Turbulence and dissipation in the solar wind

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The solar wind
Measurements in the solar wind

Wind velocity measured by the Helios 2 spacecraft during the year 1976
Magnetic fluctuations in the solar wind

Magnetic field measurements by the Helios 2 spacecraft during the year 1976
Turbulence in the solar wind
Turbulence in the solar wind

Power spectrum of the magnetic fluctuations
Turbulent spectrum in the solar wind

Magnetic power spectra measured by the Helios 2 spacecraft at different distances
Examples of power spectra

Brownian noise $S_{ww}(\nu) = (2\pi\nu)^{-2}$

Sawtooth wave $S_{ss}(\nu) = (2\pi\nu)^{-2}$
Examples of power spectra

Brownian noise $S_{ww}(\nu) = (2\pi \nu)^{-2}$

Sawtooth wave $S_{ss}(\nu) = (2\pi \nu)^{-2}$

Same spectrum but different type and distribution of singularities

⇒ Necessity to look at other quantities... Higher-order moments?
Irreversibility of time

![Graph showing radial velocity over time]

- **Radial velocity, \( u_R \) (km/s)**
- **Time, \( t \) (days of 1976)**
Irreversibility of time

Statistics of gradients or increments are not invariant by time-reversal
Energy injection, transfer and dissipation

Ando Hiroshige, The Naruto rapids
Dissipation mechanisms

Consider the velocity difference \( u_\ell \) between two points separated by a distance \( \ell \). How can it be annihilated?
Dissipation mechanisms

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- **Dissipation by viscosity**

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\partial_t u = \nu \partial_x^2 u \quad \Rightarrow \quad \tau_D(\ell) \sim \frac{\ell^2}{\nu}
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Dissipation mechanisms

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- **Non-linearity**

  $$\partial_t u + u \partial_x u = 0 \quad \Rightarrow \quad \tau_{NL}(\ell) \sim \frac{\ell}{u_\ell}$$
Dissipation mechanisms

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  \partial_t u + u \partial_x u = 0 \quad \Rightarrow \quad \tau_{NL}(\ell) \sim \frac{\ell}{u_\ell}
  \]

The ratio of these two times at large scale is the Reynolds number

\[
\frac{\tau_D(L)}{\tau_{NL}(L)} = \mathcal{R} = \frac{UL}{\nu}
\]
Energy cascade

Injection

\[ \varepsilon_D = \frac{U^2}{\tau_D(L)} \]
Energy cascade

\[ \varepsilon_D = \nu U^2 / L^2 \]
Energy cascade

Injection

\[ \varepsilon_L = \frac{U^2}{\tau_{NL}(L)} \]
Energy cascade

Injection

\[ \varepsilon_L = \frac{U^3}{L} \]
Energy cascade

Injection

\[ \varepsilon_L = \frac{U^3}{L} \]

Transfer

\[ \varepsilon_\ell = \frac{u_\ell^3}{\ell} \]
Energy cascade

Injection

$$\varepsilon_L = \frac{U^3}{L}$$

Transfer

$$\varepsilon_\ell = \frac{u_\ell^3}{\ell} = \varepsilon_L$$
**Energy cascade**

**Injection**

\[ \varepsilon_L = \frac{U^3}{L} \]

**Transfer**

\[ \varepsilon_\ell = \frac{u_\ell^3}{\ell} = \varepsilon_L \]

**Dissipation**

\[ \varepsilon_\eta = \frac{u_\eta^3}{\eta} \]
Energy cascade

Injection

\[ \varepsilon_L = \frac{U^3}{L} \]

Transfer

\[ \varepsilon_\ell = \frac{u_\ell^3}{\ell} = \varepsilon_L \]

Dissipation

\[ \varepsilon_\eta = \frac{u_\eta^3}{\eta} = \frac{u_\eta^2}{\tau_D(\eta)} \]
Energy cascade

**Injection**

\[ \varepsilon_L = \frac{U^3}{L} \]

**Transfer**

\[ \varepsilon_\ell = \frac{u_\ell^3}{\ell} = \varepsilon_L \]

**Dissipation**

\[ \varepsilon_\eta = \frac{u_\eta^3}{\eta} = \nu \frac{u_\eta^2}{\eta^2} \]
Energy cascade

Injection
\[ \varepsilon_L = \frac{U^3}{L} \]

Transfer
\[ \varepsilon_\ell = \frac{u_\ell^3}{\ell} = \varepsilon_L \]

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Cascade down to the Kolmogorov scale

\[ \eta = \left( \frac{\nu^3}{\varepsilon} \right)^{1/4}, \quad u_\eta = (\nu \varepsilon)^{1/4}, \quad \mathcal{R}_\eta = \left( \frac{u_\eta \eta}{\nu} \right) = 1 \]
Yaglom and Kolmogorov relations

Take the difference of Navier-Stokes equations

\[ \partial_t u + u \cdot \nabla_x u = \frac{-1}{\rho} \nabla_x P + \nu \nabla^2_x u + f \]

at two points \( x \) and \( x' \equiv x + r \) to get an equation for the velocity difference \( \Delta u(r; x, t) \equiv u(x + r, t) - u(x, t) \)
Yaglom and Kolmogorov relations

Take the difference of Navier-Stokes equations

\[ \partial_t u + u \cdot \nabla_x u = -\frac{1}{\rho} \nabla_x P + \nu \nabla^2_x u + f \]

at two points \( x \) and \( x' \equiv x + r \) to get an equation for the velocity difference \( \Delta u(r; x, t) \equiv u(x + r, t) - u(x, t) \)
then multiply by \( 2 \Delta u \) and average over the space \( x \) to get

\[ \partial_t \langle |\Delta u|^2 \rangle + \nabla_r \cdot \langle |\Delta u|^2 \Delta u \rangle = 2 \nu \nabla^2_r \langle |\Delta u|^2 \rangle - 4 \nu \langle |\nabla u|^2 \rangle \]
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so that in stationary conditions and in the limit \( \nu \to 0 \)

\[ \nabla_r \cdot \langle |\Delta u|^2 \Delta u \rangle = -4 \varepsilon \quad \varepsilon \equiv -\frac{dE}{dt} = \nu \langle |\nabla u|^2 \rangle \]
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which in case of isotropy implies \(\langle |\Delta u|^2 \Delta u \rangle = -4/3 \varepsilon r\)  \(\text{(Yaglom)}\)
Yaglom and Kolmogorov relations

Take the difference of Navier-Stokes equations

$$\partial_t u + u \cdot \nabla_x u = \frac{-1}{\rho} \nabla_x P + \nu \nabla_x^2 u + f$$

at two points $x$ and $x' \equiv x + r$ to get an equation for the velocity difference $\Delta u(r; x, t) \equiv u(x + r, t) - u(x, t)$ then multiply by $2 \Delta u$ and average over the space $x$ to get

$$\partial_t \langle |\Delta u|^2 \rangle + \nabla_r \cdot \langle |\Delta u|^2 \Delta u \rangle = 2 \nu \nabla_r^2 \langle |\Delta u|^2 \rangle - 4 \nu \langle |\nabla u|^2 \rangle$$

so that in stationary conditions and in the limit $\nu \to 0$

$$\nabla_r \cdot \langle |\Delta u|^2 \Delta u \rangle = -4 \varepsilon \quad \varepsilon \equiv -\frac{dE}{dt} = \nu \langle |\nabla u|^2 \rangle$$

which in case of isotropy implies $\langle |\Delta u|^2 \Delta u \rangle = -4/3 \varepsilon r$ (Yaglom) and using local isotropy again $\langle [\Delta u_i]^3 \rangle = -4/5 \varepsilon r_i$ (Kolmogorov)
"Exact" conservation laws

\[
\frac{12}{d(d+2)} \varepsilon r_i \left\{ \begin{array}{l}
\frac{4}{d} \varepsilon r \\
4 \varepsilon
\end{array} \right\} = \left\{ \begin{array}{l}
- \left\langle [\Delta u_i]^3 \right\rangle \\
- \left\langle |\Delta u|^2 \Delta u \right\rangle \\
- \nabla_r \cdot \left\langle |\Delta u|^2 \Delta u \right\rangle
\end{array} \right\} + \left\{ \begin{array}{l}
2 d \nu \partial_i \left\langle |\Delta u_i|^2 \right\rangle \\
2 \nu \nabla_r \left\langle |\Delta u|^2 \right\rangle \\
2 \nu \nabla_r^2 \left\langle |\Delta u|^2 \right\rangle
\end{array} \right\}
\]

Total dissipation
Transfer
Viscous dissipation

at scale \( r \)
to scales \(< r \)
at scale \( r \)

\( \text{Exp} \)

- Scale by scale energy conservation \( \Rightarrow \) no pileup of energy
- Increasing generality (Monin’s law valid for anisotropic flows)
- Increasing experimental difficulty
The Kolmogorov spectrum

In the inertial range

\[ L \ll \ell \ll \eta \]

\[ \langle (\Delta u_\parallel (\ell))^3 \rangle = -\frac{4}{5} \varepsilon \ell \]
The Kolmogorov spectrum

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By self-similarity

\[ \langle (\Delta u_\parallel(\ell))^p \rangle = C_p (\varepsilon \ell)^{p/3} \]

\[ \langle (\Delta u_\parallel(\ell))^2 \rangle = C_2 (\varepsilon \ell)^{2/3} \]
The Kolmogorov spectrum

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\[ E(k) = C_K \varepsilon^{2/3} k^{-5/3} \]
The Kolmogorov spectrum

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Magneto-Hydrodynamics

For a charged fluid at velocities $u \ll c$ in a mean magnetic field $B_0$ velocity $u$ and magnetic field $b$ perturbations obey the MHD equations

\[
\nabla \cdot u = \nabla \cdot b = 0 \quad B = B_0 + b
\]

\[
\partial_t u + u \cdot \nabla u = -\nabla p + \frac{1}{4\pi \rho} (\nabla \times b) \times B + \nu \nabla^2 u
\]

\[
\partial_t b = \nabla \times (u \times B) + \eta \nabla^2 b
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Magneto-Hydrodynamics

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$$\partial_t u + u \cdot \nabla u = -\nabla p + \frac{1}{4\pi \rho} (\nabla \times b) \times B + \nu \nabla^2 u$$

$$\partial_t b = \nabla \times (u \times B) + \eta \nabla^2 b$$

Consider the Elsässer variables $z^\pm = u \pm (4\pi \rho)^{-1/2} b$

$$\nabla \cdot z^\pm = 0$$

$$\partial_t z^\pm + z^\mp \cdot \nabla z^\pm = -\nabla p^* + c_A \cdot \nabla z^\pm + \nu^\pm \nabla^2 z^\pm + \nu^\mp \nabla^2 z^\mp$$

where $c_A$ is the Alfvén velocity $(4\pi \rho)^{-1/2} B_0$ and $\nu^\pm = (\nu \pm \eta)/2$
Yaglom equations for MHD turbulence

Derivation as for the Navier-Stokes equations, excepted that one of the Elsässer variables $z^\pm$ is transported by the other $z^\mp$

$$Y^\pm(\ell) \equiv \left\langle |\Delta z^\pm|^2 \Delta z^\mp \right\rangle = -\frac{4}{3} \varepsilon^\pm \ell$$
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which in terms of velocity $u$ and magnetic field $b$ reads

$$\left< \left[ |\Delta u|^2 + |\Delta b|^2 \pm 2 \Delta u \cdot \Delta b \right] \left( \Delta u^\parallel \mp \Delta b^\parallel \right) \right> = -\frac{4}{3} \varepsilon^\pm \ell$$

coupling energy $|u|^2 + |b|^2$ and cross-helicity $u \cdot b$ cascades
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coupling energy $|u|^2 + |b|^2$ and cross-helicity $u \cdot b$ cascades

Does it hold in the solar wind ???
Verification in the Ulysses data

The Ulysses mission first north polar pass during the year 1996 at solar minimum
The solar wind as seen by Ulysses

High-latitude $\theta > 35^\circ$
fast $|u| > 700 \text{ km/s}$
polar solar wind
near solar minimum
1995 – 1996
The solar wind as seen by Ulysses

High-latitude $\theta > 35^\circ$
fast $|u| > 700 \text{ km/s}$
polar solar wind
near solar minimum
1995 – 1996

Use 8-min average data of $\rho$, $u$ and $b$
to build the Elsässer variables $z^\pm$
Data processing

- Reconstruction of the spatial (radial) dependence using the Taylor’s frozen-flow method

\[
\begin{align*}
    u(x, t + \tau) & \approx u(x - \bar{u} \, \tau, t) \\
    z^\pm(x, t + \tau) - z^\pm(x, t) & \approx z^\pm(x - \bar{u}_r \, \tau \, 1_R, t) - z^\pm(x, t) \\
    & \approx \Delta z^\pm(-\bar{u}_r \, \tau \, 1_R, t)
\end{align*}
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  & \approx \Delta z^\pm(-\overline{u_r} \tau 1_R, t)
\end{align*}
\]

- Use 11-days (≈ 2000 data points) time-average moving window to build up statistical averages on stationary data sets, and avoid radial and latitudinal variations
Yaglom law is observed

First direct evidence of an MHD turbulent energy cascade in the solar wind
Measurements of the cascade rate

First measurements of the energy transfer rates $\varepsilon^\pm \approx 200 \text{ J s}^{-1} \text{ kg}^{-1}$
Effect of compressibility

- Incompressibility is not always satisfied in the solar wind

\[ \frac{v}{c_s} \approx \frac{v}{c_A} \approx 5 - 10 \quad \Rightarrow \quad \rho \neq \text{const} \]
Effect of compressibility

- Incompressibility is not always satisfied in the solar wind
  \[ \frac{v}{c_s} \approx \frac{v}{c_A} \approx 5 - 10 \Rightarrow \rho \neq \text{const} \]
  \[ \Rightarrow \text{the dissipation per unit volume} \quad \varepsilon_V \equiv \rho \varepsilon = \rho \frac{u^3}{\ell} \]

should be statistically constant rather than the dissipation per unit mass \( \varepsilon \)
Effect of compressibility

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  should be statistically constant rather than the dissipation per unit mass \( \varepsilon \)

  \[ \Rightarrow \text{Check for a Yaglom-type law using density-weighted Elsässer fields} \]

  \[ \mathbf{w}^\pm \equiv \rho^{1/3} z^\pm \]

  and weighted flux \[ W^\pm(\ell) \equiv \frac{\left\langle |\Delta \mathbf{w}^\pm|^2 \Delta w^\mp \right\rangle}{\langle \rho \rangle} \propto -\frac{4}{3} \varepsilon^\pm \ell \]
Compressible scaling is observed but the compressible pseudo-dissipations $\varepsilon_\text{C}^{\pm} \approx 3.5 \text{kJ s}^{-1} \text{kg}^{-1}$ are much larger than the incompressible dissipations $\varepsilon_\text{I}^{\pm} \approx 200 \text{J s}^{-1} \text{kg}^{-1}$

$\Rightarrow$ Correlations between density gradients and velocity/magnetic field
Solar wind heating

Solar wind radial temperature profile \( T(R) \) decreases as a power law \( T(R) \sim R^{-\xi} \) \( \xi \approx 0.7 – 1.0 \) but slower than adiabatic spherical cooling \( T(R) \sim R^{-4/3} \)
Solar wind heating

Solar wind radial temperature profile $T(R)$ decreases as a power law $T(R) \sim R^{-\xi}$, $\xi \approx 0.7 \text{ – } 1.0$ but slower than adiabatic spherical cooling $T(R) \sim R^{-4/3}$

⇒ Necessity of in-situ heating . . . Turbulence?
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$\Rightarrow$ Necessity of in-situ heating . . . Turbulence?
Measurements in the ecliptic

The dissipation rate $\varepsilon$ depends on the type (fast/slow) of solar wind and is much higher for slow wind in the ecliptic.
Conclusions

- The solar wind is a real turbulent magneto-hydrodynamic plasma
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• Turbulence can account for (some of) the heating during the solar wind expansion
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Thanks to everybody, especially to the organizers . . .

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Alain Arneodo † 2019
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