



**HAL**  
open science

# Adaptive mesh procedure for the unstructured inductive PEEC formulation

Kouceila Alkama, Gérard Meunier, Olivier Chadebec, Jean-Michel Guichon,  
Bertrand Bannwarth, Enrico Vialardi, Jonathan Siau, Rémy Perrin-Bit

► **To cite this version:**

Kouceila Alkama, Gérard Meunier, Olivier Chadebec, Jean-Michel Guichon, Bertrand Bannwarth, et al.. Adaptive mesh procedure for the unstructured inductive PEEC formulation. 23rd Conference on the Computation of Electromagnetic Fields COMPUMAG 2021, Jan 2022, Cancun (Online), Mexico. hal-03553885

**HAL Id: hal-03553885**

**<https://hal.science/hal-03553885>**

Submitted on 10 Nov 2022

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# Adaptive Mesh Procedure for the Unstructured Inductive PEEC Formulation

K. Alkama<sup>1,2</sup>, G. Meunier<sup>1</sup>, O. Chadebec<sup>1</sup>, J-M. Guichon<sup>1</sup>, B. Bannwarth<sup>1</sup>, E. Vialardi<sup>2</sup>, J. Siau<sup>2</sup>, R. Perrin-Bit<sup>2</sup>

<sup>1</sup>Univ. Grenoble Alpes, CNRS, Grenoble INP, G2Elab, 38000 Grenoble, France

<sup>2</sup>Altair Engineering France, 38240 Meylan, France, kalkama@atair.com

An adaptive mesh refinement procedure is presented in order to address efficiently low frequencies electromagnetic problems with the unstructured inductive PEEC method. An *a posteriori* error estimator based on an equilibrated energy criterion and on a method of calculating a second admissible solution are proposed. TEAM Workshop problem no<sup>o</sup>7 has been treated in order to show the good efficiency of the procedure.

*Index Terms*—Unstructured Partial Element Equivalent Circuit (PEEC) method, Error Estimator, Adaptive mesh refinement.

## I. INTRODUCTION

IN the process of designing power electronics devices, the correct representation of physical phenomena is crucial to ensure, for example, the electromagnetic compatibility. The sub-discretization of the studied domain can be a major cause of numerical error in electromagnetic simulations, and a good discretization requires having a good knowledge of the studied phenomena, which is generally not the case. The main motivation of this work is to develop an efficient error estimator for the unstructured Partial Element Equivalent Circuit (PEEC) method [1], and to couple it with an adaptive mesh refinement procedure. These developments will improve the accuracy without excessively increasing the number of degrees of freedom (DOF). This will make *in fine* the method much more functional and attractive to model complex devices.

The approach consists in using an *a posteriori* equilibrated error estimator [2]. It is based on the non-verification of the material constitutive relationship (e.g. Ohm's law) discretization in the active domain using two complementary solutions, the initial unstructured PEEC solution  $\mathbf{J}$  and an another admissible solution  $\mathbf{E}'$ . The admissible solution is obtained by projecting  $\mathbf{J}$  interpolated in the facet function space and reconstructing it in the edge function space using the appropriate tools as shown in [2]. This approach has been well validated on the TEAM workshop problem 7 [3] using a refinement mesh algorithm of the Altair **Flux**<sup>TM</sup> software [4].

## II. UNSTRUCTURED INDUCTIVE PEEC FORMULATION

Considering the integral representation of the electric field  $\mathbf{E}$  and the linear constitutive law  $\mathbf{J} = \sigma \cdot \mathbf{E}$  in the conductive domain  $\Omega$ , we can write

$$\frac{\mathbf{J}(P)}{\sigma(P)} = -j\omega \frac{\mu_0}{4\pi} \int_{\Omega} \frac{\mathbf{J}(Q)}{r} d\Omega - \nabla V_{(P)} \quad (1)$$

where  $r$  is the distance between  $P$  and the integration point  $Q$ . As proposed in [5],  $\mathbf{J}$  is interpolated with first-order face shape

functions  $w_f$  and with applying a standard Galerkin projection procedure to (1), a matrix system is obtained.

$$([\mathbf{R}] + j\omega [\mathbf{L}]) \{I\} = \{\delta U\} \begin{cases} [\mathbf{R}]_{i,j} = \int_{\Omega} \frac{w_{f_i} w_{f_j}}{\sigma} d\Omega \\ [\mathbf{L}]_{i,j} = \frac{\mu_0}{4\pi} \int_{\Omega} w_{f_i} \int_{\Omega} \frac{w_{f_j}}{r} d\Omega d\Omega \\ \{\delta U\}_i = - \int_{\Omega} w_{f_i} \nabla V d\Omega \end{cases} \quad (2)$$

It is seen as a circuit matrix system  $\mathbf{Z}\mathbf{I} = \mathbf{U}$ , where each face of the mesh element represents one resistance and one self inductance and is completed by all mutual inductances, and  $I_i$  is the current flowing through the face  $i$ . This equivalent circuit is solved using a classical circuit analysis approach i.e. an independent loop solver or an independent node solver. In this work, the solution  $\mathbf{J}$  obtained by the system (2) is considered as a primal solution. As the facet functional space is used, the continuity of the normal component of  $\mathbf{J}$  is naturally ensured, while the continuity of the tangential component of  $\mathbf{E}$  would not be without the use of the edge functional space.

## III. COMPUTING THE ADMISSIBLE SOLUTION $\mathbf{E}'$

In order to calculate the admissible solution  $\mathbf{E}'$  which ensures the continuity of its tangential component, a projection and reconstruction method of  $\mathbf{E}$  is used, as proposed for the magnetic field  $\mathbf{H}$  in [2].  $\mathbf{J}$  can be scaled and projected from the facet to the edge functional space, so this procedure is done by solving a finite element problem, according to:

$$\sum_j \int_{\Omega_e} w_{e_i} \cdot w_{e_j} \cdot \mathbf{E}'_j d\Omega_e = \int_{\Omega_e} w_{e_i} \cdot \frac{\mathbf{J}}{\sigma} d\Omega_e \quad (3)$$

where  $w_e$  are the edge shape functions used for the interpolation, and  $\frac{\mathbf{J}}{\sigma}$  represents values of the electric field  $\mathbf{E}$  at Gauss points of each mesh element. The field  $\mathbf{E}'$  is obtained and the continuity of its tangential component on the mesh edges is ensured. This method is very efficient because a complete dual problem does not need to be solved, but only a FEM limited to the active material, i.e. very few DOF and a low memory are needed compared to PEEC approach to get the initial solution.

## IV. ERROR ESTIMATION CRITERION

The error in the discretization of the constitutive relation can be estimated using the energy-based estimator [6]. The local

error estimator is written in a normalized way according to a global quantity in the domain  $\Omega$ . This normalization helps to highlight the contribution of the local error of a mesh element to the global error.

$$\eta_{elem} = N \times \left| \frac{\int_{\Omega_E} \mathbf{J}^2 d\Omega_E - \int_{\Omega_E} (\boldsymbol{\sigma} \cdot \mathbf{E}')^2 d\Omega_E}{\int_{\Omega} \mathbf{J}^2 d\Omega} \right| \quad (4)$$

And the global error estimator is written as follows

$$\eta_{global} = \left| \frac{\int_{\Omega} \mathbf{J}^2 d\Omega - \int_{\Omega} (\boldsymbol{\sigma} \cdot \mathbf{E}')^2 d\Omega}{\int_{\Omega} \mathbf{J}^2 d\Omega} \right| \quad (5)$$

$N$  is the number of mesh elements, and  $\Omega_E$  a volume element.

## V. APPLICATION EXAMPLE

The TEAM Workshop problem 7 device is presented in Fig. 1. The plate is meshed with a initial tetrahedral conform mesh composed of 380 elements and solved at 50 Hz. The Joule losses are compared with the references obtained in [5] with a dense FEM mesh of 2,000,000 elements. Fig. 1 shows the Joule losses in the plate versus the iterations of the adaptive and the uniform refinement (each element is divided into four sub-elements). Table I presents a summary of the comparison between the results obtained over the adaptive refinement iterations with the FEM reference.

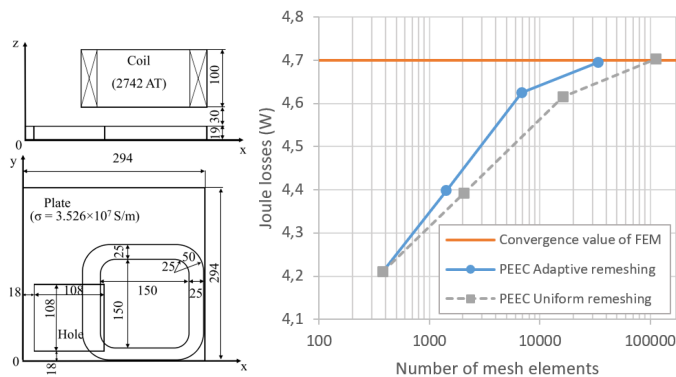


Fig. 1. (Left) TEAM problem 7 geometry (mm) (Right) Joule losses evolution

TABLE I

RELATIVE ERROR OF THE ADAPTIVE REMESHING COMPARED WITH FEM

	Elements	Joule losses (W)	Diff
FEM	2.000.000	4,70	Ref
PEEC 2 <sup>nd</sup> mesh iteration	6.868	4,63	1,48%
PEEC 3 <sup>rd</sup> mesh iteration	33.977	4,70	0,08%

It is noticed that a solution close to the reference is quickly obtained, even if the starting mesh is coarse as in this case. At the 3<sup>rd</sup> iteration, the same value of Joule loss is obtained for uniform and adaptive refinement, but with a large difference in terms of number of elements, as shown in Fig. 2, and also in terms of computation time: the refinement procedure takes 12 min and 103 min for the adaptive and the uniform strategies, respectively. If we have a look at the uniform refinement procedure, the 4<sup>th</sup> iteration could not be solved with our computer due to the large number of elements (around 800,000), unlike

adaptive refinement where the number is around 165,000. If the frequency is increased i.e skin depth highly reduced or if we need to deal with a more complex geometry, basic uniform mesh refinement becomes unsustainable for the PEEC method making the adaptive mesh technique very interesting.

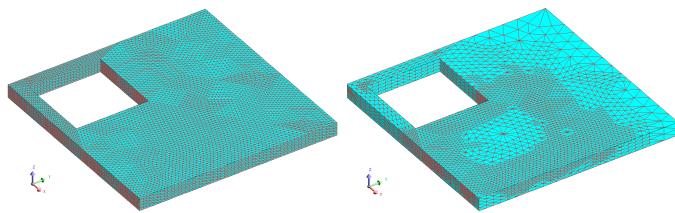


Fig. 2. The mesh obtained for the 3<sup>rd</sup> iteration of refinement at 50 Hz: (left) uniform (112.000 elements), (right) adaptive (34.000 elements)

In Fig. 3, a comparison is done between the estimated global errors and real error, i.e. the Joule losses by taking the final mesh as a reference. The global estimated error has globally the same behavior compared to the real, which proves that the calculating method of the second admissible solution and the error estimator proposed are very efficient for dealing with this type of Eddy current problem.

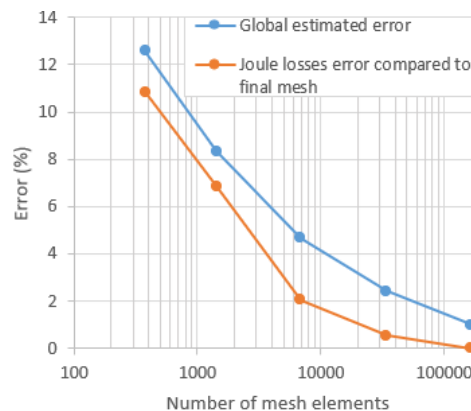


Fig. 3. Global errors on the plate at 50 Hz

## REFERENCES

- [1] J. Siau, G. Meunier, O. Chadebec, J.-M. Guichon, and R. Perrin-Bit, "Volume Integral Formulation Using Face Elements for Electromagnetic Problem Considering Conductors and Dielectrics," *IEEE Trans. Electromagn. Compat.*, vol. 58, no. 5, pp. 1587–1594, oct 2016.
- [2] J.-F. Remacle, C. Geuzaine, P. Dular, H. Hedia, and W. Legros, "Error estimation based on a new principle of projection and reconstruction," *IEEE Transactions on Magnetics*, vol. 34, no. 5, pp. 3264–3267, 1998. [Online]. Available: <http://ieeexplore.ieee.org/document/717766/>
- [3] L. R. Turner, K. Davey, N. Ida, D. Rodger, A. Kameari, A. Bossavit, and C. R. I. Emson, "Workshops and problems for benchmarking eddy current codes," United States, Tech. Rep., 1988. [Online]. Available: [http://inis.iaea.org/search/search.aspx?origi={\\_}q=RN:20009769](http://inis.iaea.org/search/search.aspx?origi={_}q=RN:20009769)
- [4] Altair Engineering, "Altair Flux™," 2021. [Online]. Available: <https://www.altair.com/flux/>
- [5] T.-T. Nguyen, G. Meunier, J.-M. Guichon, O. Chadebec, and T.-S. Nguyen, "An Integral Formulation for the Computation of 3-D Eddy Current Using Facet Elements," *IEEE Transactions on Magnetics*, vol. 50, no. 2, pp. 549–552, feb 2014. [Online]. Available: <http://ieeexplore.ieee.org/document/6749118/>
- [6] D. Ladas, V. Mazauric, G. Meunier, O. Chadebec, M. Ebene-Ebene, Y. Maréchal, and P. Wendling, "An energy based approach of electromagnetism applied to adaptive meshing and error criteria," *IEEE Trans. Magn.*, vol. 44, no. 6, pp. 1246–1249, 2008.