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► **To cite this version:**

Smail Lebbal, Frédéric Alizard, Benoît Pier. Linear instabilities of pulsatile plane channel flow between compliant walls. 2022. hal-03559481v1

**HAL Id: hal-03559481**

**<https://hal.science/hal-03559481v1>**

Preprint submitted on 6 Feb 2022 (v1), last revised 25 Sep 2022 (v3)

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# 1 **Linear instabilities of pulsatile plane channel flow** 2 **between compliant walls**

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7 (6 February 2022)

8 The linear dynamics of perturbations developing in a channel with compliant walls is  
9 investigated for pulsatile base flows. It is found that the flow dynamics is mainly governed by  
10 four control parameters: the Reynolds number  $Re$ , the reduced velocity  $V_R$ , the Womersley  
11 number  $Wo$  and the amplitude of the base flow modulation  $\tilde{Q}$ . Linear stability analyses are  
12 carried out within the framework of Floquet theory, implementing an efficient approach for  
13 removing spurious eigenmodes. The characteristics of flow-based (Tollmien–Schlichting)  
14 and wall-based (both travelling-wave flutter and divergence) modes are investigated over  
15 a large control-parameter space. It is shown that travelling-wave flutter (TWF) modes are  
16 predominantly influenced by the reduced velocity and that the Reynolds number has only a  
17 marginal effect. The critical reduced velocity (corresponding to onset of linear instability)  
18 is demonstrated to depend both on the Womersley number and modulation amplitude for a  
19 given set of wall parameters. The Tollmien–Schlichting (TS) mode is only weakly affected  
20 by the flexibility of the wall. Finally, the classification given by Benjamin (*J. Fluid Mech.* **16**  
21 436–450, 1963) is found to be too restrictive in the case of pulsatile base flows. In particular,  
22 a new type of transition mode is identified that results from the coalescence of two Floquet  
23 eigenmodes: interaction between TS and TWF modes due to coupling of the different Floquet  
24 harmonics, a phenomenon specific to time-periodic base flows.

25 **Key words:** Fluid–structure interactions, instability, Floquet theory

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## 26 **1. Introduction**

27 Pulsating flows in pipe or channel flows are laminar provided that Reynolds numbers are  
28 sufficiently low as is largely the case for vast parts of the cardiovascular system. In the main  
29 arteries, however, blood flow may experience instability waves, generating large fluctuating  
30 shear stresses, which are a possible cause for cardiovascular diseases (Chiu & Chien 2011).  
31 The compliance of arteries plays a major role in blood transport, such as maintaining blood  
32 pressure and regularizing the flow rate (Ku 1997). The flexibility of the aorta is also a  
33 key element in minimizing pressure fluctuations of blood provided by the left ventricle

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34 and distributing oxygen-rich blood through capillaries (O'Rourke & Hashimoto 2007). For  
35 these reasons, both flexible walls and pulsatile flow are ubiquitous in the physiological  
36 context. When a pulsatile flow interacts with compliant walls, a better understanding of the  
37 development of instabilities is therefore required in order to improve the analysis of the link  
38 between wall-shear stress distributions and flow dynamics.

39 The theory of viscous flow interacting with compliant walls has come a long way from Gray  
40 (1936)'s initial observations of the outstanding performance of dolphin skins in delaying  
41 turbulence, to the recent review of Kumaran (2021) enlightening the various instability  
42 mechanisms. In the 50s, Kramer conducted pioneering tests in water by towing a dolphin-  
43 shaped object covered with viscoelastic materials of varying compliance (Kramer 1957). The  
44 author shows that the compliant coating leads to a significant drag reduction and suggests  
45 that the dolphin's secret originates in the laminarisation of the flow due to its skin material.

46 On one hand, several researchers tried and failed to replicate Kramer's experiments; see  
47 Gad-el-Hak (1986, 1996) for reviews. On the other hand, theoretical results of Carpenter  
48 & Garrad (1985) extend the first analytical studies developed by Benjamin (1959, 1960,  
49 1963) and Landahl (1962) and demonstrate that a suitable choice of wall properties could  
50 control the onset of the primary instability mode of a flat-plate boundary layer, the so-called  
51 Tollmien–Schlichting (TS) mode. However, it is also suggested that the emergence of solid-  
52 based instability modes due to fluid–structure interactions (also referenced as flow–structure  
53 instabilities, FSI) can limit the potential of laminarisation of the flow (Carpenter & Garrad  
54 1986). The FSI modes can be divided into two categories: the travelling-wave flutter (TWF)  
55 modes and the (almost static) divergence (DIV) modes. While the physics of TWF modes  
56 is fairly well understood using an analogy with the onset of water-waves (Miles 1957),  
57 scientists are still arguing about the physical mechanism behind the divergence mode. The  
58 first successful experimental attempt to reproduce Kramer's findings was given by Gaster  
59 (1988). In particular, he found that the measured growth rates of TS waves developing along  
60 a simple compliant coating match those obtained by Carpenter & Garrad (1985).

61 Several attempts to classify instability modes in the presence of fluid–structure interactions  
62 were made since the seminal study of Benjamin (1963). In particular, three types of instability  
63 mechanisms have been considered: TS modes belong to class A, TWF modes are associated  
64 with class B and class C modes correspond to almost steady waves, i.e. the divergence mode  
65 (see Davies & Carpenter (1997*a,b*) for the channel flow case). Apart from these modes, a  
66 transition mode is also found by Sen & Arora (1988), resulting from the coalescence between  
67 a TS mode and a TWF mode. For instance, Davies & Carpenter (1997*a*) have shown that the  
68 transition mode could develop inside a flow between a compliant channel for a sufficiently  
69 high level of wall damping. For the same flow case, we have recently shown that while  
70 class B modes are mainly driven by the reduced velocity, which corresponds to the ratio  
71 of characteristic wall and advection time scales, class C mode is influenced by both the  
72 Reynolds number and the reduced velocity (Lebbal *et al.* 2022).

73 Independently of studies assessing optimal properties of wall coating to delay transition to  
74 turbulence in wall-bounded flows, the stability of pulsatile flow with respect to viscous shear  
75 instability modes has been theoretically addressed since the middle of the 70s (Davis 1976). In  
76 comparison with steady flows, pulsatile flows are governed by additional control parameters:  
77 the pulsation amplitudes and the pulsating frequency, of which the Womersley number  $Wo$   
78 is a non-dimensional measure (see its definition in (3.6) below). In physiological situations,  
79 typical Womersley numbers for large blood vessels are in the range 5–15 (Ku 1997). Within  
80 a Floquet theory framework, von Kerczek (1982) shows that the sinusoidally pulsating flow  
81 developing between two flat plates is more stable than the steady plane Poiseuille flow for  
82 Womersley numbers in excess of  $Wo = 12$ . This result was confirmed by direct numerical  
83 simulations carried out by Singer *et al.* (1989). Using linear Floquet stability analyses and

84 nonlinear numerical simulations, Pier & Schmid (2017) explored a large parameter space  
 85 for the same flow configuration, confirming and extending the earlier results given by von  
 86 Kerczek (1982).

87 On the other hand, several authors (Straatman *et al.* 2002; Blennerhassett & Bassom  
 88 2006) have found that the perturbations may experience a strong increase in kinetic energy  
 89 during the deceleration phase of the pulsatile base flow. This suggests that transient growth  
 90 mechanisms and nonlinear effects come likely into play during this part of the pulsation  
 91 cycle and that the flow could possibly break down to turbulence. Recently such a scenario  
 92 has been further supported by nonmodal stability analyses, experiments and direct numerical  
 93 simulations for both pipe and channel flows (Xu *et al.* 2020, 2021; Pier & Schmid 2021).

94 In spite of major successes achieved so far in the understanding of the dynamics prevailing  
 95 for either pulsatile base flows or wall flexibility, only few studies address these two effects in  
 96 combination. For the channel flow case, using Floquet stability analyses, Tsigklifis & Lucey  
 97 (2017) show that wall flexibility has a stabilising effect for some range of the Womersley  
 98 number. The combined effect of wall damping and Womersley number is also illustrated by  
 99 these authors. However, the influence of the reduced velocity on travelling-wave flutter modes  
 100 is not considered when the pulsatile base-flow component comes into play. Furthermore, it is  
 101 not completely clear if the classification made by Benjamin (1963) still holds for the pulsatile  
 102 flow case. Finally, the transition mode that emerges due to an increase in wall damping is not  
 103 investigated when considering a pulsatile flow component.

104 To provide further understanding to the above points, the present study addresses the linear  
 105 stability properties of small-amplitude perturbations developing in pulsatile flows through  
 106 compliant channels. This paper is organized as follows. In the section 2, we introduce the  
 107 coupled fluid–structure system, and the base flow and non-dimensional control parameters are  
 108 given in section 3. The mathematical formulation of the linear stability problem is presented  
 109 in section 4. The numerical methods to solve and reduce the generalised eigenvalue problem  
 110 are explained in section 5. Section 6 devoted to the results constitutes the main contribution  
 111 of the paper: discussion of the spectra, influence of the control parameters, spatio-temporal  
 112 structure of the eigenmodes, and a variety of critical curves associated with onset of linear  
 113 instability.

## 114 2. Fluid–structure interaction model and interface conditions.

115 In the present study, the analysis is restricted to the two-dimensional case. We introduce the  
 116 Cartesian coordinate system  $(x, y)$  with unit vectors  $(\mathbf{e}_x, \mathbf{e}_y)$  and consider an incompressible  
 117 Newtonian fluid with dynamic viscosity  $\mu$  and density  $\rho$  between two spring-backed  
 118 deformable plates located at  $y = \zeta^\pm(x, t)$  which are allowed to move only in the wall-  
 119 normal direction (see figure 1). The flow between the walls is governed by the Navier–Stokes  
 120 equations

$$121 \quad \rho \frac{\partial \mathbf{u}}{\partial t} + \rho(\mathbf{u} \cdot \nabla) \mathbf{u} = -\nabla p + \mu \Delta \mathbf{u}, \quad (2.1)$$

$$122 \quad 0 = \nabla \cdot \mathbf{u}, \quad (2.2)$$

123 where  $\mathbf{u} = (u, v)$  is the velocity field, with streamwise ( $u$ ) and wall-normal ( $v$ ) velocity  
 124 components, respectively, and  $p$  the pressure field.

125 The movement of the flexible plates obeys the following equations:

$$126 \quad m \frac{\partial^2 \zeta^\pm}{\partial t^2} + d \frac{\partial \zeta^\pm}{\partial t} + \left( B \frac{\partial^4}{\partial x^4} - T \frac{\partial^2}{\partial x^2} + K \right) \zeta^\pm = f^\pm, \quad (2.3)$$

127 where  $m$  denotes the mass per unit area of the plates,  $d$  their damping coefficient,  $B$  the

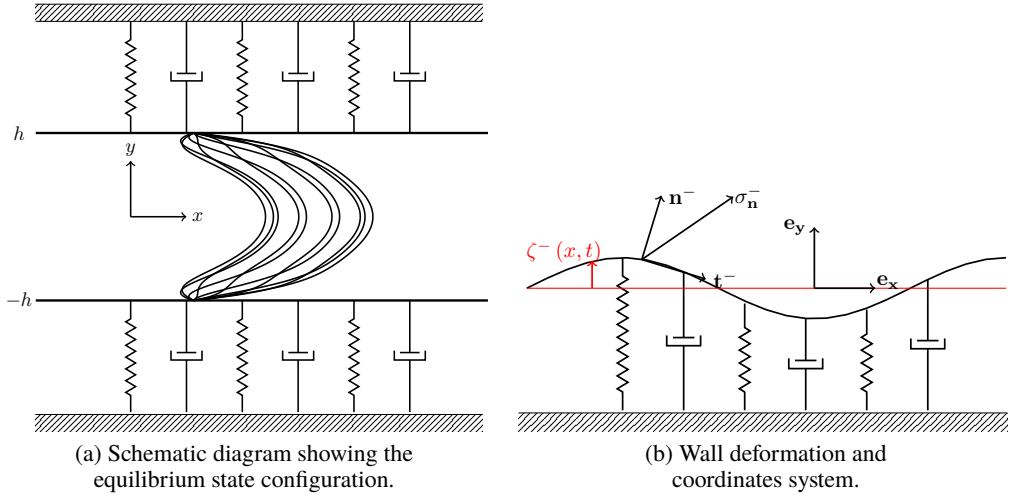


Figure 1: Channel flow with infinite spring-backed flexible walls.

128 flexural rigidity,  $T$  the wall tension,  $K$  the spring stiffness and  $f^\pm$  represents the  $y$ -component  
 129 of the hydrodynamic forces acting on the plates. These forces are obtained as

$$130 \quad f^\pm = \mathbf{e}_y \cdot \mathbf{f}^\pm \quad \text{with} \quad \mathbf{f}^\pm = \left( \bar{\boldsymbol{\tau}}^\pm - \delta p^\pm \mathbf{I} \right) \cdot \mathbf{n}^\pm. \quad (2.4)$$

131 Here  $\bar{\boldsymbol{\tau}}^\pm$  denotes the viscous stress tensor at the walls,  $\delta p^\pm$  the transmural surface pressure  
 132 and  $\mathbf{n}^\pm = (n_x^\pm, n_y^\pm)$  is the unit vector normal to the walls pointing towards the fluid. The  
 133  $y$ -component of the normal forces acting on the plate then reads

$$134 \quad f^\pm = \mu \left( \frac{\partial u}{\partial y} \Big|_{y=\zeta^\pm} + \frac{\partial v}{\partial x} \Big|_{y=\zeta^\pm} \right) n_x^\pm + 2\mu \frac{\partial v}{\partial y} \Big|_{y=\zeta^\pm} n_y^\pm - \delta p^\pm n_y^\pm, \quad (2.5)$$

135 with

$$136 \quad n_x^\pm = \pm \frac{\partial \zeta^\pm}{\partial x} \frac{1}{\sqrt{1 + \left( \frac{\partial \zeta^\pm}{\partial x} \right)^2}} \quad \text{and} \quad n_y^\pm = \mp \frac{1}{\sqrt{1 + \left( \frac{\partial \zeta^\pm}{\partial x} \right)^2}}. \quad (2.6)$$

137 Finally, the no-slip conditions on both walls lead to the kinematic conditions:

$$138 \quad u = 0 \quad \text{and} \quad v = \frac{\partial \zeta^\pm}{\partial t} \quad \text{for} \quad y = \zeta^\pm. \quad (2.7)$$

139 The fluid–structure interaction problem is thus fully defined by the coupling of the fluid  
 140 equations (2.1,2.2), the wall equations (2.3,2.5) and the boundary conditions (2.7).

### 141 3. Base flows and non-dimensional control parameters

142 A pulsatile base flow, of frequency  $\Omega$ , is considered. Such a flow is driven by a spatially  
 143 uniform and temporally periodic streamwise pressure gradient and is obtained as an exact  
 144 solution of the Navier–Stokes equations. The solution consists of a velocity field in the  
 145 streamwise direction with profiles that only depend on the wall-normal coordinate and time.

146 It can be expanded as a temporal Fourier series

$$147 \quad \mathbf{U}(y, t) = U(y, t)\mathbf{e}_x \quad \text{with} \quad U(y, t) = \sum_n U^{(n)}(y) \exp(in\Omega t). \quad (3.1)$$

148 Similarly, the pressure gradient that drives the flow is expanded as

$$149 \quad G(t) = \sum_n G^{(n)} \exp(in\Omega t) \quad (3.2)$$

150 and is associated with a pulsatile flow rate

$$151 \quad Q(t) = \sum_n Q^{(n)} \exp(in\Omega t). \quad (3.3)$$

152 The velocity profile is analytically obtained for each harmonic component. The mean-flow  
153 component  $U^{(0)}(y)$  corresponds to a parabolic steady Poiseuille flow solution. For  $n \neq 0$ , the  
154 profiles  $U^{(n)}(y)$  are obtained in terms of exponential functions (Womersley 1955). Analytical  
155 expressions are detailed in the appendix of Pier & Schmid (2021). In this work, we focus on  
156 pulsatile base flows with a single oscillating component. We can thus write the flow rate as:

$$157 \quad Q(t) = Q^{(0)} \left( 1 + \tilde{Q} \cos \Omega t \right), \quad (3.4)$$

158 with the relative pulsating amplitude  $\tilde{Q}$  defined as

$$159 \quad \tilde{Q} = 2 \frac{Q^{(1)}}{Q^{(0)}}. \quad (3.5)$$

160 The problem is then characterized by 11 dimensional parameters: the mean flow rate  
161  $[Q^{(0)}] = \text{m}^3 \text{s}^{-1}$ , the half-channel width  $[h] = \text{m}$ , the fluid density  $[\rho] = \text{kg m}^{-3}$ , the  
162 viscosity  $[\mu] = \text{kg s}^{-1} \text{m}^{-1}$ , the mass of the plate per unit area  $[m] = \text{kg m}^{-2}$ , the damping  
163 coefficient of the wall  $[d] = \text{kg m}^{-2} \text{s}^{-1}$ , the bending stiffness of the plate  $[B] = \text{kg m}^2 \text{s}^{-2}$ ,  
164 the wall tension  $[T] = \text{kg s}^{-2}$ , the spring stiffness  $[K] = \text{kg m}^{-2} \text{s}^{-2}$ , the pulsation frequency  
165  $[\Omega] = \text{s}^{-1}$  and the amplitude of the oscillating flow component  $[Q^{(1)}] = \text{m}^3 \text{s}^{-1}$ . Hence, the  
166 present configuration may be described by 8 non-dimensional control parameters.

167 The base flow is characterized by three non-dimensional parameters:

$$168 \quad \text{Re} = \frac{Q^{(0)}}{\nu}, \quad \text{Wo} = h \sqrt{\frac{\Omega}{\nu}} \quad \text{and} \quad \tilde{Q} = 2 \frac{Q^{(1)}}{Q^{(0)}}. \quad (3.6)$$

169 Here, the Reynolds number  $\text{Re}$  is based on the average fluid velocity, the channel diameter  
170 and the kinematic viscosity  $\nu = \mu/\rho$ ; the Womersley number  $\text{Wo}$  is a measure of the pulsation  
171 frequency and can be interpreted as the ratio of the channel half-diameter to the thickness  
172  $\delta = \sqrt{\nu/\Omega}$  of the oscillating Stokes boundary layers.

173 The parameters associated with the walls are non-dimensionalised with respect to the  
174 spring stiffness  $K$ , which leads to

$$175 \quad B_* = \frac{B}{Kh^4}, \quad T_* = \frac{T}{Kh^2} \quad \text{and} \quad d_* = \frac{d}{\sqrt{mK}}. \quad (3.7)$$

176 Finally, two non-dimensional parameters account for the coupling between the fluid and  
177 the compliant walls

$$178 \quad V_R = \frac{Q^{(0)}}{4h^2} \sqrt{\frac{m}{K}} \quad \text{and} \quad \Gamma = \frac{m}{\rho h}. \quad (3.8)$$

179 The reduced velocity  $V_R$  represents the ratio of the wall characteristic time scale  $\tau_K = \sqrt{\frac{m}{K}}$ ,

180 associated with the spring stiffness, to the characteristic flow advection time scale  $\tau_Q = \frac{4h^2}{Q^{(0)}}$   
 181 (de Langre 2000). The parameter  $\Gamma$  is the mass ratio between the walls and the fluid.

182 Unperturbed base configurations are thus completely specified by the 8 non-dimensional  
 183 control parameters (3.6–3.8). We further use  $\rho = 1$ ,  $h = 1$  and  $Q^{(0)} = 1$  to uniquely determine  
 184 dimensional quantities. Hereafter, to reduce the dimensionality of control-parameter space,  
 185 the mass ratio is kept constant at  $\Gamma = 2$  and we consider walls without tension  $T = 0$ .

#### 186 4. Linear stability analysis

187 For the stability analysis, the total flow fields are decomposed as the superposition of base  
 188 and small-amplitude perturbation fields:

$$189 \quad \mathbf{u}(x, y, t) = \mathbf{U}(y, t) + \mathbf{u}'(x, y, t), \quad (4.1)$$

$$190 \quad p(x, y, t) = G(t)x + p'(x, y, t). \quad (4.2)$$

191 The wall displacement is similarly written as

$$192 \quad \zeta^\pm(x, t) = \pm h + \eta^{\pm'}(x, t). \quad (4.3)$$

193 Since the base configuration is homogeneous in  $x$ , perturbation fields may be expressed as  
 194 spatial normal modes

$$195 \quad \mathbf{u}'(x, y, t) = \tilde{\mathbf{u}}(y, t)e^{i\alpha x} + c.c., \quad (4.4)$$

$$196 \quad p'(x, y, t) = \tilde{p}(y, t)e^{i\alpha x} + c.c., \quad (4.5)$$

$$197 \quad \eta^{\pm'}(x, t) = \tilde{\eta}^\pm(t)e^{i\alpha x} + c.c., \quad (4.6)$$

198 where  $\alpha$  denotes the streamwise wave number and *c.c.* stands for the complex conjugate.  
 199 Introducing this decomposition into the governing equations (2.1–2.3) and neglecting the  
 200 quadratic terms leads to the following system of coupled linear partial differential equations:

$$201 \quad \frac{\partial \tilde{u}}{\partial t} = -i\alpha U(y, t) \tilde{u} - \frac{\partial U}{\partial y}(y, t) \tilde{v} - \frac{1}{\rho} i\alpha \tilde{p} + \nu \left( \frac{\partial^2}{\partial y^2} - \alpha^2 \right) \tilde{u}, \quad (4.7)$$

$$202 \quad \frac{\partial \tilde{v}}{\partial t} = -i\alpha U(y, t) \tilde{v} - \frac{1}{\rho} \frac{\partial \tilde{p}}{\partial y} + \nu \left( \frac{\partial^2}{\partial y^2} - \alpha^2 \right) \tilde{v}, \quad (4.8)$$

$$203 \quad 0 = i\alpha \tilde{u} + \frac{\partial \tilde{v}}{\partial y}, \quad (4.9)$$

$$204 \quad m \frac{\partial \tilde{\gamma}^\pm}{\partial t} = -d\tilde{\gamma}^\pm - \left( B\alpha^4 + T\alpha^2 + K \right) \tilde{\eta}^\pm \pm \tilde{p}(y, t) \Big|_{\pm h} \mp \mu \frac{d\tilde{v}}{dy} \Big|_{\pm h}, \quad (4.10)$$

205 where we have introduced the additional functions  $\tilde{\gamma}^\pm = \partial_t \tilde{\eta}^\pm$  in order to reduce the system  
 206 to first-order differential equations in time. Note that the wall equations (4.10) assume a  
 207 pressure outside the channel walls always equal to the unperturbed pressure  $G(t)x$  prevailing  
 208 inside (see Lebbal *et al.* (2022) for further details). The boundary conditions at the perturbed  
 209 interface are expanded in a Taylor series about their equilibrium values at  $y = \pm h$  (Shankar  
 210 & Kumaran 2002). At linear order, the flow velocities at the walls are obtained as

$$211 \quad \mathbf{u}(x, y = \zeta^\pm, t) = \mathbf{u}'(x, y = \pm h, t) + \eta^{\pm'}(x, t) \frac{dU}{dy} \Big|_{(y=\pm h, t)} \mathbf{e}_x. \quad (4.11)$$

212 Thus, the kinematic boundary conditions (2.7) become

$$213 \quad \tilde{u}(y = \pm h, t) + \tilde{\eta}^\pm(t) \frac{dU}{dy} \Big|_{(y=\pm h, t)} = 0 \quad \text{and} \quad \tilde{v}(y = \pm h, t) = \tilde{\gamma}^\pm(t). \quad (4.12)$$

214 Since the base flow is time-periodic, the linear stability analysis proceeds by following  
 215 Floquet theory, where the eigenfunctions are assumed to have the same temporal periodicity  
 216 as the base flow. The perturbations are therefore further decomposed as

$$217 \quad \tilde{\mathbf{u}}(y, t) = \left[ \sum_n \hat{\mathbf{u}}^{(n)}(y) \exp(in\Omega t) \right] \exp(-i\omega t), \quad (4.13)$$

$$218 \quad \tilde{p}(y, t) = \left[ \sum_n \hat{p}^{(n)}(y) \exp(in\Omega t) \right] \exp(-i\omega t), \quad (4.14)$$

$$219 \quad \tilde{\eta}^\pm(t) = \left[ \sum_n \hat{\eta}^{\pm(n)} \exp(in\Omega t) \right] \exp(-i\omega t), \quad (4.15)$$

$$220 \quad \tilde{\gamma}^\pm(t) = \left[ \sum_n \hat{\gamma}^{\pm(n)} \exp(in\Omega t) \right] \exp(-i\omega t), \quad (4.16)$$

221 where the complex frequency  $\omega = \omega_r + i\omega_i$  is the eigenvalue, with  $\omega_i$  the growth rate and  
 222  $\omega_r$  the circular frequency. After substitution of these expansions, the linearized equations  
 223 governing the dynamics of small perturbations take the following form, for each integer  $n$ ,

$$224 \quad \omega \hat{u}^{(n)}(y) = \left[ n\Omega + iv \left( \partial_{yy} - \alpha^2 \right) \right] \hat{u}^{(n)}(y) + \frac{\alpha}{\rho} \hat{p}^{(n)}(y) \\ 225 \quad + \sum_k \left[ \alpha U^{(k)}(y) \hat{u}^{(n-k)}(y) - i \frac{dU^{(k)}}{dy} \hat{v}^{(n-k)}(y) \right], \quad (4.17)$$

$$226 \quad \omega \hat{v}^{(n)}(y) = \left[ n\Omega + iv \left( \partial_{yy} - \alpha^2 \right) \right] \hat{v}^{(n)}(y) + \frac{1}{\rho} \frac{d\hat{p}^{(n)}}{dy} \\ 227 \quad + \sum_k \left[ \alpha U^{(k)}(y) \hat{v}^{(n-k)}(y) \right], \quad (4.18)$$

$$228 \quad 0 = i\alpha \hat{u}^{(n)}(y) + \frac{\partial \hat{v}^{(n)}}{\partial y}, \quad (4.19)$$

$$229 \quad \omega \hat{\eta}^{\pm(n)} = n\Omega \hat{\eta}^{\pm(n)} + i\hat{\gamma}^{\pm(n)}, \quad (4.20)$$

$$230 \quad \omega \hat{\gamma}^{\pm(n)} = n\Omega \hat{\gamma}^{\pm(n)} - i \frac{d}{m} \hat{\gamma}^{\pm(n)} - \frac{i}{m} \left( B\alpha^4 + T\alpha^2 + K \right) \hat{\eta}^{\pm(n)} \\ 231 \quad \pm \frac{i}{m} \left( \hat{p}^{(n)}(\pm h) - \mu \frac{d\hat{v}^{(n)}}{dy} \Big|_{\pm h} \right), \quad (4.21)$$

232 together with the kinematic wall conditions

$$233 \quad \hat{u}^{(n)}(\pm h) = - \sum_k \frac{dU^{(k)}}{dy} \Big|_{\pm h} \hat{\eta}^{\pm(n-k)}, \quad (4.22)$$

$$234 \quad \hat{v}^{(n)}(\pm h) = \hat{\gamma}^{\pm(n)}. \quad (4.23)$$

235 The system of coupled linear differential equations (4.17–4.21) with boundary condi-  
 236 tions (4.22,4.23) forms the generalised eigenvalue problem that governs the dynamics of  
 237 small-amplitude perturbations developing in this time-periodic fluid–structure interaction  
 238 system.



## 239 5. Numerical methods

240 In this section, we outline the numerical strategy that has been implemented for solving the  
 241 generalised Floquet eigenvalue problem derived in the previous section. The main objectives  
 242 in this implementation are the elimination of spurious (nonphysical) eigenvalues and the  
 243 reduction of the required computational efforts. To that purpose, we follow the general  
 244 framework described by Manning *et al.* (2007).

245 The velocity and pressure components are discretized in the wall-normal direction using  
 246 a Chebyshev collocation method. To suppress spurious pressure modes, we consider the  
 247  $(\mathbb{P}_N, \mathbb{P}_{N-2})$ -formulation where the pressure is approximated with a polynomial of degree  
 248  $N - 2$  while the velocity is discretized with a polynomial of degree  $N$  (Schumack *et al.* 1991;  
 249 Boyd 2001; Peyret 2002). In classical fashion, velocity fields are therefore represented by  
 250 their values over  $N$  Gauss–Lobatto collocation points spanning the entire channel diameter  
 251 and including the boundary points, while the pressure fields use only the  $N - 2$  interior points.  
 252 We note the vectors containing the unknown velocity and pressure components at the interior  
 253 points for each Fourier mode:

$$254 \quad \mathbf{V}_I^{(n)} = \left( \hat{u}_2^{(n)}, \dots, \hat{u}_{N-1}^{(n)}, \hat{v}_2^{(n)}, \dots, \hat{v}_{N-1}^{(n)} \right), \quad (5.1)$$

$$255 \quad \mathbf{P}_I^{(n)} = \left( \hat{p}_2^{(n)}, \dots, \hat{p}_{N-1}^{(n)} \right). \quad (5.2)$$

256 Similarly, wall displacements and wall velocities are denoted by

$$257 \quad \mathbf{W}^{(n)} = \left( \hat{\eta}_1^{(n)}, \hat{\eta}_N^{(n)}, \hat{\gamma}_1^{(n)}, \hat{\gamma}_N^{(n)} \right). \quad (5.3)$$

258 The kinematic conditions (4.22,4.23) may be used to express the velocity values at the  
 259 boundaries in terms of the wall variables. As a consequence, the variables  $\hat{u}_1^{(n)}$ ,  $\hat{v}_1^{(n)}$ ,  $\hat{u}_N^{(n)}$   
 260 and  $\hat{v}_N^{(n)}$  may be directly eliminated from the problem together with the boundary conditions.  
 261 Then, using

$$262 \quad \hat{\mathbf{X}}^{(n)} = \left( \mathbf{V}_I^{(n)}, \mathbf{P}_I^{(n)}, \mathbf{W}^{(n)} \right) \quad (5.4)$$

263 for each harmonic of the Floquet eigenvector, the system (4.17–4.21) is recast as a generalised  
 264 algebraic eigenvalue problem of the form

$$265 \quad \hat{\mathbf{A}}^{(n)} \hat{\mathbf{X}}^{(n)} + \sum_k \hat{\mathbf{S}}^{(k)} \hat{\mathbf{X}}^{(n-k)} = i\omega \hat{\mathbf{B}}^{(n)} \hat{\mathbf{X}}^{(n)}. \quad (5.5)$$

266 Here, the square matrices  $\hat{\mathbf{A}}^{(n)}$ ,  $\hat{\mathbf{S}}^{(k)}$  and  $\hat{\mathbf{B}}^{(n)}$  are of size  $(3N - 2)^2$  and may be written in  
 267 bloc structure as

$$268 \quad \hat{\mathbf{A}}^{(n)} = \begin{pmatrix} \hat{\mathbf{A}}_{\mathbf{V}\mathbf{V}}^{(n)} & \hat{\mathbf{G}}_{\mathbf{V}} & \hat{\mathbf{A}}_{\mathbf{V}\mathbf{W}} \\ \hat{\mathbf{D}}_{\mathbf{V}} & \mathbf{0} & \hat{\mathbf{D}}_{\mathbf{W}} \\ \hat{\mathbf{A}}_{\mathbf{W}\mathbf{V}} & \hat{\mathbf{G}}_{\mathbf{W}} & \hat{\mathbf{A}}_{\mathbf{W}\mathbf{W}}^{(n)} \end{pmatrix}, \quad \hat{\mathbf{S}}^{(k)} = \begin{pmatrix} \hat{\mathbf{S}}_{\mathbf{V}\mathbf{V}}^{(k)} & \mathbf{0} & \hat{\mathbf{S}}_{\mathbf{V}\mathbf{W}}^{(k)} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{pmatrix} \quad \text{and} \quad \hat{\mathbf{B}}^{(n)} = \begin{pmatrix} \mathbb{I}_{\mathbf{V}\mathbf{V}} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbb{I}_{\mathbf{W}\mathbf{W}} \end{pmatrix}. \quad (5.6)$$

269 Their decomposition in terms of square blocs along the diagonal and rectangular blocs off  
 270 diagonal reflects the structure of the vectors  $\hat{\mathbf{X}}^{(n)}$  (5.4), with  $2N - 4$  variables for  $\mathbf{V}_I^{(n)}$ ,  
 271  $N - 2$  variables for  $\mathbf{P}_I^{(n)}$ , and 4 variables for  $\mathbf{W}^{(n)}$ . In the above expressions,  $n$  may in theory  
 272 take all integer values (positive and negative) but in practice the Fourier series are truncated  
 273 at  $|n| \leq N_f$  for some cut-off value  $N_f$ . Note also that the blocs for which the superscript  
 274  $(n)$  is not indicated in (5.6) do not depend on a specific harmonic and that the matrix  $\hat{\mathbf{B}}^{(n)}$   
 275 only consists of two identity blocs. The matrices  $\hat{\mathbf{S}}^{(k)}$  account for the advection terms due  
 276 to the pulsating base flow component  $U^{(k)}(y)$  and are responsible for the coupling of the

277 different Fourier components of the Floquet eigenfunctions. Since we consider pulsating  
 278 flows with only a single oscillating component  $U^{(\pm 1)}(y)$ , only the matrices  $\hat{\mathbf{S}}^{(k)}$  with  $|k| \leq 1$   
 279 are here non zero. As a result, the eigenvalue problem (5.5) has bloc-tridiagonal structure,  
 280 which allows the use of efficient solution methods such as a generalised form of the Thomas  
 281 algorithm.

282 The next step consists in eliminating the pressure by using the discrete version of the  
 283 divergence-free condition

$$284 \quad \hat{\mathbf{D}}_{\mathbf{V}} \mathbf{V}_{\mathbf{I}}^{(n)} + \hat{\mathbf{D}}_{\mathbf{W}} \mathbf{W}^{(n)} = \mathbf{0}. \quad (5.7)$$

285 Hence, applying this divergence operator to the parts of the algebraic system (5.5) corre-  
 286 sponding to the momentum equations yields:

$$287 \quad -\left(\hat{\mathbf{D}}_{\mathbf{V}} \hat{\mathbf{G}}_{\mathbf{V}} + \hat{\mathbf{D}}_{\mathbf{W}} \hat{\mathbf{G}}_{\mathbf{W}}\right) \mathbf{P}_{\mathbf{I}}^{(n)} = \left(\hat{\mathbf{D}}_{\mathbf{V}} \quad \hat{\mathbf{G}}_{\mathbf{V}}\right) \begin{pmatrix} \hat{\mathbf{A}}_{\mathbf{V}\mathbf{V}}^{(n)} & \hat{\mathbf{A}}_{\mathbf{V}\mathbf{W}}^{(n)} \\ \hat{\mathbf{A}}_{\mathbf{W}\mathbf{V}}^{(n)} & \hat{\mathbf{A}}_{\mathbf{W}\mathbf{W}}^{(n)} \end{pmatrix} \begin{pmatrix} \mathbf{V}_{\mathbf{I}}^{(n)} \\ \mathbf{W}^{(n)} \end{pmatrix} \\
 288 \quad + \hat{\mathbf{D}}_{\mathbf{V}} \sum_k \left(\hat{\mathbf{S}}_{\mathbf{V}\mathbf{V}}^{(k)} \mathbf{V}_{\mathbf{I}}^{(n-k)} + \hat{\mathbf{S}}_{\mathbf{V}\mathbf{W}}^{(k)} \mathbf{W}^{(n-k)}\right). \quad (5.8)$$

289 The operator  $(\hat{\mathbf{D}}_{\mathbf{V}} \hat{\mathbf{G}}_{\mathbf{V}} + \hat{\mathbf{D}}_{\mathbf{W}} \hat{\mathbf{G}}_{\mathbf{W}})$  is a square matrix of size  $(N - 2)^2$ , independent of the  
 290 harmonic  $n$  and non singular. By inverting it, the pressure components  $\mathbf{P}_{\mathbf{I}}^{(n)}$  are obtained as  
 291 the result of linear operators acting on the components  $\mathbf{V}_{\mathbf{I}}^{(k)}$  and  $\mathbf{W}^{(k)}$ .

292 Thus eliminating the pressure, the system (5.5) is recast as

$$293 \quad \mathbf{A}^{(n)} \mathbf{X}^{(n)} + \sum_k \mathbf{S}^{(k)} \mathbf{X}^{(n-k)} = i\omega \mathbf{X}^{(n)}. \quad (5.9)$$

294 Now, the components of the eigenvector

$$295 \quad \mathbf{X}^{(n)} = \left(\mathbf{V}_{\mathbf{I}}^{(n)}, \mathbf{W}^{(n)}\right) \quad (5.10)$$

296 contain  $2N$  variables and the new matrices  $\mathbf{A}^{(n)}$  and  $\mathbf{S}^{(k)}$  are of size  $(2N)^2$ . Note also  
 297 that, through the elimination of the pressure, the generalised eigenproblem (5.5) has been  
 298 transformed into a regular eigenproblem.

299 The system (5.9) may be further reduced when  $\alpha \neq 0$ . Indeed, using the discretized  
 300 version of the divergence-free condition  $\tilde{u} = \frac{i}{\alpha} \partial_y \tilde{v}$ , allows to eliminate the longitudinal  
 301 velocity components by expressing them in terms of the wall-normal velocities. This leads  
 302 to an eigenvalue problem of the same form as (5.9) where the components of the eigenvector  
 303 are of size  $N + 2$ , with  $N - 2$  wall-normal velocity values and 4 wall variables. In practice,  
 304 this system is solved using an Arnoldi algorithm that exploits the bloc-tridiagonal structure  
 305 of the matrices.

306 The transformations that have led from the initial generalised eigenvalue problem of size  
 307  $3N + 2$  to a regular eigenproblem of size  $N + 2$  may appear tedious. However, it is largely  
 308 worth the effort: the final formulation is not only free of spurious eigenmodes, it is also  
 309 drastically more efficient in terms of numerical computations. Finally, the method may be  
 310 further improved by considering separately perturbations of sinuous or varicose symmetries  
 311 and using only half of the channel together with derivative operators appropriate for the  
 312 symmetry of each component of the different flow fields. Thus the complete problem may  
 313 be addressed by carrying out two eigenvalue computations (sinuous and varicose) of half  
 314 size, which further speeds up the process and directly provides the information about the  
 315 symmetry of the different modes.

316 The numerical method has been validated using the results given by Pier & Schmid (2017)

317 for the pulsatile flow inside a rigid channel and those provided by Davies & Carpenter  
 318 (1997a) for the steady base flow between compliant walls.

## 319 6. Stability analysis

320 In fluid–structure interaction problems, perturbations are classically identified as either fluid-  
 321 based or solid-based instabilities, depending on the nature of the main driving mechanism.  
 322 For steady flows in compliant channels, Tollmien–Schlichting (TS) modes are known as  
 323 the prevailing fluid-based instabilities: they are driven by a viscous instability within the  
 324 bulk of the flow and only weakly affected by wall compliance. On the other hand, solid-  
 325 based instabilities are due to the interaction between a compliant wall and the fluid flow  
 326 and are often labelled as fluid–structure interaction (FSI) modes since their dynamics is  
 327 predominantly governed by the wall parameters. Among these FSI modes, one may further  
 328 distinguish between travelling-wave flutter (TWF) and divergence (DIV) modes: while DIV  
 329 modes are almost stationary, TWF modes display phase velocities of the same order as those  
 330 of elastic waves due to the wall only. However, such a mode classification is not always  
 331 possible since one type may continuously transform into another. For example, when the  
 332 phase speed of TWF modes approaches that of TS modes, merging may occur to form a  
 333 transition instability mode.

334 The purpose of the present study is to identify the main instability modes in the presence  
 335 of a pulsating base flow and to document their dynamics for a wide range of flow and wall  
 336 parameters. We first discuss how the eigenvalue spectra are modified due to the time-periodic  
 337 flow component. Then, the influence of the main control parameters on the dominant modes  
 338 is investigated, with special attention given to possible cross over between different mode  
 339 types. Close monitoring of the spatial and temporal structure of the eigenfunctions reveals  
 340 the rich dynamics of perturbations interacting with the pulsating flow and the compliant  
 341 wall. Finally, the multi-dimensional parameter space is mapped out using a variety of critical  
 342 curves for instability onset.

### 343 6.1. The Floquet eigenspectrum

344 The linear stability properties of time-periodic flow configurations is addressed by resorting  
 345 to Floquet theory, as explained in section §4. In order to introduce the specific features of  
 346 Floquet eigenspectra that are essential to this entire investigation, we illustrate them for a  
 347 situation with small pulsation amplitude  $\tilde{Q}$ , and compare them with the corresponding steady  
 348 case.

349 Figure 2 shows the eigenvalue spectrum computed with  $\alpha = 1$  for a pulsating base  
 350 configuration characterized by  $V_R = 1$ ,  $Wo = 10$ ,  $\tilde{Q} = 0.02$ ,  $Re = 10000$ ,  $B_* = 4$  and  $d_* = 0$ .  
 351 This plot reveals the characteristic feature of any Floquet spectrum: multiple eigenvalues  
 352 of same growth rate  $\omega_i$  and frequencies  $\omega_r$  separated by integer multiples of the base  
 353 frequency  $\Omega$ . This is due to the fact that if  $\omega$  is a complex eigenvalue associated with an  
 354 eigenfunction of the form (4.13–4.16), then all frequencies  $\omega_* = \omega + k\Omega$  (for any positive or  
 355 negative integer  $k$ ) are also among the eigenvalues and their associated eigenfunctions are  
 356 simply obtained by similarly shifting the Fourier components in the Floquet expansion as,  
 357 for example,  $\hat{\mathbf{u}}_*^{(n)}(\mathbf{y}) = \hat{\mathbf{u}}^{(n-k)}(\mathbf{y})$ . In theory, the Fourier expansions (4.13–4.16) are infinite  
 358 series, and the infinite number of eigenvalues  $\omega + k\Omega$  correspond thus all to the same physical  
 359 perturbation. In practice, however, the Fourier expansions are truncated to a finite number of  
 360 components, leading to a finite set of eigenvalues  $\omega_*$ . These are then no longer exactly equal  
 361 to  $\omega + k\Omega$  and the associated normal modes also differ since they correspond to different  
 362 truncations of the Fourier series.

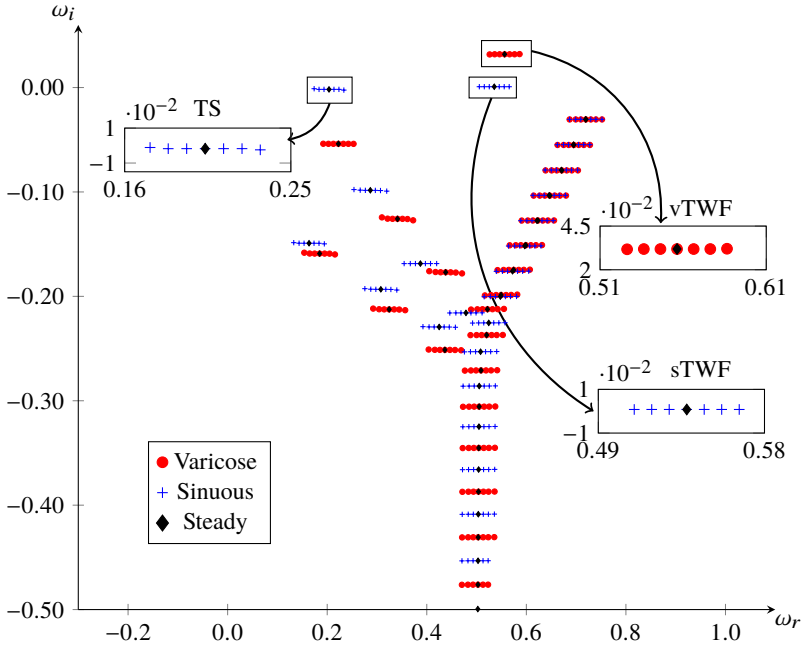


Figure 2: Spectrum for  $\tilde{Q} = 0.02$ ,  $Wo = 10$ ,  $V_R = 1$ ,  $Re = 10000$ ,  $\alpha = 1$ ,  $B_* = 4$  and  $d_* = 0$ . The steady case corresponds to  $\tilde{Q} = 0$ .

363 In figure 2, the Floquet eigenspectrum for  $\tilde{Q} = 0.02$  is shown together with the eigenspec-  
 364 trum of the corresponding steady configuration (at  $\tilde{Q} = 0$ ). This superposition illustrates the  
 365 close similarity of both spectra and reveals that each of the steady eigenvalues is located very  
 366 near one of the Floquet eigenvalues (see insets). Here, the Floquet spectrum corresponds to  
 367 a weakly modulated base flow, for which the oscillating base-flow component  $U^{(\pm 1)}(y)$  is  
 368 much smaller than the Poiseuille component  $U^{(0)}(y)$ . Thus, the magnitude of the off-diagonal  
 369 blocs  $\mathbf{S}^{(\pm 1)}$  in the Floquet eigenproblem (5.9) is small in comparison with the diagonal blocs  
 370 and the adjacent Fourier components in the Floquet eigenfunction are therefore only weakly  
 371 coupled. As a result, the growth rates in the eigenspectrum here closely follow those prevailing  
 372 for the equivalent steady flow. For weakly modulated base flows, as is the case in figure 2,  
 373 it seems thus natural to choose the eigenvalue  $\omega + k\Omega$  nearest its steady counterpart as the  
 374 most representative frequency of the Floquet normal mode.

375 For larger pulsation amplitudes  $\tilde{Q}$ , however, this similarity with the steady spectrum no  
 376 longer holds and a more robust criterion is required to lift the formal degeneracy of the Floquet  
 377 eigenspectrum. In order to identify the most representative frequency among the multiple  
 378 Floquet eigenvalues for each normal mode, we consider the magnitude of the different Fourier  
 379 components of the Floquet eigenfunctions, defined as

$$\begin{aligned}
 380 \quad E_n &= \rho \int_{-h}^{+h} |\hat{\mathbf{u}}^{(n)}(y)|^2 dy \\
 381 \quad &+ m \left( |\hat{\gamma}^{+(n)}|^2 + |\hat{\gamma}^{-(n)}|^2 \right) + \left( B\alpha^4 + T\alpha^2 + K \right) \left( |\hat{\eta}^{+(n)}|^2 + |\hat{\eta}^{-(n)}|^2 \right). \quad (6.1)
 \end{aligned}$$

382 By using this energy-based norm, it is possible to single out the dominant component in  
 383 the eigenfunction Fourier expansion and also to check if the truncation contains enough  
 384 harmonics for an accurate representation of the normal mode. This process is illustrated in

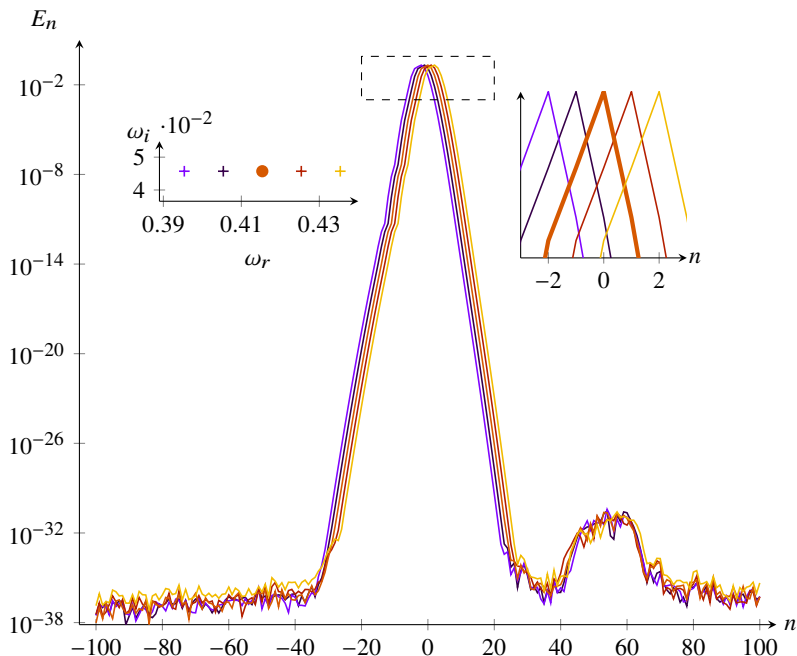


Figure 3: Fourier density for FSI varicose mode associated with  $\alpha = 0.8$ ,  $\text{Re} = 10000$ ,  $\tilde{Q} = 0.2$ ,  $\text{Wo} = 10$ ,  $V_R = 1$ ,  $B_* = 4$  and  $d_* = 0$ .

385 figure 3, where the magnitudes  $E_n$  are plotted for five consecutive eigenvalues corresponding  
 386 to the varicose TWF mode associated with  $\alpha = 0.8$ ,  $\text{Re} = 10000$ ,  $\tilde{Q} = 0.2$ ,  $\text{Wo} = 10$ ,  $V_R = 1$ ,  
 387  $B_* = 4$  and  $d_* = 0$ . It is observed that the  $E_n$ -distribution peaks at  $n = 0$  for the eigenvalue  
 388  $\omega = 0.415 + 0.046i$ , while the distributions associated with the surrounding eigenfrequencies  
 389  $\omega + k\Omega$  peak at  $n = k$  since they correspond to similarly shifted Fourier components. It  
 390 follows that  $\omega = 0.415 + 0.046i$  is the dominant frequency of this eigenmode. Throughout this  
 391 paper we will therefore always consider that, for a given mode, the dominant frequency is  
 392 derived by this energy-based criterion and choose the eigenvalue for which the Fourier series  
 393 is centered on the  $n = 0$  component, i.e. for which  $E_0$  is largest. For the rigid wall case, this  
 394 method has been proven to be effective in recovering the TS mode frequency obtained using  
 395 linearized direct numerical simulation (results are given in Pier & Schmid (2017)). Note also  
 396 that the plots in figure (3) demonstrate that we are using more than enough harmonics to  
 397 fully resolve the Floquet eigenfunctions.

398 The Floquet eigenfunctions correspond to either sinuous or varicose modes, depending  
 399 on the symmetry or antisymmetry of the different flow fields with respect to the mid-plane  
 400  $y = 0$ . As explained in section §5, they may be efficiently computed by taking advantage  
 401 of these symmetry properties. In the spectrum of figure 2, the sinuous eigenfrequencies are  
 402 given in blue and the varicose frequencies in red. Despite the multiplicity of the eigenvalues  
 403 due to the time-periodic base flow, the spectrum still displays the familiar structure made of a  
 404 large number of Orr–Sommerfeld modes (as A, P and S branches) together with two isolated  
 405 TWF modes (one sinuous and one varicose). Note that the two DIV modes are here out of  
 406 the range of this plot.

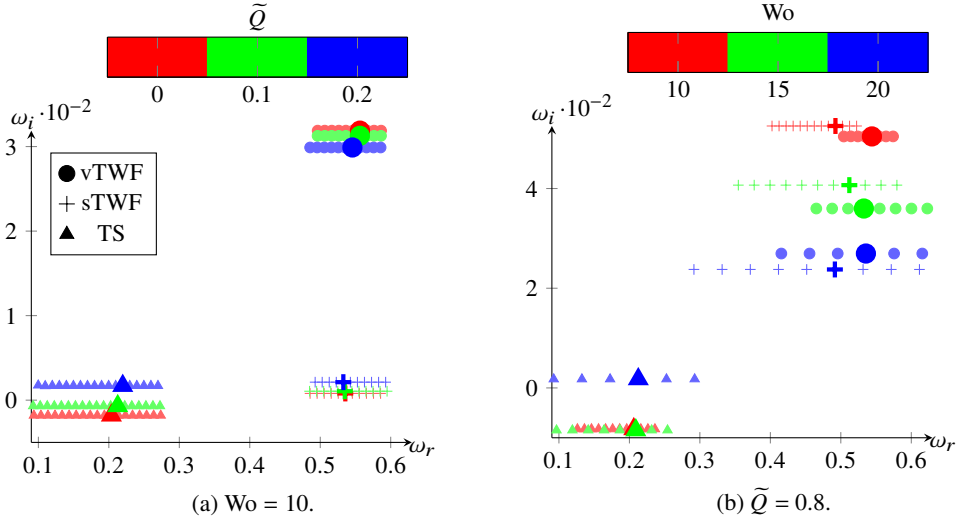


Figure 4: Spectra with  $V_R = 1$ ,  $Re = 10000$ ,  $\alpha = 1$ ,  $B_* = 4$  and  $d_* = 0$ .

407

### 6.2. Influence of some parameters on the spectrum

408 The figure 4a displays spectra for  $V_R = 1$ ,  $Wo = 10$ ,  $d_* = 0$  and  $\tilde{Q}$  varying from 0 (i.e. the  
 409 steady flow case) to  $\tilde{Q} = 0.2$ . The figure 4b illustrates the effect of the base-flow frequency  
 410 by varying  $Wo$  from 10 to 20. As discussed in the previous section, the figure shows modes  
 411 that exhibit equispaced eigenfrequencies where the gap between two successive frequencies  
 412 corresponds to the base frequency  $\Omega$ , which scales as the square root of the Womersley  
 413 number. In the figure 4, the bold eigenvalues are associated with the dominant frequency for  
 414 each mode, obtained by considering the magnitude of the Floquet harmonics as explained in  
 415 the previous subsection. Concentrating on FSI modes, dominant frequencies for both sinuous  
 416 and varicose symmetries have finite  $\omega_r$  values. These FSI modes are thus connected to TWF  
 417 instability waves. They are referenced hereafter as sTWF (sinuous TWF) or vTWF (varicose  
 418 TWF) depending on their symmetry with respect to the midplane  $y = 0$ .

419 For flow and wall parameters that are considered and  $\tilde{Q} = 0$ , the most amplified TWF  
 420 mode is of varicose type (see figure 4a). For this steady base flow case, the sTWF is seen to  
 421 be marginally stable and the temporal growth rate of the TS mode is damped. For  $Wo = 10$ ,  
 422 an increase of  $\tilde{Q}$  tends to destabilise TS wave (see figure 4a). This is reminiscent of the  
 423 results of Pier & Schmid (2017) where TS modes for a pulsatile base flow between rigid  
 424 walls have been computed. By contrast, the TWF modes exhibit distinct behaviours whether  
 425 the sinuous or varicose symmetry is considered. While an increase of  $\tilde{Q}$  leads to a reduction  
 426 of the temporal amplification rate for the varicose type, the opposite behaviour is observed  
 427 for the sinuous TWF mode. Figure 4b shows the effect of the Womersley number  $Wo$  onto  
 428 TS and TWF modes for the same case. An increase of  $Wo$  has a stabilising effect on TWF  
 429 modes for both symmetries. The opposite role of  $Wo$  is seen for the TS mode. This reflects  
 430 the richness of physical processes that are involved, in comparison to the rigid wall case.

431 Finally, figure 5 shows the effect of wall compliance on TS and TWF modes for  $\tilde{Q} =$   
 432  $0.2$ ,  $Wo = 10$ ,  $Re = 10000$ ,  $B_* = 4$  and  $\alpha = 1$ . Only Floquet modes that match the  
 433 dominant frequency for each mode are shown. As  $V_R$  is approaching zero, the phase speed  
 434 of TWF modes tends to infinity which is consistent with the rigid wall case. An increase of  
 435  $V_R$  has a stabilising effect on the TS mode. The opposite behaviour is observed for TWF  
 436 modes whatever the symmetry considered. However, the figure suggests a preferred varicose

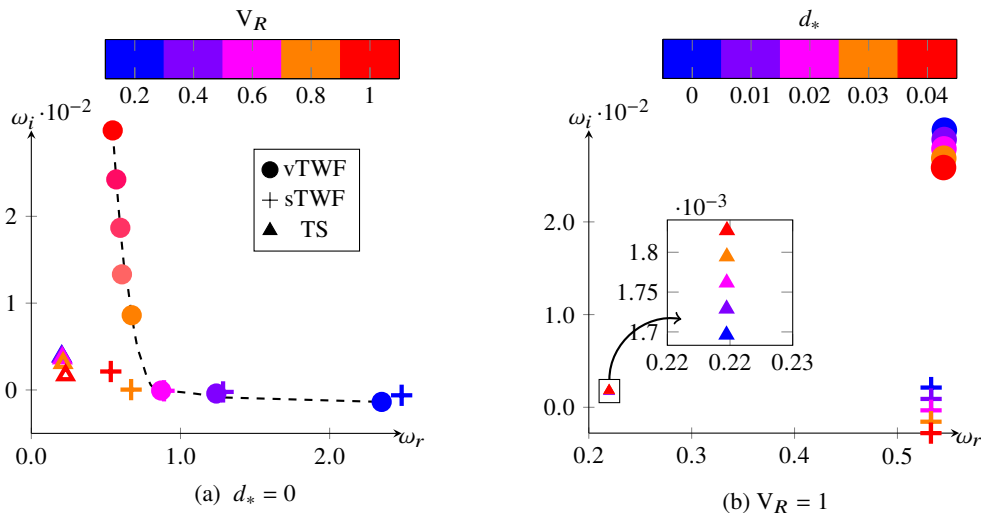


Figure 5: Spectra with  $\tilde{Q} = 0.2$ ,  $Wo = 10$ ,  $Re = 10000$ ,  $\alpha = 1$  and  $B_* = 4$ .

437 symmetry for large  $V_R$ . Parenthetically, one can see in figure 5a that the phase speed tends to a finite value ( $\approx 0.5$ ) for both sTWF and vTWF as wall compliance is increasing. The  
 438 influence of the wall dissipation is illustrated in figure 5b for small values of  $d_*$ . The figure  
 439 shows that the temporal amplification rate of the TS mode is slightly enhanced by increasing  
 440  $d_*$ . In contrast, growth rates of both sTWF and vTWF modes are significantly reduced by  
 441 wall dissipation.  
 442

443 These behaviours suggest similarities between the steady case and our results. Especially,  
 444 for the parameters that have been considered, the distinction made between class A and B  
 445 modes by Benjamin (1963), Landahl (1962) and Carpenter & Garrad (1985) still holds for  
 446 our pulsatile flow case. However, we will see in the next section that this classification is  
 447 clearly too restrictive for pulsatile base flows.

### 448 6.3. Waves superposition for sinuous Floquet mode

449 The Fourier density distributions for a sinuous Floquet mode associated with  $V_R = 1$ ,  $Wo =$   
 450  $10$ ,  $Re = 10000$ ,  $\alpha = 1$ ,  $B_* = 4$  and  $d_* = 0$  are shown in figure 6 together with the associated  
 451 eigenvalues for  $\tilde{Q} = 0.349$  and  $0.350$ . Both series of eigenvalues have positive growth rates.  
 452 The figure shows the existence of two distinct peaks in the Fourier density distribution. It  
 453 suggests that two different mechanisms influence this mode. To further illustrate this scenario,  
 454 the different contributions of the total energy per Fourier mode are also reported in figure 6.  
 455 For the  $E_n$ -distribution at  $\tilde{Q} = 0.349$  (left curves), the main peak is due to the fluid kinetic  
 456 energy contribution, associated with a dominant frequency of  $\omega_r \approx 0.26$ . In contrast, for  $\tilde{Q} =$   
 457  $0.350$ , the wall contributions take over, leading to a dominant frequency of  $\omega_r \approx 0.52$ . For  
 458 both cases, the  $E_n$ -distributions are very similar, but the exchange in dominant peaks due  
 459 to a continuous modification of the distribution results in a sudden jump of the dominant  
 460 frequency. This behaviour indicates that the intracyclic mechanism involves the interference  
 461 between fluid-based (TS) and solid-based (TWF) modes. By contrast with the steady base  
 462 flow case, we can therefore no longer distinguish here between class A and class B modes.  
 463 Moreover, figure 6 also shows that for  $\tilde{Q} < 0.35$ , the Floquet mode is mainly driven by its  
 464 TS component. When  $\tilde{Q}$  is increased up to  $0.35$ , the intracyclic growth is mainly due to its  
 465 sTWF part. This new type of mode will be called hereafter a two-waves mode.



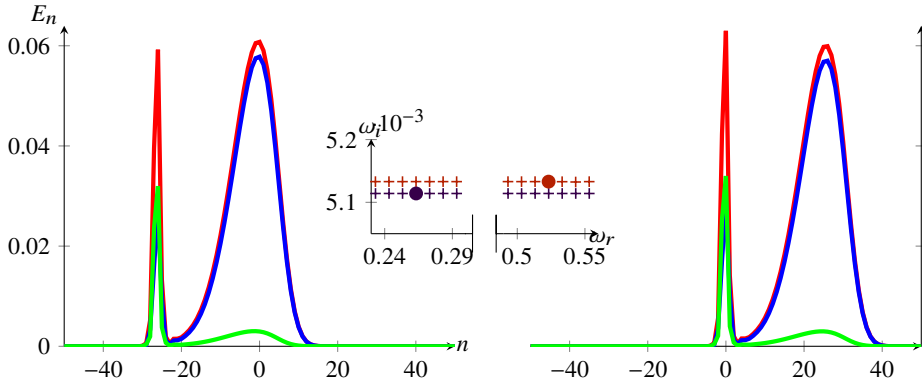


Figure 6: Energy per Floquet modes (red for total energy, blue for fluid kinetic energy, green for the wall potential and kinetic energy) at  $\tilde{Q} = 0.349$  (left) and  $\tilde{Q} = 0.350$  (right) with  $V_R = 1$ ,  $Wo = 10$ ,  $Re = 10000$ ,  $\alpha = 1$ ,  $B_* = 4$  and  $d_* = 0$ . The frequency associated with the dominant peak is indicated by  $\bullet$ .

466 For the same set of parameters, the influence of  $\tilde{Q}$  on the stability of the system is shown  
 467 in figure 7. For comparison purposes, the rigid case is also reported. We restrict our analysis  
 468 to the sinuous symmetry. For the compliant wall case, the evolution of both temporal growth  
 469 rates and circular frequencies are displayed for the first and second most amplified Floquet  
 470 modes. The dominant Floquet frequency for each mode is selected using the methodology  
 471 mentioned in the previous section. The TS mode distribution is seen to closely follow its  
 472 rigid wall counterpart up to  $\tilde{Q} = 0.3$ . When  $\tilde{Q}$  exceeds this value, however, its  $E_n$ -distribution  
 473 exhibits two peaks and the mode consists of the superposition of TS and TWF waves (as  
 474 illustrated in figure 6). For  $\tilde{Q}$  greater than 0.35 the energy peak is connected to the TWF  
 475 wave. At this point, the dominant frequency for this mode is associated with wall dynamics as  
 476 shown by the energy contributions in figure 6. For  $\tilde{Q}$  up to 0.5, we observe the co-existence  
 477 of TWF and two-waves modes. For  $0.5 < \tilde{Q} < 0.6$ , the two-waves modes is seen to be  
 478 temporally damped. By contrast, the growth rate of TWF mode is increased for this range  
 479 of  $\tilde{Q}$ .

480 For larger values of  $\tilde{Q}$ , the spectrum features a single unstable mode that shares the main  
 481 characteristics of TWF modes.

482 In the next section, we will describe the spatio-temporal behaviour of Floquet eigenfunc-  
 483 tions, for TWF, TS and two-waves modes.

#### 484 6.4. Spatial structure of eigenmodes

485 The structure of TS and FSI Floquet modes are investigated in more detail by monitoring  
 486 the wall-normal distribution of their flow kinetic energy. To that purpose we define the fluid  
 487 kinetic energy of the Floquet mode (4.13–4.16) as

$$488 \hat{E}(y, t) = \frac{1}{2} \rho \left| \sum_n \hat{\mathbf{u}}^{(n)}(y) \exp(in\Omega t) \right|^2, \quad (6.2)$$

489 which is periodic in time and may be used to characterize the intracyclic dynamics, since it  
 490 does not contain the long-term exponential growth (or decay) part.



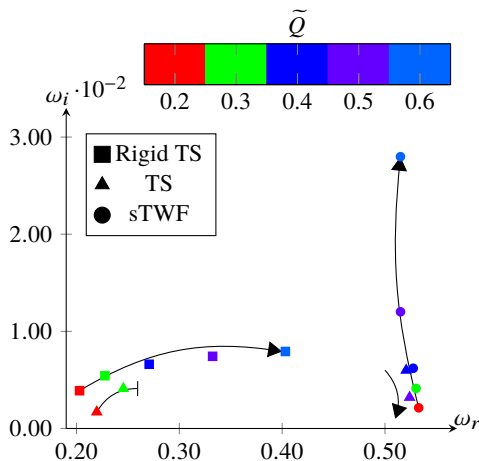


Figure 7: Spectra for different  $\tilde{Q}$  with  $V_R = 1$ ,  $Wo = 10$ ,  $Re = 10000$ ,  $\alpha = 1$ ,  $B_* = 4$  and  $d_* = 0$ .

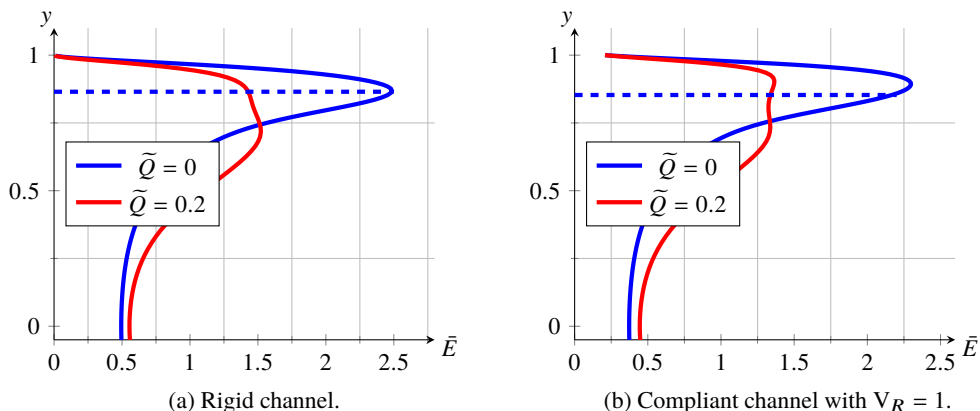


Figure 8: The wall-normal distribution  $\bar{E}(y)$  of averaged kinetic energy for the TS perturbations with  $\tilde{Q} = 0, 0.2$ ,  $Wo = 10$ ,  $Re = 10000$ ,  $\alpha = 1$ ,  $B_* = 4$  and  $d_* = 0$ . The critical layer position is indicated by blue dashed lines.

491 First, we will discuss the temporally averaged energy distribution, obtained as

$$492 \quad \bar{E}(y) = \frac{1}{T} \int_0^T \hat{E}(t, y) dt. \quad (6.3)$$

493 In figure 8b,  $\bar{E}(y)$  is shown for the TS mode for  $Wo = 10$ ,  $\tilde{Q} = 0.2$ ,  $Re = 10000$ ,  $\alpha = 1$ ,  
494  $B_* = 4$ ,  $d_* = 0$  and  $V_R = 1$ . For comparison purposes, the rigid wall and steady base flow  
495 cases are also reported in figure 8a.

496 We recall that for the case of a Poiseuille flow between flat rigid walls,  $\bar{E}(y)$  should  
497 peak around the critical layer, i.e. the wall-normal position where the phase speed equals  
498 the base-flow velocity (Drazin & Reid 1981), as shown in figure 8a. For compliant walls  
499 and the steady flow case, a similar behaviour is observed (Davies & Carpenter 1997a), see  
500 figure 8b. However, a slight shift near the wall is observed for the peak in kinetic energy as  
501 a consequence of the stabilising effect of the compliant wall onto TS modes.

502 The distribution of  $\bar{E}(y)$  for the time-periodic base flow exhibits a double peak structure

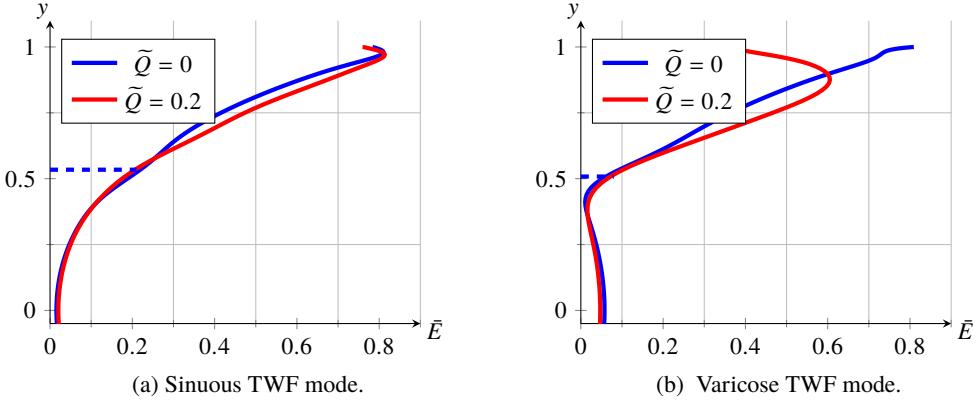


Figure 9: The wall-normal distribution of energy for TWF modes are shown for  $\tilde{Q} = 0$  and  $0.2$ ,  $V_R = 1$ ,  $Wo = 10$ ,  $Re = 10000$ ,  $\alpha = 1$ ,  $B_* = 4$  and  $d_* = 0$ .

503 for both the rigid wall and compliant wall cases (see red curves in figure 8). This behaviour  
 504 has also been observed by Singer *et al.* (1989) for the same flow case by using linearized  
 505 direct numerical simulations and assuming rigid walls. They have shown that in a certain  
 506 moment of the cycle, the mean flow profile exhibits an inflection point. These authors came  
 507 to the conclusion that this second peak is a consequence of changes in the base flow profile  
 508 during the cycle. Interestingly, figure 8 shows that the wall compliance enhances the first  
 509 peak. More recently, Tsigklifis & Lucey (2017) also noticed this tendency for a similar case.  
 510 The inner peak is also shifted closer to the wall due to wall compliance. It suggests that the  
 511 stabilising effect of elastic walls onto TS modes still holds for pulsatile base flows.

512 The wall-normal distributions of  $\bar{E}(y)$  for both varicose and sinuous TWF modes are  
 513 shown in figure 9 for the same set of parameters. While for the sinuous symmetry, the TWF  
 514 mode exhibits no significant changes in comparison with the steady flow case, the varicose  
 515 TWF mode displays clearly a different structure near the walls. Indeed, the amplitude of  
 516 vTWF mode peaks near  $y = 0.9$  for the pulsatile base flow case, while it exhibits its  
 517 maximum at the wall for the steady case. The consequences onto stability properties will be  
 518 discussed in the next sections.

### 519 6.5. Temporal dynamics of Floquet eigenmodes

520 In this section, we investigate the intracyclic dynamics of the perturbations for the same  
 521 configurations as in the previous section. For that purpose, we consider the instantaneous  
 522 total perturbation energy, defined as

$$523 \quad \tilde{E}(t) = \frac{1}{2h} \int_{-h}^h \hat{E}(y, t) dy + \tilde{E}_{WK}(t) + \tilde{E}_{WP}(t). \quad (6.4)$$

524 This quantity  $\tilde{E}(t)$  is the sum of the instantaneous fluid kinetic energy and the kinetic and  
 525 potential energies of the walls:

$$526 \quad \tilde{E}_{WK}(t) = m \left| \sum_n \hat{\gamma}^{\pm(n)} \exp(in\Omega t) \right|^2, \quad (6.5)$$

$$527 \quad \tilde{E}_{WP}(t) = (B\alpha^4 + T\alpha^2 + K) \left| \sum_n \hat{\eta}^{\pm(n)} \exp(in\Omega t) \right|^2. \quad (6.6)$$

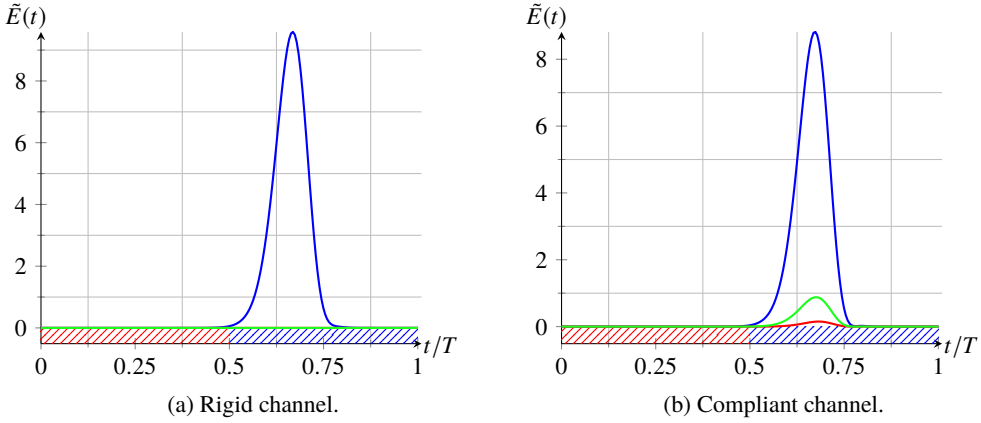


Figure 10: The intracyclic modulation of energy for the TS perturbations with  $\tilde{Q} = 0.2$ ,  $Wo = 10$ ,  $Re = 10000$  and  $\alpha = 1$ . Color : blue for the fluid kinetic energy, green for the wall kinetic energy, red for wall potential energy. (a) Rigid channel, (b) compliant channel with  $B_* = 4$ ,  $d_* = 0$  and  $V_R = 1$ .

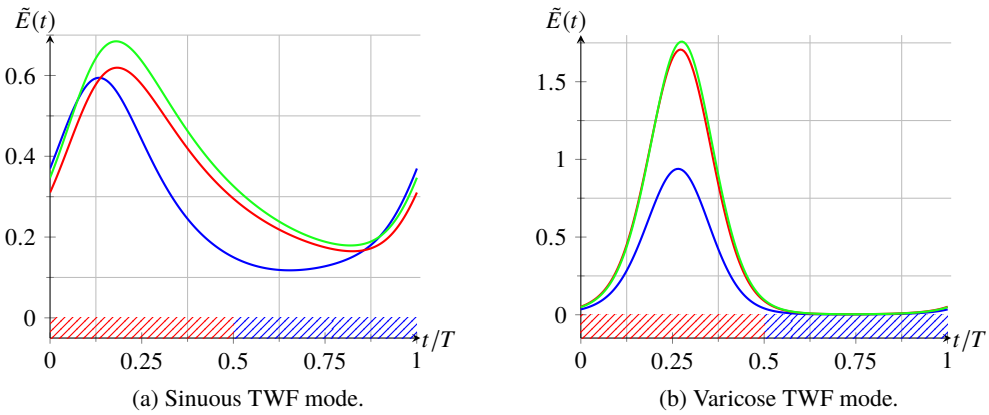


Figure 11: The intracyclic modulations of energy for the TWF perturbations with  $\tilde{Q} = 0.2$ ,  $V_R = 1$ ,  $Wo = 10$ ,  $Re = 10000$ ,  $\alpha = 1$ ,  $B_* = 4$  and  $d_* = 0$ .

528 Recall that all these quantities are temporally periodic (with period  $\Omega$ ) since the complex  
529 exponential term  $\exp(-i\omega t)$  has been removed.

530 We first consider the TS mode. Figure 10 shows the different contributions to  $\tilde{E}(t)$  for  
531  $Wo = 10$ ,  $\tilde{Q} = 0.2$ ,  $Re = 10000$ ,  $B_* = 4$ ,  $d_* = 0$  and  $\alpha = 1$  for both the rigid case and  
532 compliant walls with  $V_R = 1$ . As noted by Pier & Schmid (2017) for the rigid case, the  
533 growth of  $\tilde{E}(t)$  occurs in the deceleration phase of the base flow (indicated by regions  
534 hatched in red along the  $t$ -axis), while decay occurs during the acceleration phase (hatched  
535 in blue). This remains true for the compliant wall configuration (figure 10b). In particular,  
536 the dynamics of the perturbation is mainly driven by the flow kinetic energy while the wall  
537 energy is almost negligible. A similar behaviour has also been observed by Tsigklifis &  
538 Lucey (2017).

539 The intracyclic dynamics associated with TWF disturbances exhibits a markedly different  
540 behaviour as shown in figure 11. In contrast with the TS mode, the growth in total energy  
541 occurs during the acceleration phase of the base flow for both symmetries. Especially, walls

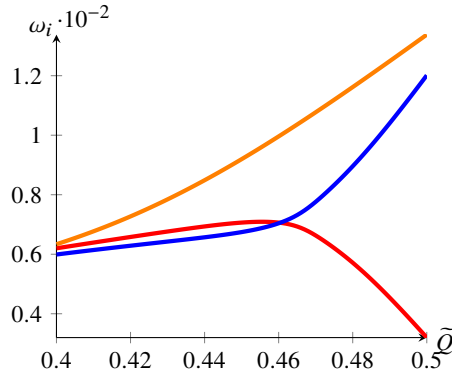


Figure 12: Dispersion relation with  $V_R = 1$ ,  $Wo = 10$ ,  $Re = 10000$ ,  $\alpha = 1$ ,  $B_* = 4$  and  $d_* = 0$  (blue for sTWF, red for TS and orange for sTWF at  $Re = 2000$ ).

542 mainly contribute to the total energy growth whatever the symmetry that is considered.  
 543 However, the fluid kinetic energy is less significant for the varicose case than the sinuous  
 544 one.

#### 545 6.6. Interaction between TS and sTWF waves

546 As mentioned earlier, a two-waves mode (i.e. where both FSI and TS waves interact) may  
 547 emerge for a given set of parameters.

548 Here we will further investigate the occurrence of this Floquet mode. To that purpose, the  
 549 parameters  $Wo = 10$ ,  $B_* = 4$ ,  $d_* = 0$ ,  $V_R = 1$  and  $\alpha = 1$  are considered. Two Reynolds  
 550 numbers will be studied  $Re = 2000$  and  $Re = 10000$  (i.e. corresponding to either subcritical  
 551 or supercritical regime with respect to TS mode associated with the rigid case.)

552 In figure 12, the variation with  $\tilde{Q}$  of the temporal growth rates for both two-waves mode  
 553 and sTWF mode are shown. For  $Re = 10000$ , the figure shows that these modes merge at  
 554  $\tilde{Q} \approx 0.46$ . Beyond this value, we observe that both modes diverge away from one other. Then,  
 555 one mode exhibits a strong increase in temporal growth rate. This behaviour is similar to  
 556 modes coalescence found in the steady case between TS and sTWF modes when increasing  
 557 wall dissipation, giving rise to the so-called transition mode. Interestingly, for pulsatile flow,  
 558 this phenomenon also appears for  $d_* = 0$ . Then, for  $\tilde{Q} \approx 0.46$ , we can associate this mode  
 559 to a transition mode. For comparison purpose, the case for  $Re = 2000$  is also reported in  
 560 figure 12. The merging is not observed here because TS wave is temporally damped for this  
 561 Reynolds number.

562 To better illustrate the temporal dynamics associated with the two-waves mode and  
 563 transition mode, we show in figure 13 the intracyclic evolution of the total perturbation  
 564 energy  $\tilde{E}(t)$  for different pulsation amplitudes  $\tilde{Q}$  and the sinuous symmetry. On one hand,  
 565 the TWF mode exhibits almost no effect as  $\tilde{Q}$  is increased in the range 0.38–0.48 for both  
 566 Reynolds numbers. On the other hand, the two-waves mode presents an interesting intracyclic  
 567 dynamics.

568 At  $\tilde{Q} = 0.38$ , growth occurs for both the acceleration and deceleration phases of the base  
 569 flow. It is consistent with the fact that this Floquet mode shares common features with both  
 570 TS and TWF waves. When increasing  $\tilde{Q}$ , the growth associated with the acceleration phase  
 571 is increasing. On the contrary, figure 13 shows that the energy peak in the deceleration  
 572 phase is damped with  $\tilde{Q}$ . It means that the two-waves mode is mainly driven by its TWF  
 573 contribution as  $\tilde{Q}$  is increased from 0.38 to 0.48. In particular, beyond  $\tilde{Q} = 0.46$ , the TS wave

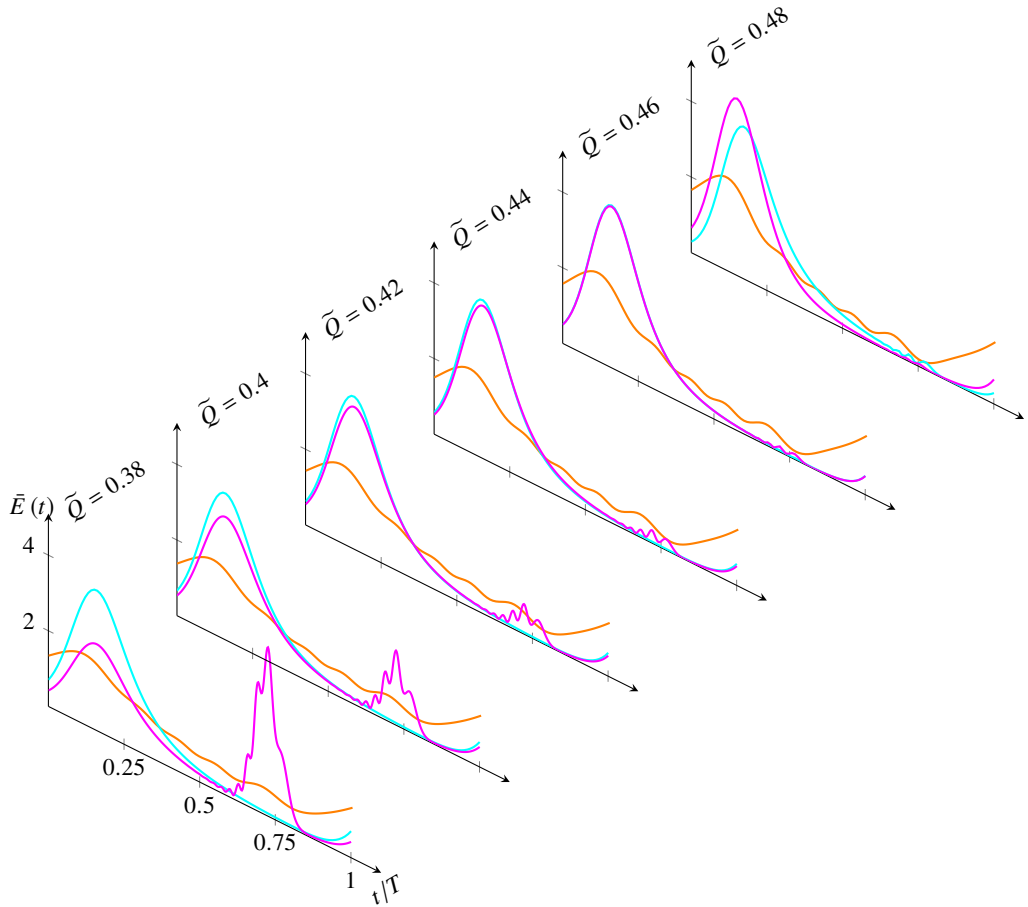


Figure 13: Intracyclic evolution of the total perturbation energy  $\tilde{E}(t)$  with  $V_R = 1$ ,  $Wo = 10$ ,  $Re = 10000$ ,  $\alpha = 1$ ,  $B_* = 4$  and  $d_* = 0$  (Cyan for sTWF, Fuchsia for TS and orange for sTWF at  $Re = 2000$ ).

574 contribution is negligible. As a consequence, at  $\tilde{Q} = 0.46$ , the variation of  $\tilde{E}(t)$  for TWF  
 575 and two-waves modes are almost undistinguishable. This corresponds to the coalescence  
 576 observed in figure 12. Beyond  $\tilde{Q} = 0.46$ , this transition mode shows the same characteristics  
 577 as the TWF instabilities, namely, a growth of energy in the acceleration phase of the base  
 578 flow.

579 The intracyclic behaviour associated with the two-waves mode displays a low-frequency  
 580 beating during the deceleration phase of the base flow for  $0.38 \leq \tilde{Q} \leq 0.46$  (see figure 13).  
 581 This phenomenon results from an interference between two waves of slightly different  
 582 frequencies. Especially, for this range of  $\tilde{Q}$ , TS and sTWF modes merge to form a wavepacket  
 583 whose frequency is the difference of the two frequency peaks. The Fourier density distribution  
 584 of the two-waves mode for  $\tilde{Q} = 0.38$  is shown in figure 14a. The difference between the two  
 585 peaks indeed corresponds exactly to the frequency beating observed in figure 14b.

586 To further illustrate this point,  $\tilde{E}(t)$  has been computed by filtering the Fourier components  
 587 in the neighbourhood of either the TS wave or the sTWF wave, using components from the  
 588 ranges hatched respectively in fuchsia or orange in figure 14a. The plots of figure 14b show

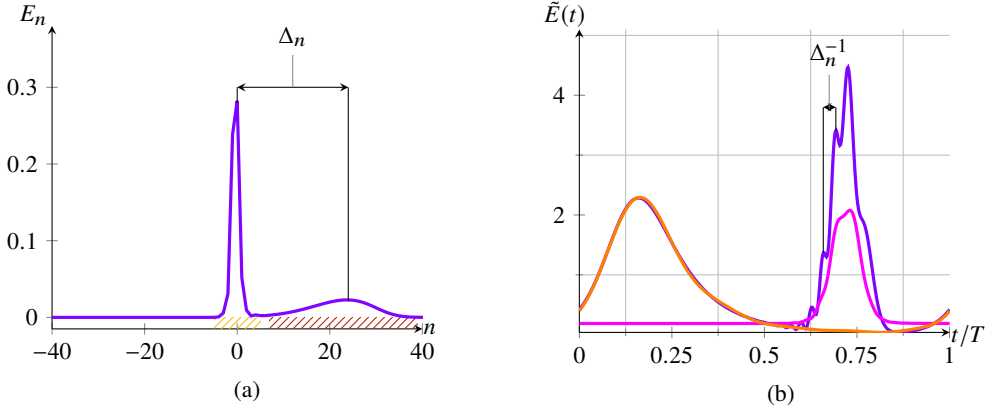


Figure 14: (a) Fourier density distribution and (b) filtered intracyclic total energy for the two-waves mode with  $\bar{Q} = 0.38$ ,  $V_R = 1$ ,  $Wo = 10$ ,  $Re = 10000$ ,  $\alpha = 1$ ,  $B_* = 4$  and  $d_* = 0$ . Blue: full dynamics, fuchsia: TS wave dynamics and orange: sTWF wave dynamics.

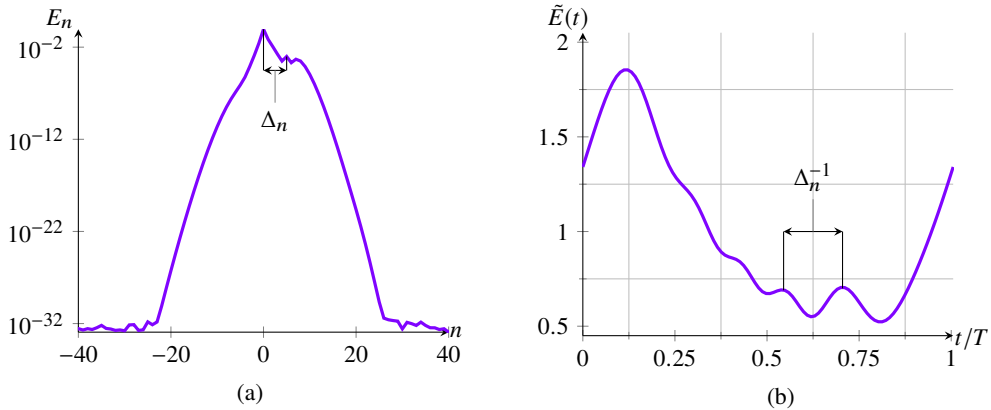


Figure 15: (a) Fourier density distribution and (b) intracyclic energy variation for the sTWF mode with  $\bar{Q} = 0.38$ ,  $V_R = 1$ ,  $Wo = 10$ ,  $Re = 2000$ ,  $\alpha = 1$ ,  $B_* = 4$  and  $d_* = 0$ .

589 that intracyclic dynamics pertaining to either the TS wave or the sTWF wave is recovered  
 590 and the beating phenomenon is then suppressed.

591 To conclude this section, we show in figure 15 the case of the sinuous TWF mode at  
 592  $Re = 2000$ . As noted earlier, at this subcritical Reynolds number, the TS wave is damped.  
 593 Therefore, the interference appears only as small amplitude oscillations (see figure 15b). The  
 594 harmonics of the two waves are hard to distinguish in figure 15a, which results in a greater  
 595 period for the secondary oscillations.

596

### 6.7. Neutral curves

597 The purpose of this section is to identify the critical parameters for the onset of instabilities  
 598 over a wide parameter space. To this end, we first compute the neutral boundaries (i.e.  
 599 corresponding to  $\omega_i = 0$ ). For the rigid case, the neutral stability curves are commonly  
 600 presented in the  $(Re, \alpha)$  plane within a temporal framework. However, it has been shown by  
 601 Lebbal *et al.* (2022) that the Reynolds number has only a weak effect onto the TWF instability  
 602 modes for a steady base flow. Hence, neutral stability curves associated with TWF Floquet  
 603 modes will be displayed in the  $(V_R, \alpha)$ -plane hereafter.

604 Figure 16 shows neutral boundaries associated with the TS Floquet mode for  $\tilde{Q} = 0, 0.1$   
 605 and 0.2,  $Wo = 10, 12, 14$  and  $V_R = 0.2, 0.4, 0.6$  and 0.8. One may recall that for the rigid  
 606 case the oscillating component reduces instability for  $Wo > 13$ , while it has a destabilising  
 607 effect at lower frequencies (Pier & Schmid 2017). The effect of wall compliance is therefore  
 608 here investigated in a neighbourhood of  $Wo = 13$ .

609 Increasing the wall compliance (i.e. increasing  $V_R$ ) leads to a reduction of the range of  
 610 unstable streamwise wave numbers and an increase of the critical Reynolds numbers for all  
 611 flow parameters. Similar behaviour is also observed for the steady base flow (Lebbal *et al.*  
 612 2022). In addition, the stabilising effect of  $\tilde{Q}$  onto TS Floquet modes for  $Wo > 13$  is also  
 613 recovered for the compliant walls case. Finally, for moderate pulsation amplitudes  $\tilde{Q}$ , the  
 614 stabilisation mechanism caused by wall flexibility seems not to be affected by the base flow  
 615 modulation.

616 For  $Wo = 20$ , neutral boundaries in the  $(V_R, \alpha)$ -plane for the TWF Floquet modes are  
 617 shown in figure 17 for both symmetries. On one hand, the upper bound of the unstable wave  
 618 number range grows lineary with the amplitude of the pulsation. On the other hand, the lower  
 619 bound is almost unchanged by the pulsatile base flow components.

620 Near the critical reduced velocity, referenced as  $V_R^c$  hereafter (i.e. the value of  $V_R$  that  
 621 corresponds to the onset of instability), the varicose TWF is weakly stabilised for  $\tilde{Q} < 0.4$ .  
 622 For larger values of  $\tilde{Q}$ , a destabilisation effect is observed. For the sinuous symmetry, a  
 623 decrease in  $V_R^c$  is observed when the amplitude of the base flow modulation is increased, for  
 624 all values of  $\tilde{Q}$  that are considered.

625 Neutral stability curves are also displayed in figure 18 for the same parameters except that  
 626  $Wo$  is fixed to 10. Comparison of figures 17 and 18 shows that the Womersley number has a  
 627 weak effect onto the neutral boundaries for these TWF Floquet modes.

### 628 6.8. Influence of $\tilde{Q}$ and $Wo$ onto temporal growth rates

629 In this section, we investigate the combined effect of wall flexibility and pulsatile base flow  
 630 parameters ( $\tilde{Q}$  and  $Wo$ ) onto the temporal growth rates of TWF and TS Floquet modes.  
 631 Results are conveniently summarized by monitoring the growth rate  $\omega_i^{\max}$  associated with  
 632 the most unstable streamwise instability (i.e.  $\omega_i^{\max} = \max_{\alpha} \omega_i$ ) for a given set of fluid and  
 633 wall parameters. For illustration purposes,  $B_* = 4$  and  $d_* = 0$  are fixed.

634 Figures 19a and 19b show the variation of the maximum temporal growth rates with  $\tilde{Q}$   
 635 and  $Wo$  of the TS mode for both the rigid case in (a) and compliant walls ( $V_R = 0.2$ ) in (b).

636 The TS Floquet modes exhibit similar dynamics whatever the case considered (either rigid  
 637 or compliant walls). For  $Wo \geq 14$ , the temporal growth rate decays with  $\tilde{Q}$  for both rigid  
 638 and flexible cases while it increases for  $Wo \leq 13$ . One may recall that a similar behaviour is  
 639 observed for the rigid case (Pier & Schmid 2017).

640 Figure 20 illustrates the effect of  $Wo$  and  $\tilde{Q}$  onto travelling-wave flutter Floquet modes for  
 641  $V_R = 1$ . The temporal growth rate for the varicose vTWF mode presents two distinguishable  
 642 phases (figure 20a). For small and moderate values of  $\tilde{Q}$ ,  $\omega_i^{\max}$  is damped. Then, one may  
 643 observe a growth of  $\omega_i^{\max}$  as  $\tilde{Q}$  is increasing. The turning point depends on the Womersley  
 644 number. Especially, the corresponding  $\tilde{Q}$  is seen to increase with  $Wo$ .

645 The distribution of  $\omega_i^{\max}$  for the sinuous symmetry exhibits a different behaviour. For  
 646 weakly pulsatile base flows ( $\tilde{Q} < 0.2$ ), the sTWF mode is destabilised whatever the  $Wo$   
 647 number that is considered. In particular, this instability is strongly enhanced for the small  
 648 frequencies of modulation  $Wo$ . For moderate values of  $\tilde{Q}$  ( $0.2 < \tilde{Q} < 0.5$ ), a more complex  
 649 behaviour is observed. For this range of amplitudes, the sTWF mode interacts with TS

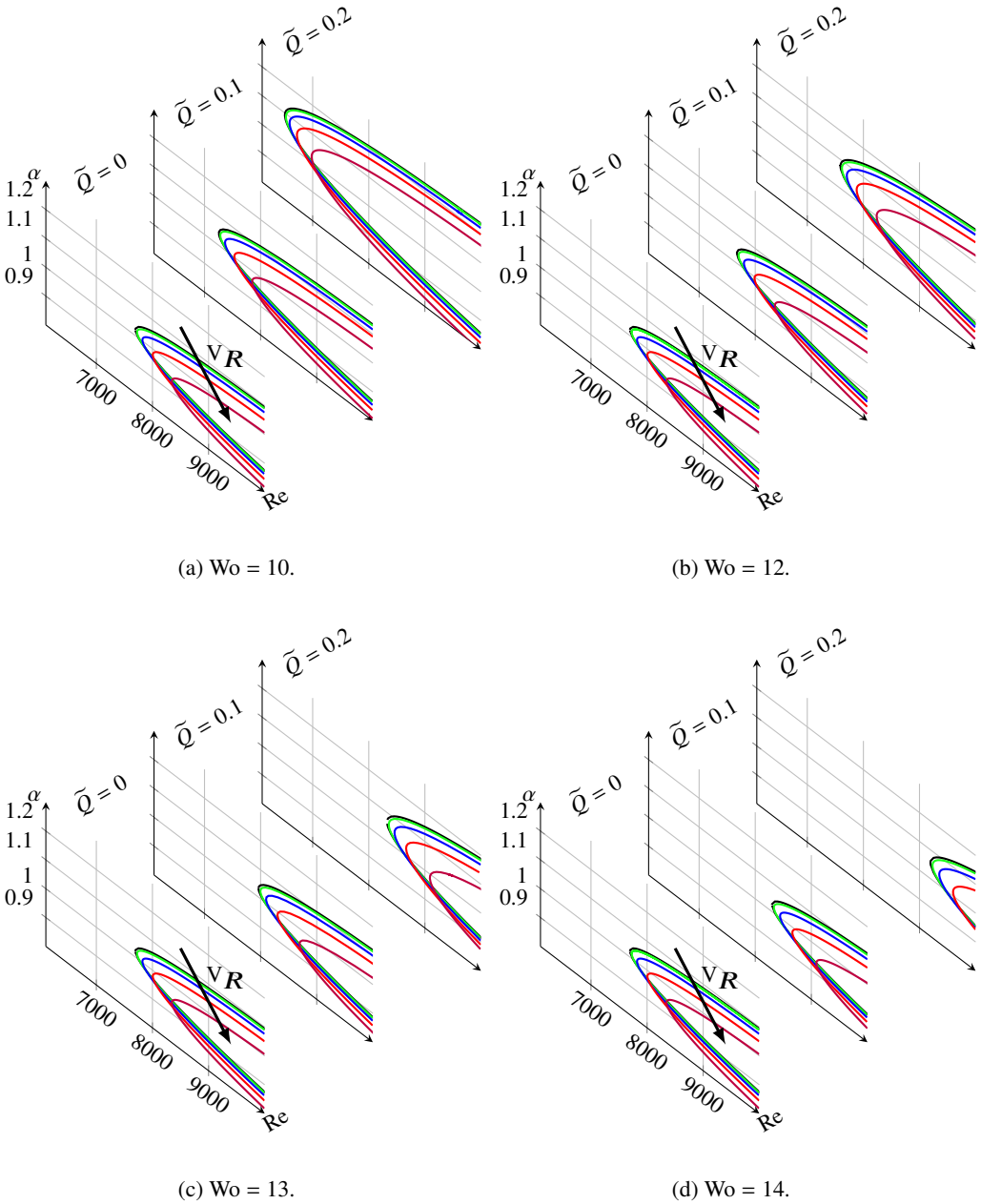


Figure 16: Neutral curves in the  $(\alpha, Re)$ -plane for TS mode with  $B_* = 4$  and  $d_* = 0$  for different amplitudes  $\bar{Q} = 0, 0.1, 0.2$  and frequencies  $Wo = 10, 12, 13, 14$ , with  $V_R$  varying from 0. to 0.8. The associated colors are respectively black, green, blue, red and brown.

650 Floquet mode, and we can no longer distinguish between these two waves. Beyond this point,  
 651  $\omega_i^{\max}$  strongly increases and reaches similar values as its varicose counterpart.  
 652 An intracyclic modulation amplitude  $E_{\min}^{\max}$ , defined as the ratio of the maximum to the  
 653 minimum of  $\tilde{E}(t)$ , is computed for the same parameter range as in figure 20, for travelling-



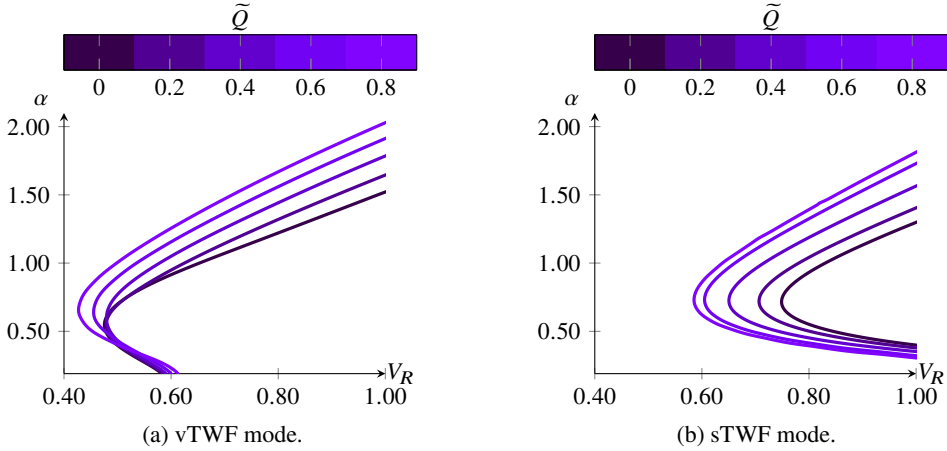


Figure 17: Neutral curves in the  $(\alpha, V_R)$ -plane for TWF modes with  $Wo = 20$ ,  $Re = 10000$ ,  $B_* = 4$  and  $d_* = 0$ .

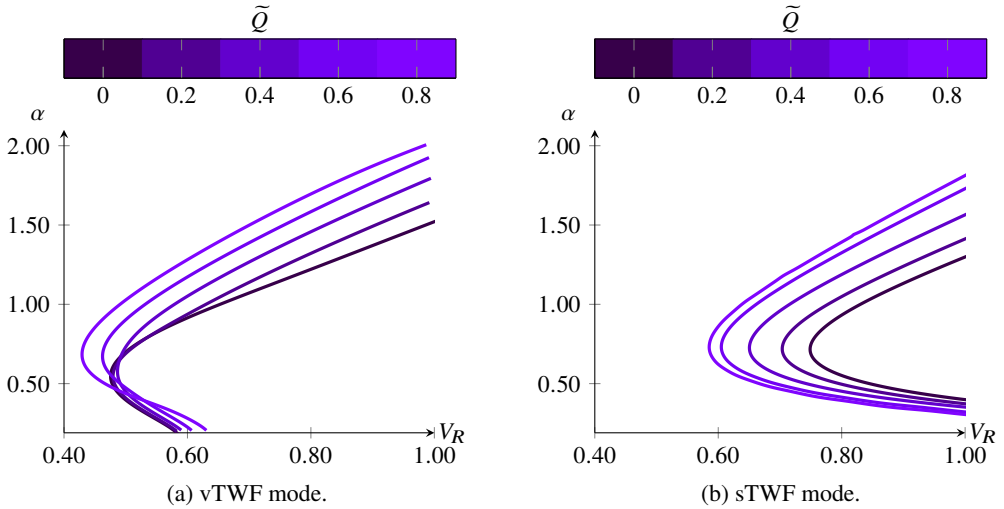


Figure 18: Neutral curve in the  $(\alpha, V_R)$ -plane for TWF modes with  $Wo = 10$ ,  $Re = 10000$ ,  $B_* = 4$  and  $d_* = 0$ .

654 wave flutter Floquet modes only. For the varicose symmetry, the plots in figure 21a show  
 655 that  $E_{\min}^{\max}$  increases for all  $Wo$  under consideration. The evolution of  $E_{\min}^{\max}$  for the sinuous  
 656 symmetry is displayed in figure 21b. The figure shows that the sinuous TWF Floquet modes  
 657 exhibit smaller amplitude variations than their varicose counterparts. In addition, for  $\tilde{Q} >$   
 658  $0.5$ , a saturation of  $E_{\min}^{\max}$  is observed. Such a behaviour occurs beyond the collapse of TWF  
 659 and TS Floquet modes. One may thus suggest that it is a consequence of the emergence of a  
 660 transition mode. Comparison with figure 11 also reveals that the very large values of  $E_{\min}^{\max}$   
 661 observed for the vTWF modes (figure 21a) are mainly due to the fact that  $\tilde{E}(t)$  drops to  
 662 extremely low levels during the acceleration phase of the pulsating cycle (figure 11b).

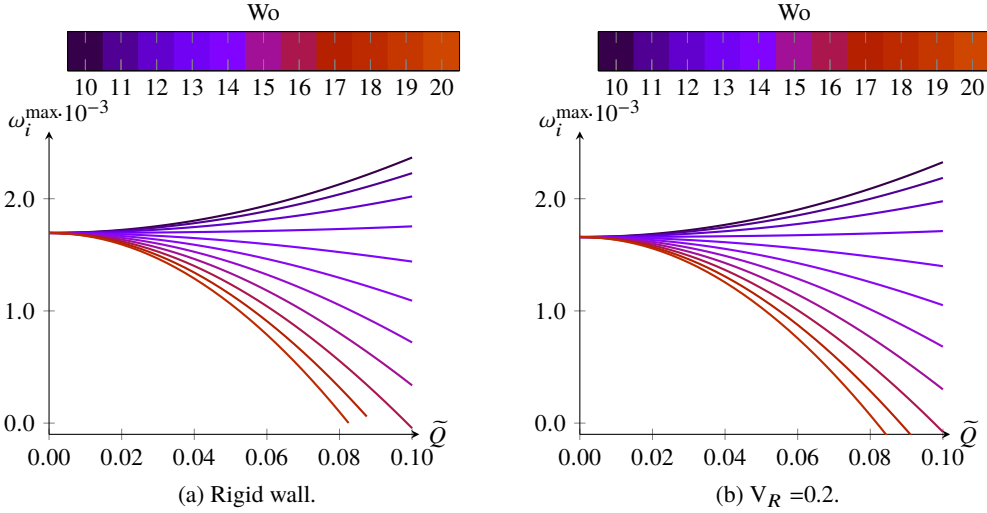


Figure 19: TS Floquet mode. Maximum temporal growth rates  $\omega_i^{\max}$  for different amplitudes  $\tilde{Q} = 0, \dots, 0.1$  and frequencies  $Wo = 10, \dots, 20$ , with  $Re = 10000$ ,  $B_* = 4$  and  $d_* = 0$ .

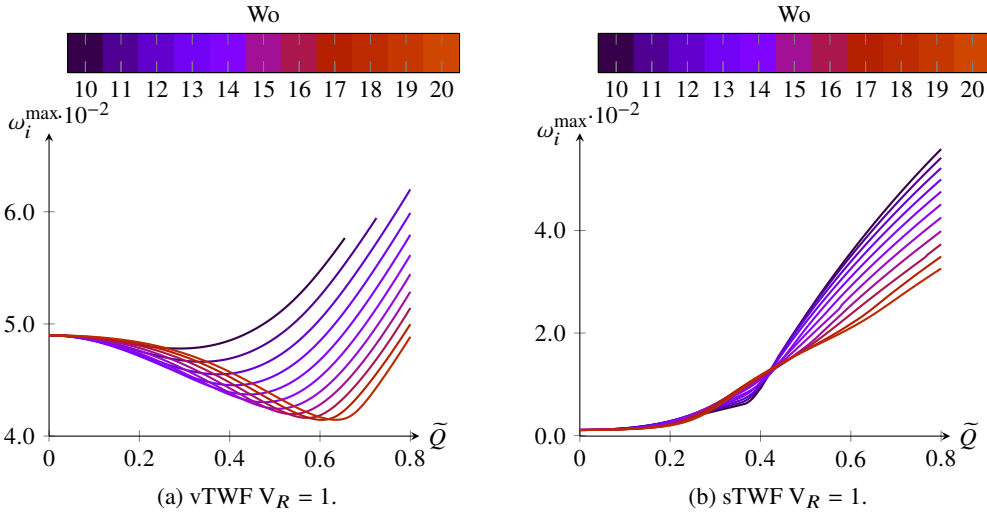


Figure 20: Maximal temporal growth rates  $\omega_i^{\max}$  for different amplitudes  $\tilde{Q} = 0, \dots, 0.7$  and frequencies  $Wo = 10, \dots, 20$ , with  $Re = 10000$ ,  $B_* = 4$  and  $d_* = 0$ .

663

### 6.9. Critical parameters for onset of instability

664 A complete two-dimensional instability analysis is now performed by exploring a wide range  
 665 of wall and flow parameters. In a effort to summarize the different results only critical  
 666 Reynolds numbers ( $Re^c$ ) and critical reduced velocities ( $V_R^c$ ) are monitored (corresponding  
 667 to the onset of TS or TWF Floquet modes, respectively).

668 The variations of  $Re^c$  for the TS mode are computed for  $V_R = 0.2$  and  $V_R = 0.6$  for different  
 669 pulsatile flow parameters in figure 22. Beyond  $Wo = 13$ , the TS Floquet modes are stabilised  
 670 by the pulsatile flow component. For lower frequencies, the opposite behaviour is observed.  
 671 For example, at  $Wo = 20$ , the critical Reynolds number for  $\tilde{Q} = 0.10$  is already about 50%

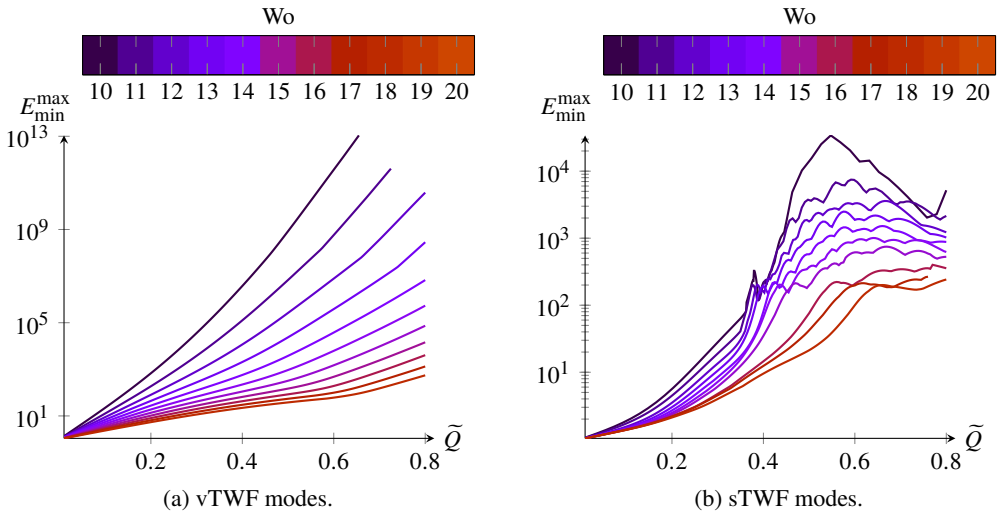


Figure 21: The amplitude of the modulation ratio  $E_{\min}^{\max}$  for different amplitudes  $\tilde{Q} = 0, \dots, 0.7$  and frequencies  $Wo = 10, \dots, 20$ , with  $Re = 10000$ ,  $B_* = 4$ ,  $d_* = 0$  and  $V_R = 1$ .

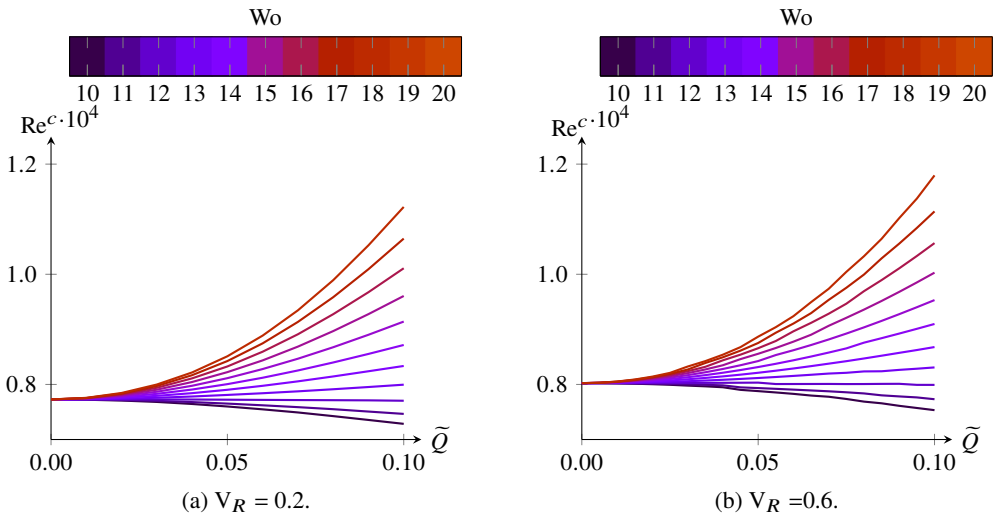


Figure 22: Critical Reynolds number  $Re^c$  for TS mode with  $B_* = 4$  and  $d_* = 0$ .

672 larger than the value found for the Poiseuille flow case ( $\tilde{Q} = 0$ ). The dynamics including  
 673 compliant walls is thus found to be very similar to the rigid walls case (see Pier & Schmid  
 674 (2017)).

675 The critical reduced velocity  $V_R^c$  for the TWF modes are shown in figure 23. The varicose  
 676 travelling-wave flutter displays two phases. For moderate pulsation amplitudes ( $\tilde{Q} < 0.4$ ),  
 677 the instability is weakly stabilised. For higher pulsation amplitudes, the vTWF mode is  
 678 destabilised for all the frequencies studied. Unlike the vTWF, the sinuous TWF mode shows  
 679 a monotonic destabilisation as the pulsation amplitude is increased. The Womersley numbers  
 680 considered here have almost no effect on the critical curves. Note that even for highly pulsatile  
 681 flows, onset of TWF instability is always due to the varicose symmetry.

682 In order to systematically study the linear stability over the entire parameter space, the

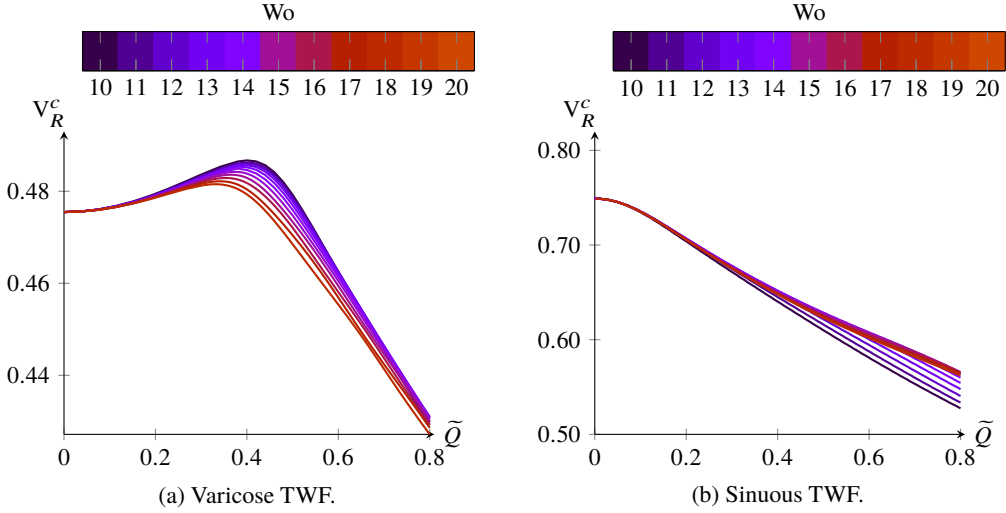


Figure 23: Critical reduced velocities  $V_R^c$  for TWF modes and different  $Wo$  numbers. The other parameters are:  $Re = 10000$ ,  $B_* = 4$  and  $d_* = 0$ .

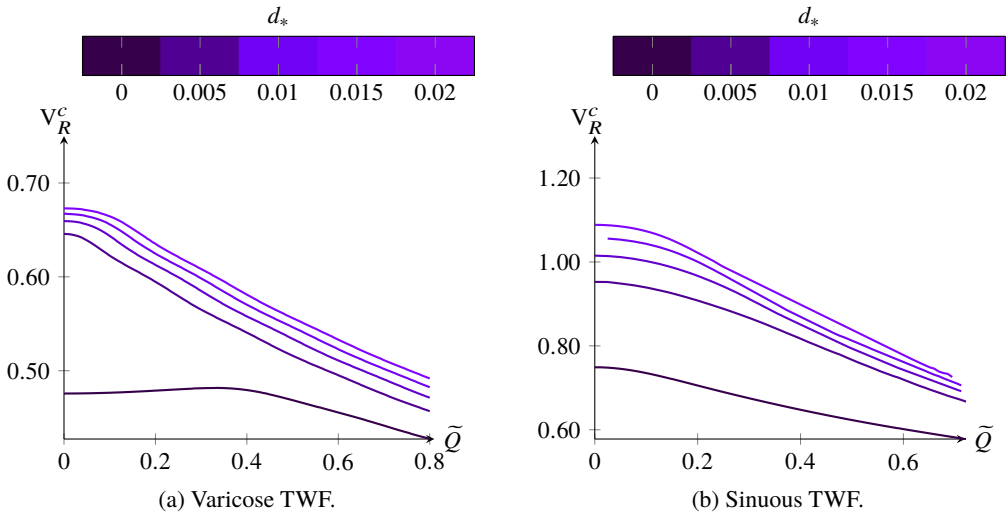


Figure 24: Critical reduced velocities  $V_R^c$  for TWF modes for different  $d_*$ . The other parameters are:  $Wo = 20$ ,  $Re = 10000$  and  $B_* = 4$ .

683 critical reduced velocity  $V_R^c$  is also computed for various wall dissipations  $d_*$  (figure 24),  
 684 flexural rigidities  $B_*$  (figure 25) and Reynolds numbers  $Re$  (figure 26).

685 According to the energy classification of Benjamin (1963) and Landahl (1962), the  
 686 dissipation has a stabilising effect on the travelling-wave flutter instabilities. The plots in  
 687 figure 24 show the variation of the critical reduced velocity with wall dissipation  $d_*$ . For a  
 688 Poiseuille base flow ( $\tilde{Q} = 0$ ), the critical  $V_R^c$  is almost multiplied by a factor 2 when  $d_*$  is  
 689 varied from 0 to 0.02.

690 An increase in  $\tilde{Q}$  leads to stabilise the flow for all values of  $d_*$  that have been considered  
 691 (see figure 24). In particular, the overall behaviour is quite similar for  $d_*$  varying from 0.005  
 692 to 0.04. The critical  $V_R^c$  is nearly constant for  $\tilde{Q}$  up to 0.05. Beyond this value,  $V_R^c$  decreases

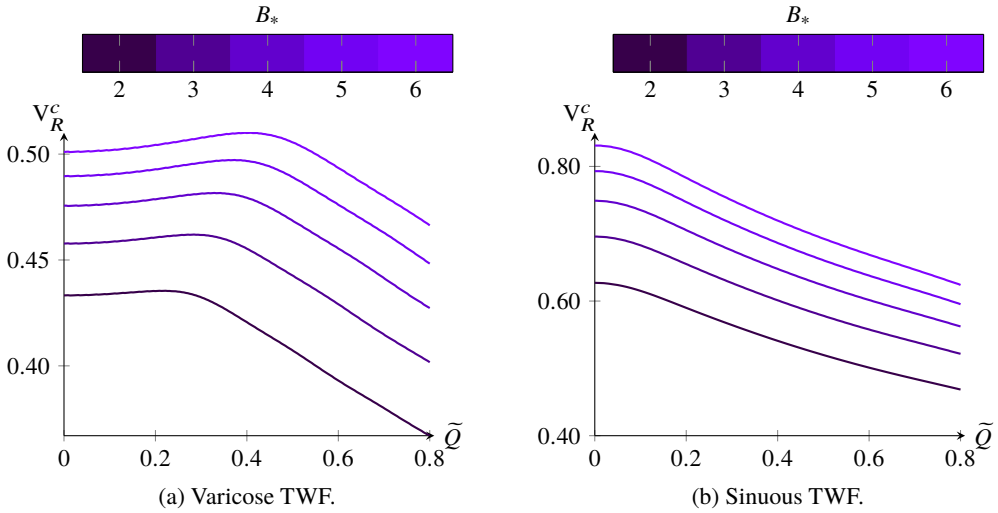


Figure 25: Critical  $V_R^c$  for TWF modes for different  $B_*$ . The other parameters are:  $Wo = 20$ ,  $Re = 10000$  and  $d_* = 0$ .

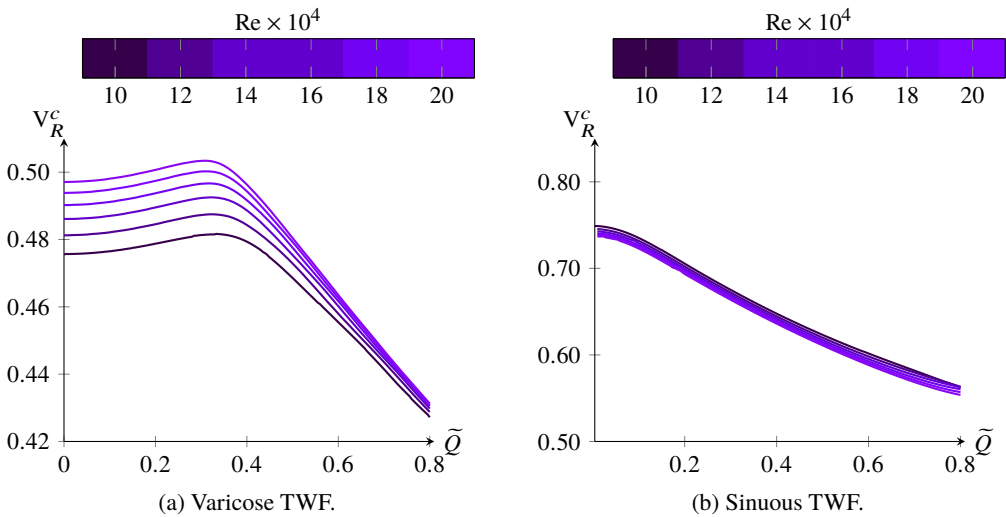


Figure 26: Critical  $V_R^c$  for TWF modes for different  $Re$  numbers. The other parameters are:  $Wo = 20$ ,  $B_* = 4$  and  $d_* = 0$ .

693 almost linearly with  $\tilde{Q}$  at a similar rate of change. Interestingly, the symmetry of the Floquet  
 694 mode appears to have a negligible effect onto  $V_R^c$  for this range of parameters.

695 The effect of flexural rigidity is illustrated in figure 25. Increasing  $B_*$  results in stabilising  
 696 the TWF Floquet modes for all  $\tilde{Q}$  that are considered. However, the overall shape of these  
 697 curves is almost unaffected by  $B_*$  for both sinuous and varicose cases. For the varicose case,  
 698 a nearly constant value of  $V_R^c$  is observed up to  $\tilde{Q} = 0.3$ . Beyond  $\tilde{Q} = 0.4$ , the critical  $V_R^c$  is  
 699 seen to decrease almost linearly with  $\tilde{Q}$  for all flexural rigidities that have been investigated.  
 700 In particular, the slope seems to be independent of  $B_*$ .

701 The sinuous case appears to be more stable than its varicose counterpart. As for the varicose

702 symmetry, the overall tendency is not affected by  $B_*$ . A slight decrease in  $V_R^c$  is observed for  
 703  $\tilde{Q}$  up to 0.1. Beyond  $\tilde{Q} = 0.4$ ,  $V_R^c$  exhibits an almost linear behaviour with  $\tilde{Q}$ .

704 Finally, the influence of the Reynolds number onto  $V_R^c$  is shown in figure 26. The onset  
 705 of Floquet TWF modes are almost unchanged by the Reynolds number for both the varicose  
 706 and sinuous TWF modes. The insensitivity to  $Re$  is more pronounced when  $\tilde{Q}$  is increased to  
 707 large amplitudes. This weak influence has already been reported for the steady case (Lebbal  
 708 *et al.* 2022).

## 709 7. Conclusions

710 In this paper, we have systematically investigated the rich dynamics resulting from perturba-  
 711 tions developing in harmonically pulsating flows between two compliant walls. The stability  
 712 analysis is restricted to the time-asymptotic behaviour of the flow perturbation and to the  
 713 two-dimensional case within the framework of Floquet theory. A numerical solution strategy  
 714 has been implemented that is free of spurious modes and greatly reduces the computational  
 715 effort.

716 When accounting for wall compliance, we show that the most relevant control parameter is  
 717 the reduced velocity  $V_R$  for travelling-wave flutter Floquet modes. Especially, the Reynolds  
 718 number appears to have a negligible influence on these modes. As already observed for the  
 719 steady case (Lebbal *et al.* 2022), the most unstable modes are associated with the varicose  
 720 symmetry. For the pulsatile flow configurations, we show that the instability onset for these  
 721 modes is mainly driven by the amplitude of the pulsation rather than its frequency. For  $\tilde{Q}$  in  
 722 the range 0–0.4 and varicose perturbations, the pulsatile base flow is seen to weakly stabilise  
 723 the TWF Floquet modes (*i.e.* the critical reduced velocity increases) with respect to the  
 724 steady flow case. The opposite behaviour is observed for  $\tilde{Q}$  larger than 0.4. For the sinuous  
 725 symmetry, we always observe a flow destabilisation with an increase of  $\tilde{Q}$ . When accounting  
 726 for the wall dissipation, we show that an increase of  $d_*$  tends to stabilise the travelling-  
 727 wave flutter Floquet modes for both symmetries whatever the value of  $\tilde{Q}$  in agreement with  
 728 Benjamin’s classification (Benjamin 1963). For the Tollmien–Schlichting Floquet modes, the  
 729 intracyclic dynamics exhibits strong similarities with the pulsatile flow case in a rigid channel  
 730 (Pier & Schmid 2017). However, a stronger stabilisation is observed when wall flexibility  
 731 comes into play.

732 On one hand, it has been shown that Benjamin’s classification still holds for a wide range  
 733 of parameters. Especially, for fluid-structure interaction modes, similar general trends are  
 734 observed for steady and pulsatile flow configurations. But on the other hand, this study  
 735 has also revealed a more complex flow dynamics that is not found when wall flexibility or  
 736 pulsating base flow are studied independently. In particular, a new type of transition mode  
 737 has been discovered that emerges due to the coalescence of two Floquet modes, even in the  
 738 absence of wall dissipation. The first Floquet mode combines properties of both Tollmien–  
 739 Schlichting modes and travelling-wave flutter modes. This coupling leads to interference that  
 740 generates a low-frequency beating during the intracyclic dynamics. The second Floquet mode  
 741 shares the main properties of a travelling-wave flutter mode. When increasing the pulsation  
 742 amplitude, these two modes merge into a more unstable Floquet mode.

743 Extension of the present study to nonmodal stability analyses can be considered in a future  
 744 work, continuing the investigations of Tsigklifis & Lucey (2017) and Pier & Schmid (2021).  
 745 Finally, it would also be interesting to generalise our analyses to pipe geometries which cover  
 746 more biologically significant phenomena. The theoretical developments and numerical tools  
 747 that have been used in the present investigation can be easily adapted to a formulation in  
 748 cylindrical coordinates, following the same approach used by Pier & Schmid (2021).

749 **Acknowledgements.**

750 Fédération Lyonnaise de modélisation et sciences numériques (FLMSN) is gratefully acknowledged for  
 751 providing access to high-performance computing resources.

752 **Declaration of interests.**

753 The authors report no conflict of interest.

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 759 Accepted Manuscript (AAM) version arising from this submission.

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