

The interaction between the SEIS seismometer of the InSight Martian mission and a regolith simulant

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1 The interaction between the SEIS seismometer of the InSight Martian

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mission and a regolith simulant

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- 19

20 Abstract

21 A detailed investigation of the interaction between a Martian regolith simulant and the foot 22 of a seismometer (SEIS) recently deployed on the surface of Mars within the NASA InSight 23 mission has been conducted. A specific device used to investigate the SEIS/ground 24 interaction was improved to provide accurate measurements of low forces and 25 displacements, with a higher system stiffness and appropriate thermal insulation. A series 26 of tests were carried out with a 60 mm diameter disk and the SEIS foot (disk with a spike in 27 the disk centre). The maximum disk penetration in the loose sand used as simulant under 28 the SEIS weight (10 N) was between 400 and 600 µm, with a tiny effect of the spike. Load 29 cycles under various forces were performed to investigate the elastic interaction, with good 30 reversibility and a linear change of the Young modulus with respect to the average vertical 31 stress. The tests provided comparable values, showing that the Young modulus was around 20 MPa, compatible with that of loose terrestrial sands and agreeing well with the seismic 32 33 wave velocities at surface (from laboratory experiments and from measuring on the surface 34 of Mars the travel times of waves received by the SEIS seismometer).

35

36 Keywords: Laboratory tests, sands, soil structure interaction

38 1. Introduction

39 The Seismic Experiment for Interior Structure (SEIS, Lognonné et al. 2019, 2020) of the NASA 40 (National Aeronautics and Space Administration) InSight mission on Mars (Interior 41 Exploration using Seismic Investigations, Geodesy and Heat Transport, Banerdt et al. 2020) 42 is the first seismometer ever deployed in direct contact with the surface of the planet Mars, 43 in the footsteps of the Apollo Lunar Surface Experiments Package on the Moon (e.g. Latham et al. 1969, 1970; Bates et al. 1979) and 44 years after the landing of the two Viking 44 45 seismometers in 1976 (these seismometers, placed on the landers with no direct contact 46 with the ground, didn't provide any exploitable data). The InSight mission is a geophysical 47 mission aimed at further understanding the structure of Mars and of the other terrestrial 48 planets of the solar system. The main geophysical instruments deployed on Mars are a very 49 high sensitivity seismometer (SEIS) funded by the French space agency (CNES - Centre 50 National d'Etudes Spatiales) and the HP³ instrument (Heat flow and Physical Properties 51 Package), a self-penetrating dynamic probe (called the mole, 39.6 cm long and 2.7 cm 52 diameter) funded by the German Institute for Planetary Research (DLR - Deutsches Zentrum 53 für Luft- und Raumfahrt). The HP³ instrument is designed to perform thermal conductivity 54 measurements along the first 3 – 5 meters below the surface. The InSight lander that landed 55 on Mars on 26 November 2018, is represented in the artist view of Figure 1, with both the 56 SEIS (covered by a wind and thermal shield) and the HP³ instruments deployed. Both are 57 linked to the lander by tethers that transmit energy and data.

58 The Figure also shows the 2 m long Instrument Deployment Arm (IDA) and its grapple that deployed both SEIS and HP³ on the ground. The IDA also carries a high-resolution 59 60 Instrument Deployment (colour) Camera (IDC). The lander is also equipped with an 61 Instrument Context Camera (ICC), a meteorological station including temperature, 62 windspeed and pressure sensors, two RISE antennas (Rotation and Interior Structure 63 Experiment) providing a precise location of the lander to accurately monitor the 64 movements of the planet from the Earth and an Ultra High Frequency antenna for data transmission. Energy is provided by two twin 1.8 m diameter solar panels with a power of 65 66 700 W each, on clear days.

67 The average size of the regolith particles on Mars was determined from thermal inertia 68 measurements by the Viking lander and the Themis orbiter (Arvidson et al. 1989). The 69 microscopic imagers of the Mars Exploration Rovers (MER - Spirit and Opportunity, 2004), 70 Mars Science Laboratory (MSL - Curiosity, 2012) and Phoenix lander (2008) evidenced 71 sorted dark grey basaltic sub-rounded to rounded fine sand particles (due to wind saltation 72 under a 750 Pa atmospheric pressure), with diameter between 80 and 200 µm. This range 73 is compatible with the average diameter (170 μ m) derived from thermal inertia measured 74 at the InSight landing site (160–230 J m⁻² K⁻¹ s^{-1/2}, Golombek et al., 2020). Bright red dust 75 particles $(2 - 5 \mu m)$ of slightly chemically altered basalt are also observed (Arvidson et al. 76 2004a et b, Golombek et al. 2006a and b and Goetz et al. 2010). Orbiter observations 77 indicated that dust has been expelled by the rockets during landing at an average distance 78 of 20 m around the InSight lander (Golombek et al. 2020).

Figure 2 shows a high-definition IDC photo of the Martian surface in the InSight landing site,
showing the sand-like appearance of the regolith, with also some pebbles on the surface
around.

A detailed geological description of the so-called "Homestead hollow" where the lander is located was presented by Golombek et al. (2020), from which the interpretative cross section of the landing site of Figure 3 has been extracted. It shows a typical regolith profile, with an around 3 m thick layer of relatively fine-grained impact generated regolith (3) that likely grades with depth into coarse, blocky ejecta (2) that overlies fractured basalt flows (1), with an estimated 10 m thick layer of blocky ejecta.

88 This paper is devoted to further investigating the elastic interaction between the SEIS foot 89 and a Martian regolith simulant, by using a specific device working at very low stresses and 90 strains. In a first approach, the investigation was carried out under terrestrial gravity (9.81 91 m/s^{2}), higher than Mars' gravity (3.72 m/s^{2}). The aim of the program is to provide an 92 estimation of the value of the axial elastic spring constant of the regolith simulant to allow 93 for a theoretical modelling of the elastic interaction between the SEIS and the Martian 94 regolith (see Fayon et al. 2018). From a seismology point of view, the elastic parameters of 95 the surface regolith that can be derived from this experiment are also interesting to 96 estimate the wave velocity in the near surface.

97 2. Material and methods

98 2.1.Introduction

99 The shape of the SEIS foot, designed based on a first series of tests carried out in a device 100 developed at Ecole des Ponts ParisTech (Karakostas et al. 2013) is presented in Figure 4. The 101 foot consists of a disk with 60 mm diameter and a 20 mm long conic spike (10 mm diameter 102 at its base) in its centre. As further detailed in Section 2.3, the device was improved to get 103 a better accuracy in terms of displacement and force measurements.

104 2.2.Martian simulant

105 The selection of a relevant Martian regolith simulant is a difficult task (Seiferlin et al. 2008). 106 Given the data provided by the MER, MSL and Phoenix missions and by thermal inertia 107 measurement (see above), it was decided to adopt the Fontainebleau sand (NE34), a well 108 sorted rounded sand with an average grain diameter D_{50} = 220 µm (a little bit larger than 109 the 170 μ m estimated value at the InSight site) and a uniformity coefficient C_u = 1,57 (the 110 maximum and minimum void ratios are $e_{min} = 0.54$ and $e_{max} = 0.86$, respectively, Andria-111 Ntoanina 2011). The grain size distribution curve and a SEM photo showing the rounded 112 shape of the grains are presented in Figure 5. As observed in Figure 2, it is possible that the 113 InSight regolith be less well sorted with more fine particles, but it is presently difficult to get 114 further precision about the exact in-situ grain size distribution. Observations from previous 115 missions also indicated a loose state of the regolith (Golombek et al. 2008) recently confirmed by local thermal measurements conducted at the surface by the HP³ mole (Grott 116 117 et al. 2021).

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120 2.3.Experimental set-up

121 The precision of the experimental device used by Karakostas et al. (2013) was significantly 122 improved by adopting high precision sensors for the measurements of both force and 123 displacement $(0.1\mu m)$, together with a stiff device designed to support the displacement (6x10⁻⁴ N) transducer and a thermal insulation device of the whole system. The device 124 125 (Figure 6) consists of a sand container of diameter 240 mm and height 120 mm placed on 126 the plateau of a standard 100 kN triaxial press, allowing for upwards displacement at 127 constant speed at 0.080 mm/min. The diameter of the container was constrained by the 128 space available between the two rods of the triaxial press.

129 The relevance of the container dimensions with respect to boundary conditions was 130 examined through elastic calculations based on Sneddon (1946)'s solution of a rigid circular 131 plate on a semi-infinite elastic homogeneous space (see Appendix), with the following 132 expression of the vertical displacement Δz :

133
$$\Delta z = \frac{\pi}{2} (1 - \nu^2) \frac{\sigma_V R}{F} = \frac{F}{2RF} (1 - \nu^2)$$
(1)

134 where *R* is the disk radius, *F* the applied force, $\sigma_v = F / \pi R^2$ the average applied vertical 135 stress, *E* the Young modulus and ν the Poisson coefficient. A Young modulus typical of loose 136 sands (*E* = 20 MPa, Massarsch 2015) was adopted and ν was taken equal to 0.22 (Delage et 137 al. 2017). Equation 1 allows analysing stresses on the boundaries of the mould, as shown in 138 Figure 7. This analysis reveals slight changes in stress at the bottom (5% for σ_z and 2% for 139 τ_{zx} of the vertical stress applied on the surface) and at the periphery, leading to a vertical 140 strain $\varepsilon_z = 0.001\%$ at bottom (compared to 0.018% below the disk). 141 Finite element elastic calculations were also carried out with the Plaxis code to account for 142 possible effects due to the dimensions of the mould. Calculations made with the mould (240 143 mm diameter and 120 mm height) were compared to those with a much larger container 144 comparable to a semi-infinite space (600 mm diameter and 550 mm depth). The differences obtained between the vertical displacements were 5.8 x $10^{-4} \mu m/N$ on the lateral boundary 145 146 and 0.04 µm/N in the middle axis. They were considered small enough to conclude that the 147 semi-infinite hypothesis used in Sneddon's approach was satisfactory for the mould used. 148 Compared to the 220 μ m average diameter of the sand grains, the small displacements 149 calculated would result in a tiny movement of the grains at the interface with the container, 150 with no significant effect on the disk penetration in the centre.

151 The option adopted in the work was to keep the real (Martian) foot dimensions by reusing 152 the foot model used during the SEIS foot design (Karakostas et al. 2013), by putting it in 153 contact with a simulant with grain size and shape comparable (albeit slightly larger) to those 154 of the InSight regolith, and by loading it up to the force supported by each foot on Mars. 155 The length scale used is 1 (e.g., $L^m/L^p=1$, where L^m is the length of the model and L^p is the 156 prototype's length), leading to a disagreement in the similitude scale for stresses, that can 157 be analysed by splitting stresses into those due to the seismometer weight σ_s (with a scale relationship of 1, e.g., $\sigma_s^{m}/\sigma_s^{p} = 1$) and those due to the self-weight of the soil σ_g , which is 158 159 2.64 times greater than on Mars. The match is not perfect along the earlier part of 160 penetration, where the stresses due to soil self-weight predominates. However, once the 161 load grows, the issue with similitude on stresses decreases because the induced stresses 162 beneath the foot dominates over the self-weight stresses.

163 The SEIS foot is fixed at the bottom of a cylindrical mass of 1 kg, corresponding to the load 164 supported by one SEIS foot under Mars' gravity, fixed to the horizontal rigid top beam of 165 the press through a force gauge. The experiment is carried out by putting the spike of the 166 foot in contact with the surface of the sand, prior to activating the upwards movement of 167 the press. This allows for the progressive penetration, at a constant displacement rate, of 168 the foot within the sand mass, until reaching the contact between the disk and the sample 169 surface. The spike penetration is monitored through the displacement rate of the press that 170 has been carefully calibrated. Once the disk contacts the sand, a high accuracy in 171 displacement monitoring is achieved by using a LVDT of 4.5 mm range with 0.1 µm 172 resolution.

173 The force measurement was improved by using a 25 N range force gauge (Interface SMT1-174 25N). To reduce the noise in displacement measurements, a stiffer fixation system for the 175 LVDT was adopted. As seen in Figure 6, the system is made up of two vertical rods strongly 176 fixed on a thick aluminium plate placed on the plateau of the press along a diameter of the 177 container, and of a horizontal rod strongly fixed to the two vertical ones. The 4.5 mm LVDT was fixed on this system, with its stem in contact with the upper side of the 1 kg mass. 178 179 Another LVDT with a larger range of 20 mm, seen in Figure 6, has also been used for tilt 180 verification. To the same aim, an insulation system from temperature changes and air 181 movements has been set up, with particular attention paid to thermally insulate the two 182 vertical rods of the triaxial press, because of the possible perturbations due to their thermal 183 expansion/contraction on the displacement measurements. The insulation system 184 consisted of a light prismatic box made up of 5 cm thickness expanded polystyrene and cork

layer, placed around the device, as seen in Figure 8a, with a small access made through thebox for the control system.

The performance of the insulating system was verified by performing several day/night 187 188 temperature measurements carried out by means of a thermocouple (able to detect change 189 in temperature smaller than 0.1°C, see Figure 9) inside the box during up to three days. As 190 shown in Figure 8b, thermal stabilization was attained one day after the box was placed 191 around the device. Also, the sand container was insulated from possible thermal 192 perturbations from the press by placing a thick polystyrene plate between the plateau and 193 the thick plate. As observed in Figure 8b and Figure 9, the resulting stabilization after 14 194 hours reduces temperature variations to the order of 0.1°C, which is found quite 195 satisfactory.

196 2.4.Setting-up the low-density simulant specimen

Given that the low unit mass of the regolith (1400 kg/m³ or even less, Golombek et al. 2008, 197 198 Delage et al. 2017, Morgan et al. 2018, Grott et al. 2021), a dry funnel deposition procedure, 199 already used by various authors (including Tatsuoka et al. 1979, Zlatovic and Ishihara 1997, 200 Lade and Yamamuro 1997, Yamamuro and Wood 2004, Flitti et al. 2019) was adopted. To 201 do so, the sand was gently poured at the centre of a container by using a funnel, keeping a 202 zero falling height between the bottom end of the funnel and the top of the cone. The 203 funnel was gently lifted by hand as the pile rises. An upper Plexiglas ring (8 cm in height) 204 with the same diameter was placed on top of the container. Once the sand pile reached the 205 right height, the ring was removed and the sand in excess was carefully erased to get a 206 smooth horizontal upper plane surface. To minimise any disturbance, the sample was 207 prepared close to the press and to a scale, on a mechanically insulating polystyrene layer. 208 Once prepared, it was very carefully placed (no walking) on the scale for weighing, prior to 209 be placed on the platen of the press. Once on the platen, the erased top surface of the sand 210 was inspected to make sure that it stayed at the top of the container, which was the case. 211 The unit mass of the deposited sand samples was found to be between 1.403 and 212 1436 kg/m³ (see Table 1). It corresponds to an average density index I_d = 6 %, indicating a 213 very loose state of the specimen.

214 2.5. Starting the test

215 Once the sample placed on the platen of the press, the thermal insulation box was placed 216 and data acquisition started. As already mentioned, almost one day was necessary to reach 217 temperature stabilization, a mandatory condition for proper displacement measurements. 218 The plateau was moved at a low velocity of 0.08 mm/min to adequately monitor all relevant 219 parameters. Various loading and unloading cycles were applied at different forces to 220 investigate the dependence of the elastic response with respect to stress (see Table 1).

221 **3. Experimental results**

222 Most preliminary tests were aimed at establishing the validity of the device and at checking 223 repeatability. To do so, it was found simpler to carried out tests with the 60 mm disk of 224 diameter (without spike), because it allowed for an easier detection of the first sand/disk 225 contact. As seen in Table 1, 10 tests were conducted with the disk. Once the performance 226 of the system assessed, it was found that only 3 tests with the foot (disk + spike) were 227 enough. The data obtained are presented in Table 1, including the values of the spring 228 coefficient K_c determined from load cycles under various forces, and K_f from the final 229 unloading and the Young moduli (E_c and E_f). The latter was derived under the hypothesis of 230 elastic semi-infinite homogeneous half-space, as commented later on.

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Table 1. Experimental program carried out.

N°	Unit mass kg/m ³	Max. p _t * μm	Max. force N	Load cycles min – max N	Slope K _c cycle MN/m	Slope K _{fu} final unload	Young modulus <i>E</i> c cycle	Young modulus E _{fu} final unload	
		(± 0.1)	(±0.01)	(±0.01)	(±0.07)	MN/m (+ 0.07)	MPa (+ 1 1)	MPa (+ 1 1)	
D1	1421	412.6	10.48	8.14 - 9.63	1.32	1.19	20.89	18.8	
D2	1432	534.1	10.39	2.72 - 1.57	0.28	1 47	4.46	23.31	
			_0.00	7.51 - 5.53	1.31		20.76		
D3	1413	419.6	10.47	9.30 - 8.06	1.33	1.09	21.11	17.21	
D4	1389	605.9	10.15	4.00 - 2.83	0.37	1.43	5.84	22.68	
DE	DF 1401	10E E		3.01 - 2.07	0.27	1 26	4.23	21.57	
03	1421	403.3	10.27	5.67 - 3.32	0.82	1.50	12.94		
D6	1436	438.2	10.	9.46 - 7.87	1.23	1.47	19.53	23.31	
D7	1423	392.5	10.01	6.92 - 5.62	1.03	0.93	16.38	14.71	
50	1400		10.07	3.12 - 1.77	0.63	0.76	10.03	11.97	
D8	1409	605.4	10.97	5.53 - 3.02	1.15		18.17		
D9	1428	348.6	9.80	8.77 - 7.44	1.16	1.53	18.37	24.27	
D10	1435	408.3	10.31	8.98 - 6.37	1.47	1.37	23.25	21.73	
Disk + spike									
DS1	1428	607.4	10.6	8.55 6.49	1.11	1.087	17.59	17.24	
DS2	1403	598.4	9.91	8.81 - 6.77	1.26	0.825	20.02	13.09	
DS3	1418	611.3	10.56	- 7.40	1.18	0.936	18.65	14.85	

232

* *p*_t: penetration

3.1. Penetration tests with the disc 233

234 10 tests were carried out with the disk up to a maximum force of 10 N, i.e., the force applied 235 on the ground by each SEIS foot on Mars. Half of them were performed to determine, 236 through a load cycle, the elastic response under 10 N. 5 others were performed under 237 smaller stresses to characterize the variation of the elastic response with stress. All final unloading sequences were also used to determine the *K*_f coefficients and the *E*_f Young
moduli, as indicated in Table 1.

Figure 10 shows the penetration curves of the 10 tests in terms of force (N) with respect to the penetration distance (μ m) monitored by the 4.5 mm range LVDT. The Figure shows some variability in the maximum penetration observed that ranges between around 400 μ m for 7 tests and around 600 μ m for the other 3. The responses at the beginning of the series of the 7 tests show a stronger increase in force with respect to the displacement. A comparable slope is however reached for the three other tests after a penetration of 300 μ m.

All curves show many spikes corresponding to sudden decreases in force, followed by a force recovery along several micrometres that brings back the curve along the initial one that are typical of slip and stick phenomena (Cain et al. 2001).

250 As indicated in Table 1, the elastic response under the SEIS weight (10 N) and thus very small 251 stresses was investigated through cycles in force. Figure 11 shows a cycle performed during 252 test D10 between 8.98 and 6.37 N (force amplitude of 2.61 N), with a resulting penetration 253 response between 395.65 and 397.4 μm (amplitude of 1.75 μm). The curve shows that the 254 decrease in force, resulting from stopping and reversing the direction of the movement of 255 the plateau, is not instantaneous, with a progressive decrease in force between 9.67 and 256 9.29 N occurring along 0.8 μ m, followed by a frank linear decrease in both force and 257 displacement. A very slight hysteresis is observed, with a difference of 0.2 μ m between the 258 two points at 8 N, showing quite a good reversibility and linearity of the response. A value 259 of an elastic spring constant $\Delta F / \Delta d$ of 1.47 x 10⁶ N/m is derived from the graph.

260 3.2.Penetration tests with the foot (disk + spike)

261 The difficulty was to accurately determine the first contact between the spike and the sand, 262 an impossible task if based on force measurement. Since it was easier to detect the contact 263 between the disk and the sand, it was decided to set this point at zero and to plot back the 264 penetration of the spike, as shown in Figure 12a. The curve shows that the penetration of 265 the spike along the first 17 mm (17 000 μ m to compare to the 400 – 600 μ m of disk 266 penetration) provides no significant added effect of the spike in the loose sand (added 267 contribution between 0.14 and 0.54 N). The Force/Penetration curve of test DS1 is 268 presented in Figure 12 a and b, showing no significant force mobilisation below 3 mm and 269 a maximum force of 0.14 N obtained just before the sand/disk contact is reached. The 270 monitored forces also give an idea of the force gauge accuracy that can be estimated \pm 0.01 271 N. The sudden drops in force previously observed with the disk and typical of stick and slip 272 are also seen with the cone (at 5 200 and 9 100 μ m).

The curves illustrating the interaction between the foot and the sand (Figure 13) are similar to those obtained without the spike, confirming the tiny effect of the spike. A maximum penetration of 600 µm was obtained in these tests, like in three of the tests with the disk only (Figure 10). Reversible load cycles similar to that presented in Figure 11 were also observed, confirming little effect of the spike on the elastic response.

278 **4. Interpretations and discussion**

279 4.1.General considerations

280 The challenge of setting up an experimental device able to deal with very low stress and 281 strain has been met, thanks to special care devoted to the thermal insulation and to the stiffness of the device, resulting in a precision of around 0.1 µm in displacements. The 282 283 maximum disk penetration obtained with our device under the weight of the SEIS (under 284 Mars gravity - around 10 N) is between 400 and 600 µm, whereas the average grain 285 diameter of the Fontainebleau sand, 220 µm, is around two to three times smaller. The low 286 stress in the area affected by the disk penetration hence results in quite small plastic strains 287 that correspond to the displacement of a limited number of grains below the sample 288 surface, illustrated by the slope of the curves of Figure 10. The data of Figure 10 and Figure 289 13 show that the force reached at a displacement corresponding to the average grain 290 diameter is between 2 and 7.5 N. This shows that the beginning of the curve is influenced 291 by the roughness of the surface, that is controlled by the arrangement of the grains. Besides 292 possible effects of local changes in density, this may be the explanation for the less 293 pronounced start of the curves reaching 600 µm. Conversely, the reversible load cycles 294 illustrate a small elastic response of $1.3 - 2 \mu m$ in vertical displacement, to be compared to 295 the 220 µm average diameter of the grains, indicating a limited reorganisation of the grain 296 assembly. The elastic response should mainly be governed by the reversible stress 297 release/recompression at grain contacts, following the Hertz-Mindlin contact theory (Hertz 298 1882, Mindlin 1949).

When comparing the penetration curves and the close values of the elastic spring constants of the foot and the disk only (Table 1), no significant added effect of the spike was observed, showing that there is space enough in the loose contracting sand to accommodate the thin

302 spike without any added strength. This probably also holds for the InSight regolith, the low 303 density of which has been confirmed by both the development of a pit around the mole at 304 the start of the penetration of the HP³ instrument (Golombek et al. 2020) and by some in-305 situ thermal conductivity measurements that provided estimated unit mass values around 1200 kg/m³ (Grott et al. 2021). Some pebbles, like those observed in Figure 2, have also 306 307 been observed in the excavations created by the rockets during landing and in the mole pit. 308 The possibility of not having full penetration of one of the SEIS feet due blocking by a pebble 309 cannot be completely eliminated. However, the thin shape of the spike is optimum in this regard, allowing easier deviation of the spike along a small enough pebble, and full 310 311 penetration in the loose regolith.

312 4.2.Elastic interpretation of the response

Based on the measurements of elastic spring constants K_v presented in Table 1 and on the simplifying assumption of homogeneous linear and isotropic behaviour of the sand, one derived the Young moduli for all the cycles performed (see Table 1) based on the following expression derived from Sneddon's Equation (1):

317
$$E = \frac{k_{\nu}(1-\nu^2)}{2R}$$
(2)

The elastic strain increment corresponding to the force cycle of Figure 11 is equal to 6×10^{-5} showing that the Young moduli are determined at small deformations, not far from the range of those derived from bender element measurements ($10^{-5} - 10^{-6}$), as done in Delage et al. (2007). The changes in Young modulus with respect to the average vertical stress at 322 surface are presented in Figure 14 (given the negligible effect of the spike, the calculations323 for the foot have also been made by using Sneddon's solution).

324 In the area corresponding to the SEIS weight (around 3.5 kPa), some dispersion is observed, 325 with E values derived from load cycles between 17.59 and 23.25 MPa. This is an indication 326 of possible changes in local density of the sample at the contact area with the disk/spike, 327 probably enhanced by the small number of grains mobilised by the low applied stress, or 328 for other reasons that are not yet clear, including possible hysteretic material behaviour of 329 the sand. Averaging the values between 3 and 3.5 kPa yields an average E value of 20.07 330 MPa (note that, for some reasons, some of the monotonic unloading coefficients at 331 maximum force stay well below the line, with values as small as 11.97 – 14.85 MPa). Possible 332 effects of changes in Young modulus with depth, suggested by the data of Figure 14, were 333 investigated by carrying out finite elements calculations with a linear change of E with 334 depth, in a model composed of 12 layers of 1 cm thickness. The difference in settlement at 335 surface was found equal to 0.0468 μ m/m, indicating negligible effects of changes in Young 336 modulus with depth.

The linear changes in Young modulus with respect to changes in average vertical stress (a value easy to calculate under a rigid plate) observed in Figure 14 are distinct from the wellknown power law that relates the Young modulus with the confining stress, defined by the two following equations (e.g. Santamarina et al. 2001):

341

$$V_p = \alpha \left(\frac{p}{1 \, kPa}\right)^{\beta} \tag{3}$$

342 and

343
$$E = \rho V_p^2 \left(\frac{3 - \frac{4}{l^2}}{l^2 - 1}\right)$$
(4)

344 with $l = \sqrt{\frac{2(1-\nu)}{1-2\nu}}$

345 where p is the mean stress, and α (m/s) and β (-) two fitted parameters. Delage et al. (2017) 346 obtained β = 0.3 and v = 0.22 on three different (more angular) Martian simulants. Following 347 Santamarina et al. (2011), it was found more relevant to adopt, for subrounded grains, a 348 slightly smaller value of 0.28, which results in having a power law with β = 0.56, slightly larger than the 0.5 value commonly adopted for sands (e.g., Hardin and Black 1966, 349 350 Oztoprak and Bolton 2013). The significant difference with the linear changes observed in Figure 14 is related to the use of average vertical stress σ_z instead of mean stress ((σ_z + 2 σ_x 351 352 /3, since $\sigma_x = \sigma_y$). Inspection of the Sneddon's expressions of σ_z , σ_x and σ_y (see Appendix) 353 shows that the relation between σ_z and p is far from being simple, with the added difficulty 354 of properly estimating the relevant layer thickness needed to estimate p under the disk.

Knowing *E* allows for the calculation of the other parameters governing the SEIS/ground interaction used in Fayon et al. (2019) in their modelling of the SEIS ground interaction, i.e., the horizontal spring coefficient k_h and the torque C_h (Poulos and Davis 1974):

358
$$k_h = \frac{16 \text{ ER}(1-\nu)}{(7-8\nu)(1+\nu)} = 1.17 \times 10^6 N/M$$
(5)

359
$$C_h = \frac{4 \text{ ER}^3}{3(1-\nu^2)} = 756.6 \text{ N}. m$$
(6)

Given that the Young modulus has been derived at a strain of 6 x 10⁻⁵, another interesting seismic parameter that can be derived is the compression wave velocity at surface, according to the following equation:

363
$$V_p = \sqrt{\frac{E(1-\nu)}{\rho(1+\nu)(1-2\nu)}}$$
 (7)

in which ρ is the regolith unit mass. Adopting E = 20 MPa, ρ = 1.400 kg/m³ and v = 0.22 yields a value of compression wave velocity of 128 m/s, in the expected range for sands under low confinement. Interestingly, this is also in a good agreement with the in-situ velocities at the surface of Mars derived from analysing the seismic signals received by SEIS from HP³ hammering at around 0.3 m deep, and 1.1 m away, equal to 118 ± 34 m/s for a surface layer approximately 30 cm thick (Lognonné et al. 2020).

370 However, note that these satisfactory agreements do not necessarily prove that our 371 experiment exactly reproduces the situation on Mars. Some limitations have been discussed 372 above in this regard, both in terms of simulant and scale effects. The calculations here are 373 also made based on a simplifying assumption of an equivalent homogeneous isotropic semi-374 infinite medium, whereas it is known that there is an increase of the elastic modulus of 375 granular media with increased mean stress, as shown in Figure 14. Beside the detailed 376 geological examination of the surface (Golombek et al. 2020), further investigations are 377 presently carried out in this regard within the Near Surface Working Group of the InSight 378 Science Team, based on visual and geological analyses, on the local measurements of 379 thermal inertia, on the seismic signals received by SEIS and on the thermal data provided by HP³. Based on thermal inertia measurement, a possibility could be to have a finer 380

381 material with some cohesion, which would also be compatible with the very low density382 suspected.

383 **5. Conclusions**

384 Some improvements were made on the specific device that allowed to design the final shape of the SEIS foot (Lognonné et al. 2019) and to provide the first estimation of the 385 386 elastic parameters governing the interaction between a Martian simulant and the SEIS foot 387 as analysed in Fayon et al. (2018). To successfully carry out accurate tests in conditions of 388 very low stresses (< 4 kPa) and displacements (< 600 μ m), particular care was taken in 389 optimising measurements by using a stiff system to support the container and the LVDT 390 gauges, by using an adapted force gauge and a complete thermal insulation device, allowing 391 to get rid of any perturbation that could result from the thermal expansion/contraction of 392 the loading system, provided a time period of 24h was waited for to ensure a constant 393 temperature with variations smaller than 0.5°C. This high precision device allowed to 394 successfully carry out interaction tests between both a disk and the foot. The tests showed 395 that, under the SEIS weight, the maximum penetration in the simulant adopted was around 396 600 μm, i.e., less that the thickness of a layer made up of 3 grains of Fontainebleau sand, a 397 well graded rounded sand with an average grain diameter of 220 µm. As a consequence, 398 the start of the penetration curves was affected by the roughness of the specimen surface, 399 who resulted in having maximum penetration between 400 and 600 μ m, i.e., around twice 400 the diameter of a grain. The large number of tests conducted on loose specimens with dry 401 unit mass around 1400 kg/m³ (density index of 6%) showed reasonable repeatability, with

402 however some dispersion in measured modulus related to possible changes in local density. 403 The elastic response was investigated by conducting stress cycles at various force levels that provided fairly reversible responses with small hysteresis. A simplified analysis considering 404 405 a homogeneous elastic isotropic semi-infinite medium was carried out to determine the 406 dependency of the Young modulus with respect to the average vertical stress. The linearity 407 observed was distinct to the power law observed with respect to the mean stress, a value 408 difficult to estimate in our case. It was also shown that the effect of the spike in a loose regolith was not significant. The order of magnitude of the Young modulus under the SEIS 409 410 foot is 20 MPa, a reasonable value for loose sand on the Earth. This value was found 411 compatible with previous laboratory estimations of the surface compression wave velocity 412 and also with in-situ seismic measurements carried out on the InSight landing site by SEIS, from the hammering sessions of HP³. The data obtained from this improved system will 413 414 allow for a better determination of the SEIS – regolith interaction and of the resulting 415 resonance frequencies (Fayon et al. 2018). They of course could be improved in the future, 416 once better knowledge about the regolith simulant is obtained. They can also be used for 417 the resonance frequencies of the lander itself, that appeared to also have significant effects 418 on the SEIS measurements.

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8. List of Figures

Figure 1. InSight NASA Lander concept and main instruments. Image Credit: https://mars.nasa.gov/insight/spacecraft/instruments/summary/

Figure 2. IDC photo showing the scoop of the Instrument Deployment Arm compressing the regolith above a hole developed around the self-penetrating mole during hammering. The scoop width is 7.6 cm (image credit NASA – CalTech).

Figure 3. Interpretative cross-section of the shallow surface beneath the InSight lander (Golombek et al. 2020, Creative Commons CC BY license). 1: Fractured basalt flow; 2: Blocky ejecta; 3: Finegrained impact generated regolith; 4, 5: Overlapping craters; 6: Rockier area; 7: Rocks embedded in regolith; 8: Pits opened by retro rockets during landing; 9: Surface divots; 10: Lens of ejecta from other craters.

Figure 4. The shape of the SEIS foot. The conical spike has a maximum diameter of 10 mm.

Figure 5. NE 34 Fontainebleau sand: a) Grain size distribution; b) Scanning electron microscope photo of the showing the subrounded grains (after Andria-Ntoanina 2011).

Figure 6. Experimental set-up, showing more details the fixation of the mass to the force gauge and the LVDTs providing the vertical displacement.

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Figure 8. a) Thermal insulation of the device; b) temperature stabilisation is reached after 10 hours for tests DS1, DS2 and DS3 (see Table 1).

Figure 9. Temperature stabilization in tests DS1, DS2 and DS3 (see Table 1) made possible by the thermal insulation of the device, is of the order of 0.1°C.

Figure 10. Force-penetration curves for the 10 tests performed with the disc.

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Figure 12. a) Force-penetration curves for the 3 tests performed with the foot (disk+spike). Note that the force significantly increases once the disk contacts the sand surface. The start of the disk/sand contact is taken as 0 for the x-axis, and the curves are started at a point obtained by subtracting the length of the spike (- 20 000 μ m), b) Detailed force penetration curve of the spike alone for test DS1, with a maximum force of 0.14 N reached just before sand/disk contact. Two stick-slips are observed at around 4800 and 8600 μ m.

Figure 13. Force penetration curves for the 3 tests carried out with the foot (disk + spike) once the spike has fully penetrated the sand and the disk is in contact with the sand.

Figure 14. Changes in Young modulus with respect to the average vertical stress at surface. Data are derived from load cycles tests and from the final unloading step.

9. Appendix

The normal and stress fields presented in Figure 7 are based on Sneddon's solution for a

cylindrical shaped indenter. The expressions are presented in more detail below.

Sneddon (1946) provided, from an elastic analysis based on Boussinesq's solution, the following expressions for the stress and strain field under a rigid loaded plate:

$$\sigma_z = \frac{3F}{2\pi} \cdot \frac{z^3}{R^5} \tag{8}$$

$$\sigma_{\chi} = \frac{3F}{2\pi} \cdot \frac{x^2 z}{R^5} - \frac{m-2}{3m} \left[-\frac{1}{R \cdot (R+z)} + \frac{(2R+z)x^2}{(R+z)^2 R^3} + \frac{z}{R^3} \right]$$
(9)

$$\sigma_y = \frac{3F}{2\pi} \cdot \frac{y^2 z}{R^5} - \frac{m-2}{3m} \left[-\frac{1}{R \cdot (R+z)} + \frac{(2R+z)y^2}{(R+z)^2 R^3} + \frac{z}{R^3} \right]$$
(10)

$$\tau_{xy} = \frac{3F}{2\pi} \cdot \frac{xyz}{R^5} - \frac{m-2}{3m} \left[\frac{(2R+z)xy}{(R+z)^2 \cdot R^3} \right]$$
(11)

$$\tau_{xz} = \frac{3F}{2\pi} \cdot \frac{xz^2}{R^5} \tag{12}$$

$$\epsilon_z = \sigma_z / E \tag{13}$$

$$p_t = \frac{1+\nu}{2\pi E} \cdot \left[\frac{z^2}{R^3} - \frac{(1-2\nu)x}{R}\right] \cdot F \tag{14}$$

in which *F* is the force applied on the disk, *z* the depth, x and y the horizontal distance from the axis, $R = \sqrt{x^2 + y^2 + z^2}$, p_t the penetration and m = 1/v.