

Analysis of small scale fluctuations in Hall effect thrusters using virtual Thomson scattering on PIC simulations

Tarek Ben Slimane, Cyrille Honoré, Thomas Charoy, Anne Bourdon, Pascal Chabert

► To cite this version:

Tarek Ben Slimane, Cyrille Honoré, Thomas Charoy, Anne Bourdon, Pascal Chabert. Analysis of small scale fluctuations in Hall effect thrusters using virtual Thomson scattering on PIC simulations. Physics of Plasmas, 2022, 29, 10.1063/5.0073215. hal-03551302

HAL Id: hal-03551302 https://hal.science/hal-03551302

Submitted on 1 Feb 2022

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers. L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

¹ Analysis of small scale fluctuations in Hall effect thrusters using virtual Thomson ² scattering on PIC simulations

- Tarek Ben Slimane,^{1, a)} Cyrille Honoré,¹ Thomas Charoy,¹ Anne Bourdon,¹ and Pascal Chabert¹
- Laboratoire de Physique des Plasmas (LPP), CNRS, Sorbonne Université,
- École Polytechnique, Institut Polytechnique de Paris, 91120 Palaiseau,
 - France

8

9

10

11

12

13

14

15

16

17

18

19

20 21

22

23

24

25

(Dated: 10 January 2022)

Hall effect thrusters are yet not fully understood due to multiple open questions, among which the anomalous electron transport is the most challenging. Particle-In-Cell (PIC) simulations suggested that electron transport is strongly affected by small-scale instabilities which can be investigated experimentally using Collective Thomson Scattering (CTS). In this paper, we perform a virtual CTS diagnostic on PIC data in order to investigate these small scales and to relate simulation and experiments. The virtual diagnostic is performed on the electron density maps from a simplified 2D axial-azimuthal PIC simulation. Analysis along the channel axis showed two dominant modes at 8 rad.mm^{-1} and 3 rad.mm^{-1} . These modes exhibit ion-acoustic wave features and appear to be selectively affected by the acceleration of the ions when transitioning from the ionization zone to the acceleration zone. A numerical study of the electron density fluctuation rate from the virtual CTS is also performed and the obtained profiles are compared to PIC. This reveals that the virtual CTS integrates both axial and azimuthal fluctuations of the electron density due to the spatial extension of the scattering volume. Moreover, it also highlights the importance of a good estimation of the electron density in the scattering volume for an accurate estimation of the fluctuation rate.

PACS numbers: 0

a)tarek.ben-slimane@lpp.polytechnique.fr

Physics of Plasmas

This is the author's peer reviewed, accepted manuscript. However, the online version of record will be different from this version once it has been copyedited and typeset

PLEASE CITE THIS ARTICLE AS DOI: 10.1063/5.0073215



26 I. INTRODUCTION

Since their development in the mid-60s, Hall effect thrusters (HET) have been successfully used 27 for many space missions either as a secondary propulsion system destined for station keeping and 28 attitude control or as a primary propulsion system for interplanetary missions^{1,2}. The propellant, 29 ommonly a noble gas, is injected from the anode side into an annular chamber while an electron-30 mitting cathode provides electrons to ionize the propellant and produce plasma. A high voltage 31 applied across the chamber and accelerates ions downstream while electrons are trapped in a 32 $\times B$ configuration using a strong radial magnetic field. However, the rate at which electrons flow E 33 from the cathode to the anode exceeds the rates predicted by classical theory by several orders of 34 magnitude, which impede a full understanding of HET operation. 35 Several contributions to this anomalous mobility have been extensively studied throughout the 36

years^{3–5} and recently the electron cyclotron drift instability or ECDI was identified as a key driver 37 or electron transport. Indeed, the asymmetry between the electron fast azimuthal motion and the 38 ons' axial motion creates a kinetic, MHz (millimeter scale) azimuthal instability that was observed PIC simulations⁶⁻¹⁰ as well as in experiments¹¹⁻¹⁵ and was associated with enhanced electron ansport. The linear theory of ECDI instability is well developed, however, nonlinear saturation 41 still the subject of active research and debate as discussed in the recent review by Kaganovich 42 al.16. Given the complexity of mode saturation in the 3D geometry of Hall thrusters, only 43 large-scale 3D PIC simulations can provide a full understanding of the mechanism of anomalous 44 transport. However, these simulations are still out of reach and most PIC simulation studies are 45 carried out in simplified 1D and 2D configurations. 46

Experimentally, the investigation of instabilities at millimeter scales in a 3D thruster is challenging and was done so far using coherent Thomson scattering (CTS)^{13,17–19} and different probe techniques^{12,15,16}. Furthermore, most measurements are performed outside the narrow thruster channel, in the vicinity of the exit plane, and downstream in the plume.

In 2011, a previous work by Honoré et al.²⁰ used an analysis method that combines 2D PIC axial-azimuthal simulation results with a virtual CTS. The approach consisted of generating a theoretical scattering signal from PIC data and showed an excellent agreement with CTS observations of the plume on the static form factor which is a measure of the intensity of the electron density fluctuations. However, the time resolution of the PIC simulation used was limited to $\Delta t = 1 \,\mu s$ so that no spectral comparison was performed.

This is the author's peer reviewed, accepted manuscript. However, the online version of record will be different from this version once it has been copyedited and typeset

PLEASE CITE THIS ARTICLE AS DOI: 10.1063/5.0073215

PLEASE CITE THIS ARTICLE AS DOI: 10.1063/5.0073215



In this paper, we apply a virtual CTS method to the highly resolved PIC simulations of a 2D PIC 57 axial-azimuthal benchmark test case developed recently in Ref.¹⁰ to identify the main parameters 58 impacting the evaluation of the electron density fluctuation rate between PIC simulations and CTS 59 experiments. This is achieved by comparing the reference fluctuation rate derived from PIC data 60 with the one obtained from virtual CTS analysis. 61

CTS experiments usually measure an electron density fluctuation rate of the order of 1%¹⁷, 62 whereas PIC simulations tend to produce large electron density fluctuations at around 10%-15% 63 with significant axial variations⁸. This difference is often explained by stating that, as aforemen-64 oned, 2D simulations are unable to account for the 3D structure of the ECDI. Nevertheless, 65 experiment-wise, the determination of the absolute electron fluctuation rate in CTS is also chal-66 enging since it requires a precise measurement of the electron density in the scattering volume and 67 absolute calibration of the setup²¹. Still, recent work using incoherent Thomson scattering to mea-68 ure this density reported a local electron density lower than previous estimates and a fluctuation 69 density rate of the order of 8%²², closer to the value reported by simulations. 70

In this respect, the local electron density inside the scattering volume seems to have a significant 71 impact on the value of the fluctuation rate. The probed volume, which is defined by the intersection 72 of two laser beams, depends on the waist of the laser. Experimentally, it is very burdensome to 73 vary this parameter and keep a good-quality signal, since it directly impacts the measurement 74 esolution. On the other hand, since PIC simulations are only limited by the mesh resolution, it is 75 possible to numerically produce the scattering signal from PIC data for different waists and assess 76 the impact of the waist on the estimation of the electron density fluctuation rate. Furthermore, PIC 77 results offer more flexibility in terms of the accessible zones for the diagnostic. Indeed, contrary to 78 CTS experiments where the diagnostic has only been done outside the thruster channel, by basing 79 virtual CTS on PIC results, it is also possible to explore the thruster channel. Interestingly, a а 80 similar virtual diagnostic was also performed within an incoherent Thomson scattering setup in 81 Ref^{23} to investigate the impact of the laser intensity on the measurement of the electron density. 82 The study also highlighted the valuable flexibility and insight that virtual diagnostics can bring to 83 experiments. 84

The organization of the paper is as follows. The 2D axial-azimuthal PIC simulation and the 85 virtual collective Thomson scattering analysis are presented in Section II. Then, in section III, 86 first, we present the virtual CTS observation at a fixed time to identify the dominant modes in 87 the plume as well as in the thruster channel. Then, we perform a temporal-Fourier analysis to 88

PLEASE CITE THIS ARTICLE AS DOI: 10.1063/5.0073215

89

90

91

92

93

hysics of Plasmas

density fluctuations. **METHODS** II. **Axial-Azimuthal PIC Simulation** у Cathode emission plane Thruster Periodicity exit plane вŤ Anode (200V) $L_y = 12.8 \, mm$ E **•** B x=7.5mm L_x=25mm (a) Cross section

identify the main frequencies and derive the dispersion relations for the dominant propagating

modes. Finally, we discuss the impact of the waist of laser beams on the derivation of electron

tion (b) Simulation domain

FIG. 1: Schematic of a transversal cross-section of the thruster and the simulation domain adapted from Ref¹⁰. *x* is the axial direction, *y* is the (periodic) azimuthal direction. Black dashed line at x = 7.5 mm corresponds to the position of the maximum radial magnetic field and the exit plane of the thruster. Green dashed line at x = 24 mm is the plane from which electrons are emitted.

Х

As mentioned in the introduction, an absolute comparison with CTS experimental results would 94 require a full highly-resolved 3D PIC simulation of the whole thruster. However, the latter is 95 still out of reach due to its computational cost. Currently, 2D PIC simulation results in both 96 xial-azimuthal and radial-azimuthal simulation planes $^{6-10,24}$ are the most complete data to study 97 azimuthal instabilities. In particular, the axial-azimuthal plane allows the study of the development 98 azimuthal instabilities and their impact on axial electron transport. Therefore in this work, the 99 virtual CTS analysis was applied to the axial-azimuthal PIC benchmark results from Charoy et 100 al.¹⁰. Although the chosen simulation case is non-self consistent, it was subject to an international 101 benchmark effort and the instabilities involved in the simulation are well characterized making it a 102



FIG. 2: Typical results from PIC simulations. This snapshot of the electron density is taken at $t = 12.5 \,\mu\text{s}$. The dashed red line at $x = 7.5 \,\text{mm}$ indicates the limit between the ionization zone and the acceleration zone which also corresponds to the position of the maximum radial magnetic field and the exit plane of the thruster

solid analysis framework to test the virtual CTS. In the following, a summary of the conditions of 103 the 2D benchmark and key results are presented, but additional details are given in Refs. ^{10,25}. The 104 2D simulation domain consists of a 500×256 Cartesian mesh in which the x-axis corresponds to 105 the axial direction while the y-axis corresponds to the azimuthal direction, as illustrated in Fig 1. 106 The curvature of the domain was hence neglected. The axial length L_x was fixed to 25 mm while 107 the azimuthal length L_{y} was fixed to 12.8 mm. A periodic boundary condition was imposed in 108 the azimuthal direction to circumvent the finite azimuthal length and to reduce the computational 109 time. 110

Initially, ions and electrons are uniformly distributed over the simulation domain at a density 111 of $5 \times 10^{16} \text{m}^{-3}$ and are subject to a 200 V voltage, which was applied between the anode and the 112 athode. The radial magnetic field profile, as well as the ionization profile, were fixed to remove 113 the low-frequency breathing mode oscillations and highlight the high-frequency instabilities in-114 cluding the ECDI. To maintain the discharge current balance, electrons were injected from the 115 cathode side from an emission line positioned at 1 mm from the right boundary of the simulation 116 domain (Green dashed line in Fig.1), and its potential is adjusted at each step to ensure a zero 117 azimuthally averaged potential at this position^{10,25}. This allows keeping a constant averaged po-118

Physics of Plasmas

This is the author's peer reviewed, accepted manuscript. However, the online version of record will be different from this version once it has been copyedited and typeset

PLEASE CITE THIS ARTICLE AS DOI: 10.1063/5.0073215



PLEASE CITE THIS ARTICLE AS DOI: 10.1063/5.0073215



FIG. 3: 2D FFT of the electron density at x = 3 mm and x = 20 mm. Solid white line: ion acoustic dispersion relation.

tential drop between the anode and the emission line. With this method, the right boundary plane has a varying negative potential but this drop in potential between the emission plane and the right boundary plane does not have any useful physical meaning and does not affect the main discharge physics^{10,25}. The integration used an explicit scheme with a grid size $\Delta x = 50 \,\mu\text{m}$ and a time step $\Delta t = 5 \,\text{ps}$. A total of 4×10^6 time steps were simulated resulting in a discharge simulation up to around 20 μ s. Nevertheless, due to limited storage capacity, the plasma parameters are only stored every 5000-time steps, thus yielding a sampling time for the electron density of $T_s = 25 \,\text{ns}$.

¹²⁷ A quasi-steady-state was reached at around 12 µs featuring an almost constant discharge cur-¹²⁸ rent. In Fig 2, a typical 2D map of the electron density at t = 12.5µs is shown. It highlights mil-¹²⁹ limetric fluctuations of the electron density propagating mainly along the azimuthal y-axis. These ¹³⁰ are marked with a change in the wavelength around the exit plane of the thruster at x = 7.5 mm. In ¹³¹ the ionization zone which extends from x = 2 to 7.5 mm, the fluctuations have a small wavelength





Publishing

PLEASE CITE THIS ARTICLE AS DOI: 10.1063/5.0073215

146

and their wavevector has both an axial and an azimuthal component. From x = 7.5 to 25 mm, 132 which corresponds to the acceleration zone, the structures have a larger wavelength and are almost 133 azimuthal. 134

A spatial and temporal Fourier analysis of these maps between 16 and 20 µs highlights a con-135 tinuous dispersion relation in the ionization zone (Iz) that shows a good correspondence with the 136 modified ion-acoustic wave dispersion relation from Lafleur et al.^{7,8}, as illustrated in Fig 3. The 137 mode corresponding to the maximum growth rate under the ion-acoustic wave approximation is 138 identified at $k_{Iz} \approx 8 \text{ rad.mm}^{-1}$ and has a high frequency $f_{Iz} \approx 5 \text{ MHz}$. In the acceleration zone 139 Acc), the fit with the modified ion-acoustic dispersion relation is less satisfying. Still, the dynam-140 ics is dominated by a single mode at $k_{Acc} \approx 3 \text{ rad.mm}^{-1}$ at a lower frequency $f_{Acc} \approx 3 \text{ MHz}$. The 141 nderlying reason for this change in the dynamics is yet not clearly understood and, in the follow-142 ing, we mainly focus on the estimation of the fluctuation rate from the PIC simulation results. 143

The fluctuation rate from PIC data is assessed by calculating the fluctuation in time and aver-144 aging the results azimuthally for each axial position. It is expressed as it follows: 145

$$\left. \frac{\delta n_e}{n_e} \right|_{PIC} (x) = \left\langle \frac{\sqrt{\langle n_e^2 \rangle_t - \langle n_e \rangle_t^2}}{\langle n_e \rangle_t} \right\rangle_y. \tag{1}$$

The blue curve in Fig 4 shows the axial evolution of the fluctuation rate between 12 µs and 20 µs for 147 the simulated benchmark case. The profile exhibits a large axial variation as already observed by 148 T.Lafleur et al.⁸ in the first microseconds of a self-consistent 2D (axial-azimuthal) PIC simulation, 149 that seems to be linked to the ionization profile. Indeed, the fluctuation rate decreases from 10% 150 close to the anode to a minimum of 2.6% at x = 6.4 mm. This coincides with the maximum 151 ionization rate imposed in the simulation. Then, it increases with small oscillations in the plume 152 to stabilize at around 7% after x = 20 mm. 158

In the following, the blue curve will be used as a reference for the comparison with the virtual 155 CTS results. 156

Collective Thomson Scattering Analysis В. 157

Collective Thomson scattering describes a scattering phenomenon in which the wavelength of 158 scattering λ is larger or comparable to the Debye length λ_D^{26-28} . An incident laser beam at a fixed 159 wave vector ki is applied upon the plasma and accelerated electrons behave like dipoles and scat-160 ter a part of the incident electromagnetic field at the same wavelength as the incoming field. The 161

Publishing

This is the author's peer reviewed, accepted manuscript. However, the online version of record will be different from this version once it has been copyedited and typeset

PLEASE CITE THIS ARTICLE AS DOI: 10.1063/5.0073215



FIG. 4: The axial variation of the electron density fluctuation rate in the simulation case from Charoy et al.¹⁰. The blue curve corresponds to the reference fluctuation rate from PIC results using Eq 1. The red curves corresponds to the synthetical fluctuation rate from the virtual CTS at a waist w = 0.1 mm based on Eq 7. Black dashed line at x = 7.5 mm corresponds to the position of the maximum radial magnetic field and the exit plane of the thruster.

scattered fields add up to yield the total scattered field. This sum can be constructive or destruc-162 tive depending on the phase correlation between the electrons' positions. This correlation might 163 occur if coherent structures are propagating through the medium. If so, these can be highlighted 164 by varying the observation length scale of the scattering experiment to match the length scale of 165 the propagating structures. The wave vector \mathbf{k} along which the CTS is performed is called the 166 cattering vector and is determined using Bragg's relation, relating the wave vector \mathbf{k} to the differ-167 ence between the laser wave vector \mathbf{k}_i and the scattered wave vector \mathbf{k}_s , as it follows: $\mathbf{k} = \mathbf{k}_s - \mathbf{k}_i$. 168 Hence, by changing the angle between these vectors, it is possible to change the norm of the scat-169 tering wave vector \mathbf{k} and thus to probe the medium at different scales. In practice, the scattering 170 wave vector \mathbf{k} is changed using a translating mirror to vary the angle between \mathbf{k}_s and \mathbf{k}_i . 171

For a fixed **k**, the resulting signal is proportional to the Fourier transform of the electron density along **k** in the scattering volume \mathscr{V} and is recorded as a function of time, such as:

$$\mathbf{s}(\mathbf{k},t) = \int_{\mathcal{V}} e^{i\mathbf{k}\cdot\mathbf{r}} n_e(r,t) d^3 \mathbf{r}.$$
 (2)



FIG. 5: 2D configuration for the virtual CTS calculation. The scattering volume is a Gaussian profile with a waist w = 3 mm. The phase factor in the Fourier transform is directed along the scattering vector **k**, here expressed in polar coordinates with $\alpha = 120^{\circ}$ and k = 3 rad.mm⁻¹

Further frequency analysis is usually performed over the total acquisition duration T and provides the spectral density of the scattering signal $S(\mathbf{k}, \boldsymbol{\omega})$. This quantity is proportional to the dynamic form factor and is expressed here as it follows:

$$S(\mathbf{k},\boldsymbol{\omega}) = \left| \int_{\mathcal{V},T} e^{i(\mathbf{k}\cdot\mathbf{r}-\boldsymbol{\omega}t)} n_e(r,t) d^3\mathbf{r} dt \right|^2.$$
(3)

The integral over all frequencies of Eq3 is noted $S(\mathbf{k})$ and is proportional to the static form factor, usually measured in CTS experiments. $S(\mathbf{k})$ is written as:

$$S(\mathbf{k}) = \int_{\omega} S(\mathbf{k}, \omega) d\omega.$$
(4)

Finally, the total fluctuation of the electron density can be related directly to the static factor, using Parceval's theorem, yielding the following relation:

$$\langle n_e^2 \rangle = \int_{\mathbf{k}} S(\mathbf{k}) d\mathbf{k},$$
 (5)

185

184

178

181

To perform a virtual CTS on PIC results, we adapt the former integral to a 2D geometry. This is illustrated in Fig 5. The scattering volume V_s , defined as the intersection between the laser beam and the 2D map, is modeled using a Gaussian intensity profile u in polar coordinates with a fixed waist w, such as:

$$u(r) = \frac{1}{\pi w^2} e^{-\frac{r^2}{w^2}}.$$

This is the author's peer reviewed, accepted manuscript. However, the online version of record will be different from this version once it has been copyedited and typeset

PLEASE CITE THIS ARTICLE AS DOI: 10.1063/5.0073215

AIP Publishing



A 2D local spatial Fourier transform is applied to the electron density maps from the PIC simulation along a fixed scattering vector **k** given in polar coordinates (k, α) . This enables to probe structures at a given scale *k* and that are propagating at a given angle α . The virtual scattered field is thus expressed as:

$$S_{PIC}(\mathbf{k},\mathbf{r_0},t) = \int_{\mathscr{A}} e^{i\mathbf{k}\cdot(\mathbf{r}-\mathbf{r_0})} u(\mathbf{r}-\mathbf{r_0}) n_e(r,t) d^2\mathbf{r},$$
(6)

where \mathbf{r}_0 is the center of the 2D equivalent of the scattering volume \mathscr{A} , i.e the center of the Gaussian profile.

The laser intensity profile is applied at each point $r_0 = (x_0, y_0)$ of a rectangular grid at a step $\Delta x_0 = \Delta y_0 = 1$ mm. Each position represents a different Thomson scattering experiment where the scattering vector $\mathbf{k} = (k, \alpha)$ is varied by changing both the norm k and angle α . The angle was varied with a resolution of 1° from 0° to 180°, whereas the norm k is varied with a resolution of 0.3 rad.mm⁻¹ from 0.5 rad.mm⁻¹ to 12.5 rad.mm⁻¹. The laser waist is fixed to 3 mm for the reference case as usually used in CTS experiments. Besides, this value offers good analysis features for simulations, considering the size of the structures we want to investigate.

Virtual CTS can be understood as a windowed Fourier transform, for which the chosen window is a 2D Gaussian function, centered at r_0 and which has a standard deviation $\sigma = \frac{w}{\sqrt{2}}$. Similar to windowed Fourier transform, virtual CTS has two main advantages. First, it limits the analysis to an area proportional to the laser beam waist w^2 . Second, it introduces a diagnostic length scale $\lambda = \frac{2\pi}{|k|}$ to highlight fluctuations occurring at the same scale as λ . This offers both spatial localization and selectivity over the k-component within the uncertainty inequality limit.

The electron density fluctuation rate is calculated also from the static form factor, using Eq5. However, the calculation yields the root mean square (rms) of the electron density, weighted by the square of the laser intensity profile u^2 , denoted as $\langle n_e^2 u^2 \rangle_{\mathscr{A},T}$. The operator $\langle . \rangle_{\mathscr{A},T}$ corresponds to the average over the area delimited by the weighting function u^2 , such as :

$$< n_e^2 u^2 >_{\mathscr{A},T} = \frac{\int_{\mathscr{A},T} n_e^2 u^2 d^2 r dt}{\int_{\mathscr{A},T} u^2 d^2 r dt}$$

which ensures that $\langle n_e^2 u^2 \rangle_{\mathscr{A},T}$ is homogeneous to a density.

The u^2 stems from Parceval's theorem since the scattering integral is also the Fourier transform of a Gaussian weighted electron density map. Ergo, the fluctuation rate is expressed as:

210

191

$$\frac{\delta n_e}{n_e}\Big|_{CTS} = \left\langle \frac{\sqrt{\langle n_e^2 u^2 \rangle_{\mathscr{A},T} - \langle n_e u^2 \rangle_{\mathscr{A},T}^2}}{\langle n_e u^2 \rangle_{\mathscr{A},T}} \right\rangle_{y}.$$
(7)

10

PLEASE CITE THIS ARTICLE AS DOI: 10.1063/5.0073215

211

212

the axial profile of the weighted electron density in the scattering volume converges also to the reference PIC profile, as it is illustrated in Fig 4 for a waist of w = 0.1 mm.

This definition of the fluctuation rate is consistent with Eq1 for small waists. Indeed as the

waist tends to 0, the normalized laser profile converges to the Dirac distribution. Consequently,

215 C. Pre-Processing of the PIC results

To obtain the results presented in the following sections, PIC results were pre-processed before 216 generating a virtual Thomson scattering signal. The time average was subtracted from the typical 217 2D map to highlight the fluctuations of the density. This was possible because the benchmark 218 est case reaches a quasi-steady state. Moreover, to reduce the computational cost, the spatial 219 esolution of the PIC simulation was divided by a factor of 2. Namely, the 2D electron density 220 maps were interpolated over a 251 × 128 grid instead of the initial 501 × 256 grid. We have checked 221 that the percent error introduced by this interpolation was less than 0.03% on the averaged plasma 222 parameters and the reference fluctuation rate calculated for PIC results with Eq1. 223

A total of N = 150 time-samples were analyzed using this method, resulting in a temporal 224 signal that extends on 3.75 µs. In the azimuthal direction y, using the periodic boundary condition, 225 ach 2D map of the electron density was extended by a length equal to L_y (Figure 1) to reduce 226 the edge effect in case the waist extends beyond the simulation domain in that direction. Along 227 ne axial direction x, the data was not extended when applying the virtual CTS. Then, edge effects 228 lose to the axial boundaries will impact virtual CTS results. In the following, for waists of 229 = 0.1, 1 and 3 mm, the comparison of results will be carried out mostly for $3 \le x \le 21$ mm. The 230 time step of the temporal scattering signal is $\Delta t = 25$ ns which corresponds to a sampling rate of 231 40MHz. A complex time-Fourier transform was performed over the sample. The modulus of the 232 spectrum was averaged along the y-direction, to reduce the numerical noise. This was possible 233 since the spectral properties of the azimuthal modes in the benchmark model do not depend on the 234 azimuthal positions. 235



Physics of Plasmas

PLEASE CITE THIS ARTICLE AS DOI: 10.1063/5.0073215



FIG. 6: 2D polar maps of the intensity of the scattering signal at different axial positions

236 III. RESULTS

237 A. Amplitude variation of the dominant modes

First, we examine the virtual CTS result at a fixed time t to identify the dominant structures present in the discharge.

Fig 6 shows the variation of the scattering signal along the wave vector k_x and k_y at different 240 axial position, at $t = 12.5 \,\mu$ s, when the simulation has reached a quasi-steady state, for |**k**| between 241 .5 rad.mm⁻¹ and 12.5 rad.mm⁻¹. Fig 6a reports a snapshot of the 2D polar map of the scattering 242 signal at x = 3 mm and shows multiple modes propagating inward, to the anode, at an angle of 243 110°. First, there is an intense and highly peaked mode at $k_y = 8 \text{ rad.mm}^{-1}$ that is identified as the 244 ion-acoustic wave based on the plasma parameters from the PIC simulation. Then, there is a less 245 intense but broad mode at around $k_y \approx 3 \text{ rad.mm}^{-1}$ These two modes (at $k_y = 8 \text{ rad.mm}^{-1}$ and $k_y =$ 246 3 rad.mm⁻¹) correspond to the dominant coherent structures propagating in the thruster's channel 247



Physics of Plasmas

PLEASE CITE THIS ARTICLE AS DOI: 10.1063/5.0073215

and in the following, we will focus on their axial variation. Downstream in the ionization zone, at 248 = 7 mm, the intensity of the $k_y = 8 \text{ rad.mm}^{-1}$ mode weakens favouring the second mode at $k_y =$ 249 х rad.mm⁻¹ which becomes dominant as shown in Fig 6b. In the acceleration zone, at x = 10 and 3 250 20 mm, (Fig 6c and 6d), the 3 rad.mm⁻¹ mode increases in intensity and its direction shifts slowly 251 from 110° to 90°. The change of direction occurs mainly at the beginning of the acceleration 252 zone where the ion velocity increases greatly. This is consistent with previous observations in 253 Honoré et al.²⁰ suggesting that the modes are convected with ions as they are accelerated. The 254 inward negative component k_x , in the ionization zone, is cancelled to an almost zero value, in the 255 observation frame of reference, as the positive outward velocity component of the ions begins to 256 increase in the acceleration zone. 258 Moreover, this change of direction concurs with a change in the intensity of the modes. This is 259

illustrated in Fig 7 which shows the intensity evolution of the two dominant modes as a function 260 of the axial position x. In addition to the previous observations, the intensity of the 3 rad.mm⁻¹ 261 mode increases linearly along the thruster axis. The change in the direction of the modes happens 262 at a constant k_y while the magnitude of the k_x component slowly decreases to 0. The decrease of 263 the intensity at x = 21 mm is due to the edge effects near the simulation boundaries and the local 264 extremum at x = 3 mm is mainly due to the presence of multiple small peaks at small k that are 265 difficult to resolve. The mode at 8 rad.mm^{-1} demonstrates also similar dynamics concerning the 266 wave vector variation. Its decrease at the beginning of the acceleration zone is however very sharp 267 and the k_x component blue converges quickly to 0, contrasting with the smooth behavior of the 268 small k modes. 269

270 B. Dispersion Relation

In the previous section, two coherent modes at $k_y = 8 \text{ rad.mm}^{-1}$ and $k_y = 3 \text{ rad.mm}^{-1}$ were identified in the simulation. To determine the frequencies of these modes, a spectral analysis of the time-dependent scattered signal is performed in the following paragraph.

Fig 8 shows the normalised Fourier transform of the time-dependent scattered signal in the ionization zone, at x = 3 mm and at an angle $\alpha = 110^{\circ}$ and in the acceleration zone, at x = 20 mmand at an angle $\alpha = 90^{\circ}$. The spectra illustrate one well-defined peaks in each case for both modes. Fig 8a shows that for the $k_y = 3 \text{ rad.mm}^{-1}$ the frequency is different with respect to the observation zone. The mode's mean frequency increases from 1 MHz in the ionization zone to 3 MHz



This is the author's peer reviewed, accepted manuscript. However, the online version of record will be different from this version once it has been copyedited and typeset. PLEASE CITE THIS ARTICLE AS DOI: 10.1063/5.0073215



FIG. 7: Variation of the dominant modes along the axial direction. The blue curves show the intensity of the scattering signal while, on the secondary axis, the dashed curves show the variation of the axial component k_x and the azimuthal component k_y . The dashed black curve indicates the thruster exit plane and the position of the maximum radial magnetic field.

Physics of Plasmas

Publishing



(a) Dynamic form factor for $k_v = 3 \text{ rad.mm}^{-1}$ (b) Dynamic form

(b) Dynamic form factor for $k_y = 8 \text{ rad.mm}^{-1}$

FIG. 8: Dynamic form factor for $k_y = 3 \text{ rad.mm}^{-1}$ and $k_y = 8 \text{ rad.mm}^{-1}$ at x = 3 mm and 20 mm. Blue curves correspond to x = 3 mm and an angle $\alpha = 110^{\circ}$ and the red curve correspond to x = 20 mm and an angle $\alpha = 90^{\circ}$

²⁷⁹ in the acceleration zone. The results for the mode at $k_y = 8 \text{ rad.mm}^{-1}$ are presented in Fig 8b. ²⁸⁰ The mode's mean frequency also increases from 4.5 MHz in the ionization zone to 6.5 MHz in ²⁸¹ the acceleration zone. We also observe a broad peak at a negative frequency that suggests a wave ²⁸² back-flow, however, this would need further investigation.

The dispersion relation, from the virtual Thomson scattering experiment is computed for dif-283 ferent observation angles α and at different axial positions. The results show a linear progression 284 of the frequency with respect to the wave number for all the tested angles. The maximum signal 285 obtained, though, when the angle of the observation is along the main propagation direction 286 e $\alpha = 110^{\circ}$ in the ionization zone and $\alpha = 90^{\circ}$ in the acceleration zone. Fig 9a and Fig 9b 287 show the respective dispersion relations along these privileged observation angles at x = 3 mm and 28 = 20 mm respectively. Two phase velocities are identified using a linear fit, $v_{g,Iz} = 3.5 \,\mathrm{km.s^{-1}}$ 289 and $v_{g,Acc} = 6.6 \,\mathrm{km.s^{-1}}$. These values are of the same order of magnitude as the measured phase 290 velocities from CTS experiments^{13,17}, that were conducted on a similar range of wave vectors. 291

Ohysics of Plasmas

Publishing

PLEASE CITE THIS ARTICLE AS DOI: 10.1063/5.0073215



FIG. 9: Dispersion relations derived from the virtual Thomson scattering experiment along the privileged observation angle relative to each zone: (a) corresponds to the ionization zone with $\alpha = 110^{\circ}$ while (b) corresponds to the acceleration zone with $\alpha = 90^{\circ}$. The dashed black lines correspond to the best linear fit

292 C. Electron Density Fluctuation rate

In this section, we derive the electron density fluctuation rate using the virtual CTS for different values of the waist *w* of the laser beam. The results are then compared with the reference PIC values. In Eq7, the waist appears when we calculate both the mean electron density and its rms since these values are averaged over the scattering volume.

At first, we examine the mean electron density profile. Fig 10 shows the axial profile of the electron density for two different values of the laser waist: w = 1 and 3 mm. The laser-beam

This is the author's peer reviewed, accepted manuscript. However, the online version of record will be different from this version once it has been copyedited and typeset

PLEASE CITE THIS ARTICLE AS DOI: 10.1063/5.0073215



FIG. 10: Axial profile of the electron density for different values of the laser waist: w = 1 and 3 mm in the CTS analysis and from the PIC simulations.

weighed electron density maps were averaged over 3.75 µs and the azimuthal direction. The pro-299 files diverge significantly for axial distances between x = 3 mm and x = 10 mm, where a steep 300 density gradient is observed. This is most noteworthy for a w = 3 mm waist with a maximum 11% 301 deviation from the PIC profile at around x = 5 mm. For a w = 1 mm waist, on the other hand, the 302 deviation is estimated to be a mere 2%. These observations are mainly due to the presence of gra-303 dients in the electron density profile inside the scattering volume. Indeed, if the scattering volume 304 has a sufficient spatial extension, the laser profile behaves like a Gaussian filter and smoothens the 305 electron density profile. Depending on how the electrons are distributed in the scattering volume, 306 the locally-averaged electron density is either underestimated or overestimated at a given axial po-307 sition. In the acceleration zone, between x = 10 mm and x = 214 mm, where the axial evolution of 308 the electron density is much smoother, the virtual CTS analysis gives results close to the reference 309 PIC profile for w = 1 mm and w = 3 mm. 310

Then, we examine the electron density fluctuation rate as introduced in Eq7 for two different values of the laser waist: w = 1 and 3 mm and compare it to the PIC reference value. The results are reported on Fig 11. As mentioned in Section IIC, we limit our analysis to $3 \le x \le 21$ mm. Between x = 3 mm and x = 10 mm, the virtual CTS estimates an average fluctuation rate at around



Publishing

This is the author's peer reviewed, accepted manuscript. However, the online version of record will be different from this version once it has been copyedited and typeset

PLEASE CITE THIS ARTICLE AS DOI: 10.1063/5.0073215



FIG. 11: Axial profile of the electron fluctuation rate for different values of the laser waist: w = 1 and 3 mm in the CTS analysis and from the PIC simulations.

³¹⁵ 20% for w = 3 mm, 3 times higher than what the reference PIC profile yields. In contrast, in the ³¹⁶ acceleration zone, between x = 10 mm and x = 24 mm, where the axial electron density is almost ³¹⁷ constant, the virtual CTS for w = 1 and 3 mm gives results coherent with the 7% fluctuation rate of ³¹⁸ the reference PIC profile. The aforementioned smoothing effect of the axially-averaged-electron ³¹⁹ density profile could not explain alone this difference. Indeed a factor of 2 is also observed for ³²⁰ w = 1 mm for which the difference in the mean electron density profiles is estimated to be only ³²¹ 2%.

In this regard, the difference, observed in Fig 11 stems then from the numerator of Eq7. This term corresponds, in the framework of CTS, to the integral of the form factor over all possible fluctuations in the scattering volume. Since the volume (Fig 5) has a spatial extension both in the azimuthal and the axial direction, the integrated profiles of the fluctuation rate take into account not only azimuthal fluctuations associated with the propagating modes but also the axial density standard deviation associated with the density gradient. This results in a relatively high value compared to the PIC reference value.

These observations favor the use of smaller waists in CTS experiments for positions where gradients are high to reduce the impact of the waist on the measurement of the density fluctuation

PLEASE CITE THIS ARTICLE AS DOI: 10.1063/5.0073215

Publishing

rate. However, this is more challenging in CTS experiments since a smaller w would lead to a 331 degraded k resolution. 332

CONCLUSION IV. 333

In this work, 2D axial-azimuthal PIC simulations were analyzed using a virtual CTS. It has 334 been shown that compared to the non-windowed Fourier transform, virtual CTS offers a localized 335 and directional tool to analyze PIC data. The analysis of PIC results using a virtual CTS diagnostic 336 allowed an enhanced characterization of the modes present in the stationary state of a 2D axial-337 azimuthal PIC simulation¹⁰, not only in the plume but also in the thruster channel. It permitted to 338 esolve two modes at 3 rad.mm⁻¹ and 8 rad.mm⁻¹ propagating all along the thruster axis. Their 339 direction is slightly oblique near the anode and becomes almost azimuthal at the thruster exit plane 340 they are convected by ions. The observation shows also that when the ions' convection becomes 341 ery large, the change in direction concurs with a change in the amplitudes of the modes. The 342 mplitude of the 3 rad.mm⁻¹ mode grows linearly in the acceleration zone while the 8 rad.mm⁻¹ 343 sharply damped. The dispersion relations (f, k) have been computed using this approach inside 344 the ionization zone in the thruster channel and the acceleration zone and are consistent with the observations in CTS experiments. 346

On the other hand, the examination of the electron density fluctuation rate shows a strong 347 dependence on the axial position x. Far in the thruster plume, the fluctuation rate is almost constant, 348 while in the ionization zone inside the thruster and near the exit plane, the profile exhibits strong 349 variations. Moreover, the results are shown to strongly depend on the value of the laser beam waist 350 and 3 mm in this work). The study first highlights the importance of an accurate measure of 351 the electron density inside the scattering volume. Since the electron density appears as a dividing 352 factor, a gross estimation can lead to a large difference in the order of magnitude of the fluctuation 353 rate. Recent measurements using incoherent Thomson scattering to estimate the electron density²¹ 354 also suggest a similar conclusion. The observations also highlight the impact of the waist on 355 the evaluation of the electron density root-mean-square. Because of its spatial extension, the 356 diagnostic integrates both axial and azimuthal fluctuations. This leads to a different estimation of 357 the fluctuation rate especially in areas with a strong electron density gradient. 358

Finally, this work has allowed us to clarify further the discrepancies between PIC simulations 359 and CTS experiments on the value of the electron density fluctuation rate. It also underlines the 360

PLEASE CITE THIS ARTICLE AS DOI: 10.1063/5.0073215

dependent 2D (axial-azimuthal) simulation closer to the real operation conditions of Hall effect
 thrusters.
 ACKNOWLEDGMENTS

interest to perform virtual plasma diagnostics on either PIC or fluid simulations since they allow

to relate simulations and experiments and foster common ground for comparison and discussion.

In this work, the CTS diagnostic was applied to a simplified 2D (axial-azimuthal) test case with

a stationary state. In future work, the virtual CTS will be used to analyze a self-consistent time-

This work has been partially funded by ANR (No. ANR-16-CHIN-003–01) and Safran Aircraft Engines within the project POSEIDON.

370 DATA AVAILABILITY STATEMENT

The data that support the findings of this study are available from the corresponding author upon reasonable request.

373 **REFERENCES**

361

362

363

364

- 1 A. I. Morozov and V. V. Savelyev, "Fundamentals of stationary plasma thruster theory," in *Re*-
- views of Plasma Physics, edited by B. B. Kadomtsev and V. D. Shafranov (Springer US, Boston,
 MA, 2000) pp. 203–391.
- ³⁷⁷ ²D. O'Reilly, G. Herdrich, and D. F. Kavanagh, "Electric propulsion methods for small satellites:
 ³⁷⁸ A review," Aerospace 8 (2021), 10.3390/aerospace8010022.
- ³⁷⁹ ³F. Taccogna, S. Longo, M. Capitelli, and R. Schneider, "Anomalous transport induced ³⁸⁰ by sheath instability in hall effect thrusters," Applied Physics Letters **94**, 251502 (2009), ³⁸¹ https://doi.org/10.1063/1.3152270.
- ³⁸² ⁴Y. Raitses, I. D. Kaganovich, A. Khrabrov, D. Sydorenko, N. J. Fisch, and A. Smolyakov,
- "Effect of secondary electron emission on electron cross-field current in $\mathbf{E} \times \mathbf{B}$ discharges,"
- ³⁸⁴ IEEE Transactions on Plasma Science **39**, 995–1006 (2011).
- ³⁸⁵ ⁵W. Frias, A. I. Smolyakov, I. D. Kaganovich, and Y. Raitses, "Long wavelength gradient
- drift instability in hall plasma devices. i. fluid theory," Physics of Plasmas 19, 072112 (2012),
- ³⁸⁷ https://doi.org/10.1063/1.4736997.



Physics of Plasmas

accepted manuscript. However, the online version of record will be different from this version once it has been copyedited and typeset This is the author's peer reviewed,

- PLEASE CITE THIS ARTICLE AS DOI: 10.1063/5.0073215

⁶J. C. Adam, A. Héron, and G. Laval, "Study of stationary plasma thrusters us-388 ing two-dimensional fully kinetic simulations," Physics of Plasmas 11, 295-305 (2004), 389 https://doi.org/10.1063/1.1632904. 390

- ⁷T. Lafleur, S. D. Baalrud, and P. Chabert, "Characteristics and transport effects of the electron 391 drift instability in hall-effect thrusters," Plasma Sources Science and Technology 26, 024008 392 (2017). 393
- ⁸T. Lafleur, R. Martorelli, P. Chabert, and A. Bourdon, "Anomalous electron transport in hall-394
- effect thrusters: Comparison between quasi-linear kinetic theory and particle-in-cell simula-395
- tions," Physics of Plasmas 25, 061202 (2018), https://doi.org/10.1063/1.5017626. 396
- ⁹K. Hara, "An overview of discharge plasma modeling for hall effect thrusters," Plasma Sources 397 Science and Technology 28, 044001 (2019). 398
- ¹⁰T. Charoy, J. P. Boeuf, A. Bourdon, J. A. Carlsson, P. Chabert, B. Cuenot, D. Eremin, L. Gar-399
- rigues, K. Hara, I. D. Kaganovich, A. T. Powis, A. Smolyakov, D. Sydorenko, A. Tavant, O. Ver-400 morel, and W. Villafana, "2D axial-azimuthal particle-in-cell benchmark for low-temperature 401 partially magnetized plasmas," Plasma Sources Science and Technology 28, 105010 (2019). 402
- ¹A. Lazurenko, V. Vial, M. Prioul, and A. Bouchoule, "Experimental investigation of high-
- 403 frequency drifting perturbations in hall thrusters," Physics of Plasmas 12, 013501-013501-9 404 (2005), https://doi.org/10.1063/1.1818698. 405

¹²A. Lazurenko, T. D. de Wit, C. Cavoit, V. Krasnoselskikh, A. Bouchoule, and 406 M. Dudeck, "Determination of the electron anomalous mobility through measurements 407 of turbulent magnetic field in hall thrusters," Physics of Plasmas 14, 033504 (2007), 408 https://doi.org/10.1063/1.2535813. 409

¹³S. Tsikata, N. Lemoine, V. Pisarev, and D. M. Grésillon, "Dispersion relations of electron 410 density fluctuations in a hall thruster plasma, observed by collective light scattering," Physics of 411 Plasmas 16, 033506 (2009), https://doi.org/10.1063/1.3093261. 412

¹⁴J. Cavalier, N. Lemoine, G. Bonhomme, S. Tsikata, C. Honoré, and D. Grésillon, "Hall thruster 413 plasma fluctuations identified as the $\mathbf{E} \times \mathbf{B}$ electron drift instability: Modeling and fitting on 414 experimental data," Physics of Plasmas 20, 082107 (2013), https://doi.org/10.1063/1.4817743. 415

- ¹⁵Z. A. Brown and B. A. Jorns, "Spatial evolution of small wavelength fluctuations in a hall 416 thruster," Physics of Plasmas 26, 113504 (2019), https://doi.org/10.1063/1.5116708. 417
- ¹⁶I. D. Kaganovich, A. Smolyakov, Y. Raitses, E. Ahedo, I. G. Mikellides, B. Jorns, F. Tac-418 cogna, R. Gueroult, S. Tsikata, A. Bourdon, J.-P. Boeuf, M. Keidar, A. T. Powis, M. Merino, 419



accepted manuscript. However, the online version of record will be different from this version once it has been copyedited and typeset This is the author's peer reviewed,

- PLEASE CITE THIS ARTICLE AS DOI: 10.1063/5.0073215

- M. Cappelli, K. Hara, J. A. Carlsson, N. J. Fisch, P. Chabert, I. Schweigert, T. Lafleur, 420 K. Matyash, A. V. Khrabrov, R. W. Boswell, and A. Fruchtman, "Physics of eb discharges 421 relevant to plasma propulsion and similar technologies," Physics of Plasmas 27, 120601 (2020), 422 https://doi.org/10.1063/5.0010135. 423
- ⁷S. Tsikata, C. Honoré, N. Lemoine, and D. M. Grésillon, "Three-dimensional structure of elec-424
- tron density fluctuations in the hall thruster plasma: $\mathbf{E} \times \mathbf{B}$ mode," Physics of Plasmas 17, 112110 425 (2010), https://doi.org/10.1063/1.3499350. 426
- ¹⁸S. Tsikata, C. Honoré, and D. Grésillon, "Collective thomson scattering for studying plasma 427 instabilities in electric thrusters," Journal of Instrumentation 8, C10012–C10012 (2013). 428
- ¹⁹S. Tsikata, J. Cavalier, A. Héron, C. Honoré, N. Lemoine, D. Grésillon, and D. Coulette, "An 429 axially propagating two-stream instability in the hall thruster plasma," Physics of Plasmas 21, 430 072116 (2014), https://doi.org/10.1063/1.4890025. 431
- ²⁰C. Honoré, S. Tsikata, D. Grésillon, A. Héron, N. Lemoine, and J. Cavalier, "Hall thruster small 432 scale plasma fluctuations: Qualifying 2D pic simulations against collective scattering experi-433 mental data," (The 32nd Int. Electric Propulsion Conf.11/9-5/9/2011, Wiesbaden). 434
- ²¹S. Tsikata, A. Héron, and C. Honoré, "Hall thruster microturbulence under con-435 ditions of modified electron wall emission," Physics of Plasmas 24, 053519 (2017), 436 https://doi.org/10.1063/1.4984255. 437
- ²²S. Tsikata and K. Hara, "Plasma instabilities in cross-field configuration: an analysis of the 438 relevance of different modes for electron transport," (the 36th International Electric Propulsion 439 Conference, Vienna, Austria, September 2019). 440
- ³A. T. Powis and M. N. Shneider, "Particle-in-cell modeling of laser thomson scattering in 441 low-density plasmas at elevated laser intensities," Physics of Plasmas 25, 053513 (2018), 442 https://doi.org/10.1063/1.5029820. 443
- ²⁴J.-P. Boeuf, "Tutorial: Physics and modeling of hall thrusters," Journal of Applied Physics 121, 444 011101 (2017), https://doi.org/10.1063/1.4972269. 445
- 25 J. P. Boeuf and L. Garrigues, "E \times B electron drift instability in hall thrusters: 446 Particle-in-cell simulations vs. theory," Physics of Plasmas 25, 061204 (2018), 447 https://doi.org/10.1063/1.5017033. 448
- ²⁶E. Holzhauer and J. H. Massig, "An analysis of optical mixing in plasma scattering experiments," 449
- Plasma Physics 20, 867-877 (1978). 450

Physics of Plasmas

Physics of Plasmas

PLEASE CITE THIS ARTICLE AS DOI: 10.1063/5.0073215

451

452

453

454

455

456

2011) second edition ed., pp. 103-142.

9789038624242.

²⁷D. H. Froula, S. H. Glenzer, N. C. Luhmann, and J. Sheffield, "Chapter 2 and chapter 5," in

²⁸v. d. Meiden, H.J., Thomson scattering on low and high temperature plasmas, Phd Thesis 1 (Re-

search TU/e / Graduation TU/e), Technische Universiteit Eindhoven, Eindhoven (2011), iSBN:

Plasma Scattering of Electromagnetic Radiation (Second Edition) (Academic Press, Boston,



ACCEPTED MANUSCRIPT

This is the author's peer reviewed, accepted manuscript. However, the online version of record will be different from this version once it has been copyedited and typeset. PLEASE CITE THIS ARTICLE AS DOI: 10.1063/5.0073215





ACCEPTED MANUSCRIPT



This is the author's peer reviewed, accepted manuscript. However, the online version of record will be different from this version once it has been copyedited and typeset. PLEASE CITE THIS ARTICLE AS DOI: 10.1063/5.0073215

































This is the author's peer reviewed, accepted manuscript. However, the online version of record will be different from this version once it has been copyedited and typeset. PLEASE CITE THIS ARTICLE AS DOI: 10.1063/5.0073215





This is the author's peer reviewed, accepted manuscript. However, the online version of record will be different from this version once it has been copyedited and typeset. PLEASE CITE THIS ARTICLE AS DOI: 10.1063/5.0073215

























