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# NEW FRONTIERS FOR STOCHASTIC MATCHING

JEAN MAIRESSE AND PASCAL MOYAL

## 1. INTRODUCTION : AN ECONOMY OF INTERACTIONS AND COMPATIBILITIES

In the modern peer-to-peer and collaborative economy, many applications offer their users an interface to interact and collaborate. To name a few, let us mention ride-sharing, online dating, carpooling, or jobs/internships search apps. In all these applications, the arrivals of requests are random, and their sojourn in the system is finalized to finding a *match*, identified as such according to fixed compatibility rules. Such matching mechanisms also appear in many physical systems subject to compatibility constraints, among which: blood transfusions, organ transplants, public housing allocations and assemble-to-order systems.

## 2. MATHEMATICAL MODELS

*Stochastic matching models*, through their different variants [6, 5, 9, 12], form a unified mathematical framework to study the above class of real-life systems. They share common features with classical skill-based queueing systems, but they also have very original aspects. The standard model is defined as follows: Items arrive one by one in a buffer and depart from it asap but by pairs. The items arrive according to a stochastic process, and are tagged with a finite number of *classes*. Let  $\lambda$  be the vector of arrival rates - or probabilities if time is discrete - of the various classes. The *compatibility* between classes is given by a non-directed graph  $G$ , in which the vertices represent the classes of items, and the edges represent the compatibilities. Upon arrival, an item is either matched if there is a compatible available item in storage, or itself stored in line. A matching policy  $\Phi$  determines the matches in case of multiple choices.

To properly capture the behavior of real-life systems, it is relevant to add a notion of *impatience* to the standard model. That is, after some (random) time, an unmatched item may be lost (a patient may die before getting a compatible kidney, for instance). The difficulty is that such *real-time* matching models rapidly become intractable. E.g. determining optimal policies is very intricate: contrary to classical queueing, it is easily seen that *Earliest Deadline First* is not optimal for feasibility of a given static scenario. So it would be convenient to have another representation of the real-time matching models. A possible approach, which has not yet been investigated to the best of our knowledge, would be to incorporate the impatience parameter within the standard model, by adding one or several “death” classes, compatible with the classes of perishable items.

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To define some issues at stake more precisely, let us introduce the *matching rates* matrix, that is, the symmetric matrix  $M$  whose parameter  $M_{ij}$  is the long-run proportion of  $(i, j)$  matchings executed among all matchings. It is easily seen that the  $M_{ij}$ 's solve a linear system in function of  $\lambda$ , whose coefficients are those of the adjacency matrix of  $G$ . In particular, one can then use this characterization to solve the following problem: determining the arrival rates  $\lambda$  so as to achieve a given matching rate matrix  $M$ , for a given matching policy  $\Phi$ , see [4, 7].



FIGURE 1. Left: a skill-based call center. Right: a stochastic matching model

### 3. Problem statement

Three typical problems arise:

- (i) *Stability*: Does the model operate without overload? If so, what is the steady state behavior?
- (ii) *Efficiency*: Can we compute and optimize the sojourn times in the system? If elements are perishable in the system, how to minimize loss?
- (iii) *Fairness*: Can we design a matching policy to meet some equity criteria?

The questions of *stability* and *efficiency* have been addressed in several references. The stability region can be obtained in closed form for various types of graphs and matching policies. An *explicit* product form for the stationary distribution can be obtained for various stochastic matching models, when the policy is “First Come First Matched” (FCFM), see e.g. [1, 2, 10]. The product form result yields an explicit computation of the sojourn times and matching rates. However, for most other matching policies, the model characteristics in steady state are unknown.

Characterizing and achieving *fairness* represents an acute challenge, and we want to highlight its practical importance. Consider for instance the issue of organ transplants. Both the organs and the awaiting patients have a finite lifetime in the system, so a primary goal of the matching policy is to ensure that the smallest possible fraction of resources and patients is lost. The choice of attribution of organs among waiting patients is thus driven by an emergency criterion. However, practitioners observe that giving priority to the patients in the most critical conditions, leads to disregard classes of patients in ‘intermediate’ condition, thereby increasing their mortality rate in a way that may not be acceptable. There is thus a crucial need to insure some *fairness* in the matching choices and in the loss rates. Consequently, we propose the following **Inverse problem for stochastic matching models**:

- First encode the targetted fairness into a matching rate matrix  $M$ ;
- Second, find a matching policy  $\Phi$  leading to the matching rates matrix  $M$  (with minimal loss rates if applicable).

#### 4. Discussion

It has been recently observed in [4, 8] that the matching rates matrix is *insensitive* of the matching policy for unicyclic graphs, at least for systems without impatience. So we can rephrase the above inverse problem as follows: For a general non-unicyclic graph, given the arrival rates  $\lambda$  and possibly the overall loss probability, can one determine the matching policy  $\Phi$  that is able to achieve a given matching rates matrix  $M$ ? Or else, to approximate  $M$ , or to simulate it without aprioris and bias ?

The complexity of this problem is reminiscent of the fact that no closed-form expression of the stationary state of the model is available for most models. Let us observe that a current line of literature tackles this problem by constructing optimization schemes and optimal control for matching models (see e.g. [11], [3]) leading, in some cases, to the construction of structured optimal policies. However, in such cases one faces the usual ‘simplicity vs complexity’ dilemma. On the one hand, the model and the matching policy needs to be “simple” and explainable. Indeed, there is a strong public request for “transparency of algorithms”, especially in sensitive domains like transplants access. On the other hand, the model needs to be sophisticated enough, to reflect the complexity of the model at hand and to be right on target.

#### REFERENCES

- [1] I. Adan, A. Bušić, J. Mairesse, and G. Weiss. Reversibility and further properties of fcfs infinite bipartite matching. *Mathematics of Operations Research*, 43(2):598–621, 2018.
- [2] I. Adan, I. Kleiner, R. Righter, and G. Weiss. FCFS parallel service systems and matching models. *Performance Evaluation*, 127:253–272, 2018.
- [3] A. Aveklouris, L. DeValve, A. R. Ward, and X. Wu. Matching impatient and heterogeneous demand and supply. *arXiv preprint arXiv:2102.02710*, 2021.
- [4] J. Begeot, I. Marcovici, and P. Moyal. Stability regions of systems with compatibilities, and ubiquitous measures on graphs. *arXiv preprint arXiv:2111.13231*, 2021.
- [5] A. Bušić, V. Gupta, and J. Mairesse. Stability of the bipartite matching model. *Advances in Applied Probability*, 45(2):351–378, 2013.
- [6] R. Caldentey, E. Kaplan, and G. Weiss. FCFS infinite bipartite matching of servers and customers. *Adv. Appl. Probab.*, 41(3):695–730, 2009.
- [7] C. Comte. Stochastic non-bipartite matching models and order-independent loss queues. *Stochastic Models*, 38(1):1–36, 2022.
- [8] C. Comte, F. Mathieu, and A. Bušić. Stochastic dynamic matching: A mixed graph-theory and linear-algebra approach. *arXiv preprint arXiv:2112.14457*, 2021.
- [9] J. Mairesse and P. Moyal. Stability of the stochastic matching model. *Journal of Applied Probability*, 53(4):1064–1077, 2016.
- [10] P. Moyal, A. Bušić, and J. Mairesse. A product form for the general stochastic matching model. *Journal of Applied Probability*, 58(2):449–468, 2021.
- [11] M. Nazari and A. L. Stolyar. Reward maximization in general dynamic matching systems. *Queueing Systems*, 91(1):143–170, 2019.
- [12] Y. Rahme and P. Moyal. A stochastic matching model on hypergraphs. *Advances in Applied Probability*, 53(4):951–980, 2021.