



# Homogeneity and Isotropy of the Taylor-Green Vortex: The Fate of Kolmogorov and Yaglom/Monin Relations

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Kolmogorov “K41” “law” reads

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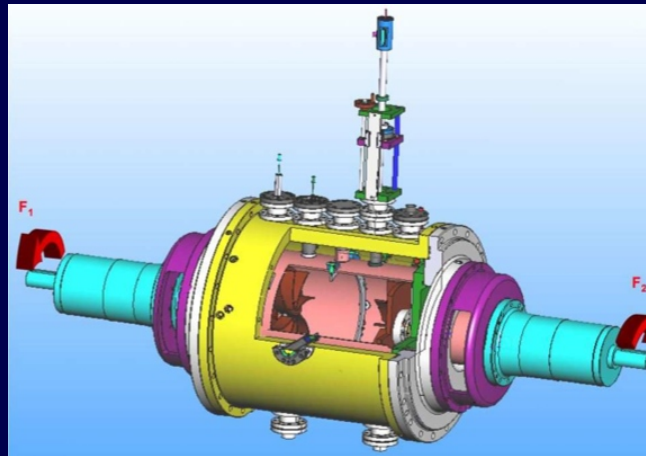
Even in the presence of a global non-homogeneous flow?

And then, what is the meaning of this  $\langle . . . \rangle$  averaging operation?



# Homogeneous and isotropic turbulence ?

The von Kármán swirling flow  
aka the “French Washing Machine”



Could Kolmogorov's law be valid in this flow ?



# Derivation of the Kolmogorov relation

Take the difference of Navier-Stokes equations

$$\partial_t \mathbf{u} + \mathbf{u} \cdot \nabla_x \mathbf{u} = \frac{-1}{\rho} \nabla_x P + \nu \nabla_x^2 \mathbf{u} + \mathbf{f}$$

at two points  $\mathbf{x}$  and  $\mathbf{x}' \equiv \mathbf{x} + \mathbf{r}$  to get an equation

for the velocity difference  $\Delta \mathbf{u}(\mathbf{r}; \mathbf{x}, t) \equiv \mathbf{u}(\mathbf{x} + \mathbf{r}, t) - \mathbf{u}(\mathbf{x}, t)$





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so that in stationary conditions (or time-averaged), we have

$$\nabla_{\mathbf{r}} \cdot \langle |\Delta \mathbf{u}|^2 \Delta \mathbf{u} \rangle = -4 \varepsilon + 2 \nu \nabla_{\mathbf{r}}^2 \langle |\Delta \mathbf{u}|^2 \rangle \quad \varepsilon \equiv - \frac{dE}{dt} = \nu \langle |\nabla \mathbf{u}|^2 \rangle$$



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and using isotropy again  $\langle [\Delta u_i]^3 \rangle = -4/5 \varepsilon r_i + 6\nu \partial_i \langle [\Delta u_i]^2 \rangle$



# "Exact" conservation laws

$$\left. \begin{array}{l} 4/5 \varepsilon r_i \\ 4/3 \varepsilon r \\ 4\varepsilon \end{array} \right\} = \left\{ \begin{array}{l} -\langle [\Delta u_i]^3 \rangle \\ -\langle |\Delta \mathbf{u}|^2 \Delta \mathbf{u} \rangle \\ -\nabla_r \cdot \langle |\Delta \mathbf{u}|^2 \Delta \mathbf{u} \rangle \end{array} \right\} + \left\{ \begin{array}{l} 6\nu \partial_i \langle [\Delta u_i]^2 \rangle \\ 2\nu \nabla_r \langle |\Delta \mathbf{u}|^2 \rangle \\ 2\nu \nabla_r^2 \langle |\Delta \mathbf{u}|^2 \rangle \end{array} \right\} \quad \begin{array}{l} (1d1c) \\ (1d3c) \\ (3d3c) \end{array}$$

Total dissipation  
at scale  $r$

Transfer  
to scales  $< r$

Viscous dissipation  
at scale  $r$

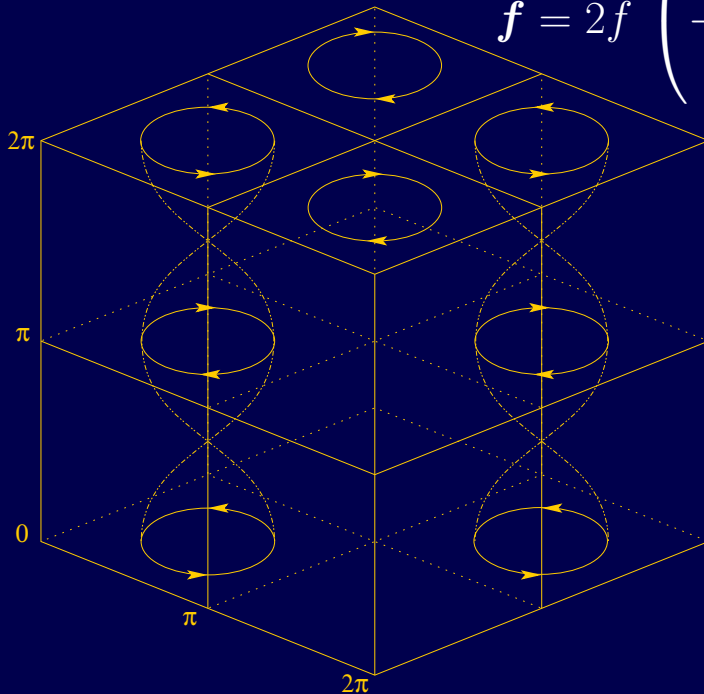
- Scale by scale energy conservation  $\Rightarrow$  no pileup of energy
- Increasing generality (Monin's law valid for anisotropic flows)
- Increasing experimental difficulty
- **Problem**:  $\lim_{\nu \rightarrow 0} \dots 0 = 0 ??? \dots$  or better?



# The Taylor-Green vortex

$$\mathbf{f} = 2f \begin{pmatrix} \sin(k_0x) \cos(k_0y) \cos(k_0z) \\ -\cos(k_0x) \sin(k_0y) \cos(k_0z) \\ 0 \end{pmatrix}$$

(here,  $k_0 = 1$ )



# Flow simulation parameters

## Pseudo-spectral DNS simulations

$$N^3 \Leftarrow 128^3$$

$$L \equiv 2\pi \quad \Lambda = 2.589 \quad \lambda = 0.487 \quad \eta = 0.035 \quad \Delta x \Leftarrow 0.049$$

$$f \Leftarrow 1.5 \quad \mathbf{U}_{\text{rms}} = 2.690 \quad \langle |\nabla \mathbf{U}|^2 \rangle = 150.5$$

$$\nu \Leftarrow 0.015 \quad \text{Re} = 440 \quad \text{Re}_\lambda = 85 \quad \varepsilon = 2.257$$

$$\Delta t \Leftarrow 1 \quad T_{\text{nl}} = 0.961 \quad dt \Leftarrow 10^{-3}$$

$10^3 \Delta t + 2 \times 10^3$  snapshots of the total velocity field  $\mathbf{U}(x, y, z, t)$   
at equispaced **uncorrelated** time intervals  $\Delta t$

$\Rightarrow$  100 GB of data to analyze !



# Space and time averages

Checks of homogeneity  $\implies$  averages over limited regions of space

Average on horizontal planes to check homogeneity in the  $z$  direction

$$\hat{\mathbf{a}}(\dots; z, t) \equiv \frac{1}{L^2} \int dy \int dx \mathbf{a}(\dots; x, y, z, t)$$





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Space average  $\quad \widetilde{\mathbf{a}}(\dots; t) \equiv \frac{1}{L^3} \int dz \int dy \int dx \mathbf{a}(\dots; x, y, z, t)$

Time average  $\quad \overline{\mathbf{a}}(\dots; x, y, z) \equiv \frac{1}{T} \int dt \mathbf{a}(\dots; x, y, z, t)$

Global average  $\quad \langle \mathbf{a} \rangle (\dots) \equiv \overline{\widetilde{\mathbf{a}}} \quad \text{or} \quad \langle \mathbf{a} \rangle (\dots; z) \equiv \overline{\widehat{\mathbf{a}}}$



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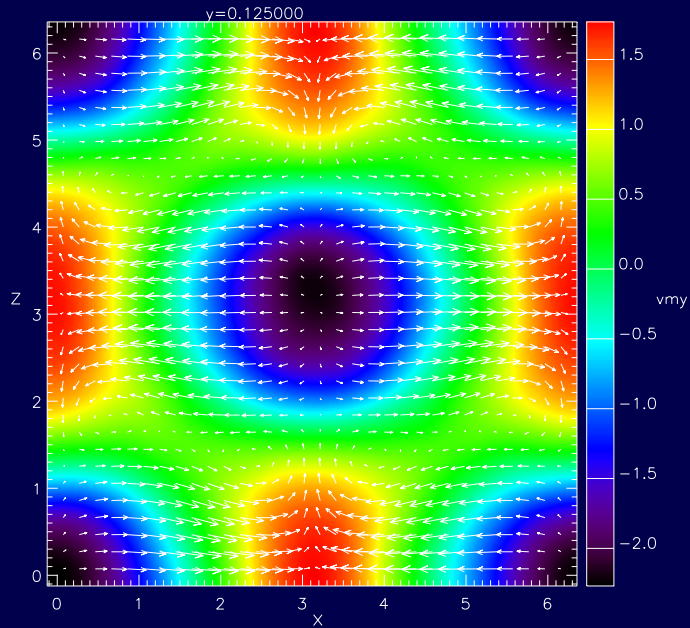
Global average  $\langle \mathbf{a} \rangle (\dots) \equiv \overline{\tilde{\mathbf{a}}}$  or  $\langle \mathbf{a} \rangle (\dots; z) \equiv \overline{\hat{\mathbf{a}}}$

Velocity  $\mathbf{U}(x, y, z, t) \equiv \overline{\mathbf{U}}(x, y, z) + \mathbf{u}(x, y, z, t)$

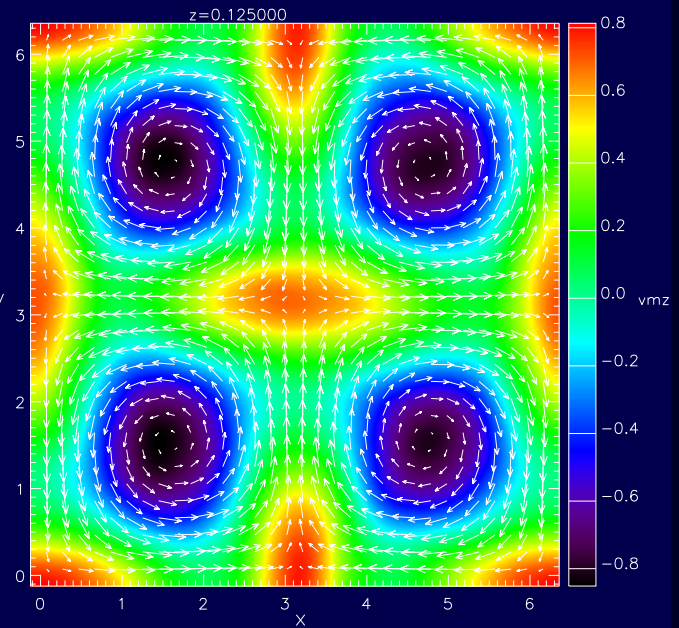
$$(U, V, W)^T(x, y, z, t) \equiv (\overline{U}, \overline{V}, \overline{W})^T(x, y, z) + (u, v, w)^T(x, y, z, t)$$



# The mean flow



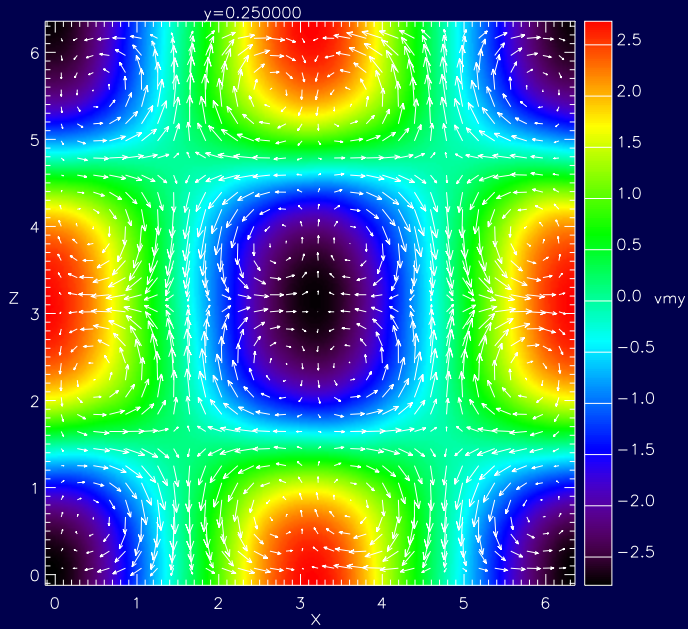
$$y = \pi/4$$



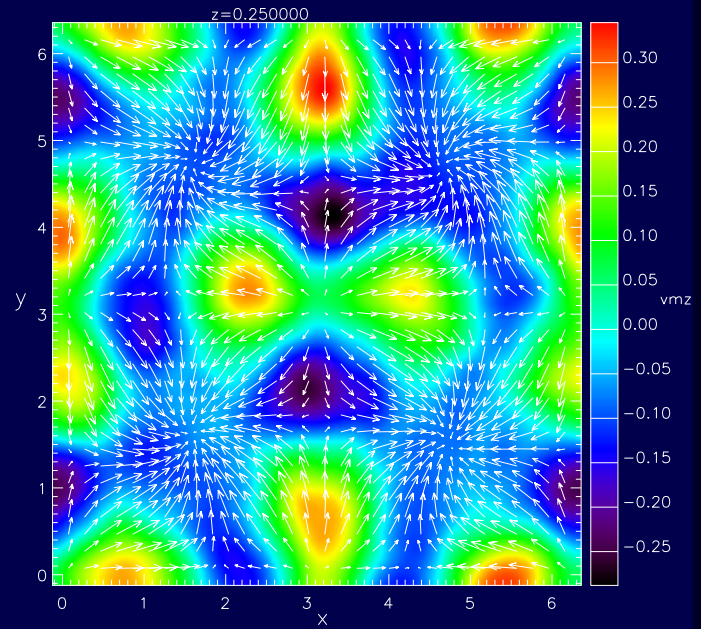
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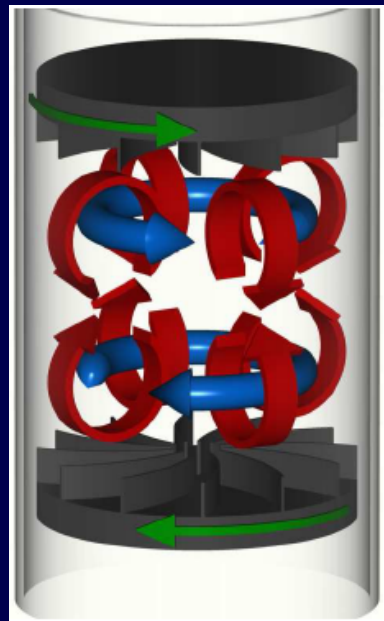
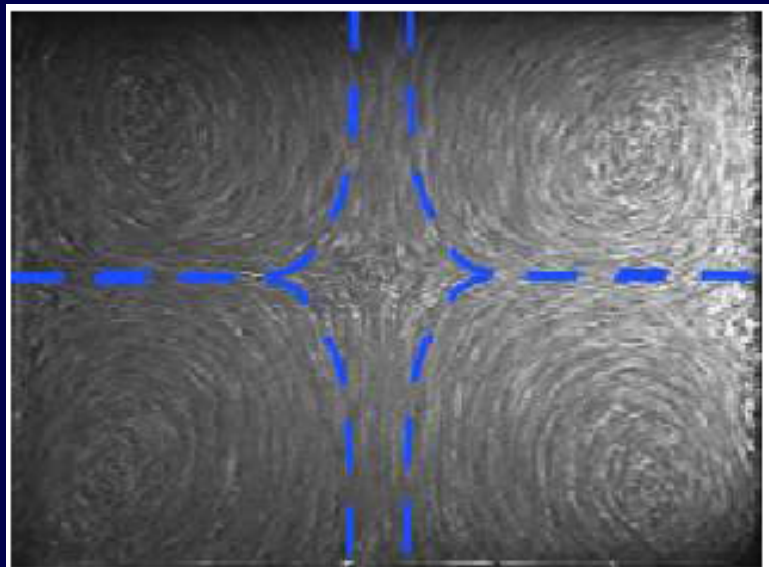
$$y = \pi/2$$



$$z = \pi/2$$



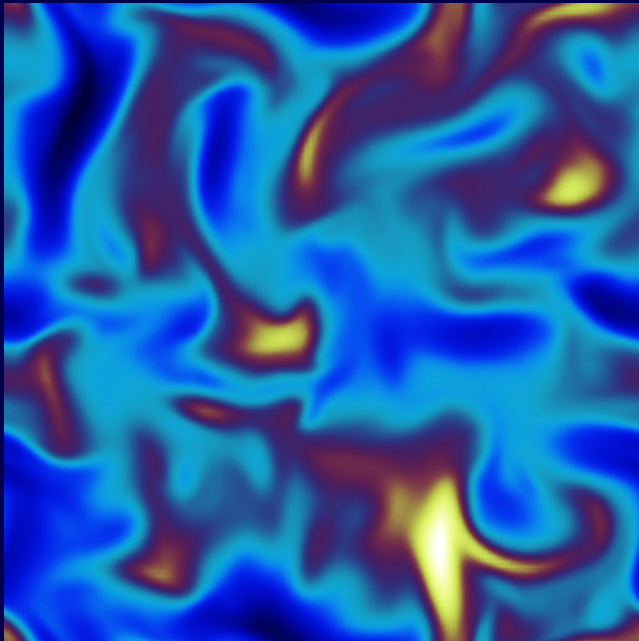
## Analogy with von Kármán flow



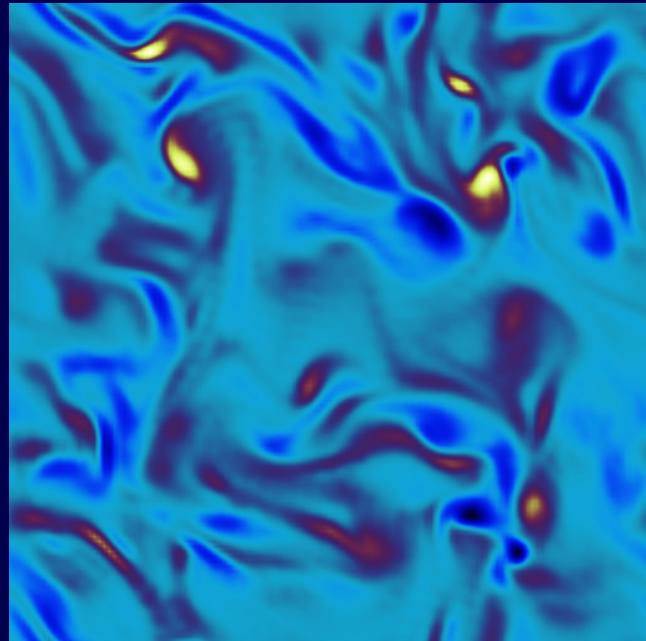
The mean flow of the Taylor-Green vortex reproduces very well the  $s2t2$  structure of the von Kármán swirling flow



# But this looks like turbulence



Vertical velocity  $U_z(x, y, \pi/2)$



Vertical vorticity  $(\nabla \times \mathbf{U})_z(x, y, \pi/2)$

at a single time  $t = 10^3$





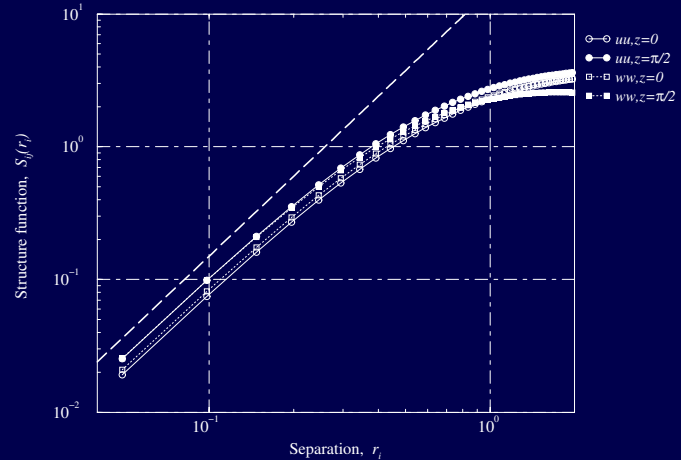
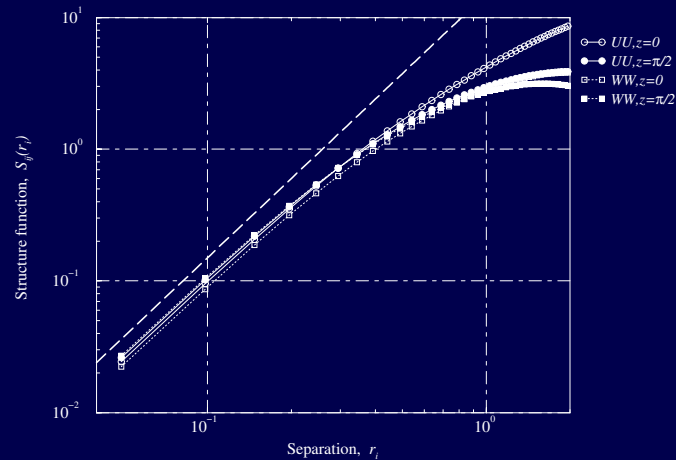
# Local statistical quantities

Total				Fluctuations			
$\langle U_i U_j \rangle$	Global	$z = 0$	$z = \pi/2$	$\langle u_i u_j \rangle$	Global	$z = 0$	$z = \pi/2$
$\langle UU \rangle$	2.875	3.590	1.988	$\langle uu \rangle$	1.766	1.668	1.904
$\langle UV \rangle$	-0.027	-0.059	-0.012	$\langle uv \rangle$	-0.007	-0.006	-0.007
$\langle UW \rangle$	-0.003	-0.007	0.001	$\langle uw \rangle$	-0.001	-0.001	0.002
$\langle VU \rangle$	-0.027	-0.059	-0.012	$\langle vu \rangle$	-0.007	-0.006	-0.007
$\langle VV \rangle$	2.851	3.549	1.964	$\langle vv \rangle$	1.752	1.628	1.880
$\langle VW \rangle$	0.001	0.008	0.003	$\langle vw \rangle$	-0.001	0.003	0.004
$\langle WU \rangle$	-0.003	-0.007	0.001	$\langle wu \rangle$	-0.001	-0.001	0.002
$\langle WV \rangle$	0.001	0.008	0.003	$\langle wv \rangle$	-0.001	0.003	0.004
$\langle WW \rangle$	1.510	1.239	1.606	$\langle ww \rangle$	1.428	1.238	1.594
$\langle U^2 \rangle$	7.235	8.377	5.559	$\langle u^2 \rangle$	4.946	4.534	5.378

- Velocity field is neither homogeneous, nor locally or globally isotropic, even in its fluctuations
- The fluctuations energy accounts for  $\approx 70\%$  of the total energy
- The mean  $\overline{U}$  and the fluctuations  $u$  are approximately uncorrelated, but their amplitudes  $\langle \overline{U}^2 \rangle$  and  $\langle u^2 \rangle$  are anticorrelated



# Second order structure functions



Longitudinal 2<sup>nd</sup> order structure functions  $S_{ii}(r_i) \equiv \langle \Delta U_i(r_i) \Delta U_i(r_i) \rangle$  are **regular** at small scales, i.e.  $S_{ii}(r_i) \propto r_i^2$

but depend on position  $z$  and orientation  $i$

and show **no** inertial range  $S_{ii}(r_i) \propto r_i^{2/3}$



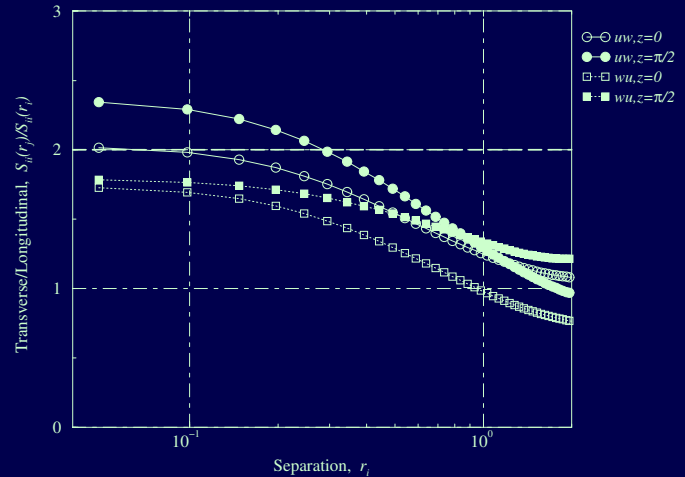
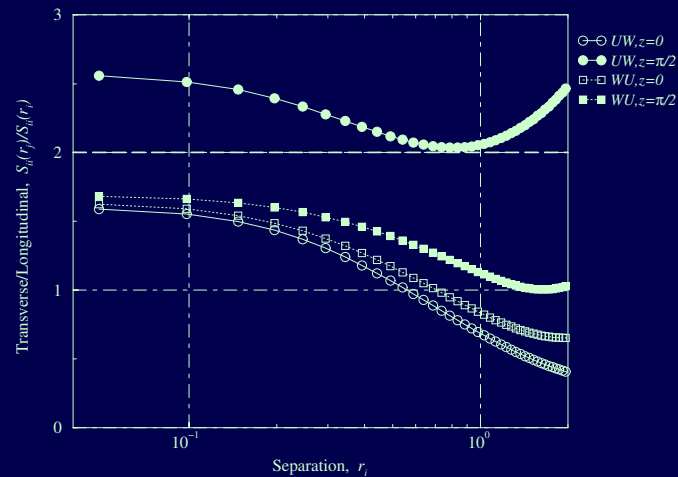
# Transverse vs longitudinal

At small scales, transverse structure functions  $S_{ii}(r_j) \equiv \langle \Delta U_i(r_j) \Delta U_i(r_j) \rangle$  should differ from the longitudinal ones  $S_{ii}(r_i) \equiv \langle \Delta U_i(r_i) \Delta U_i(r_i) \rangle$  by a factor of 2



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But this is not exactly the case, even at small scales ...



# The Kolmogorov and Yaglom laws

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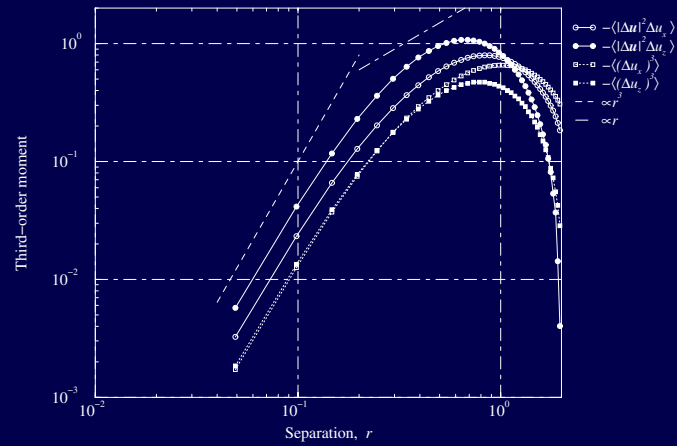
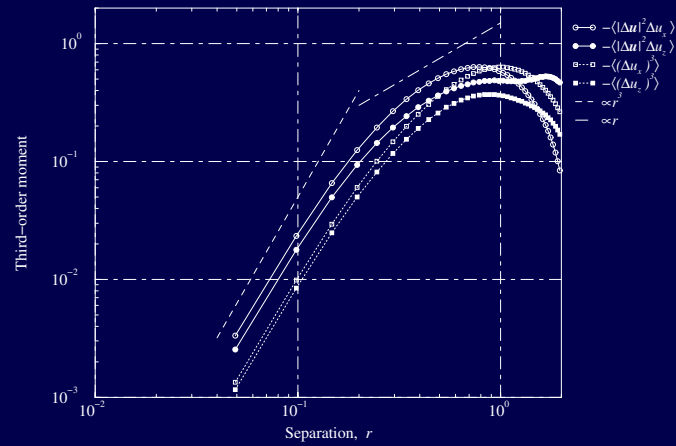
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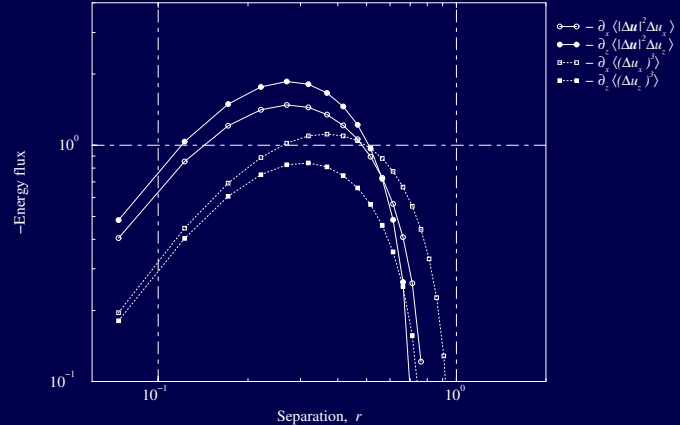
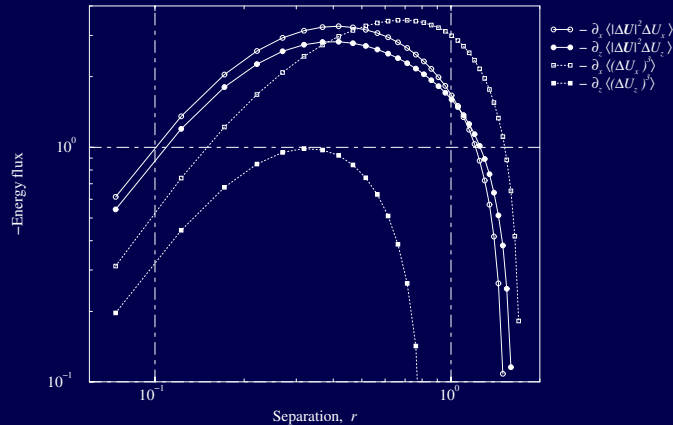
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But the Reynolds number is much too low to observe this ...  
 and neither homogeneity nor isotropy are valid for  $\mathbf{U}$  or  $\mathbf{u}$



# Viscous Kolmogorov and Yaglom laws

With the viscous correction for finite Reynolds number,  
the Yaglom “law” becomes  $\langle |\Delta \mathbf{U}|^2 \Delta \mathbf{U} \rangle = -4/3 \varepsilon \mathbf{r} + 2 \nu \nabla_r \langle |\Delta \mathbf{U}|^2 \rangle$   
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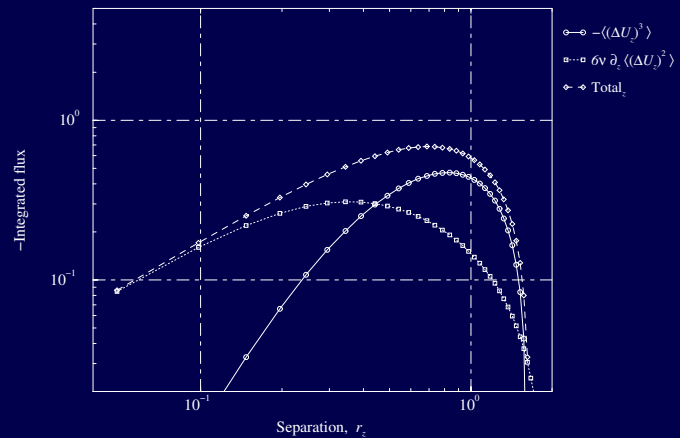
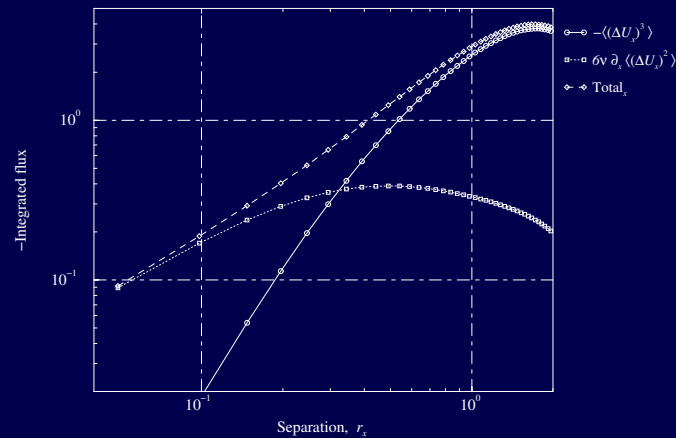
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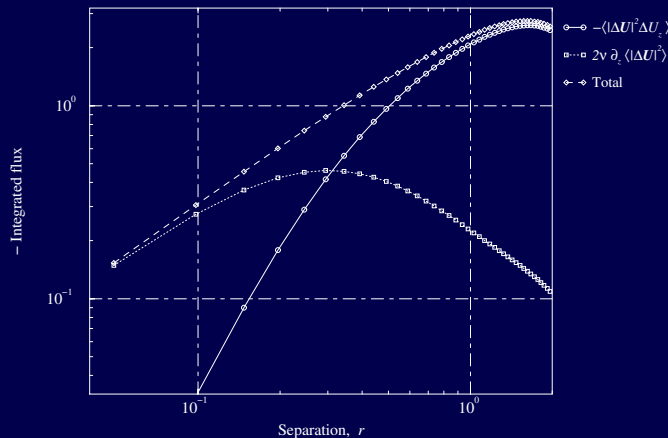
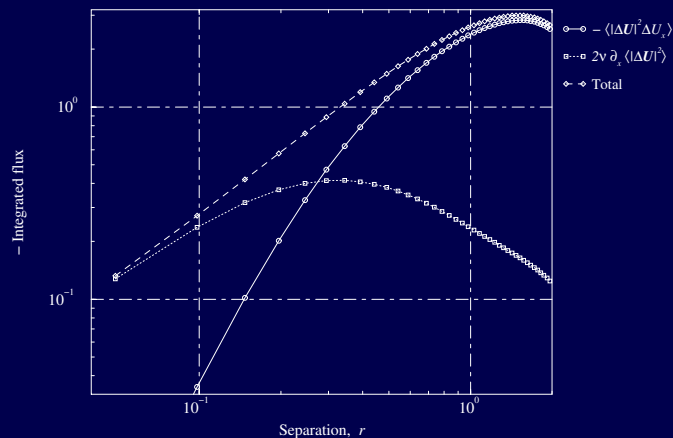
Kolmogorov relation is clearly not satisfied ...





# Viscous Kolmogorov and Yaglom laws

With the viscous correction for finite Reynolds number, the Yaglom “law” becomes  $4/3 \varepsilon r = - \langle |\Delta \mathbf{U}|^2 \Delta \mathbf{U} \rangle + 2\nu \nabla_r \langle |\Delta \mathbf{U}|^2 \rangle$  while Kolmogorov’s law reads  $4/5 \varepsilon r_i = - \langle [\Delta U_i]^3 \rangle + 6\nu \partial_i \langle [\Delta U_i]^2 \rangle$

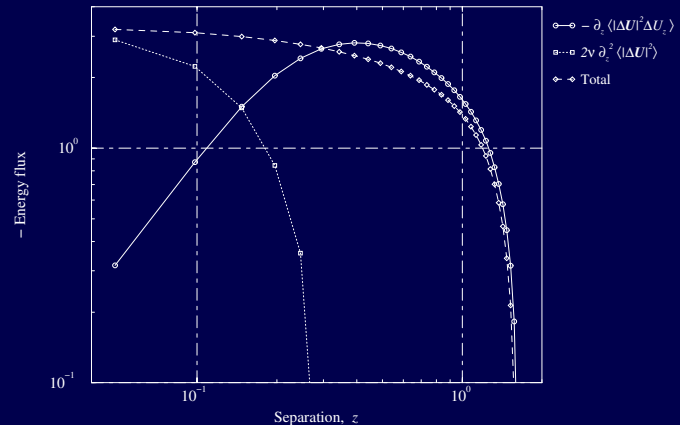
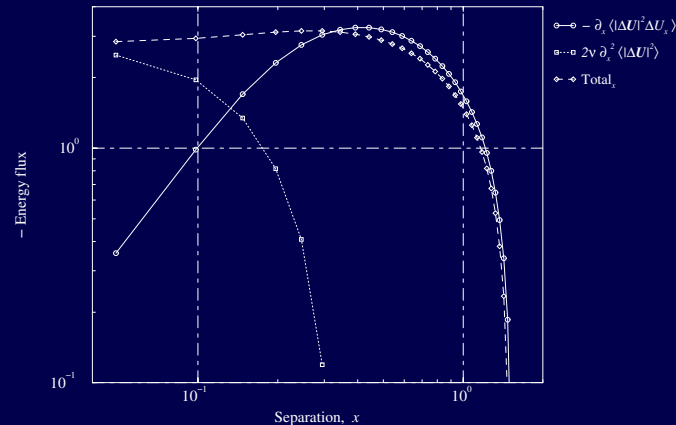


But the Yaglom relation does not look too bad ...

# Viscous Kolmogorov and Yaglom laws

... until we look at the relation in differential form

$$4/3 \varepsilon = -\partial_i \langle |\Delta \mathbf{U}|^2 \Delta U_i \rangle + 2\nu \partial_i^2 \langle |\Delta \mathbf{U}|^2 \rangle$$

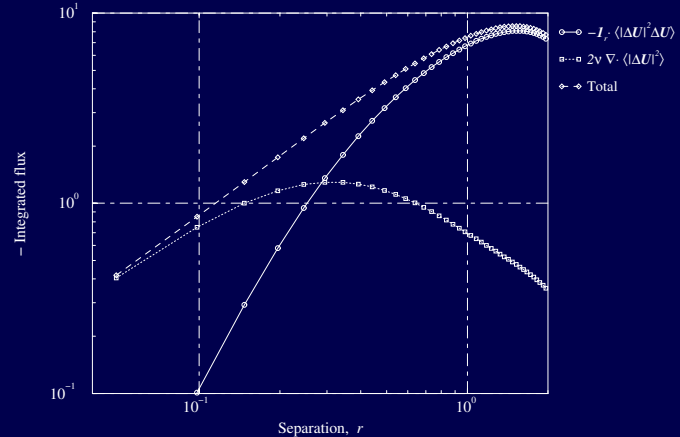
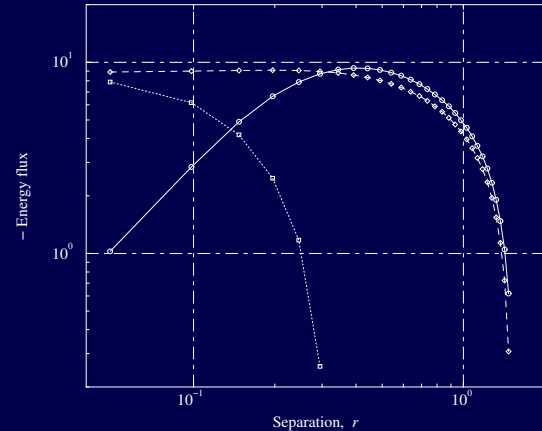


and see that it is not perfectly isotropic



# Anisotropic Monin divergence law

But the “isotropized” Monin law is valid even for anisotropic flows



both in differential

$$4 \varepsilon = -\nabla \cdot \langle |\Delta U|^2 \Delta U \rangle + 2\nu \nabla^2 \langle |\Delta U|^2 \rangle$$

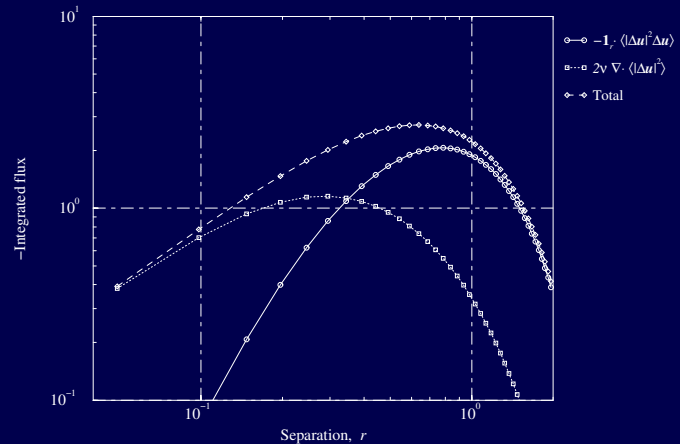
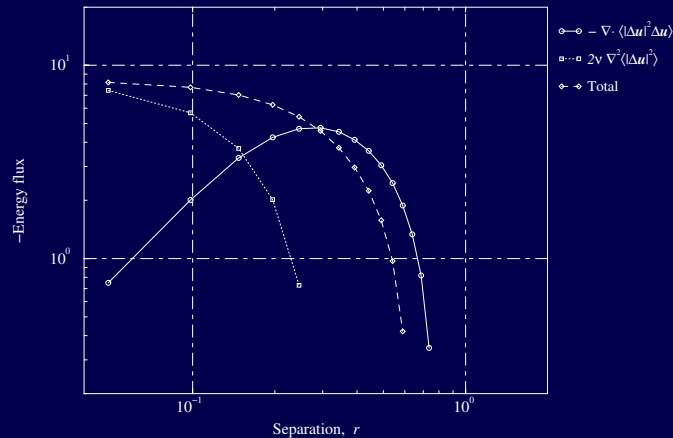
or in “isotropically integrated” form

$$4 \varepsilon \sum_{\alpha} r_{\alpha} = - \sum_{\alpha} \langle |\Delta U|^2 \Delta U_{\alpha} \rangle + 2\nu \sum_{\alpha} \partial_{\alpha} \langle |\Delta U|^2 \rangle$$



# Monin divergence law for the fluctuations

BUT note that Monin's law is valid only for the **total** velocity and **not** for its fluctuations

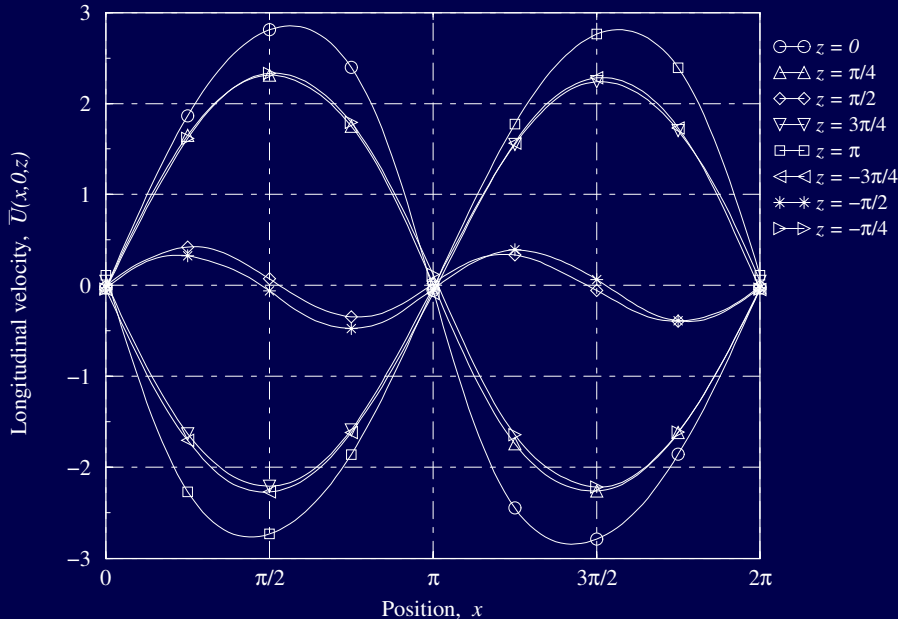


$$4 \varepsilon \neq -\nabla \cdot \langle |\Delta \mathbf{u}|^2 \Delta \mathbf{u} \rangle + 2\nu \nabla^2 \langle |\Delta \mathbf{u}|^2 \rangle$$

$$4 \varepsilon \sum_{\alpha} r_{\alpha} \neq -\sum_{\alpha} \langle |\Delta \mathbf{u}|^2 \Delta u_{\alpha} \rangle + 2\nu \sum_{\alpha} \partial_{\alpha} \langle |\Delta \mathbf{u}|^2 \rangle$$

# Mean flow skewness

The mean flow has **lost** some symmetries of the Taylor-Green forcing

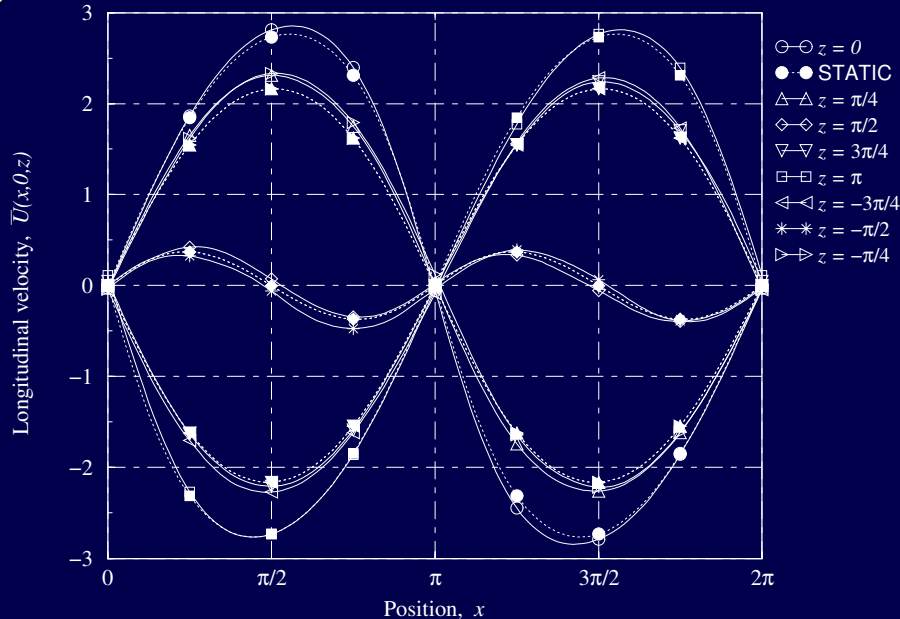


and mediates part of the energy transfer



# Static flow at low Re

At low Reynolds number Re, the flow is **static**  $\overline{U} \equiv U$



$$N^3 \Leftarrow 64^3$$

$$L \equiv 2\pi$$

$$f \Leftarrow 1.5$$

$$\nu \Leftarrow 0.3$$

$$\Delta t \Leftarrow 1$$

$$\Lambda = 3.077$$

$$U_{\text{rms}} = 1.445$$

$$\text{Re} = 14.8$$

$$T_{\text{nl}} = 2.129$$

$$\lambda = 1.076$$

$$\text{Re}_\lambda = 5.2$$

Position,  $x$

$$\eta = 0.316$$

$$\langle |\nabla U|^2 \rangle = 9.016$$

$$\varepsilon = 2.704$$

$$\Delta x \Leftarrow 0.098$$

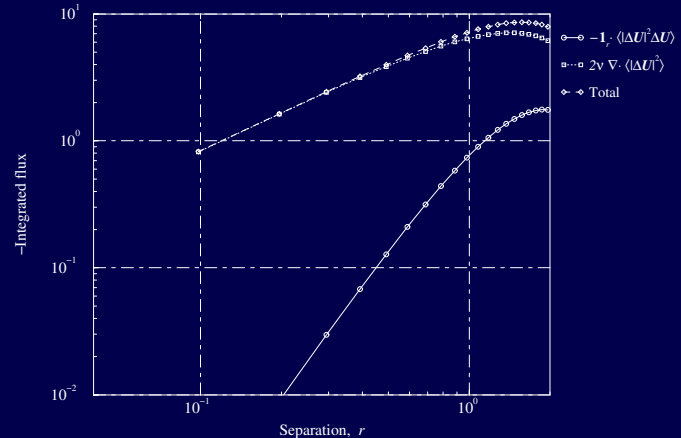
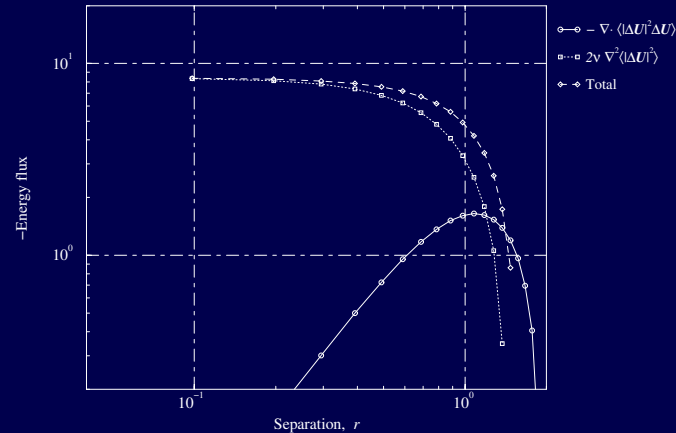
$$dt \Leftarrow 2 \cdot 10^{-3}$$





# Monin divergence law for static flow at low $Re$

Monin's law is valid for a static flow solution of Navier-Stokes equations



$$4 \varepsilon = -\nabla \cdot \langle |\Delta \mathbf{U}|^2 \Delta \mathbf{U} \rangle + 2\nu \nabla^2 \langle |\Delta \mathbf{U}|^2 \rangle$$

$$4 \varepsilon \sum_{\alpha} r_{\alpha} = -\sum_{\alpha} \langle |\Delta \mathbf{U}|^2 \Delta U_{\alpha} \rangle + 2\nu \sum_{\alpha} \partial_{\alpha} \langle |\Delta \mathbf{U}|^2 \rangle$$

# Conclusions

- Low  $Re$  Taylor-Green vortex is neither homogeneous nor isotropic, even for the velocity fluctuations or at special locations



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