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The electric location-routing problem with heterogeneous fleet: Formulation and Benders decomposition approach

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Abstract

In this paper, we focus on a problem that requires [the](#) location of recharging stations and [the](#) routing of electric vehicles in a goods distribution system. The goods are disseminated from a depot and distributed to the customers via [a heterogeneous fleet of electric vehicles](#) with limited capacity. Differently from the classical vehicle routing problem, the vehicles have battery restrictions that need to be recharged at some stations if a trip is longer than their range. The problem reduces to finding the optimal locations of the recharging stations and their number to minimize the total cost, which includes the routing cost, the recharging cost, and the fixed costs of opening stations and operating vehicles. We propose a novel mathematical formulation and an efficient Benders decomposition algorithm embedded into a two-phase general framework to solve this environmental logistics problem. Phase I solves a restricted problem to provide an upper bound for the original problem which is later solved in Phase II. Between the two phases, an intermediate processing procedure is introduced to reduce the computations of the Phase II problem. This is achieved by a combination of the Phase I upper bound and several lower bounds obtained via exploiting the underlying network structure. Our approach solves the problem in a general setting with non-identical stations and vehicles by allowing multiple visits to the stations and partial recharging. The computational study provides both managerial and methodological insights.

Keywords: Recharging Station Location, Electric Vehicle Routing, Environmental Logistics, Integer Programming, Benders Decomposition

1. Introduction

1 The transport sector is responsible, to a large extent, for energy consumption and greenhouse
2 gas emissions. According to the European Environment Agency (2018), the energy consumption

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3 of road transport increases by 32% from 1990 to 2016 in the EEA-33. To tackle environmental
4 and energy challenges, several countries are considering the prospect of carbon neutrality over the
5 next 30 years, with the objective of discouraging the sale of vehicles emitting greenhouse gases.
6 The implementation of such a strategy has already begun with the introduction of low-emission
7 zones (LEZ), where vehicles with higher emissions either cannot enter the area or have to pay a
8 high penalty. For instance, the traffic pollution charge in London LEZ is £100 per day for larger
9 vans and minibuses and rises to double this amount for lorries, buses, and coaches. Vehicles with
10 alternative fuels, such as electric vehicles (EVs) and hydrogen vehicles, provide credible solutions
11 for achieving the carbon neutrality target.

12 Unlike the hydrogen vehicle, which is currently at the experimental stage, and consequently
13 having an exorbitant cost, the EV has reached an industrial maturity that makes it competitive
14 compared to the combustion vehicle. However, as indicated by Davis and Figliozzi (2013) and Sassi
15 and Oulamara (2017), EVs are still facing weaknesses related to their availability, purchase price,
16 and battery management. From a logistics point of view, there are still weaknesses that are worth
17 pointing at. These include

18 (i) The limited choice of light duty EVs offered by the car industry. These vehicles are mainly
19 needed in the last-mile logistics.

20 (ii) The limited EV driving range. For instance, for light duty EVs, the range is between 120km
21 and 180km. Note that the range can depend on topology of the road as well as weather and driving
22 conditions.

23 (iii) The long charging time. The time to fully charge a vehicle can take up to 8 hours depending
24 on the capacity of the battery pack and chargers' level.

25 (iv) The lack of availability of charging infrastructures in existing road networks.

26 Although all these weaknesses are manageable in practice, the cost of EV presents a barrier to
27 their extensive use. An opportunity to reduce the vehicle's price is focusing on the development
28 of those markets that are ready to adopt such a green-based strategy. Such markets allow a large-
29 scale production of EVs which can consequently lead to the reduction of vehicle costs. Last-mile
30 logistics provide this opportunity to speed up the market penetration of EVs. In such markets, an
31 EV has the advantage of meeting the requirement of low-emission zones that are mainly located
32 in city centers. Here, the distances covered in last-mile logistics are either within its range or it
33 requires one charging session along the route only. Furthermore, even though the acquisition cost
34 for EVs is usually higher than the combustion engine vehicles, this difference can be offset at the
35 operational cost of EV usage. This is because a high utilization of EVs favors their TCO (Total
36 Cost Ownership) since their operating costs (maintenance, tax, fuel, and depreciation) are low
37 compared to those of their counterparts.

38 In this paper, we consider a goods distribution system that utilizes EVs. This is a system where
39 the operating companies have access to their own recharging stations (private) or subscribe to a
40 contract that warrants access (without queuing restrictions) to certain recharging stations which

41 have to be selected. Similar business models are considered by Yang and Sun (2015) for battery
42 swap stations and by Schiffer and Walther (2017a,b) for recharging stations. In these types of
43 business models, the operators need to decide on both the location and the routing aspects. As
44 location and routing decisions are interdependent, they need to be handled simultaneously to
45 operate an overall system in the most profitable way (Salhi and Rand, 1989). It may be argued
46 that it is difficult to integrate operational decisions such as routing into strategic decisions like
47 locating facilities. Though this is a critical issue, studies dealing with this dilemma showed that
48 an intelligent way of incorporating the results of the integration can be very useful. For instance,
49 Salhi and Nagy (1999) conduct a robustness analysis leading to a conclusion that integrated models
50 constantly provide higher quality solutions and they are as reliable as ‘locate first - route second’
51 methods.

52 In our study, we consider a [heterogeneous fleet of vehicles](#) to depart from a single depot. We
53 also assume there is a sufficient number of charging stations and electrical grid capacity. This is to
54 ensure that all vehicles are fully charged before their departure from the depot. However, we may
55 need to recharge them during their trips if the total energy consumption to visit certain customers
56 is larger than the battery capacity. Once a station is opened, it might be visited multiple times by
57 any vehicle. As we allow partial recharging, the vehicles do not need to be fully recharged. Besides,
58 we do not impose any restrictions on the types of stations or vehicles. In other words, we allow the
59 use of heterogeneous vehicles and stations that might have different location-dependent costs. [The
60 problem is to decide on the number and location of stations, the number of vehicles needed, the
61 amount of recharging needed for each vehicle, and the route\(s\) for visiting all the customers. The
62 objective is to minimize the total cost which includes the variable cost of routing and recharging
63 as well as the fixed costs of opening stations and operating vehicles.](#)

64 In this study, we develop an exact method based on a new formulation which utilizes disag-
65 gregated commodity flows to express sub-tour elimination and capacity restrictions (Yaman, 2006;
66 Baldacci et al., 2008; Salhi et al., 1992; Salhi and Rand, 1993). There are several applications
67 in the literature where flow based formulations with capacity constraints are successfully solved
68 using a Benders decomposition approach. These include the hub location-routing problem stud-
69 ied by de Camargo et al. (2013) and network design problems by Fortz and Poss (2009), Botton
70 et al. (2013), and Calik et al. (2017). See also other relevant Benders decomposition applications
71 for the location of EV recharging stations in car sharing systems (Çalik and Fortz, 2019), under
72 probabilistic travel range (Lee and Han, 2017), and with plug-in hybrid EVs (Arslan and Karaşan,
73 2016); in the survey by Costa (2005) for fixed-charge network design problems; and in the book
74 by Birge and Louveaux (2011) for stochastic programming problems. This motivates us to apply
75 a Benders decomposition algorithm leading to very successful results for solving the small size in-
76 stances which are shown to be challenging by the preceding study of Schiffer and Walther (2017b).
77 To the best of our knowledge, the heterogeneous fleet feature which makes the problem relatively
78 much more challenging to handle by exact or heuristic methods has not been considered within the

79 location routing framework of combined recharging station location and EV routing problems. The
80 mathematical models introduced by Schiffer and Walther (2017) for the homogeneous fleet vari-
81 ants cannot be utilized or easily adapted to solve the heterogeneous fleet variants. Given that the
82 models for homogeneous variants have difficulty in solving even small instances with 5 customers,
83 there is a clear need for more efficient exact methods which can solve relatively larger instances
84 (e.g. 10-15 customers) and enable performance evaluations of heuristic methods to be developed
85 in the future.

86 Our methodological contributions are twofold:

- 87 - to propose a new mixed integer programming formulation for this strategic electric location-
88 routing problem and
- 89 - to develop a Benders decomposition algorithm embedded in a novel two-phase framework to
90 solve the problem to proven optimality by making use of several lower and upper bounds.

91 [The model and the algorithm developed are applicable to both versions of the problem with limited
92 and unlimited number of vehicles.](#)

93 The rest of the paper is organized as follows: Section 2 gives an informative review on the
94 related works. In Section 3, we provide the notation used throughout the paper and present
95 our mathematical formulation. In Section 4, we propose our Benders decomposition algorithm
96 followed by Sections 5 and 6 describing the implementation and the intermediate reduction process,
97 respectively. In Section 7, we provide the setting and present the results of our computational study.
98 We conclude in Section 8 with a summary of our findings and a highlight of some future research
99 directions.

100 2. Related work

101 Location of recharging stations can be seen as a facility location problem. The purpose is
102 then to decide on the optimal number and locations of facilities given the position of customers
103 to serve. In this vein, He et al. (2016) present a case study in Beijing, China. Their objectives
104 are to incorporate the local constraints of supply and demand on public EV charging stations
105 into facility location models, and to compare the optimal locations from three different location
106 models: the set covering model, the maximal covering location model, and the p-median model.
107 Liu and Wang (2017) address the optimal location of multiple types of charging facilities, including
108 dynamic wireless charging facilities and different levels of plug-in charging stations. Their tri-level
109 program first treats the model as a black-box optimization, which is then solved by an efficient
110 approximation model.

111 However, as raised in Salhi and Rand (1989), facility location and routing decisions are interde-
112 pendent and should be tackled simultaneously. In the general case where both vehicles and depots
113 are capacitated, the problem is known as the capacitated location routing problem (CLRP). The
114 aim here is to i) define which depots must be opened, ii) assign each serviced node (customer)

115 to one and only one depot and, iii) route the vehicle to serve the nodes, in such a way that the
116 sum of the depot cost and the total routing cost is minimized. Many papers appeared in the sub-
117 ject and more particularly during the last decade, as shown in surveys by Nagy and Salhi (2007);
118 Prodhon and Prins (2014), and Schneider and Drexl (2017). To solve this NP-hard problem to
119 proven optimality, branch-and-cut (Belenguer et al., 2011) and set partitioning based (Akca et al.,
120 2009; Contardo et al., 2013) algorithms are available in the literature. Additionally, several new
121 efficient metaheuristics are proposed. These include a cooperative Lagrangean relaxation-granular
122 tabu search heuristic by Prins et al. (2007), an adaptive large-neighborhood search (ALNS) by
123 Hemmelmayr et al. (2012), and a three-phase matheuristic by Contardo et al. (2014). Other stud-
124 ies cover a multiple ant colony optimization algorithm (Ting and Chen, 2013) and a two-phase
125 hybrid heuristic (Escobar et al., 2013). Very recently, a tree-based search algorithm by Schneider
126 and Löffler (2017) and a Genetic Algorithm by Lopes et al. (2016) are proposed.

127 The integration of the location of recharging stations with the routing decision, also called
128 electric location-routing problem (ELRP), is relatively recent though it can lead to massive envi-
129 ronmental benefits. To the best of our knowledge, the first study of simultaneous vehicle routing
130 and charging station location for commercial EVs is presented in a conference paper in 2012 by
131 Worley et al. (2012). Then, Yang and Sun (2015) introduce the interesting battery swap station
132 location-routing problem, where the charge is completely fulfilled at each stop. The authors develop
133 two heuristic approaches. The problem is revisited by Hof et al. (2017) who adapt an interesting
134 and powerful adaptive variable neighborhood search (AVNS) heuristic originally dedicated to the
135 vehicle routing problem (VRP) with intermediate depots. Recently, Zhang et al. (2019) introduce
136 a battery swap station location-routing problem with stochastic demand and solve this problem by
137 developing a hybrid algorithm combining binary particle swarm optimization and variable neigh-
138 borhood search. Another relevant study by Yıldız et al. (2016) [introduces](#) a branch and price
139 algorithm for routing and refueling station location problem.

140 The first paper dealing with partial recharge may come from Felipe et al. (2014), and is dedicated
141 to a Green Vehicle Routing Problem (G-VRP). In G-VRP the fleet is composed of Alternative Fuel
142 Vehicles (AFV) where, in addition to the routing of each EV, the amount of energy recharged and
143 the technology used must also be determined. However, the location aspect is not considered.
144 Constructive and improving heuristics are embedded in a Simulated Annealing framework. The
145 partial recharging policies are then reused showing that they may considerably improve the routing
146 decisions as noted by Keskin and Çatay (2016). Thus, Schiffer and Walther (2017b) extend the
147 problem by including the location of charging stations which leads to the electric location routing
148 problem with time windows and partial recharging (ELRP-TWPR). The authors focus on a problem
149 with a single type of vehicle and multiple visits to the stations. They propose a mathematical
150 formulation based on Miller-Tucker-Zemlin type constraints, supported by several preprocessing
151 steps to eliminate the arcs that violate time windows, capacity, and battery restriction constraints.
152 The Location Routing Problem with Intraroute Facilities which is a generalization of the ELRP-

153 TWPR is explored by Schiffer and Walther (2017a) where large instances are solved using an ALNS
 154 which is enhanced by local search and dynamic programming components. A lower bounding
 155 procedure integrated to this ALNS algorithm by Schiffer et al. (2018) provides improved results
 156 for solving the ELRP-TWPR.

157 Our problem can be considered as a generalization of the electric vehicle routing problem
 158 (EVRP) with location decisions or an electric location-routing problem (ELRP) with a heteroge-
 159 neous fleet, multi-type stations, multi-visit, and partial recharging. The EVRP literature is not
 160 extensively discussed here except those papers considering the location decisions. However, we
 161 refer the reader to Pelletier et al. (2016) for an overview on the EVRP studies. In the next section
 162 we provide the notation and a mathematical formulation of the problem.

163 3. Notation and Problem Formulation (PF)

164 Consider a given network with a set of customer locations and a set of potential charging station
 165 locations, from which we are required to select a subset of stations. Each customer should be served
 166 by a vehicle originating from the depot and each vehicle can perform a single trip. The vehicles
 167 have a battery restriction and they have to visit one or more among the selected charging stations
 168 before the battery is depleted if a trip longer than their range is to be traversed. In addition, we
 169 have a fleet of heterogeneous vehicles with a limited number of vehicles of each type. **Note that all**
 170 **our methods remain applicable to the special case with unlimited number of vehicles which is in**
 171 **practice equivalent to the case where the number of vehicles for each type is equal to the number**
 172 **of customers.** We first provide the notation and a scheme for allowing multiple visits which is then
 173 followed by the new formulation.

174 3.1. Notation

175 In this section, we list the parameters and the decision variables as follows:

176 *Parameters:*

177 $G = (N, A)$: the given network.

178 A : the arc set.

179 $N = I \cup J \cup \{0\}$: the set of all nodes

180 I : the set of customer locations

181 J : the set of potential locations for charging stations

182 0 : the depot node

183 K : the set of vehicles

184 $d_i > 0$: the demand of client $i \in I$

185 c_{ij} : the routing cost of traversing arc $(i, j) \in A$

186 e_{ij} : the energy consumption on arc $(i, j) \in A$ expressed in kWh

187 f_j : the fixed cost of opening a charging station at node $j \in J$

188 r_k : the unit cost of recharging of vehicle $k \in K$

189 v_k : the fixed cost of operating vehicle $k \in K$

190 Q^k : the load capacity of vehicle $k \in K$

191 β^k : the battery capacity of vehicle $k \in K$ expressed in kWh.

192 *Decision variables:*

193 $y_j = 1$ if station $j \in J$ is open, 0 otherwise

194 $x_{ij}^k = 1$ if arc (i, j) is traversed by vehicle $k \in K$, 0 otherwise

195 z_j^k is the amount of energy recharged at station $j \in J$ for vehicle $k \in K$

196 b_{ij}^k is the battery level of vehicle $k \in K$ at node $i \in N$ before leaving for node $j \in N$ expressed
197 in kWh

198 l_{ij}^k is the cumulative load of vehicle $k \in K$ at node $i \in N$ before leaving for node $j \in N$.

199 In the remaining of this paper, we assume $I \subset J$ but all the methods can be easily adapted to
200 the case where $I \setminus J \neq \emptyset$ by simply defining y_j and z_j^k variables for all $j \in I \cup J, k \in K$ and setting
201 $y_j = z_j^k = 0, \forall j \in I \setminus J$. For convenience, we define $d_j = 0$ for $j \in N \setminus I$.

202 3.2. A novel mechanism that caters for multiple visits

203 In order to allow multiple visits to a station, we perform the following interesting and powerful
204 modification on our input network. For each potential station, we create as many dummy copies
205 as the number of customers (Steps 2-3 in Algorithm 1). If this potential station is also a demand
206 node, the demand of the first copy is identical to the demand of the potential station whereas the
207 demand of the remaining copies is set to zero. Similarly, the fixed cost of the first copy is identical
208 to the fixed cost of the station whereas it is set to zero for the remaining copies (Step 4). The arc
209 set of the modified network includes all the arcs of the original network. Additionally, all distinct
210 node pairs in the modified network are connected via a direct arc except the pairs which are the
211 copies of the same station (Step 5). Finally, we define an arc set for each vehicle which contains
212 all arcs in the modified network whose energy consumption is not greater than the range of the
213 vehicle and the total demand of its endpoints is not greater than the freight capacity of the vehicle
214 (Step 6).

215 *Algorithm 1: Network modification*

216 Step 1: Let $|I|$ be the number of demand nodes.

217 Step 2: Create $|I|$ copies of each station.

218 Step 3: Form set $J_j^A = \{j_1, j_2, \dots, j_{|I|}\}$ for each $j \in J$ and $J^A = \bigcup_{j \in J} J_j^A$.

219 Step 4: For each $j \in J$, set $f_{j_1} = f_j$; $d_{j_1} = d_j$ and $f_{j_i} = d_{j_i} = 0, i = 2, \dots, |I|$ where
220 $j_1, \dots, j_{|I|} \in J_j^A$.

221 Step 5: Let $N^E = J^A \cup \{0\}$ and $A^E = A \cup \{(i, j) : i, j \in N^E; i \neq j; \neg(i, j \in J_l^A \text{ for some}$
 222 $l \in J)\}$.

223 Step 6: Define $A^k = \{(i, j) \in A^E : e_{ij} \leq \beta^k; d_i + d_j \leq Q^k\}$ for $k \in K$.

224 3.3. Mathematical formulation PF

The following is a commodity flow formulation of the problem:

$$\min \sum_{k \in K} \sum_{(i,j) \in A^k} c_{ij} x_{ij}^k + \sum_{k \in K} \sum_{j \in J^A} r_k z_j^k + \sum_{j \in J^A} f_j y_j + \sum_{k \in K} \sum_{(0,i) \in A^k} v_k x_{0i}^k \quad (1)$$

$$\text{s.t. } y_i \leq y_j, \quad i \in J_j^A : i \neq j \quad (2)$$

$$\sum_{i \in J^A} x_{0i}^k \leq 1, \quad k \in K \quad (3)$$

$$\sum_{k \in K} \sum_{(j,i) \in A^k} x_{ji}^k = 1, \quad i \in J^A : d_i > 0 \quad (4)$$

$$\sum_{(j,i) \in A^k} x_{ji}^k \leq y_i, \quad i \in J^A : d_i = 0, k \in K \quad (5)$$

$$\sum_{(i,j) \in A^k} x_{ij}^k - \sum_{(j,i) \in A^k} x_{ji}^k = 0, \quad i \in J^A, k \in K \quad (6)$$

$$\sum_{(i,j) \in A^k} (l_{ij}^k - d_i x_{ij}^k) = \sum_{(j,i) \in A^k} l_{ji}^k, \quad i \in J^A, \forall k \in K \quad (7)$$

$$\sum_{j \in J^A} l_{0j}^k = 0, \quad k \in K \quad (8)$$

$$l_{ij}^k \leq Q^k x_{ij}^k, \quad k \in K, (i, j) \in A^k \quad (9)$$

$$\sum_{(i,j) \in A^k} e_{ij} x_{ij}^k - \sum_{j \in J^A} z_j^k \leq \beta^k, \quad k \in K \quad (10)$$

$$\sum_{(i,j) \in A^k} b_{ij}^k = \sum_{(j,i) \in A^k} (b_{ji}^k - e_{ji} x_{ji}^k) + z_i^k, \quad i \in I \cup J^A, k \in K \quad (11)$$

$$z_j^k \leq \beta^k y_j, \quad j \in J^A, k \in K \quad (12)$$

$$z_j^k \leq \beta^k \sum_{(i,j) \in A^k} x_{ij}^k, \quad j \in J^A, k \in K \quad (13)$$

$$b_{0j}^k = \beta^k x_{0j}^k, \quad k \in K, (0, j) \in A^k \quad (14)$$

$$b_{ij}^k \leq \beta^k x_{ij}^k, \quad k \in K, (i, j) \in A^k \quad (15)$$

$$b_{ij}^k \geq e_{ij} x_{ij}^k, \quad k \in K, (i, j) \in A^k \quad (16)$$

$$y_j \in \{0, 1\}, \quad j \in J^A \quad (17)$$

$$x_{ij}^k \in \{0, 1\}, \quad k \in K, (i, j) \in A^k \quad (18)$$

$$l_{ij}^k \geq 0, \quad k \in K, (i, j) \in A^k \quad (19)$$

$$b_{ij}^k \geq 0, \quad k \in K, (i, j) \in A^k \quad (20)$$

$$z_j^k \geq 0, \quad j \in J^A, k \in K. \quad (21)$$

225 The objective function (1) minimizes the total sum of routing costs, charging costs, fixed costs of
 226 opening stations, and fixed cost of using vehicles. If a zero-demand copy of a station is opened,
 227 Constraints (2) force the original copy of this node to be opened and therefore, ensure that the costs
 228 of the stations are counted in the objective function. By Constraints (3), we restrict the number of
 229 trips by each vehicle to at most one. Constraints (3)-(9) together ensure that each client is served
 230 by a unique vehicle that starts its trip at the depot and the capacities of vehicles are respected.
 231 Constraints (5) ensure that a zero-demand copy of any station is visited only if that station is
 232 open. We ensure the elimination of sub-tours for each vehicle trip via the load (flow) preservation
 233 constraints (7)-(9). Note that Constraints (9) also ensure that the vehicle freight capacities are
 234 always respected. Battery restriction on the vehicles are imposed by Constraints (10) and (11).
 235 Constraints (12) and (13) avoid recharging of a vehicle at a node that has no station and that is
 236 not visited by that vehicle, respectively.

237 We initialize the battery level for each vehicle to 100% by Constraints (14). For each arc-
 238 vehicle pair, Constraints (15) restrict the amount of battery level with full battery level if the
 239 arc is traversed by the vehicle and set it to zero otherwise; Constraints (16) make sure that the
 240 battery level is larger than the energy consumption on the arc that will be traversed by the vehicle.
 241 Finally, Constraints (17)-(21) represent the binary and non-negativity restrictions on the decision
 242 variables.

243 4. Benders Decomposition Algorithm (BDA)

244 Our mathematical formulation can be solved by using a Benders decomposition (Benders, 1962)
 245 framework that we briefly described here. The details of our algorithm are presented next. The
 246 classical Benders decomposition method aims to solve a mixed integer program (MIP) with a group
 247 of integer variables and a group of continuous variables by decomposing the MIP into a master
 248 problem (MP) with all integer variables and a series of subproblems with continuous variables.
 249 For each feasible solution of MP, a subproblem (SP) is constructed by fixing the values of all the
 250 integer variables in the MIP to the value obtained from the master problem. Each extreme ray
 251 and extreme point of the dual of this SP provides a so called feasibility and an optimality cut,
 252 respectively, for the MP. Since the full enumeration of the extreme points and extreme rays is
 253 impractical, the cutting plane procedures are usually employed for the generation and the addition
 254 of these cuts.

255 The classical Benders decomposition method could suffer from slow convergence especially if
 256 the subproblem is large in size. On the other hand, the method would perform relatively efficiently
 257 if the subproblem can be decomposed further into smaller and easy-to-solve subproblems as in

258 multi-commodity, multi-period, or multi-scenario problems (Birge and Louveaux, 2011). Moti-
 259 vated by this fact, we aim to further decompose our problem into $|K|$ smaller problems, each one
 260 corresponding to a single vehicle trip. For this purpose, we decide to keep \mathbf{y} , \mathbf{x} , \mathbf{l} variables in the
 261 master problem and \mathbf{z} , \mathbf{b} variables in the subproblems. The separation method for the optimality
 262 cuts plays a crucial role in efficient Benders implementations. To speed up our implementation,
 263 we adopt the recently developed high-performing strategy of Çalı̇k and Fortz (2019) and modify
 264 our formulation accordingly to obtain only feasibility cuts from the dual subproblems. In order
 265 to achieve this, we introduce an additional non-negative decision variable $w^k, \forall k \in K$ and make a
 266 slight modification to our model to ensure that w^k takes value $\sum_{j \in J^A} z_j^k, \forall k \in K$. Moreover, we set
 267 $x_{ij}^k = 0$ for $(i, j) \notin A^k$. The modified formulation (PF2), as given in what follows, is defined by
 268 Constraints (2)-(21) and (22)-(24):

$$(PF2) \quad \min \sum_{k \in K} \sum_{i \in N^E} \sum_{j \in N^E: i \neq j} c_{ij} x_{ij}^k + \sum_{k \in K} r_k w^k + \sum_{j \in J^A} f_j y_j + \sum_{k \in K} \sum_{i \in J^A} v_k x_{0i}^k \quad (22)$$

$$\text{s.t. } w^k = \sum_{j \in J^A} z_j^k, \quad \forall k \in K \quad (23)$$

$$w^k \geq 0, \quad \forall k \in K \quad (24)$$

$$(2) - (21).$$

When solving PF2 in a Benders fashion, we employ a branch-and-cut framework which keeps \mathbf{y} , \mathbf{x} , \mathbf{l} , \mathbf{w} variables in the master problem (MP) and \mathbf{z} , \mathbf{b} variables in the subproblems.

$$(MP) \quad \min (22)$$

$$\text{s.t. } (2) - (9), (17) - (19), (24)$$

$$w^k \geq \sum_{i \in N^E} \sum_{j \in N^E: j \neq i} e_{ij} x_{ij}^k - \beta^k, \quad \forall k \in K. \quad (25)$$

Let $(\bar{\mathbf{y}}, \bar{\mathbf{x}}, \bar{\mathbf{l}}, \bar{\mathbf{w}})$ be the vector of variable values in the solution obtained from the master problem. One can easily observe that if $\bar{w}^k = 0$, then, no recharging is needed for the corresponding vehicle trip and $(\bar{\mathbf{y}}, \bar{\mathbf{x}}, \bar{\mathbf{l}}, \bar{\mathbf{w}})$ is feasible for PF2. On the other hand, if $\bar{w}^k > 0$, we construct and solve the dual of the subproblem $SP_k(\bar{\mathbf{y}}, \bar{\mathbf{x}}, \bar{\mathbf{l}}, \bar{\mathbf{w}})$ for every $k \in K$. Note that when $\bar{w}^k > 0$, an optimal solution to the original problem should satisfy Equation (32), which is helpful in the following mathematical manipulations leading to an efficient implementation.

$$SP_k(\bar{\mathbf{y}}, \bar{\mathbf{x}}, \bar{\mathbf{l}}, \bar{\mathbf{w}}) \quad \min 0 \quad (26)$$

$$\text{s.t. } \sum_{j \in J^A} z_j^k = \bar{w}^k, \quad (27)$$

$$z_j^k \leq \beta^k \bar{y}_j, \quad \forall j \in J^A \quad (28)$$

$$z_j^k \leq \beta^k \sum_{i \in N^E: i \neq j} \bar{x}_{ij}^k, \quad \forall j \in J^A \quad (29)$$

$$\sum_{j \in N^E: j \neq i} b_{ij}^k = \sum_{j \in N^E: j \neq i} (b_{ji}^k - e_{ji} \bar{x}_{ji}^k) + z_i^k, \quad \forall i \in J^A \quad (30)$$

$$b_{0j}^k = \beta^k \bar{x}_{0j}^k, \quad \forall j \in J^A \quad (31)$$

$$b_{j0}^k = e_{j0} \bar{x}_{j0}^k, \quad \forall j \in J^A \quad (32)$$

$$b_{ij}^k \leq \beta^k \bar{x}_{ij}^k, \quad \forall i, j \in N^E : i \neq j \quad (33)$$

$$b_{ij}^k \geq e_{ij} \bar{x}_{ij}^k, \quad \forall i, j \in N^E : i \neq j \quad (34)$$

$$b_{ij}^k \geq 0, \quad \forall i, j \in N^E : i \neq j \quad (35)$$

$$z_j^k \geq 0, \quad \forall j \in J^A \quad (36)$$

269 Note that Constraints (10) of PF is ensured by Constraints (25) and (27) as $\sum_{j \in J^A} z_j^k = \bar{w}^k \geq$
 270 $\sum_{i \in N^E} \sum_{j \in N^E: j \neq i} e_{ij} \bar{x}_{ij}^k - \beta^k, \forall k \in K$. Moreover, we can replace equality (27) with inequality $\sum_{j \in J^A} z_j^k \geq$
 271 \bar{w}^k due to Lemma 4.1.

272 **Lemma 4.1.** *If (28)-(36) is non-empty and $\bar{w}^k > 0$, then $\sum_{j \in J^A} z_j^k = \sum_{i \in N^E} \sum_{j \in N^E: j \neq i} e_{ij} \bar{x}_{ij}^k - \beta^k$.*

Proof. $z_i^k = \sum_{j \in N^E: j \neq i} b_{ij}^k - \sum_{j \in N^E: j \neq i} (b_{ji}^k - e_{ji} \bar{x}_{ji}^k), \forall i \in J^A$ by (30). Moreover, $\sum_{i \in J^A} \beta^k \bar{x}_{0i}^k = \beta^k$ and $b_{j0}^k = e_{j0} \bar{x}_{j0}^k$ since $\bar{w}^k > 0$.

$$\begin{aligned} \sum_{i \in J^A} z_i^k &= \sum_{i \in J^A} \sum_{j \in N^E: j \neq i} b_{ij}^k - \sum_{i \in J^A} \sum_{j \in N^E: j \neq i} (b_{ji}^k - e_{ji} \bar{x}_{ji}^k) \\ &= \sum_{i \in J^A} b_{i0}^k - \sum_{i \in J^A} b_{0i}^k + \sum_{i \in J^A} \sum_{j \in J^A: i \neq j} (b_{ij}^k - b_{ji}^k) + \sum_{i \in J^A} \sum_{j \in N^E: j \neq i} e_{ji} \bar{x}_{ji}^k \\ &= \sum_{i \in J^A} e_{i0}^k \bar{x}_{i0}^k - \sum_{i \in J^A} \beta^k \bar{x}_{0i}^k + \sum_{i \in J^A} \sum_{j \in N^E: j \neq i} e_{ji} \bar{x}_{ji}^k \\ &= \sum_{i \in N^E} \sum_{j \in N^E: j \neq i} e_{ji} \bar{x}_{ji}^k - \beta^k \end{aligned}$$

273 □

After elimination of equality constraints and a few mathematical manipulations on the remaining subproblem, we obtain the following SP_k in canonical maximization form for each $k \in K$:

$$\max 0 \quad (37)$$

$$\text{s.t.} \quad - \sum_{j \in J} z_j^k \leq -\bar{w}^k, \quad (38)$$

$$z_j^k \leq \beta^k \bar{y}_j, \quad \forall j \in J^A \quad (39)$$

$$z_j^k \leq \beta^k \sum_{i \in J^A: i \neq j} \bar{x}_{ij}^k, \quad \forall j \in J^A \quad (40)$$

$$z_j^k + \sum_{i \in J^A: i \neq j} b_{ij}^k - \sum_{i \in J^A: i \neq j} b_{ji}^k \leq e_{j0}^k \bar{x}_{j0}^k - \beta^k \bar{x}_{0j}^k + \sum_{i \in N^E: i \neq j} e_{ij} \bar{x}_{ij}^k, \quad \forall j \in J^A \quad (41)$$

$$-z_j^k - \sum_{i \in J^A: i \neq j} b_{ij}^k + \sum_{i \in J^A: i \neq j} b_{ji}^k \leq \beta^k \bar{x}_{0j}^k - e_{j0} \bar{x}_{j0}^k - \sum_{i \in N^E: i \neq j} e_{ij} \bar{x}_{ij}^k, \quad \forall j \in J^A \quad (42)$$

$$b_{ij}^k \leq \beta^k \bar{x}_{ij}^k, \quad \forall i, j \in J^A : i \neq j \quad (43)$$

$$-b_{ij}^k \leq -e_{ij} \bar{x}_{ij}^k, \quad \forall i, j \in J^A : i \neq j \quad (44)$$

$$b_{ij}^k \geq 0, \quad \forall i, j \in J^A : i \neq j \quad (45)$$

$$z_j^k \geq 0, \quad \forall j \in J^A \quad (46)$$

Let $\alpha, \delta_j, \pi_j, \gamma_j, \rho_j, \phi_{ij}, \epsilon_{ij}$ be the dual variables associated with constraints (38)-(44), respectively. Then, we can write the equivalent dual problem $D_k(\bar{\mathbf{y}}, \bar{\mathbf{x}}, \bar{\mathbf{I}}, \bar{\mathbf{w}})$ for each $k \in K$ as follows:

$$\begin{aligned} D_k(\bar{\mathbf{y}}, \bar{\mathbf{x}}, \bar{\mathbf{I}}, \bar{\mathbf{w}}) \quad \min \quad & -\bar{w}^k \alpha + \sum_{j \in J^A} \beta^k \bar{y}_j \delta_j + \sum_{i \in N^E} \sum_{j \in J^A: i \neq j} \beta^k \bar{x}_{ij}^k \pi_j \\ & + \sum_{j \in J^A} e_{j0} \bar{x}_{j0}^k \gamma_j - \sum_{j \in J^A} \beta^k \bar{x}_{0j}^k \gamma_j + \sum_{i \in N^E} \sum_{j \in J^A: i \neq j} e_{ij} \bar{x}_{ij}^k \gamma_j \\ & + \sum_{j \in J^A} \beta^k \bar{x}_{0j}^k \rho_j - \sum_{j \in J^A} e_{j0} \bar{x}_{j0}^k \rho_j - \sum_{i \in N^E} \sum_{j \in J^A: i \neq j} e_{ij} \bar{x}_{ij}^k \rho_j \\ & + \sum_{i \in J^A} \sum_{j \in J^A: i \neq j} \beta^k \bar{x}_{ij}^k \phi_{ij} - \sum_{i \in J^A} \sum_{j \in J^A: i \neq j} e_{ij} \bar{x}_{ij}^k \epsilon_{ij} \end{aligned} \quad (47)$$

$$\text{s.t.} \quad -\alpha + \delta_j + \pi_j + \gamma_j - \rho_j \geq 0, \quad \forall j \in J^A \quad (48)$$

$$-\gamma_i + \gamma_j + \rho_i - \rho_j + \phi_{ij} - \epsilon_{ij} \geq 0, \quad \forall i, j \in J^A : i \neq j \quad (49)$$

$$\alpha \geq 0, \quad (50)$$

$$\delta_j, \gamma_j, \pi_j, \rho_j \geq 0, \quad \forall j \in J^A \quad (51)$$

$$\phi_{ij}, \epsilon_{ij} \geq 0, \quad \forall i, j \in J^A : i \neq j \quad (52)$$

274 In order to avoid solving the same dual problem twice (once for detecting unboundedness and
 275 once for obtaining a feasibility cut), we solve a bounded dual problem instead. This will imply
 276 unboundedness of $D_k(\bar{\mathbf{y}}, \bar{\mathbf{x}}, \bar{\mathbf{I}}, \bar{\mathbf{w}})$ if the optimal value is negative (see Lemma 4.2). To do so, we
 277 bound variables $\alpha, \gamma_j, \rho_j, \forall j \in J^A$, and $\epsilon_{ij}, \forall i, j \in J^A : i \neq j$ by 1 from above. Let us refer to this
 278 bounded dual problem as $D_k^B(\bar{\mathbf{y}}, \bar{\mathbf{x}}, \bar{\mathbf{I}}, \bar{\mathbf{w}})$. If the optimal value of $D_k^B(\bar{\mathbf{y}}, \bar{\mathbf{x}}, \bar{\mathbf{I}}, \bar{\mathbf{w}})$ is negative valued,
 279 we add the feasibility cut (53) to MP to cut the current solution $(\bar{\mathbf{y}}, \bar{\mathbf{x}}, \bar{\mathbf{I}}, \bar{\mathbf{w}})$.

$$\begin{aligned}
& -\bar{\alpha}w + \sum_{j \in J^A} \beta^k \bar{\delta}_j y_j + \sum_{i \in N^E} \sum_{j \in J^A: i \neq j} \beta^k \bar{\pi}_j x_{ij}^k + \sum_{j \in J^A} e_{j0} \bar{\gamma}_j x_{j0}^k - \sum_{j \in J^A} \beta^k \bar{\gamma}_j x_{0j}^k + \sum_{i \in N^E} \sum_{j \in J^A: i \neq j} e_{ij} \bar{\gamma}_j x_{ij}^k \\
& + \sum_{j \in J^A} \beta^k \bar{\rho}_j x_{0j}^k - \sum_{j \in J^A} e_{j0} \bar{\rho}_j x_{j0}^k - \sum_{i \in N^E} \sum_{j \in J^A: i \neq j} e_{ij} \bar{\rho}_j x_{ij}^k \\
& + \sum_{i \in J^A} \sum_{j \in J^A: i \neq j} \beta^k \bar{\phi}_{ij} x_{ij}^k - \sum_{i \in J^A} \sum_{j \in J^A: i \neq j} e_{ij} \bar{\epsilon}_{ij} x_{ij}^k \geq 0 \tag{53}
\end{aligned}$$

280 **Lemma 4.2.** *If the optimal value of $D_k^B(\bar{\mathbf{y}}, \bar{\mathbf{x}}, \bar{\mathbf{l}}, \bar{\mathbf{w}})$ is negative, then $D_k(\bar{\mathbf{y}}, \bar{\mathbf{x}}, \bar{\mathbf{l}}, \bar{\mathbf{w}})$ is unbounded.*

281 *Proof.* Let ψ be an optimal solution to $D_k^B(\bar{\mathbf{y}}, \bar{\mathbf{x}}, \bar{\mathbf{l}}, \bar{\mathbf{w}})$ and let $g(\psi) < 0$ be the value of this solution.
282 For any positive constant v , $v\psi$ is a feasible solution for $D_k(\bar{\mathbf{y}}, \bar{\mathbf{x}}, \bar{\mathbf{l}}, \bar{\mathbf{w}})$. Then, for an arbitrarily
283 large v , $g(v\psi) = vg(\psi) < 0$ will be an arbitrarily small solution value for $D_k(\bar{\mathbf{y}}, \bar{\mathbf{x}}, \bar{\mathbf{l}}, \bar{\mathbf{w}})$ which
284 implies that $D_k(\bar{\mathbf{y}}, \bar{\mathbf{x}}, \bar{\mathbf{l}}, \bar{\mathbf{w}})$ is unbounded. □

285

286 5. Implementation Details - General Framework

287 The general framework of our algorithm mainly consists of two phases. In Phase I, we solve
288 the problem with at most one visit to each station. This is done by including only one copy of
289 each station in BDA models MP and $D_k(\cdot), k \in K$. We refer to the BDA solving this restricted
290 problem as BDA^1 . In the second phase, we focus on the general problem that allows multiple
291 visits to stations. Between the two phases, we perform an intermediate reduction procedure (See
292 Section 6) to decrease the size of the problem in Phase II. The aim is to cut as much as possible
293 without eliminating any potential solution that is better than the one in Phase I. We provide a
294 brief summary of the general framework in Section 5.3. For clarity of presentation, we use the
295 notation ‘BDA’ throughout the paper to refer to both the algorithm and the formulation.

296 Through our preliminary experiments, we observed that our algorithm has a better convergence
297 behavior if we introduce a high quality initial feasible solution to our master problem. In order to
298 achieve this, we first perform a ‘Step 0’ process where we solve our BDA formulation via a CPLEX
299 option that allows stopping after finding the first integer feasible solution. We also introduce
300 a partial *warm start* solution to CPLEX by opening all potential stations. In our experiments,
301 CPLEX usually finds a solution with all stations opened. We then improve this solution by closing
302 some of the stations. This removal process is a greedy approach based on checking the energy
303 consumption between three consecutive stations and then closing the intermediate one if the battery
304 level is sufficient to go from the first one to the third one. Finally, we introduce the set of open
305 stations of this improved solution as a partial *warm start* solution for our Phase I problem and
306 solve BDA^1 with the valid inequalities given next in Section 5.1.

307 *5.1. Valid Inequalities for Phase I*

308 Let N_{min}^V be a lower bound on the number of vehicles needed for any feasible solution. We can
 309 obtain such a lower bound by solving a bin packing problem (BPP) as follows. Define $s_k = 1$ if
 310 vehicle k is used, 0 otherwise and $a_{ik} = 1$ if the request of customer i is provided by vehicle k ,
 311 otherwise. Constraints (55) assign each customer to a vehicle while Constraints (56) ensure that
 312 these assignments respect the capacities of vehicles.

$$(BPP) \quad N_{min}^V = \min \sum_{k \in K} s_k \quad (54)$$

$$\text{s.t.} \quad \sum_{k \in K} a_{ik} = 1, \quad \forall i \in I \quad (55)$$

$$\sum_{i \in I} d_i a_{ik} \leq Q^k s_k, \quad \forall k \in K \quad (56)$$

$$s_k \in \{0, 1\}, \quad \forall k \in K \quad (57)$$

$$a_{ik} \in \{0, 1\}, \quad \forall i \in N, k \in K. \quad (58)$$

313 We can detect the infeasibility due to insufficient freight capacity by solving BPP. Our preliminary
 314 experiments revealed that introducing Constraint (59), which enforces using at least N_{min}^V vehicles,
 315 usually reduces the solving time. This observation has led us to include this constraint in our
 316 computations for every model of Phase I and Phase II.

$$\sum_{k \in K} \sum_{j \in J^A} x_{0j}^k \geq N_{min}^V \quad (59)$$

317 When we solve BDA to optimality with at most one visit to each station (BDA^1), we include
 318 the following sets of valid inequalities to our master problem:

$$\sum_{k \in K} \sum_{j: (i,j) \in A^k} x_{ij}^k \leq 1, \quad i \in J^A : d_i > 0 \quad (60)$$

$$\sum_{j: (i,j) \in A^k} x_{ij}^k - \sum_{j: (0,j) \in A^k} x_{0j}^k \leq 0, \quad i \in N^E, \forall k \in K \quad (61)$$

$$\sum_{i \in J^A} x_{i0}^k \leq 1, \quad \forall k \in K \quad (62)$$

$$\sum_{j: (j,i) \in A^k} x_{ij}^k \leq y_i, \quad i \in J^A : d_i = 0, \forall k \in K \quad (63)$$

$$y_i \leq \sum_{k \in K} \sum_{j \in N^E} x_{ij}, \quad \forall i \in J^A : d_i = 0 \quad (64)$$

$$y_j \leq \sum_{k \in K} \sum_{i \in N^E} x_{ij}, \quad \forall j \in J^A : d_j = 0 \quad (65)$$

$$w^k \leq \sum_{j \in J^A} \beta^k y_j \quad \forall k \in K. \quad (66)$$

319 Constraints (60) restrict the number of arcs entering a demand node to at most one. Constraints
 320 (61) ensure that an arc is visited by a vehicle only if that vehicle leaves the depot. Constraints
 321 (62) make sure that each vehicle enters the depot at most once. Constraints (63) forbid leaving
 322 a zero-demand copy of a station if it is not open while Constraints (64) and (65) forbid opening
 323 zero-demand station copies if they are not visited by any vehicle. Constraints (66) limit the total
 324 recharging for each vehicle by full battery charging times the number of open stations.

325 Even though most of these constraints are implied by the original constraints, their inclusion
 326 improves the time performance of our algorithm considerably.

327 Let $(\bar{\mathbf{y}}^1, \bar{\mathbf{x}}^1, \bar{\mathbf{I}}^1, \bar{\mathbf{w}}^1)$ be the solution with value Z^1 that we obtain from Phase I. After the
 328 intermediate process which will be explained in Section 6, we proceed to Phase II to solve a
 329 reduced problem via BDA with the valid inequalities of Section 5.2 below. We introduce $\bar{\mathbf{y}}^1$ as a
 330 partial *warm start* solution to the Phase II problem.

331 5.2. Valid Inequalities for Phase II

332 When we apply BDA for the last time with all possible copies of potential stations, in addition
 333 to the valid inequalities (59),(61)-(66), we also introduce the following set of valid inequalities to
 334 break the symmetry between the copies of stations:

$$\sum_{i:(i,j) \in A^k} x_{ij}^k \leq \sum_{i:((j-1),i) \in A^k} x_{(j-1)i}^k, \quad \forall k \in K, j \text{ is the } m^{\text{th}} \text{ copy of some } j_1 : d_{j_1} > 0, m \geq 3 \quad (67)$$

$$\sum_{i:(i,j) \in A^k} x_{ij}^k \leq \sum_{i:((j-1),i) \in A^k} x_{(j-1)i}^k, \quad \forall k \in K, j \text{ is the } m^{\text{th}} \text{ copy of some } j_1 : d_{j_1} = 0, m \geq 2. \quad (68)$$

335 Constraints (67) and (68) make sure that an additional copy of any station is visited by a
 336 vehicle only if the preceding copy is visited by the same vehicle. Exceptionally, the second copy
 337 (the first non-original copy), might be visited by a vehicle not serving the original copy if it is a
 338 demand node.

339 5.3. General framework

340 Below we give a brief summary of the general framework of our algorithm:

341 Step 0: Solve BDA^1 to obtain a feasible solution $(\bar{\mathbf{y}}^0, \bar{\mathbf{x}}^0, \bar{\mathbf{I}}^0, \bar{\mathbf{w}}^0)$ (not necessarily optimal).

342 Close the redundant stations of $(\bar{\mathbf{y}}^0, \bar{\mathbf{x}}^0, \bar{\mathbf{I}}^0, \bar{\mathbf{w}}^0)$ in a greedy manner and obtain $(\bar{\mathbf{y}}, \bar{\mathbf{x}}, \bar{\mathbf{I}}, \bar{\mathbf{w}})$.

343 Step 1: Phase I: Solve $BDA^1 \cup (59)-(66)$ with partial *warm start* \bar{y} to obtain the optimal solution
 344 $(\bar{y}^1, \bar{x}^1, \bar{l}^1, \bar{w}^1)$.

345 Step 2: Apply the intermediate process (see Section 6) to reduce the size of $BDA \cup (59)-(68)$.

346 Step 3: Phase II: Solve the reduced $BDA \cup (59)-(68)$ with partial *warm start* \bar{y}^1 to obtain the
 347 optimal solution.

348 6. Intermediate Reduction Process

349 Creating multiple copies of stations leads to a large-size formulation and excessive solving times.
 350 We develop a two phase method that solves our Benders formulation initially for a single copy of
 351 each station. Based on the value Z^1 of the solution obtained at this stage, we apply an intermediate
 352 processing procedure that checks the availability of a solution with multiple copies of stations that
 353 has a smaller objective value than Z^1 . This is an iterative procedure that proceeds by increasing
 354 the number of copies considered, say m , one by one and applies lower bound checking steps.

355 The aim of this procedure is to check whether there exists a solution of BDA with exactly m
 356 copies for some station j whose cost is lower than Z^1 .

357 **Lemma 6.1.** *Let $Z_{(m,j,k)}^{LB}$ be a lower bound on the cost when exactly m copies of station j is visited
 358 by vehicle k . If $Z_{(m,j,k)}^{LB} \geq Z^1, \forall j, k$, then, there exists no solution with m copies of any station
 359 whose value is less than Z^1 .*

360 *Proof.* Any feasible solution to a minimization problem provides an upper bound. Therefore,
 361 the value of any feasible solution as described in Lemma 6.1 has to be greater than or equal to
 362 $Z_{(m,j,k)}^{LB} \geq Z^1$. □

363 **Lemma 6.2.** *If there exists some lower bound $Z_{(m,j,k)}^{LB}$ such that $Z_{(m',j,k)}^{LB} \geq Z_{(m,j,k)}^{LB}, \forall m' \geq m$ and
 364 if Lemma 6.1 holds for such $Z_{(m,j,k)}^{LB}$ of m for all j, k , then, there exist no solution with more than
 365 or equal m copies of any station whose value is less than Z^1 .*

366 *Proof.* $Z_{(m',j,k)}^{LB} \geq Z_{(m,j,k)}^{LB} \geq Z^1, \forall j, k, m'$ such that $m' \geq m$ by Lemma 6.1. Then, there exists no
 367 solution with m' copies of any station whose value is less than Z^1 . □

368 Below we give the details on how we obtain a lower bound that satisfies Lemma 6.2.

369 Let us consider a potential station j . If we use exactly m copies of this station, it means that
 370 we visit at least $m - 1$ different customers with some vehicle k . This leads to a partial network
 371 structure as $0 \dots j \dots i_1 \dots j \dots i_2 \dots \dots i_{m-1} \dots j \dots 0$.

372 Let E^m and R^m be the amount of energy consumption and the amount of recharging needed,
 373 respectively, when we visit station j exactly m times by some vehicle k_1 . Now, we consider two
 374 cases:

375 Case 1: all customers are visited by k_1 .

376 Case 2: some customers are visited by other vehicle(s).

377 For any of Case 1 or Case 2, the following observation holds:

378 *Observation:*

379 (i) $E^m > E_m^{base} = (m - 1)\beta^{k_1} + e_{j_0}$ and $R^m > R_m^{base} = (m - 2)\beta^{k_1} + e_{j_0}$ if m is even.

380 (ii) $E^m > E_m^{base} = (m - 1)\beta^{k_1}$ and $R^m > R_m^{base} = (m - 2)\beta^{k_1}$ if m is odd.

381 In Figure 1, we illustrate this observation for $m = 2$ and $m = 3$. In this figure, if $E^2 \leq \beta^{k_1} + e_{j_0}$,
 382 we would not need to visit j twice. Similarly, if $E^3 \leq 2\beta^{k_1}$, it would be redundant to visit j three
 383 times.

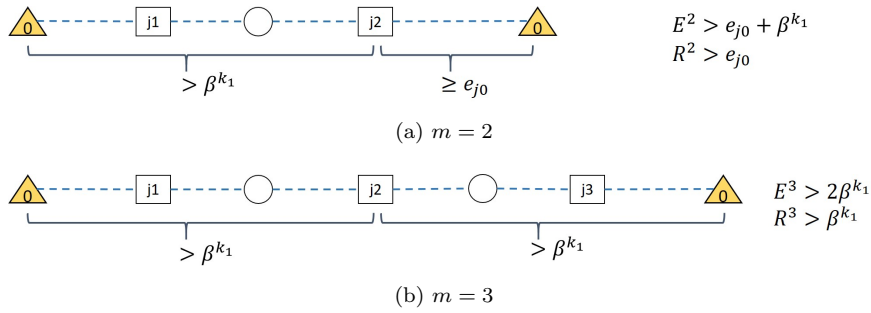


Figure 1: Illustration of minimal energy consumption and recharging need for visiting m copies of j with vehicle k for $m = 2, 3$.

384 We further check whether E^m and R^m values are much larger than E_m^{base} and R_m^{base} , respectively.

385 This is performed as follows:

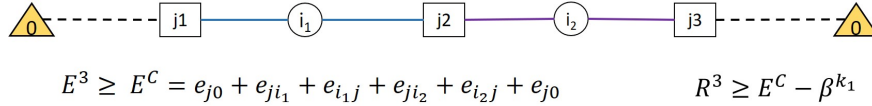


Figure 2: Calculation of minimal energy consumption E_m^C on the partial network for visiting $m = 3$ copies of j with vehicle k_1

386 In Case 1, we calculate the minimal possible energy consumption on such a partial network.
 387 More explicitly, we define $E_m^C = e_{0j} + e_{ji_1} + e_{i_1j} + e_{ji_2} + e_{i_2j} + \dots + e_{ji_{m-1}} + e_{j_0}$ where i_1, \dots, i_{m-1}
 388 are $m - 1$ closest customers to j (e.g. see Figure 2). For this particular case, we can further obtain
 389 a lower bound on the total energy consumption by constructing a 1-tree obtained via a minimum
 390 spanning tree (MST) which spans the union set of all customers and m copies of j and that is
 391 connected to the depot node with two minimal arcs. Let the energy consumption on this 1-tree
 392 be E_m^T and $E^m = \max\{E_m^C, E_m^T, E_m^{base}\}$. We obtain a lower bound C on the total routing cost
 393 similarly. Then, $R = \max\{E^m - \beta^{k_1}, 0, R^m\}$ gives us the amount of recharging needed for this
 394 partial network and $Z^{LB} = R \times r_{k_1} + C + f_j + v_{k_1}$ gives us a lower bound on the cost of routing
 395 all customers by vehicle k via visiting j m times or more.

396 When we look at Case 2, we investigate all possible vehicle combinations that need to be
397 considered by iteratively increasing the number of additional vehicles. If we find a combination
398 with \bar{k} vehicles whose lower bound is less than Z^1 , we do not check the combinations with more
399 than \bar{k} vehicles. Let us assume that in addition to k_1 , we use $K^* = \{k_2, \dots, k_l\}$. This means that
400 we are visiting a different customer by each additional vehicle. Therefore, we add $0 \dots i_{k_h} \dots 0$ as
401 a connected component to our partial network for each vehicle $k_h \in K^*$ where i_{k_h} is the closest
402 customer to depot which is not served by preceding vehicles. Let $E(k_h)$ and $R(k_h)$ denote the
403 total energy consumption and the amount of recharging needed, respectively, for the connected
404 component for vehicle $k_h \in K^*$. In a similar fashion to that of Case 1, we calculate the total
405 energy consumption $E_m^C(K^*) = E^m + \sum_{k_h \in K^*} E(k_h)$, the amount of recharging needed $R_m^C(K^*) =$
406 $R + \sum_{k_h \in K^*} R(k_h)$, the total routing cost and hence a lower bound $Z_{K^*}^{LB1}$ on the total cost with
407 the corresponding vehicle combination.

408 In order to obtain another lower bound $Z_{K^*}^{LB2}$ from the 1-tree constructed with the total energy
409 consumption $E_m^T(K^*)$, this time, we use $\beta = \sum_{k \in K^* \cup k_1} \beta^k$ as the total battery available in our
410 calculation for the amount of recharging needed, that is, $R^m(K^*) = \max\{\max\{E_m^C(K^*), E_m^T(K^*)\} -$
411 $\beta, 0\}$ and $r = \min_{k \in K^* \cup k_1} r_k$ as the unit recharging cost. Our bound $Z_{K^*}^{LB}$ for the corresponding
412 combination is defined as $Z_{K^*}^{LB} = \max\{Z_{K^*}^{LB1}, Z_{K^*}^{LB2}\}$. See Figures 3, 4, and 5 for sample 1-tree
413 constructions for visiting two copies of j with 1, 2, and 3 vehicles, respectively. When \bar{k} vehicles
414 are used, additional $\bar{k} - 1$ copies of the depot are created and added to the set of nodes to find an
415 MST. The 1-tree is then constructed by connecting this MST to the remaining (original) copy of
416 the depot via two minimal arcs. Note that there do not exist any arcs between the copies of the
417 same stations or the depot. The 1-trees constructed in this manner provide a lower bound for the
418 shortest-length Hamiltonian cycles as in Figure 6, which also provide a lower bound on the lengths
of corresponding VRP tours.

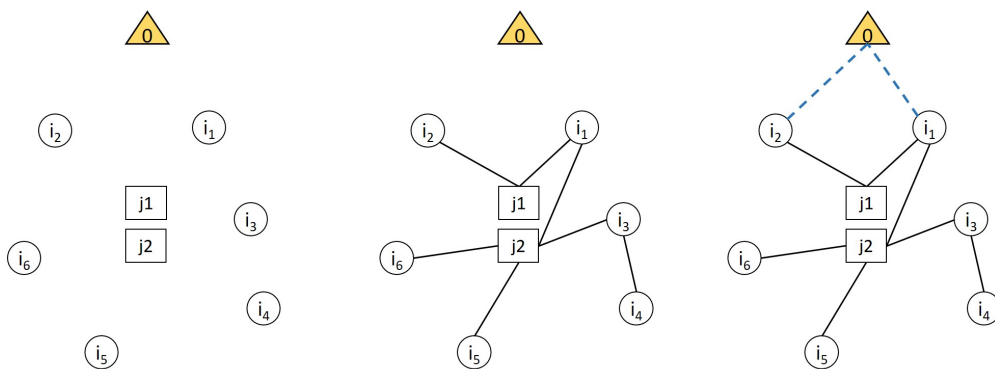


Figure 3: Construction of the 1-tree for obtaining LB2 when visiting $m = 2$ copies of j with one vehicle.

419 If $\min\{Z^{LB}, Z_{K^*}^{LB}\} \geq Z^{m-1}$ for every (k_1, K^*) combination, then, the value of any solution
420 visiting j no less than m times will be no better than Z^{m-1} . So, in further iterations, we do not
421 need more than $m - 1$ copies of j . If this holds for all stations, we can terminate the iterative
422

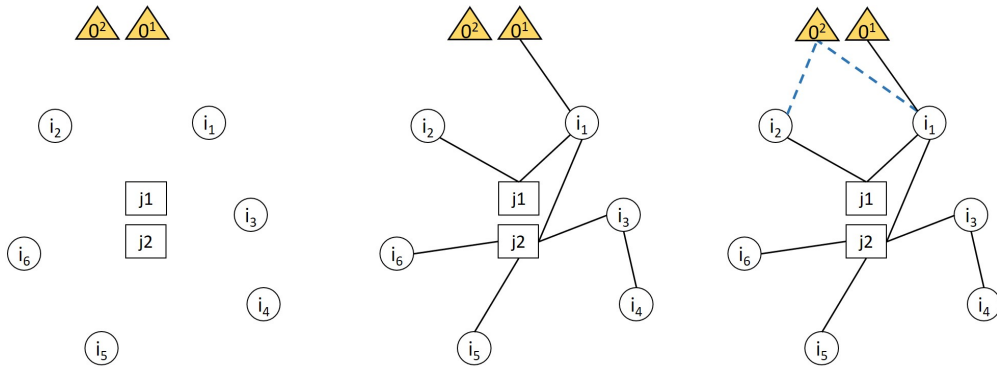


Figure 4: Construction of the 1-tree for obtaining LB2 when visiting $m = 2$ copies of j with two vehicles.

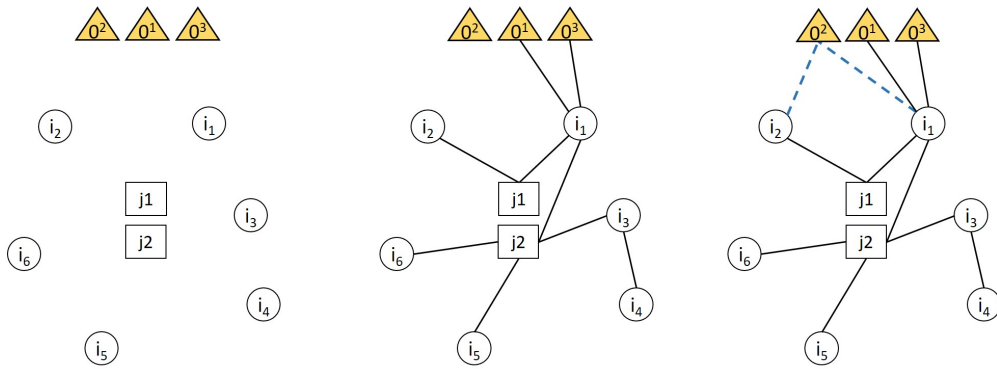


Figure 5: Construction of the 1-tree for obtaining LB2 when visiting $m = 2$ copies of j with three vehicles.

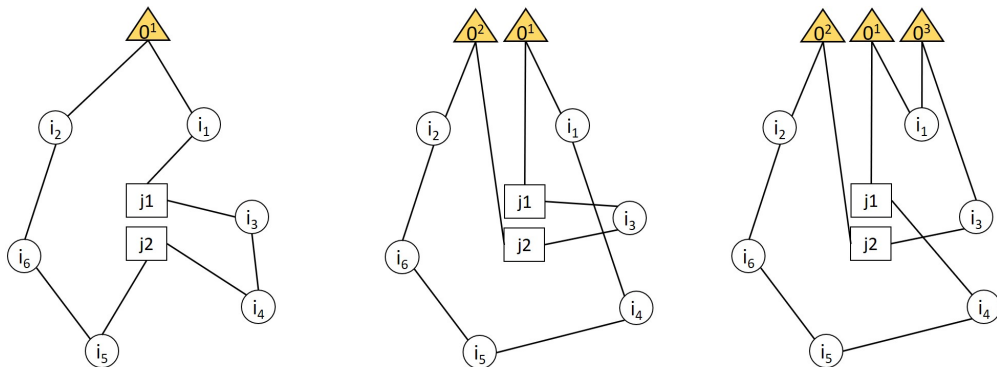


Figure 6: Illustrative Hamiltonian cycles for calculating the 1-tree lower bound LB2 on the cost of visiting $m = 2$ copies of j with 1, 2, or 3 vehicles

423 checking procedure and solve our algorithm *BDA* with at most $m - 1$ copies for each station.

424 *Additional speed-up mechanism:*

425 When solving *BDA* for this last time, we further apply variable fixing by using the information
426 we obtained from this iterative procedure. More explicitly, if it is decided that we do not need
427 more than l copies at a given station j , we fix all y values to zero for all those copies of j . Similarly,
428 if we decided that visiting more than l copies of station j with vehicle k_1 is not optimal, then we
429 set all x variables of those copies to zero for vehicle k_1 .

430 7. Computational Study

431 7.1. Experimental setting and data instances

432 In order to test our methods, we generated problem instances based on the data set provided by
433 Schneider et al. (2014) for EVRP. This data set has 36 different instances with 5, 10, and 15 cus-
434 tomers (12 instances for each customer size). The number of potential stations vary between 7 and
435 22. From these instances, we retrieved the demand and network information (node coordinates).
436 In the remainder of the paper, we will refer to these instances as ‘networks’ for clarity reasons.
437 The vehicle freight capacities are equal to 200 in the original data. We introduced additional levels
438 of capacities (80, 100), especially, to test relatively smaller instances. Similarly, for the battery
439 capacities, we conducted tests for low, medium, and high capacities (10, 16, 22 kWh) to avoid
440 extremely loose values on the tests of small problems (Sassi and Oulamara, 2017).

441 In our experiments, we use IBM ILOG CPLEX 12.8 in a Java environment. We run our tests
442 on a server with Intel(R) Xeon(R) CPU E5-2640 v3 at 2.60 GHz processor and 16 cores. For each
443 experiment, we set a memory limit of 16 GB and a time limit of 3600 seconds for instances with
444 $|I| = 5$ and 10800 seconds for the others. *BDA* and *PF* run using a single thread.

445 We assume that the system will be equipped with fast charging facilities. Note that as there are
446 no time windows or maximum travelling restrictions, introducing slow and fast charging facilities in
447 each potential station would lead to optimal solutions with only cheaper type of charging facilities.
448 As we do not have data on location-dependent fixed station costs for the instance networks, we
449 utilize a single type of stations in our experiments. The lifetime of a charging facility is estimated
450 to be 3 years and it is 5 years for the vehicles. When we calculate the fixed costs of opening
451 stations and purchasing/leasing vehicles, we divide their costs by the number of days within their
452 lifetime. We approximately obtain $f_j = 8 \text{ €}$ as the fixed cost of opening stations and $v_k = 16, 26,$
453 and 36 € as the fixed cost of low, medium, and high capacity vehicles, respectively. Let l_{ij} be the
454 distance between nodes $i, j \in N$; then, we set $c_{ij} = l_{ij} \times 0.03$ (cents/km), $r_k = r = 0.07$ (cents/km)
455 for $k \in K$, $e_{ij} = l_{ij} \times 135$ (Wh/km). In order to better tackle the precision issues of CPLEX, we
456 multiplied all the cost values by 100.

457 We conduct experiments on instances with heterogeneous fleets of 2, 3, and 4 vehicles of three
458 different types, see Table 1. In order to observe the value of using heterogeneous fleets compared to

Table 1: Freight capacity, battery capacity, and cost values for the three vehicle types considered.

Vehicle type	1	2	3
Q^k	80	100	200
β^k	10	16	22
v_k	16	26	36

459 homogeneous fleets, we also conduct experiments with homogeneous fleets of 1, 2, 3, and 4 vehicles
 460 for each vehicle type. For each network, we test the problem with heterogeneous fleets shown
 461 in Table 2 as well as the homogeneous fleets generated. Certain network-fleet combinations are
 infeasible and we exclude them from our computational analysis.

Table 2: Heterogeneous fleets tested for each network of Schneider et al. (2014).

Fleet ID	Fleet size	Vehicle types available	Fleet ID	Fleet size	Vehicle types available
K2V1-2	2	1,2	K4V1-1-1-2	4	1,1,1,2
K2V1-3	2	1,3	K4V1-1-1-3	4	1,1,1,3
K2V2-3	2	2,3	K4V1-1-2-2	4	1,1,2,2
K3V1-1-2	3	1,1,2	K4V1-2-2-2	4	1,2,2,2
K3V1-1-3	3	1,1,3	K4V1-1-3-3	4	1,1,3,3
K3V1-2-2	3	1,2,2	K4V1-1-2-3	4	1,1,2,3
K3V1-2-3	3	1,2,3	K4V1-2-2-3	4	1,2,2,3
K3V1-3-3	3	1,3,3	K4V1-2-3-3	4	1,2,3,3
K3V2-2-3	3	2,3,3			

462

463 7.2. Analysis

464 We perform two types of analysis: (1) performance analysis of the model and the algorithm in
 465 Section 7.2.1 and (2) managerial insights for selecting the fleet types in Section 7.2.2.

466 For simplicity, we provide average results for each network type in Table 3 but the detailed
 467 results can be found in the supplementary document. For illustration, we also display the results
 468 in Figures 7-11.

469 In these figures and tables, N^S and N^V show the number of stations opened and the number
 470 of vehicles used, respectively. The value ‘ g ’ represents the gap provided by CPLEX at the end
 471 of the time limit (0.00 if the problem is solved to proven optimality) for the corresponding model
 472 solved. Similarly, ‘ $t(s)$ ’ represents the total time spent in seconds for the corresponding model or
 473 algorithm if it includes additional processes. The value of the best solution obtained from a model
 474 is given under ‘Obj’. The selected vehicle types are indicated with their Q^k values. For example,
 475 [80,100] indicates that the corresponding solution selects one vehicle of type 1 and one vehicle of
 476 type 2.

477 7.2.1. Methodological observations and insights

478 Among 204 instances of $|I| = 5$ ($|N| \in \{8, 9\}$), our formulation PF can solve all but ten instances
 479 to optimality within one hour. The average time spent by PF on these instances is 523.93 seconds,

Table 3: Average solution, gap, and solving time values for BDA and PF.

Net. ID	$ I $	$ J $	BDA Phase I			BDA Phase II			N^S	N^V	PF		
			Obj	g	$t(s)$	Obj	g	$t(s)$			Obj	g	$t(s)$
c101C5	7	7127.48	0.00	1.32	7127.48	0.00	1.51	0.41	2.12	7127.48	0.00	446.07	
c103C5	7	4259.00	0.00	1.14	4259.00	0.00	1.42	0.00	1.35	4259.00	0.00	170.53	
c206C5	8	8553.95	0.00	2.55	8553.95	0.00	2.56	0.29	2.35	8553.96	0.00	819.52	
c208C5	7	5713.57	0.00	0.83	5713.57	0.00	1.03	0.18	1.35	5713.57	0.00	47.77	
r104C5	7	4214.04	0.00	1.09	4214.04	0.00	1.20	0.00	1.35	4214.04	0.00	182.98	
r105C5	7	4233.90	0.00	0.87	4233.90	0.00	1.01	0.00	1.35	4233.90	0.00	72.56	
r202C5	7	3948.66	0.00	0.92	3948.66	0.00	1.03	0.00	1.59	3948.66	0.00	27.56	
r203C5	8	6675.67	0.00	2.31	6675.67	0.00	2.28	0.06	2.12	6675.67	0.00	494.06	
rc105C5	8	7421.29	0.00	2.59	7421.29	0.00	2.80	0.06	2.12	7421.28	0.02	1077.36	
rc108C5	8	20738.84	0.00	1.97	20738.84	0.00	2.68	0.65	2.47	20738.84	0.06	815.56	
rc204C5	8	5219.75	0.00	2.61	5219.75	0.00	2.88	0.00	2.00	5219.82	0.05	1548.00	
rc208C5	7	5860.37	0.00	0.62	5860.37	0.00	0.77	0.18	1.35	5860.37	0.06	585.22	
Average			0.00	1.57		0.00	1.76	0.15	1.79		0.02	523.93	
c101e10	14	18721.65	0.00	346.81	17880.34	0.00	259.39	0.81	2.88				
c104e10	13	20193.80	0.00	259.55	20146.89	0.00	235.76	0.82	2.82				
c202e10	14	8208.00	0.00	164.15	8208.00	0.00	215.44	0.56	2.38				
c205e10	12	14160.13	0.00	182.23	14160.13	0.00	374.76	0.59	2.47				
r102e10	13	6936.08	0.00	22.18	6936.08	0.00	20.87	0.06	2.12				
r103e10	12	4247.15	0.00	7.56	4247.15	0.00	9.23	0.00	1.35				
r201e10	13	5895.34	0.00	237.09	5895.34	0.00	215.06	0.00	2.13				
r203e10	14	9780.68	0.00	151.44	9780.68	0.00	189.97	0.24	2.53				
rc102e10	13	27920.23	0.00	28.69	26254.23	0.00	32.92	1.31	3.00				
rc108e10	13	36706.26	0.00	320.01	36133.71	0.00	735.61	1.18	2.94				
rc201e10	13	14250.35	0.00	368.28	14250.35	0.00	352.18	0.47	2.59				
rc205e10	13	32217.30	0.00	175.72	31550.95	0.00	840.48	1.35	2.82				
Average			0.00	188.53		0.00	286.73	0.62	2.50				
c103e15	19	16455.88	0.07	5622.99	15546.37	0.06	4679.68	0.81	2.88				
c106e15	17	8316.83	0.01	1606.37	8316.83	0.01	1698.20	0.06	2.29				
c202e15	19	33255.84	0.09	4389.71	30922.17	0.06	4295.05	1.31	3.00				
c208e15	18	16909.28	0.08	4809.75	16396.69	0.06	4625.89	0.81	2.94				
r102e15	22	9356.76	0.03	3650.76	9356.76	0.04	4458.27	0.38	2.56				
r105e15	20	9618.99	0.07	5267.99	9618.99	0.07	5524.17	0.25	2.63				
r202e15	20	17991.78	0.11	4650.14	15944.78	0.09	5091.58	1.20	2.87				
r209e15	19	10064.97	0.02	1237.37	9981.06	0.00	1050.26	0.31	2.56				
rc103e15	19	30891.86	0.17	6556.05	30072.29	0.22	7016.98	1.38	3.00				
rc108e15	19	44373.32	0.16	4815.57	41028.85	0.28	5865.23	1.60	3.00				
rc202e15	19	33912.00	0.16	6017.79	34087.56	0.18	7061.24	1.44	3.25				
rc204e15	21	28632.75	0.22	8130.34	19652.83	0.17	9054.50	1.46	3.31				
Average			0.10	4658.65		0.10	4833.38	0.90	2.85				

480 which is much larger compared to that of BDA. On the other hand, we observe that our algorithm
 481 BDA can solve all the instances with $|I| = 5$ to optimality requiring a few seconds only. In fact,
 482 BDA is relatively faster than PF in every single instance.

483 For larger problems ($|I| = 10, 15$), optimality could not be guaranteed with PF within the
 484 time limit. In fact, PF could not even find a feasible solution for many instances. Based on these
 485 observations, for larger problems, we present the results for BDA only.

486 For the networks with 10 customers and up to 14 potential stations ($|N| \in \{13, 15\}$), BDA
 487 solves all Phase I instances to optimality in 188.53 seconds on average. The algorithm reaches
 488 either the time or memory limit before proven optimality in Phase II for 6 out of 200 instances
 489 in this category. These are instances either with fleet type K4V1-1-1-2 or K4V1-1-2-2, which are
 490 challenging also for larger instances with 15 customers, as observed in Figure 7. Figure 7 compares
 491 the average solving times (Phase I, Phase II, and Phase I+Phase II) and gaps across fleet types.
 492 In this figure, Phase I gaps and times are better indicators for challenging instances as Phase II
 493 gaps and solving times are not available due to memory limit for some instances. BDA is able to
 494 solve 65% of the instances with $|I| = 15$ ($|N| \in \{18, 19, 20, 21, 22, 23\}$) to proven optimality within
 the time limits. The average gap is 0.10.

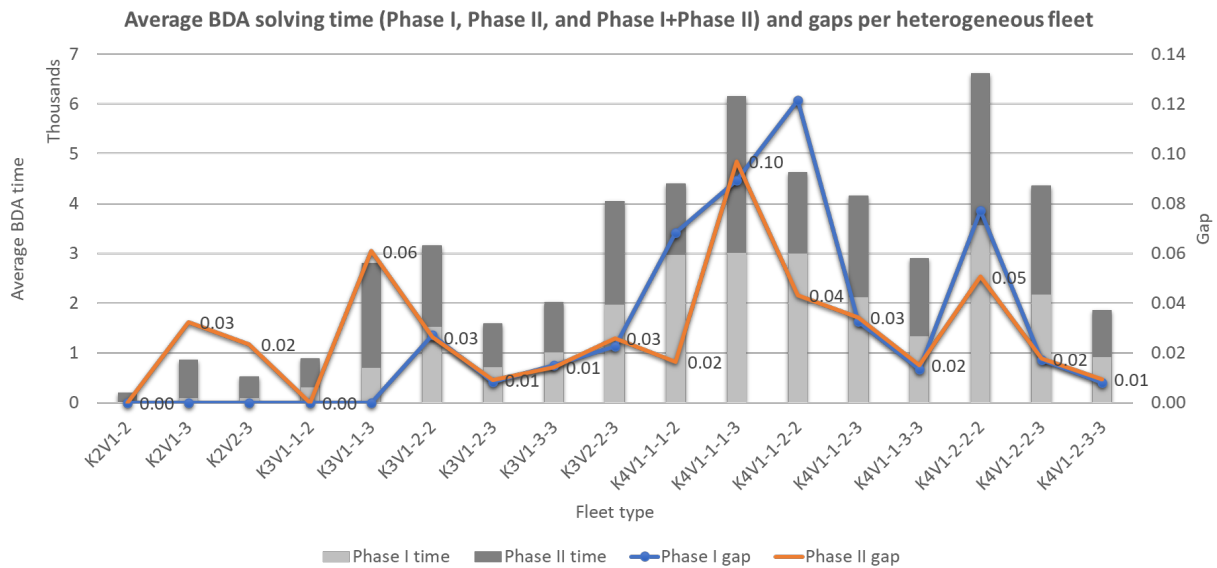


Figure 7: Average BDA solving time (Phase I, Phase II, and Phase I+Phase II) and gaps per heterogeneous fleet.

495
 496 A common pattern we observe with the instances reaching the memory limit is that (1) their
 497 fleets consist of four vehicles with either one type 3 vehicle and three type 1 vehicles or only type
 498 1 and type 2 vehicles and (2) Phase I solution uses all four vehicles. The intermediate processing
 499 procedure cannot reduce the Phase II problem sufficiently due to high level of degeneracy and
 500 low-cost and low-capacity vehicles in the fleet.

501 Overall, the algorithm is able to provide very high quality solutions within three hours. It

502 solves 88% of all instances to optimality. The average gap among all instances is 0.03 only.

503 For several instances, Phase I and Phase II costs are not identical. We interpret this as follows:

504 • If Phase II problem does not reach proven optimality within the time limit and Phase I
505 cost is less than Phase II cost, this indicates the benefit of solving a restricted version when
506 the original problem is too difficult to solve within the available time and computational
507 resources.

508 • If neither Phase I nor Phase II problems reach proven optimality within the time limit and
509 Phase I cost is less than Phase II cost, either a solution using multiple copies of a station is
510 found in Phase II, which is not feasible for Phase I problem, or Phase I solution helps Phase
511 II problem in finding a better-quality solution faster.

512 Below are some further observations and future research directions for developing algorithms
513 with improved performance.

514 • The algorithm needs more time to reach proven optimality as the size of the fleet increases.
515 This is often because the dual bound is too weak and the majority of the time is spent
516 for closing the gap. The dual bounds can be strengthened by using good valid inequalities.
517 However, valid inequalities might also make the formulation heavier and more difficult to find
518 feasible solutions. Therefore, it is in general more efficient to decompose the problem into
519 smaller problems with smaller fleet configurations and solve them iteratively by updating the
520 fleet configurations at each step. Obviously, there will be a trade-off between the number
521 of small fleet configurations and the size of each configuration as in most decomposition
522 methods.

523 • The focus in this paper was on heterogeneous-fleet problems with a limited number of vehicles.
524 For solving instances with homogeneous fleets, the efficiency of the algorithm can be improved
525 by updating the intermediate process. A similar improvement-procedure update would also
526 be helpful in solving the instances with an unlimited number of vehicles of each type.

527 *7.2.2. Managerial insights*

528 Figure 8 compares the average cost, number of stations opened and the number of vehicles
529 used for each given heterogeneous fleet. As expected, the average cost in general decreases as the
530 fleet size increases. In this Figure, most significant drops in cost occur when the fleet has a type-3
531 vehicle and another vehicle of type 2 or type 3. These are also the instances where the optimal or
532 best known solutions open fewer number of stations. The highest number of stations are opened
533 with fleets K2V1-2, K3V1-1-2, and K4V1-1-1-2 which have relatively smaller total freight capacity
534 and total range. Thus, solutions to these instances also use more vehicles on average.

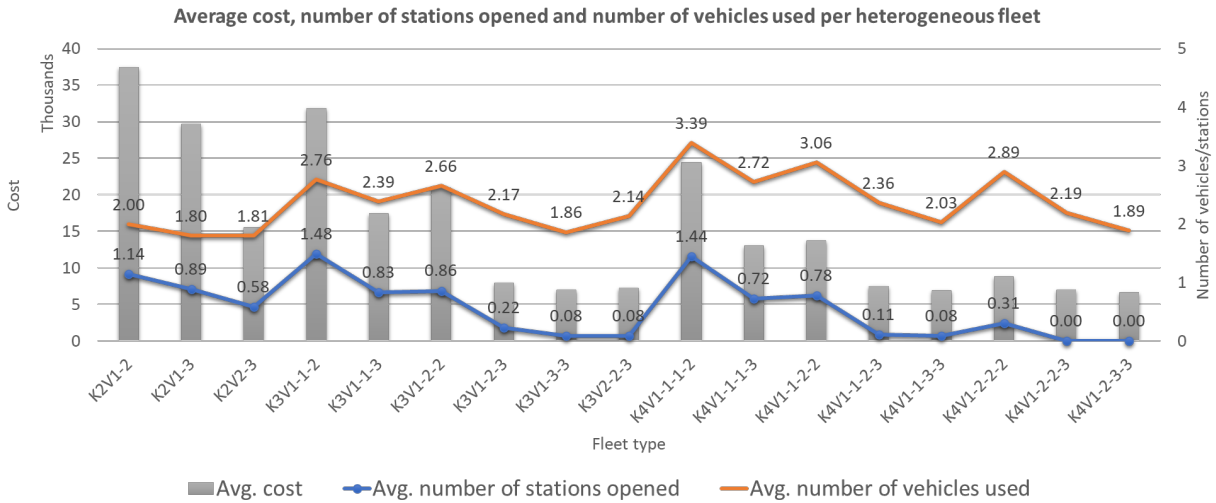


Figure 8: Average cost, number of stations opened and number of vehicles used for each heterogeneous fleet type.

535 It is, indeed, interesting to note that small vehicles result in higher costs, an observation which
 536 is not that obvious. This is because they require potentially more charging and more stations to
 537 be opened which incur very high costs.

538 Figure 9 shows the results of an analysis from another perspective where we calculate the
 539 averages over all fleet types for each network. In this figure, we can clearly observe that the
 540 number of stations needed changes a lot depending on the network type. This is then reflected
 541 in the cost. We also observe that the average cost for a smaller-network instance, for example,
 542 rc108c5, can be much higher than the cost for a larger-network instance, for example, c106c15.

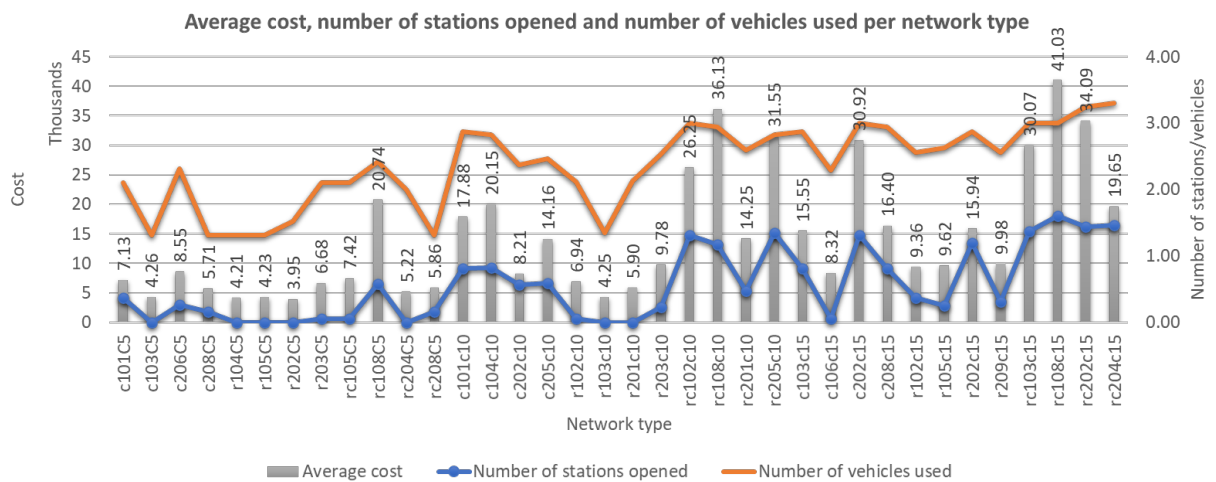


Figure 9: Average cost, number of stations opened and number of vehicles used for each network type (heterogeneous fleets).

543 Moreover, we compare the average cost, number of opened stations and vehicles used with

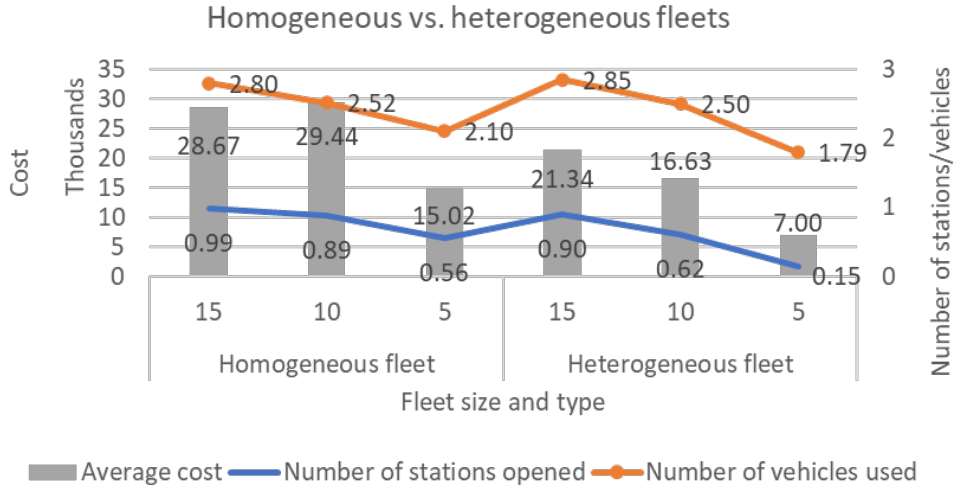


Figure 10: A comparison of homogeneous and heterogeneous fleet: average cost, number of opened stations and vehicles used for each network size.

544 homogeneous and heterogeneous fleets of at least two vehicles in Figure 10. We observe that the
 545 average cost is much higher when using homogeneous fleets compared to heterogeneous ones, it
 546 is indeed twice as much for instances with $|I| = 5$. The average number of stations opened with
 547 homogeneous fleets is also larger for each network group. And in general, a similar conclusion can
 548 be made for the average number of vehicles used.

549 In Figure 11, we also show the average cost per homogeneous fleet, including the fleets with a
 550 single vehicle. Similar to previous observations, the average cost is lower when the fleet contains
 551 larger vehicles.

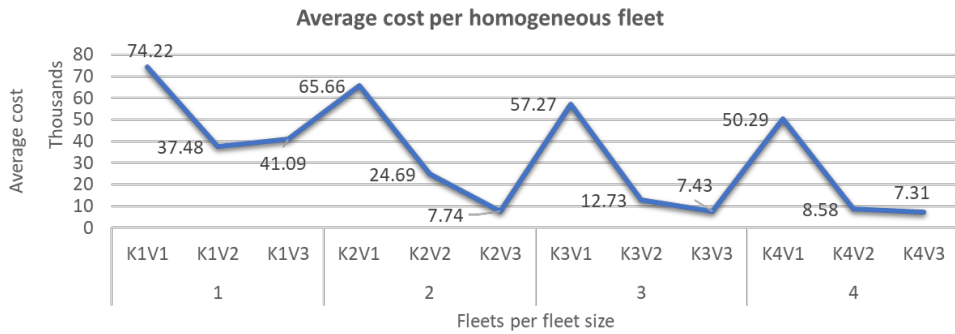


Figure 11: Average cost for each homogeneous fleet.

552 Below we provide some further key observations:

- 553 • For the majority of the instances where $|I| = 5$ and all types of vehicles are present in the
 554 heterogeneous fleet, medium-size vehicles (type 2) are not selected in the optimal solutions.

555 This is not the case with larger networks.

- 556 • When the fleets are homogeneous, no instance uses four vehicles of type 3 and only four
557 instances, two from networks rc202c15 and rc204c15 each, use three type 3 vehicles.
- 558 • When solving the instances with fleets of at least two vehicles, we observe that the optimal
559 solutions serving all the demand via a single vehicle only uses the largest vehicle type (type
560 3). Although there exist several instances where it is feasible to serve all demand with a
561 single vehicle of type 1 or type 2, such solutions are suboptimal and solutions with lower cost
562 can be obtained using multiple vehicles.

563 8. Conclusion and Future Research Directions

564 In this paper, we introduce an electric location-routing problem with heterogeneous fleet and
565 partial recharging. We initially propose a new mixed integer programming formulation for this
566 problem. This is a formulation with three-index binary routing variables where the sub-tour elimi-
567 nation is enhanced via a group of load (flow) preservation constraints. We further utilize additional
568 non-negative continuous variables to satisfy battery restrictions and energy-related constraints.

569 We test our formulation on small problem instances from the literature. Although the formu-
570 lation is able to solve instances with 5 customers and up to 8 potential stations to optimality, we
571 observe that its performance is limited when it comes to solving larger problems.

572 As we aim to solve this problem to optimality, we further develop a two-phase algorithm based
573 on the Benders decomposition of our formulation. The first phase solves a restricted version of
574 the problem that allows at most one visit to each station. By using the information obtained, the
575 second phase problem, which is the generalized problem allowing multiple visits to any station,
576 is reduced in size, making it relatively easier compared to the case with no a priori processing.
577 This enhancement step allows us to solve 88% of all the instances with up to 15 customers and 22
578 potential stations to optimality. The average optimality gap over all other instances is negligible,
579 just 0.03. In summary, our approach obtains very high quality solutions within the time limit.

580 We observe through our experimental study that the problem is usually harder to solve when
581 the vehicle capacities are smaller. We also found that using small vehicles results in higher costs.

582 Though the main focus of this study is to present an exact method with proven optimality,
583 this approach can be easily combined with additional procedures leading to powerful matheuristics
584 to obtain near optimal solutions for larger instances, see Salhi (2017). This problem can also
585 be tackled by powerful metaheuristics whose performance can be evaluated using lower bounds
586 obtained from the proposed method.

587 The current problem can be extended to cater for several deterministic and stochastic variants
588 that are worth exploring. These include the consideration of time windows, multiple depots and/or
589 additional location decisions for the selection of depots, as well as periodicity or uncertainty in the

590 customer demand. Moreover, the model and the algorithm we propose in this paper can be easily
591 modified to solve the problem variants where vehicle-dependent energy consumption and routing
592 costs are considered.

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