylinder

$w=5/1$

$\phi=0.69$



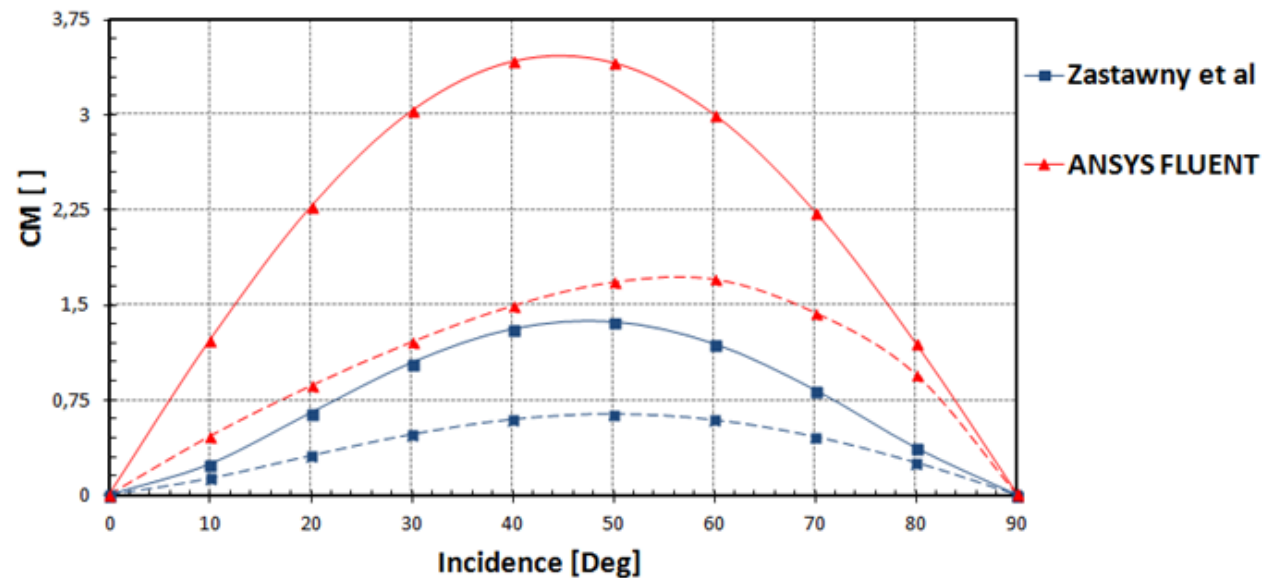
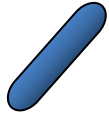
Better accordance at Re=300

## Comparison between existing correlations and the ANSYS FLUENT simulation

### Spherocylinder

$w=5/1$

$\phi=0.69$

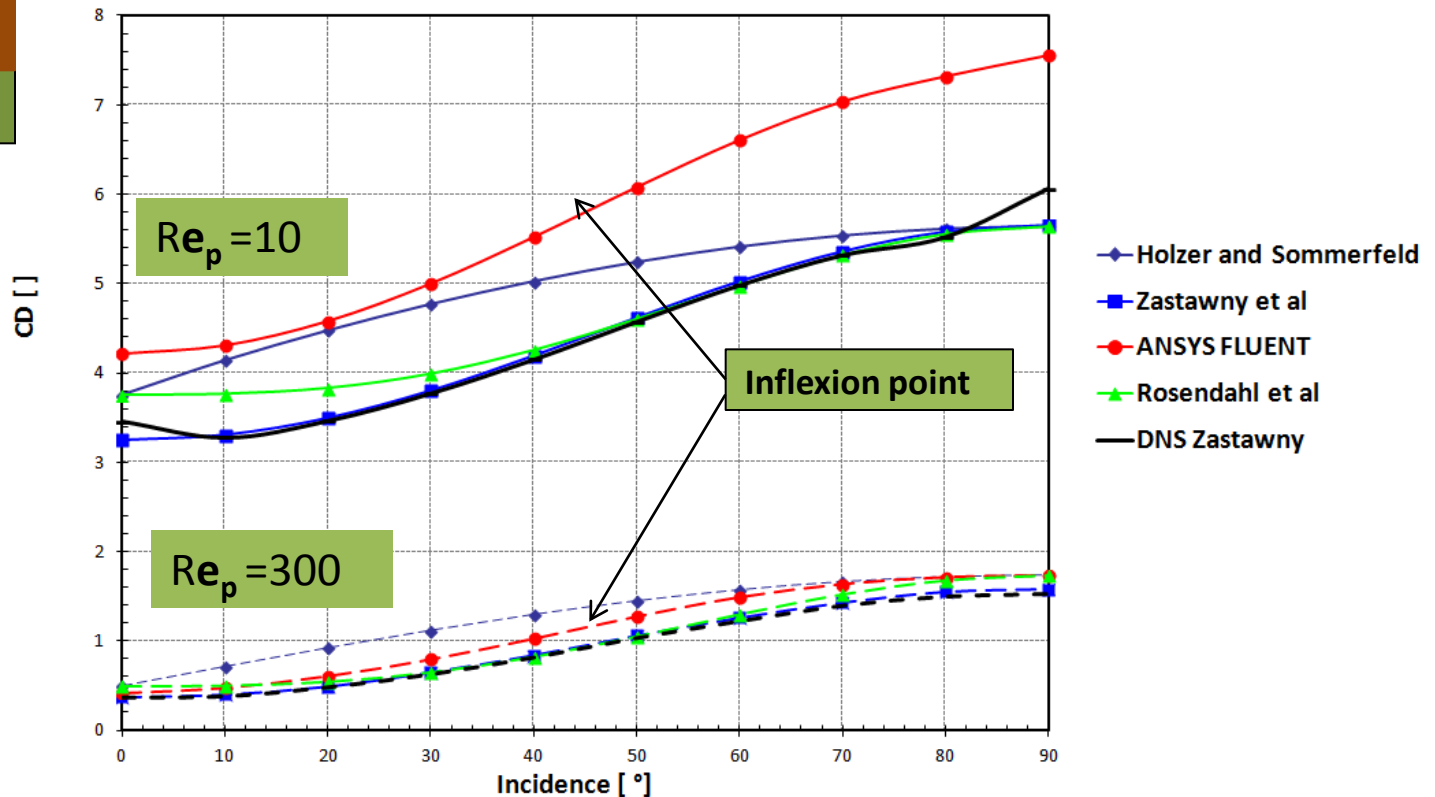
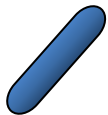


There is an important difference between our results and those by Zastawny. We think that is due to under-prediction of the forces on their simulations and this allowed for the three coefficients  $C_D$ ,  $C_L$  and  $C_M$ .

Other studies must be done, using other code CFD code for this type of the particle.

# Comparison between existing correlations and the ANSYS FLUENT simulation

**Spherocylinder**  
 $w=5/1$       $\phi=0.69$



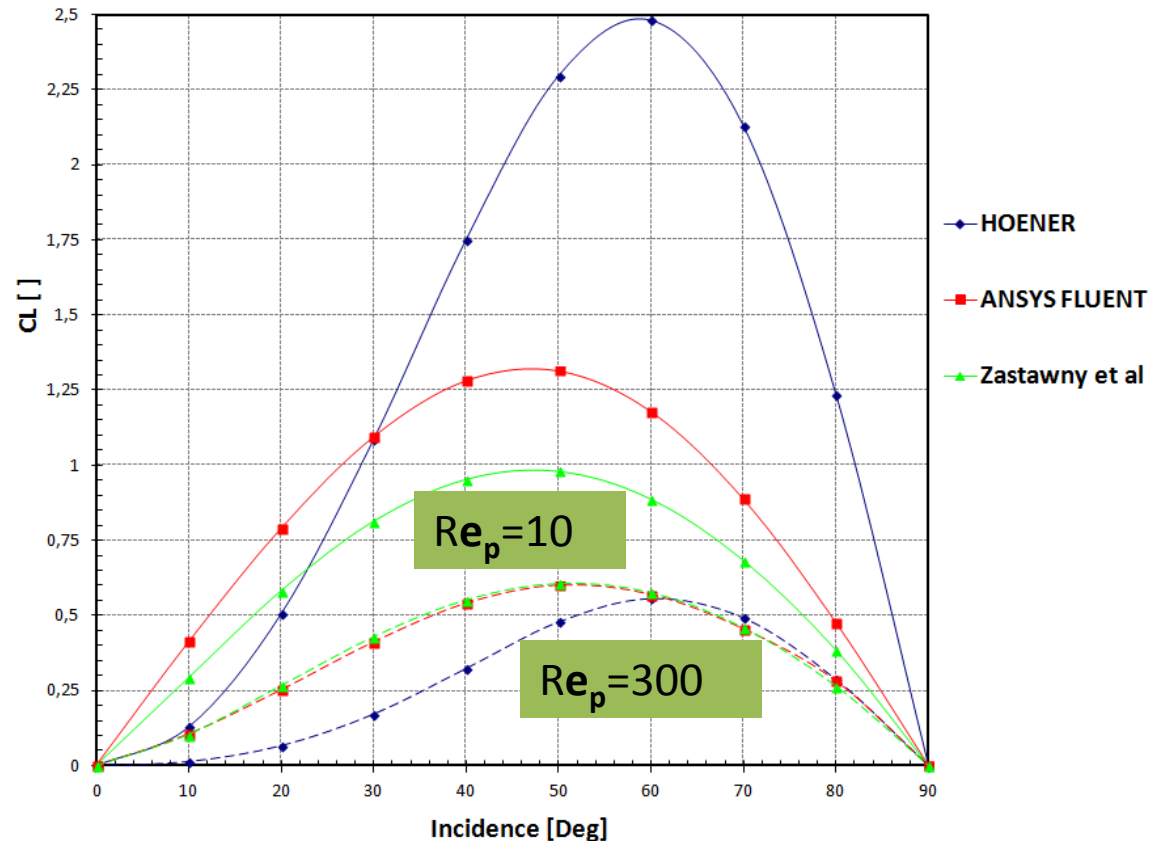
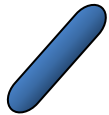
We have the same tendency between our results and those of Zastawny at the both Reynolds number. But at  $Re=10$  the results of Zastawny tend to under-predict  $C_d$  than us, as we seen for the ellipsoid at  $Re=0,1$ .

## Comparison between existing correlations and the ANSYS FLUENT simulation

Spherocylinder

$w=5/1$

$\phi=0.69$



We have the same tendency between our results and those of Zastawny at the both Reynolds number. But at  $Re_p = 10$  the results of Zastawny tend to under-predict  $C_D$  than us, as we seen for the ellipsoid at  $Re_p = 0,1$ .

