



HAL
open science

Analysis of small scale fluctuations in Hall effect thrusters using virtual Thomson scattering on PIC simulations

Tarek Ben Slimane, Cyrille Honoré, Thomas Charoy, Anne Bourdon, Pascal Chabert

► To cite this version:

Tarek Ben Slimane, Cyrille Honoré, Thomas Charoy, Anne Bourdon, Pascal Chabert. Analysis of small scale fluctuations in Hall effect thrusters using virtual Thomson scattering on PIC simulations. *Physics of Plasmas*, 2022, 29, 10.1063/5.0073215 . hal-03551302

HAL Id: hal-03551302

<https://hal.science/hal-03551302>

Submitted on 1 Feb 2022

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

This is the author's peer reviewed, accepted manuscript. However, the online version of record will be different from this version once it has been copyedited and typeset.

PLEASE CITE THIS ARTICLE AS DOI: 10.1063/5.0073215

1 **Analysis of small scale fluctuations in Hall effect thrusters using virtual Thomson**
2 **scattering on PIC simulations**

3 Tarek Ben Slimane,^{1, a)} Cyrille Honoré,¹ Thomas Charoy,¹ Anne Bourdon,¹ and Pascal
4 Chabert¹

5 *Laboratoire de Physique des Plasmas (LPP), CNRS, Sorbonne Université,*
6 *École Polytechnique, Institut Polytechnique de Paris, 91120 Palaiseau,*
7 *France*

8 (Dated: 10 January 2022)

9 Hall effect thrusters are yet not fully understood due to multiple open questions, among
10 which the anomalous electron transport is the most challenging. Particle-In-Cell (PIC)
11 simulations suggested that electron transport is strongly affected by small-scale instabili-
12 ties which can be investigated experimentally using Collective Thomson Scattering (CTS).
13 In this paper, we perform a virtual CTS diagnostic on PIC data in order to investigate these
14 small scales and to relate simulation and experiments. The virtual diagnostic is performed
15 on the electron density maps from a simplified 2D axial-azimuthal PIC simulation. Anal-
16 ysis along the channel axis showed two dominant modes at 8 rad.mm^{-1} and 3 rad.mm^{-1} .
17 These modes exhibit ion-acoustic wave features and appear to be selectively affected by
18 the acceleration of the ions when transitioning from the ionization zone to the acceleration
19 zone. A numerical study of the electron density fluctuation rate from the virtual CTS is
20 also performed and the obtained profiles are compared to PIC. This reveals that the virtual
21 CTS integrates both axial and azimuthal fluctuations of the electron density due to the spa-
22 tial extension of the scattering volume. Moreover, it also highlights the importance of a
23 good estimation of the electron density in the scattering volume for an accurate estimation
24 of the fluctuation rate.

25 PACS numbers: 0

^{a)}tarek.ben-slimane@lpp.polytechnique.fr

26 I. INTRODUCTION

27 Since their development in the mid-60s, Hall effect thrusters (HET) have been successfully used
28 for many space missions either as a secondary propulsion system destined for station keeping and
29 attitude control or as a primary propulsion system for interplanetary missions^{1,2}. The propellant,
30 commonly a noble gas, is injected from the anode side into an annular chamber while an electron-
31 emitting cathode provides electrons to ionize the propellant and produce plasma. A high voltage
32 is applied across the chamber and accelerates ions downstream while electrons are trapped in a
33 $E \times B$ configuration using a strong radial magnetic field. However, the rate at which electrons flow
34 from the cathode to the anode exceeds the rates predicted by classical theory by several orders of
35 magnitude, which impede a full understanding of HET operation.

36 Several contributions to this anomalous mobility have been extensively studied throughout the
37 years³⁻⁵ and recently the electron cyclotron drift instability or ECDI was identified as a key driver
38 for electron transport. Indeed, the asymmetry between the electron fast azimuthal motion and the
39 ions' axial motion creates a kinetic, MHz (millimeter scale) azimuthal instability that was observed
40 in PIC simulations⁶⁻¹⁰ as well as in experiments¹¹⁻¹⁵ and was associated with enhanced electron
41 transport. The linear theory of ECDI instability is well developed, however, nonlinear saturation
42 is still the subject of active research and debate as discussed in the recent review by Kaganovich
43 *et al.*¹⁶. Given the complexity of mode saturation in the 3D geometry of Hall thrusters, only
44 large-scale 3D PIC simulations can provide a full understanding of the mechanism of anomalous
45 transport. However, these simulations are still out of reach and most PIC simulation studies are
46 carried out in simplified 1D and 2D configurations.

47 Experimentally, the investigation of instabilities at millimeter scales in a 3D thruster is chal-
48 lenging and was done so far using coherent Thomson scattering (CTS)^{13,17-19} and different probe
49 techniques^{12,15,16}. Furthermore, most measurements are performed outside the narrow thruster
50 channel, in the vicinity of the exit plane, and downstream in the plume.

51 In 2011, a previous work by Honoré *et al.*²⁰ used an analysis method that combines 2D PIC
52 axial-azimuthal simulation results with a virtual CTS. The approach consisted of generating a the-
53oretical scattering signal from PIC data and showed an excellent agreement with CTS observations
54 of the plume on the static form factor which is a measure of the intensity of the electron density
55 fluctuations. However, the time resolution of the PIC simulation used was limited to $\Delta t = 1 \mu\text{s}$ so
56 that no spectral comparison was performed.

This is the author's peer reviewed, accepted manuscript. However, the online version of record will be different from this version once it has been copyedited and typeset.

PLEASE CITE THIS ARTICLE AS DOI: 10.1063/1.50073215

57 In this paper, we apply a virtual CTS method to the highly resolved PIC simulations of a 2D PIC
58 axial-azimuthal benchmark test case developed recently in Ref.¹⁰ to identify the main parameters
59 impacting the evaluation of the electron density fluctuation rate between PIC simulations and CTS
60 experiments. This is achieved by comparing the reference fluctuation rate derived from PIC data
61 with the one obtained from virtual CTS analysis.

62 CTS experiments usually measure an electron density fluctuation rate of the order of 1%¹⁷,
63 whereas PIC simulations tend to produce large electron density fluctuations at around 10%-15%
64 with significant axial variations⁸. This difference is often explained by stating that, as aforemen-
65 tioned, 2D simulations are unable to account for the 3D structure of the ECDCI. Nevertheless,
66 experiment-wise, the determination of the absolute electron fluctuation rate in CTS is also chal-
67 lenging since it requires a precise measurement of the electron density in the scattering volume and
68 absolute calibration of the setup²¹. Still, recent work using incoherent Thomson scattering to mea-
69 sure this density reported a local electron density lower than previous estimates and a fluctuation
70 density rate of the order of 8%²², closer to the value reported by simulations.

71 In this respect, the local electron density inside the scattering volume seems to have a significant
72 impact on the value of the fluctuation rate. The probed volume, which is defined by the intersection
73 of two laser beams, depends on the waist of the laser. Experimentally, it is very burdensome to
74 vary this parameter and keep a good-quality signal, since it directly impacts the measurement
75 resolution. On the other hand, since PIC simulations are only limited by the mesh resolution, it is
76 possible to numerically produce the scattering signal from PIC data for different waists and assess
77 the impact of the waist on the estimation of the electron density fluctuation rate. Furthermore, PIC
78 results offer more flexibility in terms of the accessible zones for the diagnostic. Indeed, contrary to
79 CTS experiments where the diagnostic has only been done outside the thruster channel, by basing
80 a virtual CTS on PIC results, it is also possible to explore the thruster channel. Interestingly, a
81 similar virtual diagnostic was also performed within an incoherent Thomson scattering setup in
82 Ref.²³ to investigate the impact of the laser intensity on the measurement of the electron density.
83 The study also highlighted the valuable flexibility and insight that virtual diagnostics can bring to
84 experiments.

85 The organization of the paper is as follows. The 2D axial-azimuthal PIC simulation and the
86 virtual collective Thomson scattering analysis are presented in Section II. Then, in section III,
87 first, we present the virtual CTS observation at a fixed time to identify the dominant modes in
88 the plume as well as in the thruster channel. Then, we perform a temporal-Fourier analysis to

89 identify the main frequencies and derive the dispersion relations for the dominant propagating
 90 modes. Finally, we discuss the impact of the waist of laser beams on the derivation of electron
 91 density fluctuations.

92 II. METHODS

93 A. Axial-Azimuthal PIC Simulation

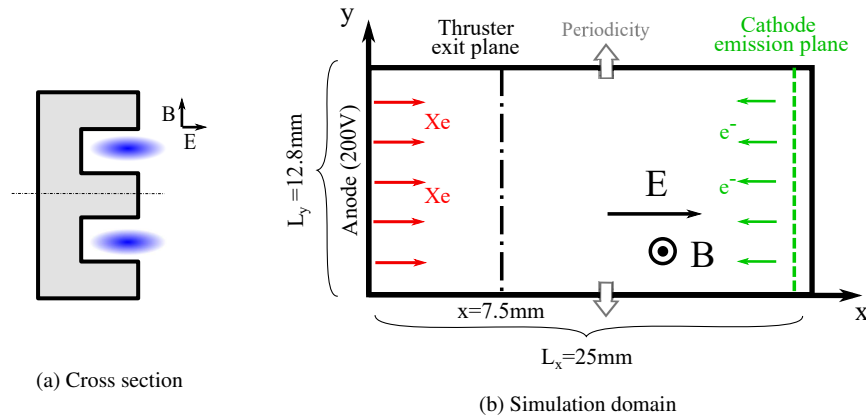


FIG. 1: Schematic of a transversal cross-section of the thruster and the simulation domain adapted from Ref¹⁰. x is the axial direction, y is the (periodic) azimuthal direction. Black dashed line at $x = 7.5$ mm corresponds to the position of the maximum radial magnetic field and the exit plane of the thruster. Green dashed line at $x = 24$ mm is the plane from which electrons are emitted.

94 As mentioned in the introduction, an absolute comparison with CTS experimental results would
 95 require a full highly-resolved 3D PIC simulation of the whole thruster. However, the latter is
 96 still out of reach due to its computational cost. Currently, 2D PIC simulation results in both
 97 axial-azimuthal and radial-azimuthal simulation planes^{6–10,24} are the most complete data to study
 98 azimuthal instabilities. In particular, the axial-azimuthal plane allows the study of the development
 99 of azimuthal instabilities and their impact on axial electron transport. Therefore in this work, the
 100 virtual CTS analysis was applied to the axial-azimuthal PIC benchmark results from Charoy et
 101 al.¹⁰. Although the chosen simulation case is non-self consistent, it was subject to an international
 102 benchmark effort and the instabilities involved in the simulation are well characterized making it a

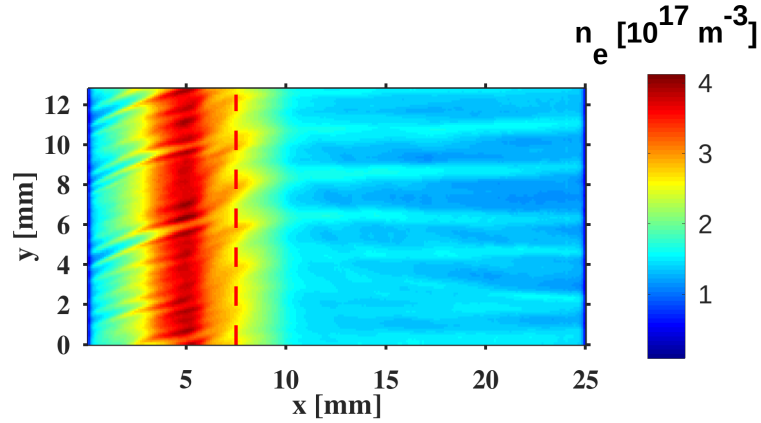


FIG. 2: Typical results from PIC simulations. This snapshot of the electron density is taken at $t = 12.5 \mu\text{s}$. The dashed red line at $x = 7.5 \text{ mm}$ indicates the limit between the ionization zone and the acceleration zone which also corresponds to the position of the maximum radial magnetic field and the exit plane of the thruster

103 solid analysis framework to test the virtual CTS. In the following, a summary of the conditions of
 104 the 2D benchmark and key results are presented, but additional details are given in Refs.^{10,25}. The
 105 2D simulation domain consists of a 500×256 Cartesian mesh in which the x -axis corresponds to
 106 the axial direction while the y -axis corresponds to the azimuthal direction, as illustrated in Fig 1.
 107 The curvature of the domain was hence neglected. The axial length L_x was fixed to 25 mm while
 108 the azimuthal length L_y was fixed to 12.8 mm. A periodic boundary condition was imposed in
 109 the azimuthal direction to circumvent the finite azimuthal length and to reduce the computational
 110 time.

111 Initially, ions and electrons are uniformly distributed over the simulation domain at a density
 112 of $5 \times 10^{16} \text{ m}^{-3}$ and are subject to a 200 V voltage, which was applied between the anode and the
 113 cathode. The radial magnetic field profile, as well as the ionization profile, were fixed to remove
 114 the low-frequency breathing mode oscillations and highlight the high-frequency instabilities in-
 115 cluding the ECDI. To maintain the discharge current balance, electrons were injected from the
 116 cathode side from an emission line positioned at 1 mm from the right boundary of the simulation
 117 domain (Green dashed line in Fig.1), and its potential is adjusted at each step to ensure a zero
 118 azimuthally averaged potential at this position^{10,25}. This allows keeping a constant averaged po-

This is the author's peer reviewed, accepted manuscript. However, the online version of record will be different from this version once it has been copyedited and typeset.

PLEASE CITE THIS ARTICLE AS DOI: 10.1063/5.0073215

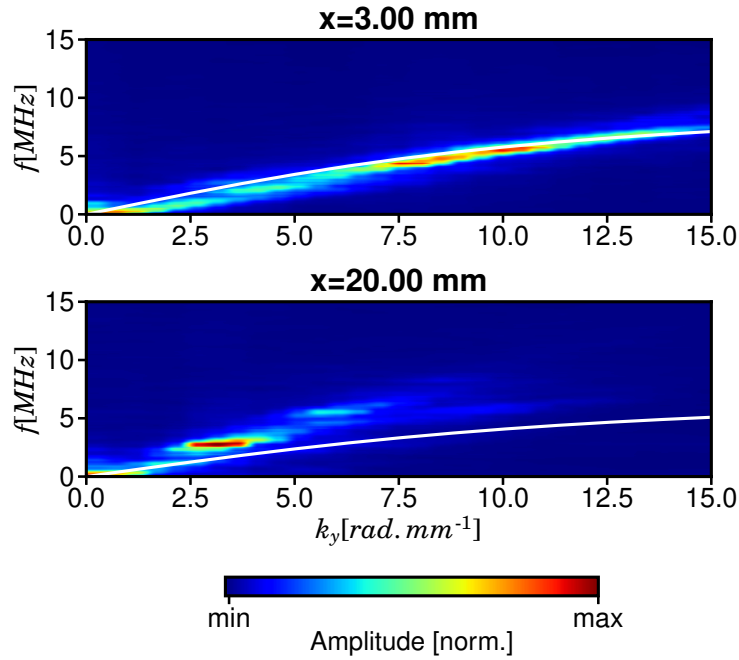


FIG. 3: 2D FFT of the electron density at $x = 3$ mm and $x = 20$ mm. Solid white line: ion acoustic dispersion relation.

119 tential drop between the anode and the emission line. With this method, the right boundary plane
 120 has a varying negative potential but this drop in potential between the emission plane and the right
 121 boundary plane does not have any useful physical meaning and does not affect the main discharge
 122 physics^{10,25}. The integration used an explicit scheme with a grid size $\Delta x = 50\mu\text{m}$ and a time step
 123 $\Delta t = 5$ ps. A total of 4×10^6 time steps were simulated resulting in a discharge simulation up to
 124 around $20\mu\text{s}$. Nevertheless, due to limited storage capacity, the plasma parameters are only stored
 126 every 5000-time steps, thus yielding a sampling time for the electron density of $T_s = 25$ ns.

127 A quasi-steady-state was reached at around $12\mu\text{s}$ featuring an almost constant discharge cur-
 128 rent. In Fig 2, a typical 2D map of the electron density at $t = 12.5\mu\text{s}$ is shown. It highlights mil-
 129 limetric fluctuations of the electron density propagating mainly along the azimuthal y -axis. These
 130 are marked with a change in the wavelength around the exit plane of the thruster at $x = 7.5$ mm. In
 131 the ionization zone which extends from $x = 2$ to 7.5 mm, the fluctuations have a small wavelength

132 and their wavevector has both an axial and an azimuthal component. From $x = 7.5$ to 25 mm,
 133 which corresponds to the acceleration zone, the structures have a larger wavelength and are almost
 134 azimuthal.

135 A spatial and temporal Fourier analysis of these maps between 16 and 20 μs highlights a con-
 136 tinuous dispersion relation in the ionization zone (Iz) that shows a good correspondence with the
 137 modified ion-acoustic wave dispersion relation from Lafleur et al.^{7,8}, as illustrated in Fig 3. The
 138 mode corresponding to the maximum growth rate under the ion-acoustic wave approximation is
 139 identified at $k_{Iz} \approx 8 \text{ rad}\cdot\text{mm}^{-1}$ and has a high frequency $f_{Iz} \approx 5 \text{ MHz}$. In the acceleration zone
 140 (Acc), the fit with the modified ion-acoustic dispersion relation is less satisfying. Still, the dynam-
 141 ics is dominated by a single mode at $k_{Acc} \approx 3 \text{ rad}\cdot\text{mm}^{-1}$ at a lower frequency $f_{Acc} \approx 3 \text{ MHz}$. The
 142 underlying reason for this change in the dynamics is yet not clearly understood and, in the follow-
 143 ing, we mainly focus on the estimation of the fluctuation rate from the PIC simulation results.

144 The fluctuation rate from PIC data is assessed by calculating the fluctuation in time and aver-
 145 aging the results azimuthally for each axial position. It is expressed as it follows:

$$146 \quad \left. \frac{\delta n_e}{n_e} \right|_{PIC} (x) = \left\langle \frac{\sqrt{\langle n_e^2 \rangle_t - \langle n_e \rangle_t^2}}{\langle n_e \rangle_t} \right\rangle_y. \quad (1)$$

147 The blue curve in Fig 4 shows the axial evolution of the fluctuation rate between 12 μs and 20 μs for
 148 the simulated benchmark case. The profile exhibits a large axial variation as already observed by
 149 T.Lafleur et al.⁸ in the first microseconds of a self-consistent 2D (axial-azimuthal) PIC simulation,
 150 that seems to be linked to the ionization profile. Indeed, the fluctuation rate decreases from 10%
 151 close to the anode to a minimum of 2.6% at $x = 6.4 \text{ mm}$. This coincides with the maximum
 152 ionization rate imposed in the simulation. Then, it increases with small oscillations in the plume
 153 to stabilize at around 7% after $x = 20 \text{ mm}$.

154 In the following, the blue curve will be used as a reference for the comparison with the virtual
 155 CTS results.

157 B. Collective Thomson Scattering Analysis

158 Collective Thomson scattering describes a scattering phenomenon in which the wavelength of
 159 scattering λ is larger or comparable to the Debye length λ_D ²⁶⁻²⁸. An incident laser beam at a fixed
 160 wave vector \mathbf{k}_i is applied upon the plasma and accelerated electrons behave like dipoles and scat-
 161 ter a part of the incident electromagnetic field at the same wavelength as the incoming field. The

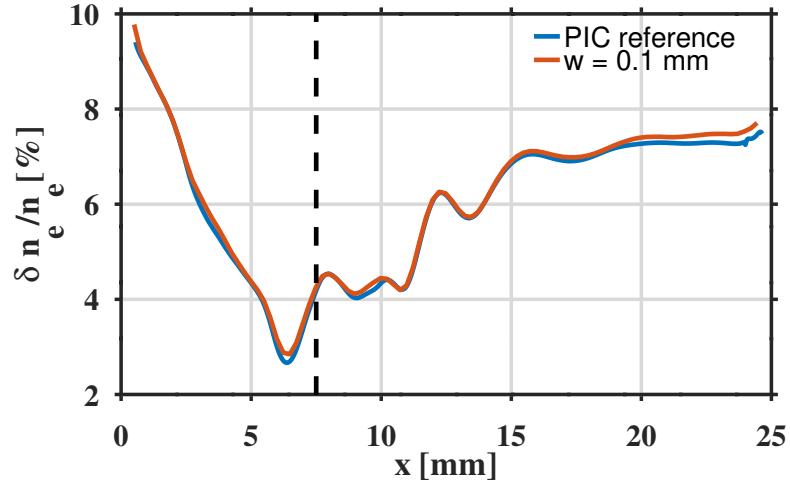


FIG. 4: The axial variation of the electron density fluctuation rate in the simulation case from Charoy et al.¹⁰. The blue curve corresponds to the reference fluctuation rate from PIC results using Eq 1. The red curves corresponds to the synthetical fluctuation rate from the virtual CTS at a waist $w = 0.1$ mm based on Eq 7. Black dashed line at $x = 7.5$ mm corresponds to the position of the maximum radial magnetic field and the exit plane of the thruster.

162 scattered fields add up to yield the total scattered field. This sum can be constructive or destruc-
 163 tive depending on the phase correlation between the electrons' positions. This correlation might
 164 occur if coherent structures are propagating through the medium. If so, these can be highlighted
 165 by varying the observation length scale of the scattering experiment to match the length scale of
 166 the propagating structures. The wave vector \mathbf{k} along which the CTS is performed is called the
 167 scattering vector and is determined using Bragg's relation, relating the wave vector \mathbf{k} to the differ-
 168 ence between the laser wave vector \mathbf{k}_i and the scattered wave vector \mathbf{k}_s , as it follows: $\mathbf{k} = \mathbf{k}_s - \mathbf{k}_i$.
 169 Hence, by changing the angle between these vectors, it is possible to change the norm of the scat-
 170 tering wave vector \mathbf{k} and thus to probe the medium at different scales. In practice, the scattering
 171 wave vector \mathbf{k} is changed using a translating mirror to vary the angle between \mathbf{k}_s and \mathbf{k}_i .

172 For a fixed \mathbf{k} , the resulting signal is proportional to the Fourier transform of the electron density
 173 along \mathbf{k} in the scattering volume \mathcal{V} and is recorded as a function of time, such as:

$$174 \quad s(\mathbf{k}, t) = \int_{\mathcal{V}} e^{i\mathbf{k}\cdot\mathbf{r}} n_e(r, t) d^3 \mathbf{r}. \quad (2)$$

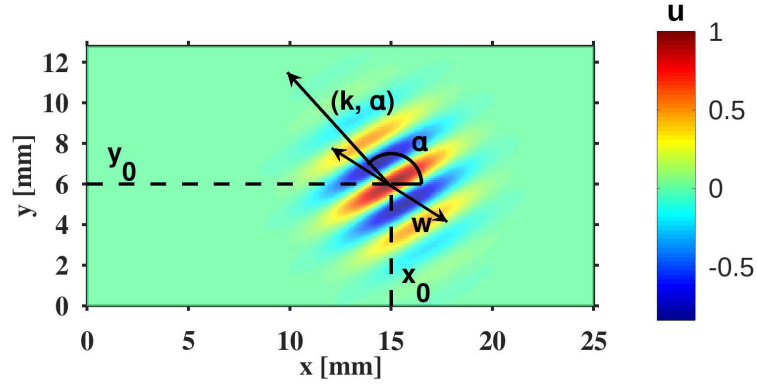


FIG. 5: 2D configuration for the virtual CTS calculation. The scattering volume is a Gaussian profile with a waist $w = 3$ mm. The phase factor in the Fourier transform is directed along the scattering vector \mathbf{k} , here expressed in polar coordinates with $\alpha = 120^\circ$ and $k = 3 \text{ rad}\cdot\text{mm}^{-1}$

175 Further frequency analysis is usually performed over the total acquisition duration T and pro-
 176 vides the spectral density of the scattering signal $S(\mathbf{k}, \omega)$. This quantity is proportional to the
 177 dynamic form factor and is expressed here as it follows:

$$178 \quad S(\mathbf{k}, \omega) = \left| \int_{\mathcal{V}, T} e^{i(\mathbf{k}\cdot\mathbf{r} - \omega t)} n_e(r, t) d^3\mathbf{r} dt \right|^2. \quad (3)$$

179 The integral over all frequencies of Eq3 is noted $S(\mathbf{k})$ and is proportional to the static form
 180 factor, usually measured in CTS experiments. $S(\mathbf{k})$ is written as:

$$181 \quad S(\mathbf{k}) = \int_{\omega} S(\mathbf{k}, \omega) d\omega. \quad (4)$$

182 Finally, the total fluctuation of the electron density can be related directly to the static factor,
 183 using Parseval's theorem, yielding the following relation:

$$184 \quad \langle n_e^2 \rangle = \int_{\mathbf{k}} S(\mathbf{k}) d\mathbf{k}, \quad (5)$$

185

To perform a virtual CTS on PIC results, we adapt the former integral to a 2D geometry. This is illustrated in Fig 5. The scattering volume V_s , defined as the intersection between the laser beam and the 2D map, is modeled using a Gaussian intensity profile u in polar coordinates with a fixed waist w , such as:

$$u(r) = \frac{1}{\pi w^2} e^{-\frac{r^2}{w^2}}.$$

187 A 2D local spatial Fourier transform is applied to the electron density maps from the PIC
 188 simulation along a fixed scattering vector \mathbf{k} given in polar coordinates (k, α) . This enables to probe
 189 structures at a given scale k and that are propagating at a given angle α . The virtual scattered field
 190 is thus expressed as:

$$191 \quad S_{PIC}(\mathbf{k}, \mathbf{r}_0, t) = \int_{\mathcal{A}} e^{i\mathbf{k} \cdot (\mathbf{r} - \mathbf{r}_0)} u(\mathbf{r} - \mathbf{r}_0) n_e(r, t) d^2\mathbf{r}, \quad (6)$$

192 where \mathbf{r}_0 is the center of the 2D equivalent of the scattering volume \mathcal{A} , i.e the center of the
 193 Gaussian profile.

194 The laser intensity profile is applied at each point $r_0 = (x_0, y_0)$ of a rectangular grid at a step
 195 $\Delta x_0 = \Delta y_0 = 1$ mm. Each position represents a different Thomson scattering experiment where the
 196 scattering vector $\mathbf{k} = (k, \alpha)$ is varied by changing both the norm k and angle α . The angle was
 197 varied with a resolution of 1° from 0° to 180° , whereas the norm k is varied with a resolution of
 198 $0.3 \text{ rad} \cdot \text{mm}^{-1}$ from $0.5 \text{ rad} \cdot \text{mm}^{-1}$ to $12.5 \text{ rad} \cdot \text{mm}^{-1}$. The laser waist is fixed to 3 mm for the ref-
 199 erence case as usually used in CTS experiments. Besides, this value offers good analysis features
 200 for simulations, considering the size of the structures we want to investigate.

201 Virtual CTS can be understood as a windowed Fourier transform, for which the chosen window
 202 is a 2D Gaussian function, centered at r_0 and which has a standard deviation $\sigma = \frac{w}{\sqrt{2}}$. Similar to
 203 windowed Fourier transform, virtual CTS has two main advantages. First, it limits the analysis
 204 to an area proportional to the laser beam waist w^2 . Second, it introduces a diagnostic length
 205 scale $\lambda = \frac{2\pi}{|k|}$ to highlight fluctuations occurring at the same scale as λ . This offers both spatial
 206 localization and selectivity over the k -component within the uncertainty inequality limit.

The electron density fluctuation rate is calculated also from the static form factor, using Eq5.
 However, the calculation yields the root mean square (rms) of the electron density, weighted by
 the square of the laser intensity profile u^2 , denoted as $\langle n_e^2 u^2 \rangle_{\mathcal{A}, T}$. The operator $\langle \cdot \rangle_{\mathcal{A}, T}$
 corresponds to the average over the area delimited by the weighting function u^2 , such as :

$$\langle n_e^2 u^2 \rangle_{\mathcal{A}, T} = \frac{\int_{\mathcal{A}, T} n_e^2 u^2 d^2 r dt}{\int_{\mathcal{A}, T} u^2 d^2 r dt}$$

207 which ensures that $\langle n_e^2 u^2 \rangle_{\mathcal{A}, T}$ is homogeneous to a density.

208 The u^2 stems from Parseval's theorem since the scattering integral is also the Fourier transform
 209 of a Gaussian weighted electron density map. Ergo, the fluctuation rate is expressed as:

$$210 \quad \left. \frac{\delta n_e}{n_e} \right|_{CTS} = \left\langle \sqrt{\frac{\langle n_e^2 u^2 \rangle_{\mathcal{A}, T} - \langle n_e u^2 \rangle_{\mathcal{A}, T}^2}{\langle n_e u^2 \rangle_{\mathcal{A}, T}}} \right\rangle_y. \quad (7)$$

This is the author's peer reviewed, accepted manuscript. However, the online version of record will be different from this version once it has been copyedited and typeset.

PLEASE CITE THIS ARTICLE AS DOI: 10.1063/1.50073215

211 This definition of the fluctuation rate is consistent with Eq1 for small waists. Indeed as the
212 waist tends to 0, the normalized laser profile converges to the Dirac distribution. Consequently,
213 the axial profile of the weighted electron density in the scattering volume converges also to the
214 reference PIC profile, as it is illustrated in Fig 4 for a waist of $w = 0.1$ mm.

215 C. Pre-Processing of the PIC results

216 To obtain the results presented in the following sections, PIC results were pre-processed before
217 generating a virtual Thomson scattering signal. The time average was subtracted from the typical
218 2D map to highlight the fluctuations of the density. This was possible because the benchmark
219 test case reaches a quasi-steady state. Moreover, to reduce the computational cost, the spatial
220 resolution of the PIC simulation was divided by a factor of 2. Namely, the 2D electron density
221 maps were interpolated over a 251×128 grid instead of the initial 501×256 grid. We have checked
222 that the percent error introduced by this interpolation was less than 0.03% on the averaged plasma
223 parameters and the reference fluctuation rate calculated for PIC results with Eq1.

224 A total of $N = 150$ time-samples were analyzed using this method, resulting in a temporal
225 signal that extends on $3.75 \mu\text{s}$. In the azimuthal direction y , using the periodic boundary condition,
226 each 2D map of the electron density was extended by a length equal to L_y (Figure 1) to reduce
227 the edge effect in case the waist extends beyond the simulation domain in that direction. Along
228 the axial direction x , the data was not extended when applying the virtual CTS. Then, edge effects
229 close to the axial boundaries will impact virtual CTS results. In the following, for waists of
230 $w = 0.1, 1$ and 3 mm, the comparison of results will be carried out mostly for $3 \leq x \leq 21$ mm. The
231 time step of the temporal scattering signal is $\Delta t = 25$ ns which corresponds to a sampling rate of
232 40 MHz. A complex time-Fourier transform was performed over the sample. The modulus of the
233 spectrum was averaged along the y -direction, to reduce the numerical noise. This was possible
234 since the spectral properties of the azimuthal modes in the benchmark model do not depend on the
235 azimuthal positions.

This is the author's peer reviewed, accepted manuscript. However, the online version of record will be different from this version once it has been copyedited and typeset.

PLEASE CITE THIS ARTICLE AS DOI: 10.1063/5.0073215

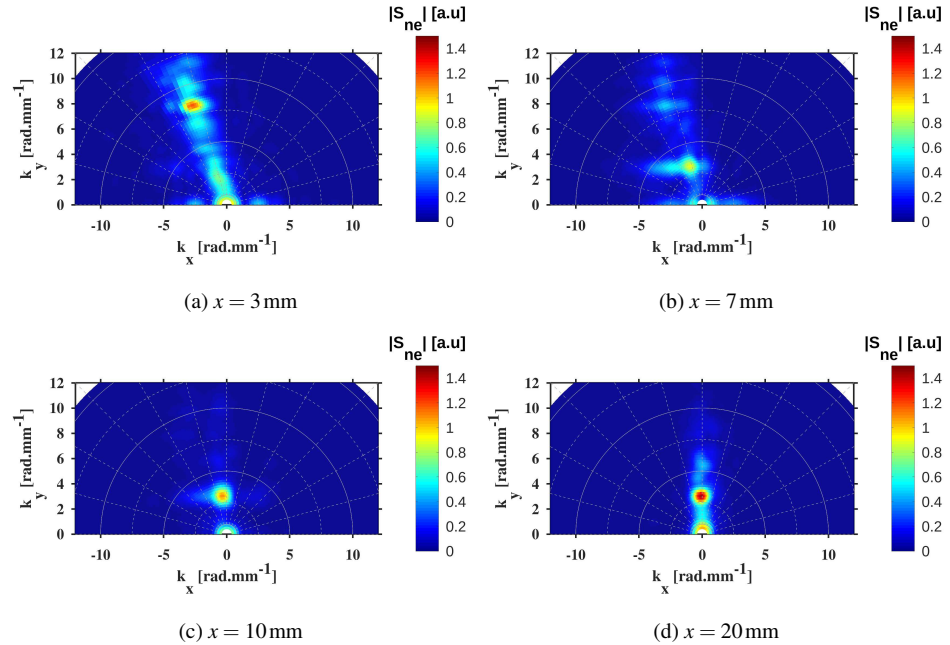


FIG. 6: 2D polar maps of the intensity of the scattering signal at different axial positions

236 III. RESULTS

237 A. Amplitude variation of the dominant modes

238 First, we examine the virtual CTS result at a fixed time t to identify the dominant structures
239 present in the discharge.

240 Fig 6 shows the variation of the scattering signal along the wave vector k_x and k_y , at different
241 axial position, at $t = 12.5 \mu\text{s}$, when the simulation has reached a quasi-steady state, for $|\mathbf{k}|$ between
242 0.5 rad.mm^{-1} and 12.5 rad.mm^{-1} . Fig 6a reports a snapshot of the 2D polar map of the scattering
243 signal at $x = 3 \text{ mm}$ and shows multiple modes propagating inward, to the anode, at an angle of
244 110° . First, there is an intense and highly peaked mode at $k_y = 8 \text{ rad.mm}^{-1}$ that is identified as the
245 ion-acoustic wave based on the plasma parameters from the PIC simulation. Then, there is a less
246 intense but broad mode at around $k_y \approx 3 \text{ rad.mm}^{-1}$. These two modes (at $k_y = 8 \text{ rad.mm}^{-1}$ and $k_y =$
247 3 rad.mm^{-1}) correspond to the dominant coherent structures propagating in the thruster's channel

248 and in the following, we will focus on their axial variation. Downstream in the ionization zone, at
249 $x = 7$ mm, the intensity of the $k_y = 8 \text{ rad.mm}^{-1}$ mode weakens favouring the second mode at $k_y =$
250 3 rad.mm^{-1} which becomes dominant as shown in Fig 6b. In the acceleration zone, at $x = 10$ and
251 20 mm, (Fig 6c and 6d), the 3 rad.mm^{-1} mode increases in intensity and its direction shifts slowly
252 from 110° to 90° . The change of direction occurs mainly at the beginning of the acceleration
253 zone where the ion velocity increases greatly. This is consistent with previous observations in
254 Honoré *et al.*²⁰ suggesting that the modes are convected with ions as they are accelerated. The
255 inward negative component k_x , in the ionization zone, is cancelled to an almost zero value, in the
256 observation frame of reference, as the positive outward velocity component of the ions begins to
257 increase in the acceleration zone.

259 Moreover, this change of direction concurs with a change in the intensity of the modes. This is
260 illustrated in Fig 7 which shows the intensity evolution of the two dominant modes as a function
261 of the axial position x . In addition to the previous observations, the intensity of the 3 rad.mm^{-1}
262 mode increases linearly along the thruster axis. The change in the direction of the modes happens
263 at a constant k_y while the magnitude of the k_x component slowly decreases to 0. The decrease of
264 the intensity at $x = 21$ mm is due to the edge effects near the simulation boundaries and the local
265 extremum at $x = 3$ mm is mainly due to the presence of multiple small peaks at small k that are
266 difficult to resolve. The mode at 8 rad.mm^{-1} demonstrates also similar dynamics concerning the
267 wave vector variation. Its decrease at the beginning of the acceleration zone is however very sharp
268 and the k_x component blue converges quickly to 0, contrasting with the smooth behavior of the
269 small k modes.

270 B. Dispersion Relation

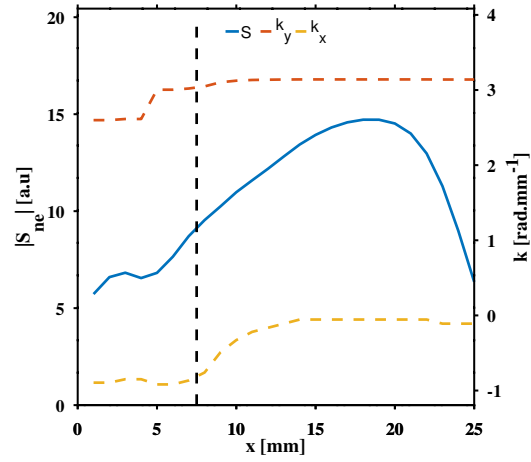
271 In the previous section, two coherent modes at $k_y = 8 \text{ rad.mm}^{-1}$ and $k_y = 3 \text{ rad.mm}^{-1}$ were
272 identified in the simulation. To determine the frequencies of these modes, a spectral analysis of
273 the time-dependent scattered signal is performed in the following paragraph.

274 Fig 8 shows the normalised Fourier transform of the time-dependent scattered signal in the
275 ionization zone, at $x = 3$ mm and at an angle $\alpha = 110^\circ$ and in the acceleration zone, at $x = 20$ mm
276 and at an angle $\alpha = 90^\circ$. The spectra illustrate one well-defined peaks in each case for both modes.

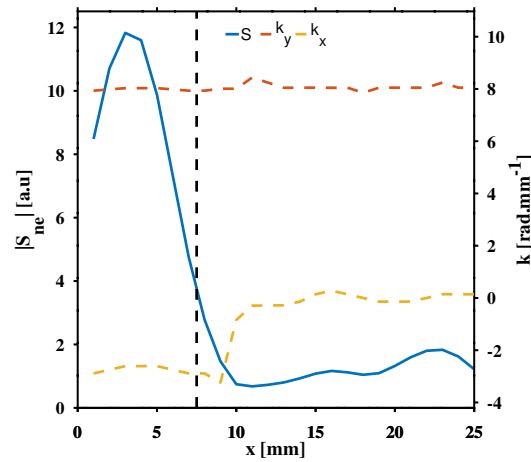
277 Fig 8a shows that for the $k_y = 3 \text{ rad.mm}^{-1}$ the frequency is different with respect to the obser-
278 vation zone. The mode's mean frequency increases from 1 MHz in the ionization zone to 3 MHz

This is the author's peer reviewed, accepted manuscript. However, the online version of record will be different from this version once it has been copyedited and typeset.

PLEASE CITE THIS ARTICLE AS DOI: 10.1063/5.0073215



(a) $k = 3 \text{ rad.mm}^{-1}$



(b) $k = 8 \text{ rad.mm}^{-1}$

FIG. 7: Variation of the dominant modes along the axial direction. The blue curves show the intensity of the scattering signal while, on the secondary axis, the dashed curves show the variation of the axial component k_x and the azimuthal component k_y . The dashed black curve indicates the thruster exit plane and the position of the maximum radial magnetic field.

This is the author's peer reviewed, accepted manuscript. However, the online version of record will be different from this version once it has been copyedited and typeset.

PLEASE CITE THIS ARTICLE AS DOI: 10.1063/5.0073215

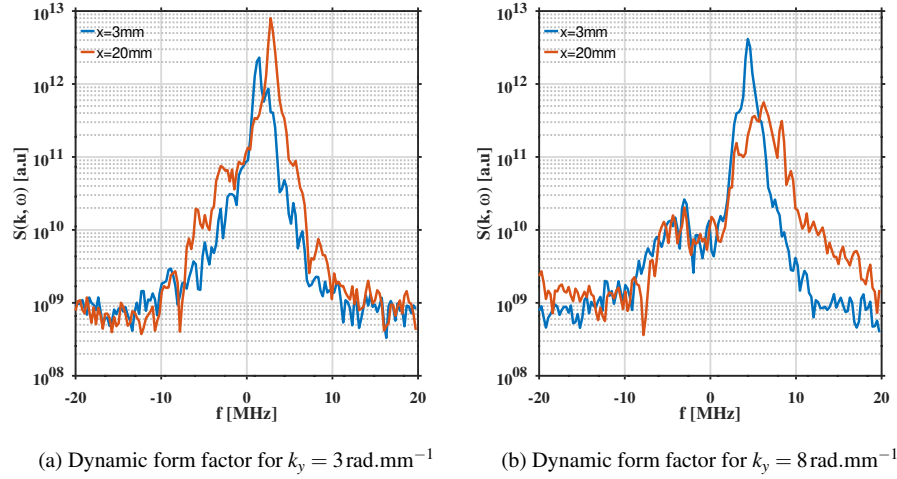


FIG. 8: Dynamic form factor for $k_y = 3 \text{ rad.mm}^{-1}$ and $k_y = 8 \text{ rad.mm}^{-1}$ at $x = 3 \text{ mm}$ and 20 mm . Blue curves correspond to $x = 3 \text{ mm}$ and an angle $\alpha = 110^\circ$ and the red curve correspond to $x = 20 \text{ mm}$ and an angle $\alpha = 90^\circ$

279 in the acceleration zone. The results for the mode at $k_y = 8 \text{ rad.mm}^{-1}$ are presented in Fig 8b.
 280 The mode's mean frequency also increases from 4.5 MHz in the ionization zone to 6.5 MHz in
 281 the acceleration zone. We also observe a broad peak at a negative frequency that suggests a wave
 282 back-flow, however, this would need further investigation.

283 The dispersion relation, from the virtual Thomson scattering experiment is computed for dif-
 284 ferent observation angles α and at different axial positions. The results show a linear progression
 285 of the frequency with respect to the wave number for all the tested angles. The maximum signal
 286 is obtained, though, when the angle of the observation is along the main propagation direction
 287 i.e $\alpha = 110^\circ$ in the ionization zone and $\alpha = 90^\circ$ in the acceleration zone. Fig 9a and Fig 9b
 288 show the respective dispersion relations along these privileged observation angles at $x = 3 \text{ mm}$ and
 289 $x = 20 \text{ mm}$ respectively. Two phase velocities are identified using a linear fit, $v_{g,Jz} = 3.5 \text{ km.s}^{-1}$
 290 and $v_{g,Acc} = 6.6 \text{ km.s}^{-1}$. These values are of the same order of magnitude as the measured phase
 291 velocities from CTS experiments^{13,17}, that were conducted on a similar range of wave vectors.

This is the author's peer reviewed, accepted manuscript. However, the online version of record will be different from this version once it has been copyedited and typeset.

PLEASE CITE THIS ARTICLE AS DOI: 10.1063/5.0073215

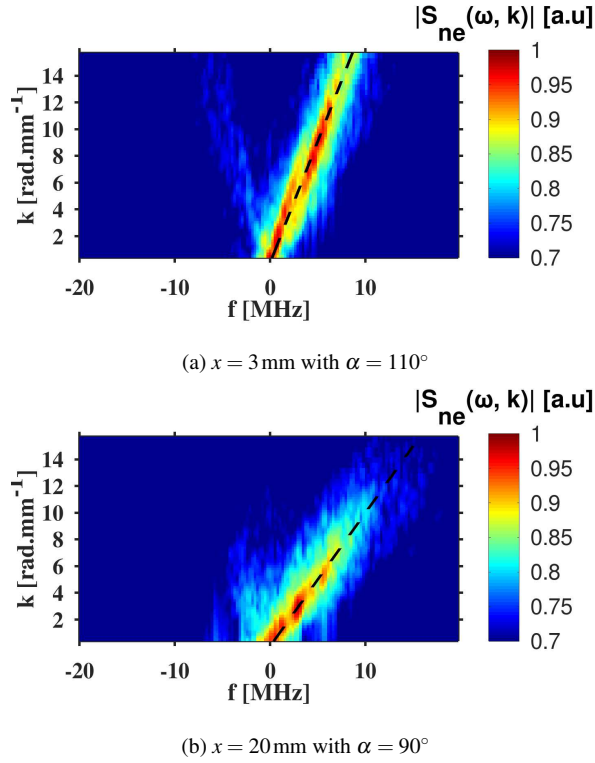


FIG. 9: Dispersion relations derived from the virtual Thomson scattering experiment along the privileged observation angle relative to each zone: (a) corresponds to the ionization zone with $\alpha = 110^\circ$ while (b) corresponds to the acceleration zone with $\alpha = 90^\circ$. The dashed black lines correspond to the best linear fit

292 C. Electron Density Fluctuation rate

293 In this section, we derive the electron density fluctuation rate using the virtual CTS for different
 294 values of the waist w of the laser beam. The results are then compared with the reference PIC
 295 values. In Eq7, the waist appears when we calculate both the mean electron density and its rms
 296 since these values are averaged over the scattering volume.

297 At first, we examine the mean electron density profile. Fig 10 shows the axial profile of the
 298 electron density for two different values of the laser waist: $w = 1$ and 3 mm . The laser-beam

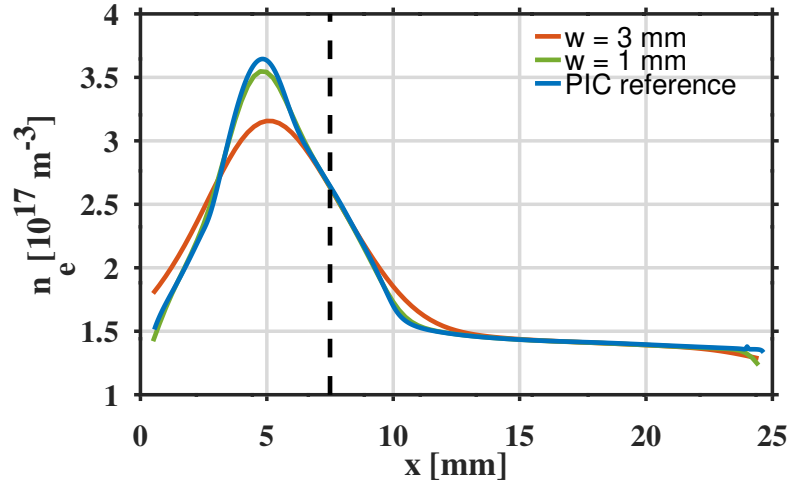


FIG. 10: Axial profile of the electron density for different values of the laser waist: $w = 1$ and 3 mm in the CTS analysis and from the PIC simulations.

299 weighed electron density maps were averaged over $3.75 \mu\text{s}$ and the azimuthal direction. The pro-
 300 files diverge significantly for axial distances between $x = 3$ mm and $x = 10$ mm, where a steep
 301 density gradient is observed. This is most noteworthy for a $w = 3$ mm waist with a maximum 11%
 302 deviation from the PIC profile at around $x = 5$ mm. For a $w = 1$ mm waist, on the other hand, the
 303 deviation is estimated to be a mere 2%. These observations are mainly due to the presence of gra-
 304 dients in the electron density profile inside the scattering volume. Indeed, if the scattering volume
 305 has a sufficient spatial extension, the laser profile behaves like a Gaussian filter and smoothens the
 306 electron density profile. Depending on how the electrons are distributed in the scattering volume,
 307 the locally-averaged electron density is either underestimated or overestimated at a given axial po-
 308 sition. In the acceleration zone, between $x = 10$ mm and $x = 214$ mm, where the axial evolution of
 309 the electron density is much smoother, the virtual CTS analysis gives results close to the reference
 310 PIC profile for $w = 1$ mm and $w = 3$ mm.

311 Then, we examine the electron density fluctuation rate as introduced in Eq7 for two different
 312 values of the laser waist: $w = 1$ and 3 mm and compare it to the PIC reference value. The results
 313 are reported on Fig 11. As mentioned in Section II C, we limit our analysis to $3 \leq x \leq 21$ mm.
 314 Between $x = 3$ mm and $x = 10$ mm, the virtual CTS estimates an average fluctuation rate at around

This is the author's peer reviewed, accepted manuscript. However, the online version of record will be different from this version once it has been copyedited and typeset.

PLEASE CITE THIS ARTICLE AS DOI: 10.1063/1.50073215

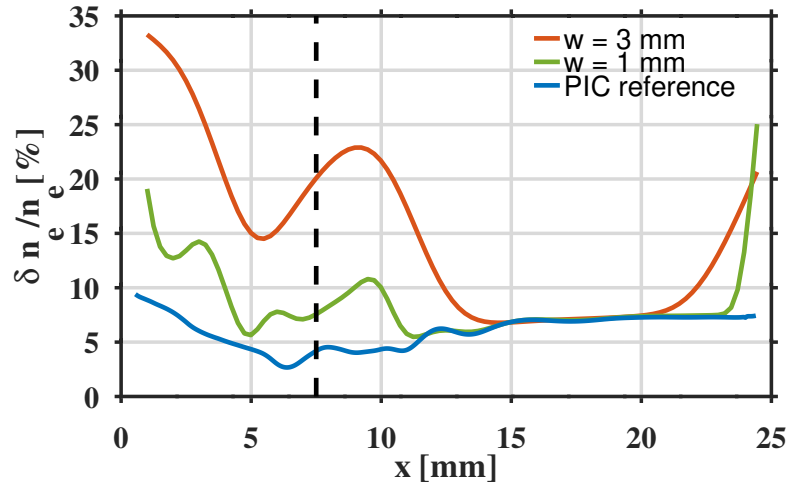


FIG. 11: Axial profile of the electron fluctuation rate for different values of the laser waist: $w = 1$ and 3 mm in the CTS analysis and from the PIC simulations.

315 20% for $w = 3$ mm, 3 times higher than what the reference PIC profile yields. In contrast, in the
 316 acceleration zone, between $x = 10$ mm and $x = 24$ mm, where the axial electron density is almost
 317 constant, the virtual CTS for $w = 1$ and 3 mm gives results coherent with the 7% fluctuation rate of
 318 the reference PIC profile. The aforementioned smoothing effect of the axially-averaged-electron
 319 density profile could not explain alone this difference. Indeed a factor of 2 is also observed for
 320 $w = 1$ mm for which the difference in the mean electron density profiles is estimated to be only
 321 2%.

322 In this regard, the difference, observed in Fig 11 stems then from the numerator of Eq7. This
 323 term corresponds, in the framework of CTS, to the integral of the form factor over all possible
 324 fluctuations in the scattering volume. Since the volume (Fig 5) has a spatial extension both in the
 325 azimuthal and the axial direction, the integrated profiles of the fluctuation rate take into account
 326 not only azimuthal fluctuations associated with the propagating modes but also the axial density
 327 standard deviation associated with the density gradient. This results in a relatively high value
 328 compared to the PIC reference value.

329 These observations favor the use of smaller waists in CTS experiments for positions where
 330 gradients are high to reduce the impact of the waist on the measurement of the density fluctuation

331 rate. However, this is more challenging in CTS experiments since a smaller w would lead to a
332 degraded k resolution.

333 IV. CONCLUSION

334 In this work, 2D axial-azimuthal PIC simulations were analyzed using a virtual CTS. It has
335 been shown that compared to the non-windowed Fourier transform, virtual CTS offers a localized
336 and directional tool to analyze PIC data. The analysis of PIC results using a virtual CTS diagnostic
337 allowed an enhanced characterization of the modes present in the stationary state of a 2D axial-
338 azimuthal PIC simulation¹⁰, not only in the plume but also in the thruster channel. It permitted to
339 resolve two modes at $3\text{ rad}\cdot\text{mm}^{-1}$ and $8\text{ rad}\cdot\text{mm}^{-1}$ propagating all along the thruster axis. Their
340 direction is slightly oblique near the anode and becomes almost azimuthal at the thruster exit plane
341 as they are convected by ions. The observation shows also that when the ions' convection becomes
342 very large, the change in direction concurs with a change in the amplitudes of the modes. The
343 amplitude of the $3\text{ rad}\cdot\text{mm}^{-1}$ mode grows linearly in the acceleration zone while the $8\text{ rad}\cdot\text{mm}^{-1}$
344 is sharply damped. The dispersion relations (f, k) have been computed using this approach inside
345 the ionization zone in the thruster channel and the acceleration zone and are consistent with the
346 observations in CTS experiments.

347 On the other hand, the examination of the electron density fluctuation rate shows a strong
348 dependence on the axial position x . Far in the thruster plume, the fluctuation rate is almost constant,
349 while in the ionization zone inside the thruster and near the exit plane, the profile exhibits strong
350 variations. Moreover, the results are shown to strongly depend on the value of the laser beam waist
351 (1 and 3 mm in this work). The study first highlights the importance of an accurate measure of
352 the electron density inside the scattering volume. Since the electron density appears as a dividing
353 factor, a gross estimation can lead to a large difference in the order of magnitude of the fluctuation
354 rate. Recent measurements using incoherent Thomson scattering to estimate the electron density²¹
355 also suggest a similar conclusion. The observations also highlight the impact of the waist on
356 the evaluation of the electron density root-mean-square. Because of its spatial extension, the
357 diagnostic integrates both axial and azimuthal fluctuations. This leads to a different estimation of
358 the fluctuation rate especially in areas with a strong electron density gradient.

359 Finally, this work has allowed us to clarify further the discrepancies between PIC simulations
360 and CTS experiments on the value of the electron density fluctuation rate. It also underlines the

This is the author's peer reviewed, accepted manuscript. However, the online version of record will be different from this version once it has been copyedited and typeset.

PLEASE CITE THIS ARTICLE AS DOI: 10.1063/1.50073215

361 interest to perform virtual plasma diagnostics on either PIC or fluid simulations since they allow
362 to relate simulations and experiments and foster common ground for comparison and discussion.
363 In this work, the CTS diagnostic was applied to a simplified 2D (axial-azimuthal) test case with
364 a stationary state. In future work, the virtual CTS will be used to analyze a self-consistent time-
365 dependent 2D (axial-azimuthal) simulation closer to the real operation conditions of Hall effect
366 thrusters.

367 ACKNOWLEDGMENTS

368 This work has been partially funded by ANR (No. ANR-16-CHIN-003-01) and Safran Aircraft
369 Engines within the project POSEIDON.

370 DATA AVAILABILITY STATEMENT

371 The data that support the findings of this study are available from the corresponding author
372 upon reasonable request.

373 REFERENCES

- 374 ¹A. I. Morozov and V. V. Savelyev, “Fundamentals of stationary plasma thruster theory,” in *Re-*
375 *views of Plasma Physics*, edited by B. B. Kadomtsev and V. D. Shafranov (Springer US, Boston,
376 MA, 2000) pp. 203–391.
- 377 ²D. O’Reilly, G. Herdrich, and D. F. Kavanagh, “Electric propulsion methods for small satellites:
378 A review,” *Aerospace* **8** (2021), 10.3390/aerospace8010022.
- 379 ³F. Taccogna, S. Longo, M. Capitelli, and R. Schneider, “Anomalous transport induced
380 by sheath instability in hall effect thrusters,” *Applied Physics Letters* **94**, 251502 (2009),
381 <https://doi.org/10.1063/1.3152270>.
- 382 ⁴Y. Raitses, I. D. Kaganovich, A. Khrabrov, D. Sydorenko, N. J. Fisch, and A. Smolyakov,
383 “Effect of secondary electron emission on electron cross-field current in $\mathbf{E} \times \mathbf{B}$ discharges,”
384 *IEEE Transactions on Plasma Science* **39**, 995–1006 (2011).
- 385 ⁵W. Frias, A. I. Smolyakov, I. D. Kaganovich, and Y. Raitses, “Long wavelength gradient
386 drift instability in hall plasma devices. i. fluid theory,” *Physics of Plasmas* **19**, 072112 (2012),
387 <https://doi.org/10.1063/1.4736997>.

This is the author's peer reviewed, accepted manuscript. However, the online version of record will be different from this version once it has been copyedited and typeset.

PLEASE CITE THIS ARTICLE AS DOI: 10.1063/1.50073215

- 388 ⁶J. C. Adam, A. Héron, and G. Laval, “Study of stationary plasma thrusters us-
389 ing two-dimensional fully kinetic simulations,” *Physics of Plasmas* **11**, 295–305 (2004),
390 <https://doi.org/10.1063/1.1632904>.
- 391 ⁷T. Lafleur, S. D. Baalrud, and P. Chabert, “Characteristics and transport effects of the electron
392 drift instability in hall-effect thrusters,” *Plasma Sources Science and Technology* **26**, 024008
393 (2017).
- 394 ⁸T. Lafleur, R. Martorelli, P. Chabert, and A. Bourdon, “Anomalous electron transport in hall-
395 effect thrusters: Comparison between quasi-linear kinetic theory and particle-in-cell simula-
396 tions,” *Physics of Plasmas* **25**, 061202 (2018), <https://doi.org/10.1063/1.5017626>.
- 397 ⁹K. Hara, “An overview of discharge plasma modeling for hall effect thrusters,” *Plasma Sources*
398 *Science and Technology* **28**, 044001 (2019).
- 399 ¹⁰T. Charoy, J. P. Boeuf, A. Bourdon, J. A. Carlsson, P. Chabert, B. Cuenot, D. Eremin, L. Gar-
400 rigues, K. Hara, I. D. Kaganovich, A. T. Powis, A. Smolyakov, D. Sydorenko, A. Tavant, O. Ver-
401 morel, and W. Villafana, “2D axial-azimuthal particle-in-cell benchmark for low-temperature
402 partially magnetized plasmas,” *Plasma Sources Science and Technology* **28**, 105010 (2019).
- 403 ¹¹A. Lazurenko, V. Vial, M. Prioul, and A. Bouchoule, “Experimental investigation of high-
404 frequency drifting perturbations in hall thrusters,” *Physics of Plasmas* **12**, 013501–013501–9
405 (2005), <https://doi.org/10.1063/1.1818698>.
- 406 ¹²A. Lazurenko, T. D. de Wit, C. Cavoit, V. Krasnoselskikh, A. Bouchoule, and
407 M. Dudeck, “Determination of the electron anomalous mobility through measurements
408 of turbulent magnetic field in hall thrusters,” *Physics of Plasmas* **14**, 033504 (2007),
409 <https://doi.org/10.1063/1.2535813>.
- 410 ¹³S. Tsikata, N. Lemoine, V. Pisarev, and D. M. Grésillon, “Dispersion relations of electron
411 density fluctuations in a hall thruster plasma, observed by collective light scattering,” *Physics of*
412 *Plasmas* **16**, 033506 (2009), <https://doi.org/10.1063/1.3093261>.
- 413 ¹⁴J. Cavalier, N. Lemoine, G. Bonhomme, S. Tsikata, C. Honoré, and D. Grésillon, “Hall thruster
414 plasma fluctuations identified as the $\mathbf{E} \times \mathbf{B}$ electron drift instability: Modeling and fitting on
415 experimental data,” *Physics of Plasmas* **20**, 082107 (2013), <https://doi.org/10.1063/1.4817743>.
- 416 ¹⁵Z. A. Brown and B. A. Jorns, “Spatial evolution of small wavelength fluctuations in a hall
417 thruster,” *Physics of Plasmas* **26**, 113504 (2019), <https://doi.org/10.1063/1.5116708>.
- 418 ¹⁶I. D. Kaganovich, A. Smolyakov, Y. Raitses, E. Ahedo, I. G. Mikellides, B. Jorns, F. Tac-
419 cogna, R. Gueroult, S. Tsikata, A. Bourdon, J.-P. Boeuf, M. Keidar, A. T. Powis, M. Merino,

This is the author's peer reviewed, accepted manuscript. However, the online version of record will be different from this version once it has been copyedited and typeset.

PLEASE CITE THIS ARTICLE AS DOI: 10.1063/5.0073215

- 420 M. Cappelli, K. Hara, J. A. Carlsson, N. J. Fisch, P. Chabert, I. Schweigert, T. Lafleur,
421 K. Matyash, A. V. Khrabrov, R. W. Boswell, and A. Fruchtman, “Physics of eb discharges
422 relevant to plasma propulsion and similar technologies,” *Physics of Plasmas* **27**, 120601 (2020),
423 <https://doi.org/10.1063/5.0010135>.
- 424 ¹⁷S. Tsikata, C. Honoré, N. Lemoine, and D. M. Grésillon, “Three-dimensional structure of elec-
425 tron density fluctuations in the hall thruster plasma: $\mathbf{E} \times \mathbf{B}$ mode,” *Physics of Plasmas* **17**, 112110
426 (2010), <https://doi.org/10.1063/1.3499350>.
- 427 ¹⁸S. Tsikata, C. Honoré, and D. Grésillon, “Collective thomson scattering for studying plasma
428 instabilities in electric thrusters,” *Journal of Instrumentation* **8**, C10012–C10012 (2013).
- 429 ¹⁹S. Tsikata, J. Cavalier, A. Héron, C. Honoré, N. Lemoine, D. Grésillon, and D. Coulette, “An
430 axially propagating two-stream instability in the hall thruster plasma,” *Physics of Plasmas* **21**,
431 072116 (2014), <https://doi.org/10.1063/1.4890025>.
- 432 ²⁰C. Honoré, S. Tsikata, D. Grésillon, A. Héron, N. Lemoine, and J. Cavalier, “Hall thruster small
433 scale plasma fluctuations: Qualifying 2D pic simulations against collective scattering experi-
434 mental data,” (The 32nd Int. Electric Propulsion Conf. 11/9-5/9/2011, Wiesbaden).
- 435 ²¹S. Tsikata, A. Héron, and C. Honoré, “Hall thruster microturbulence under con-
436 ditions of modified electron wall emission,” *Physics of Plasmas* **24**, 053519 (2017),
437 <https://doi.org/10.1063/1.4984255>.
- 438 ²²S. Tsikata and K. Hara, “Plasma instabilities in cross-field configuration: an analysis of the
439 relevance of different modes for electron transport,” (the 36th International Electric Propulsion
440 Conference, Vienna, Austria, September 2019).
- 441 ²³A. T. Powis and M. N. Shneider, “Particle-in-cell modeling of laser thomson scattering in
442 low-density plasmas at elevated laser intensities,” *Physics of Plasmas* **25**, 053513 (2018),
443 <https://doi.org/10.1063/1.5029820>.
- 444 ²⁴J.-P. Boeuf, “Tutorial: Physics and modeling of hall thrusters,” *Journal of Applied Physics* **121**,
445 011101 (2017), <https://doi.org/10.1063/1.4972269>.
- 446 ²⁵J. P. Boeuf and L. Garrigues, “ $\mathbf{E} \times \mathbf{B}$ electron drift instability in hall thrusters:
447 Particle-in-cell simulations vs. theory,” *Physics of Plasmas* **25**, 061204 (2018),
448 <https://doi.org/10.1063/1.5017033>.
- 449 ²⁶E. Holzhauser and J. H. Massig, “An analysis of optical mixing in plasma scattering experiments,”
450 *Plasma Physics* **20**, 867–877 (1978).

This is the author's peer reviewed, accepted manuscript. However, the online version of record will be different from this version once it has been copyedited and typeset.

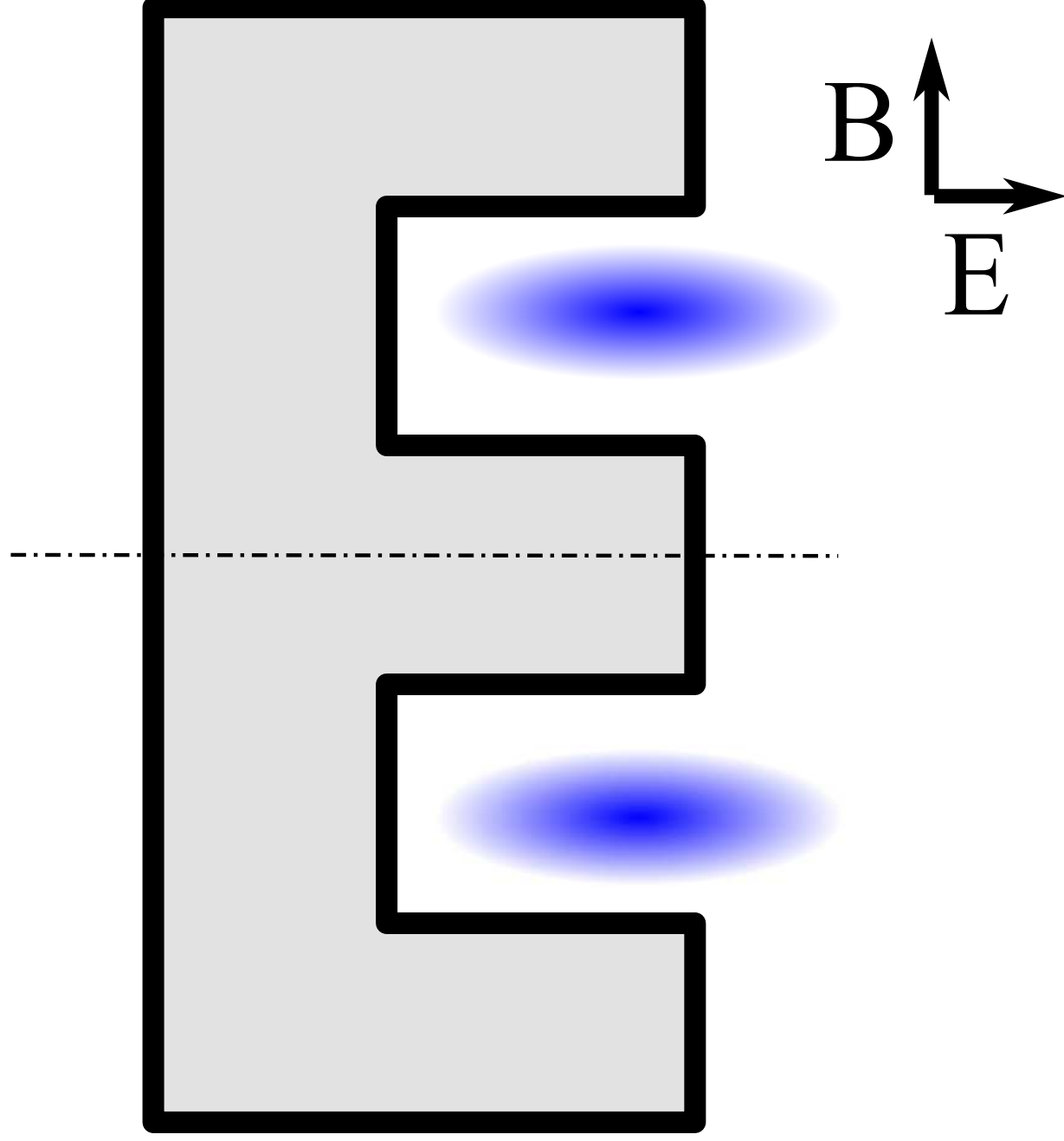
PLEASE CITE THIS ARTICLE AS DOI: 10.1063/5.0073215

451 ²⁷D. H. Froula, S. H. Glenzer, N. C. Luhmann, and J. Sheffield, “Chapter 2 and chapter 5,” in
452 *Plasma Scattering of Electromagnetic Radiation (Second Edition)* (Academic Press, Boston,
453 2011) second edition ed., pp. 103–142.

454 ²⁸v. d. Meiden, H.J., *Thomson scattering on low and high temperature plasmas*, Phd Thesis 1 (Re-
455 search TU/e / Graduation TU/e), Technische Universiteit Eindhoven, Eindhoven (2011), ISBN:
456 9789038624242.

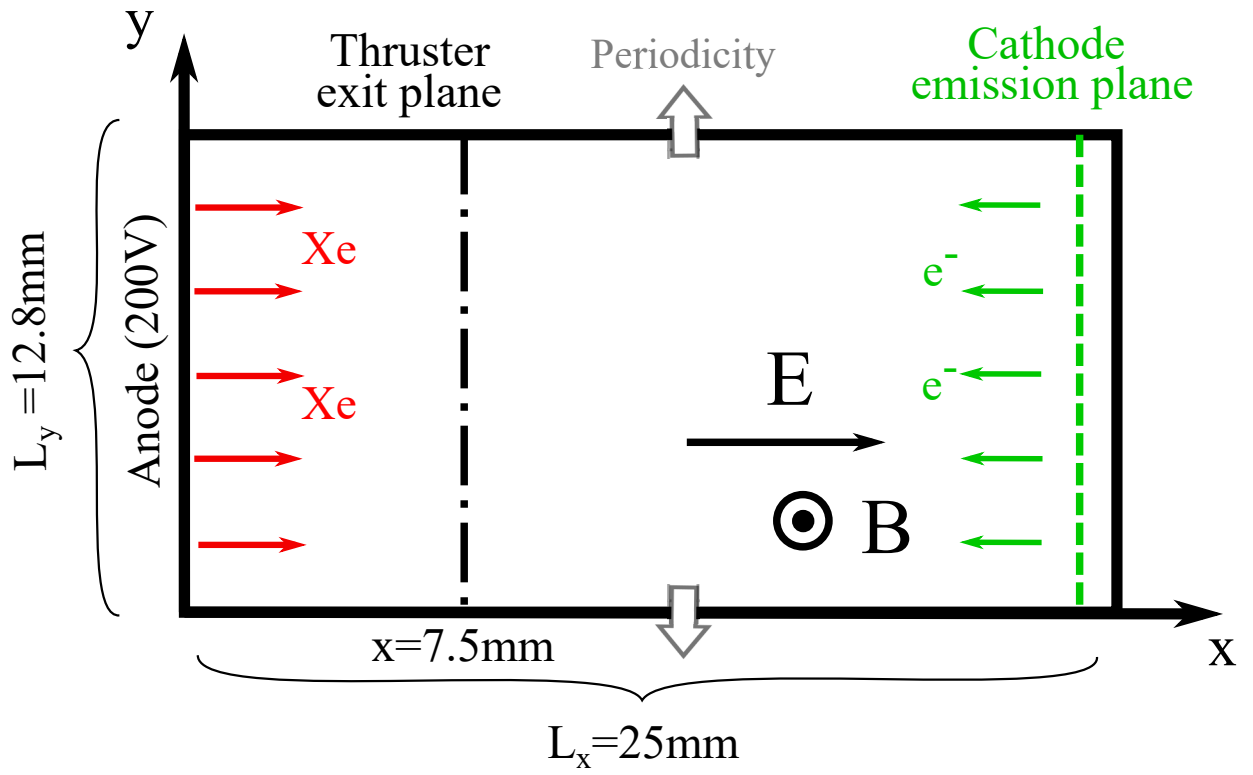
This is the author's peer reviewed, accepted manuscript. However, the online version of record will be different from this version once it has been copyedited and typeset.

PLEASE CITE THIS ARTICLE AS DOI: 10.1063/5.0073215



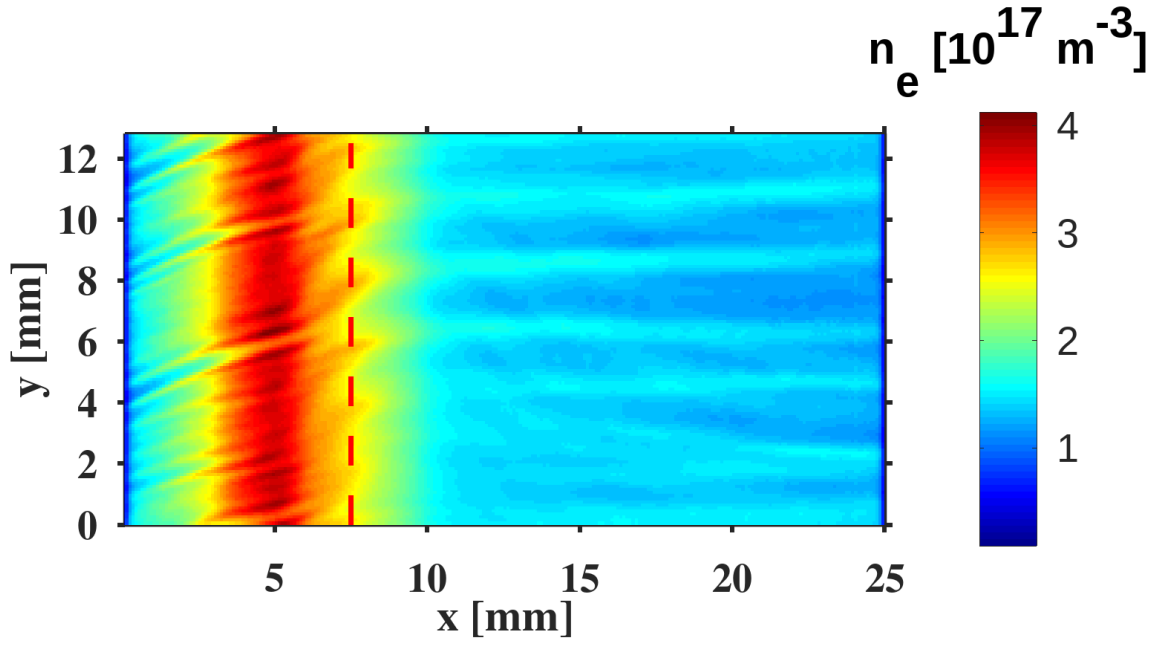
This is the author's peer reviewed, accepted manuscript. However, the online version of record will be different from this version once it has been copyedited and typeset.

PLEASE CITE THIS ARTICLE AS DOI: 10.1063/5.0073215



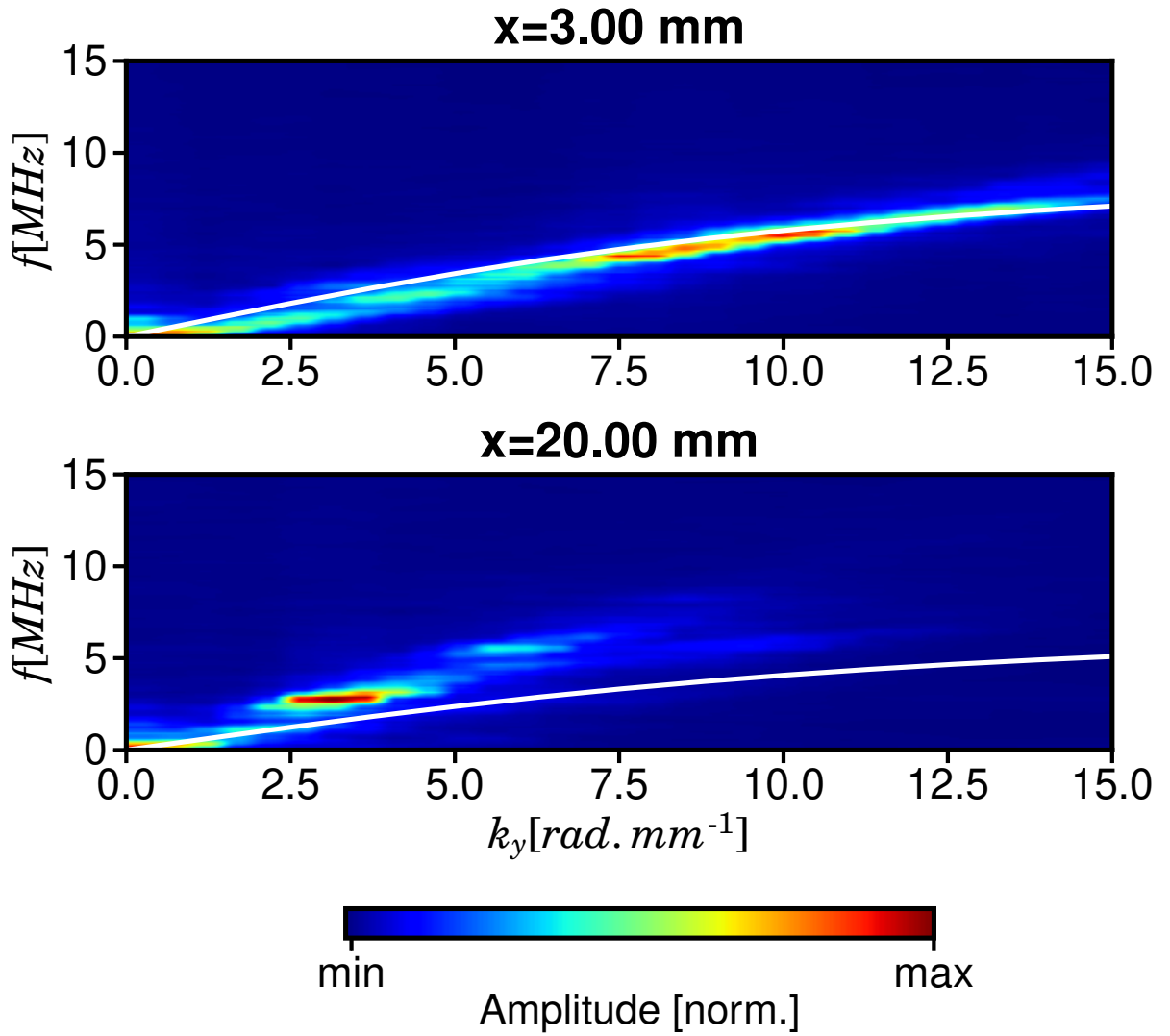
This is the author's peer reviewed, accepted manuscript. However, the online version of record will be different from this version once it has been copyedited and typeset.

PLEASE CITE THIS ARTICLE AS DOI: 10.1063/5.0073215



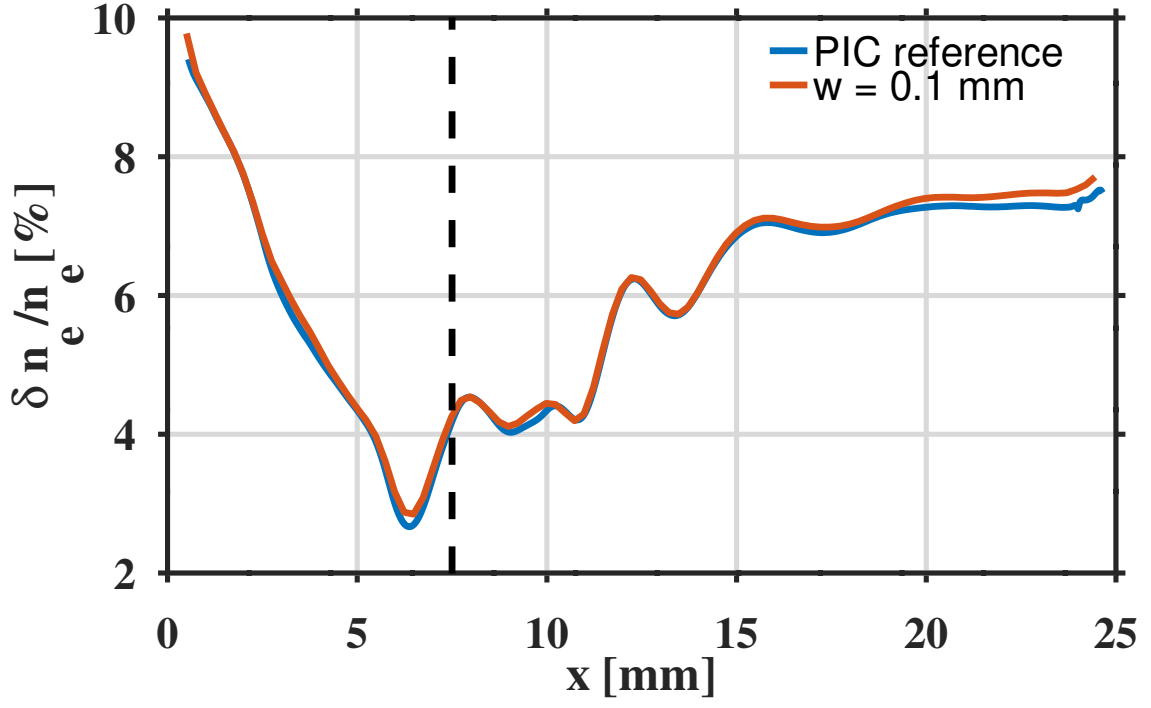
This is the author's peer reviewed, accepted manuscript. However, the online version of record will be different from this version once it has been copyedited and typeset.

PLEASE CITE THIS ARTICLE AS DOI: 10.1063/1.50073215



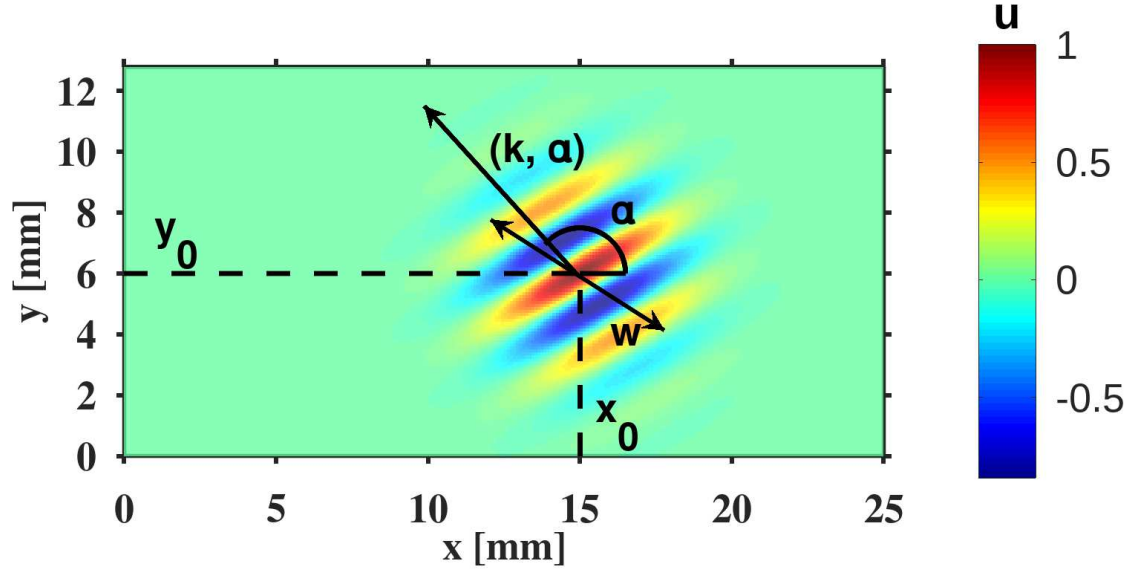
This is the author's peer reviewed, accepted manuscript. However, the online version of record will be different from this version once it has been copyedited and typeset.

PLEASE CITE THIS ARTICLE AS DOI: 10.1063/5.0073215



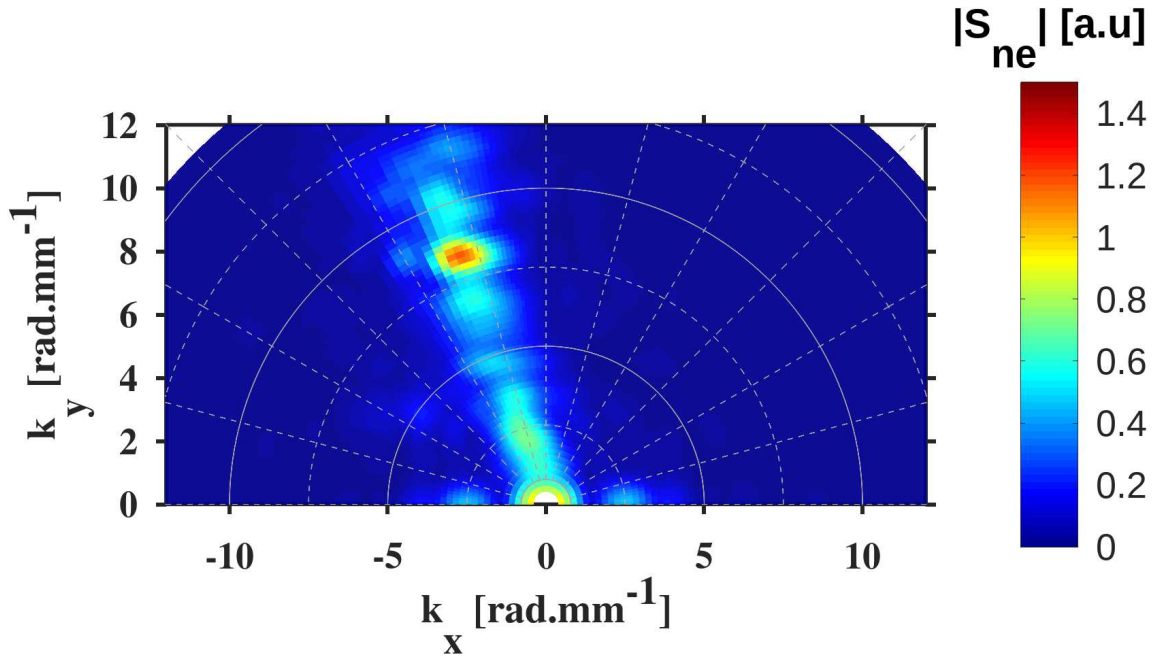
This is the author's peer reviewed, accepted manuscript. However, the online version of record will be different from this version once it has been copyedited and typeset.

PLEASE CITE THIS ARTICLE AS DOI: 10.1063/5.0073215



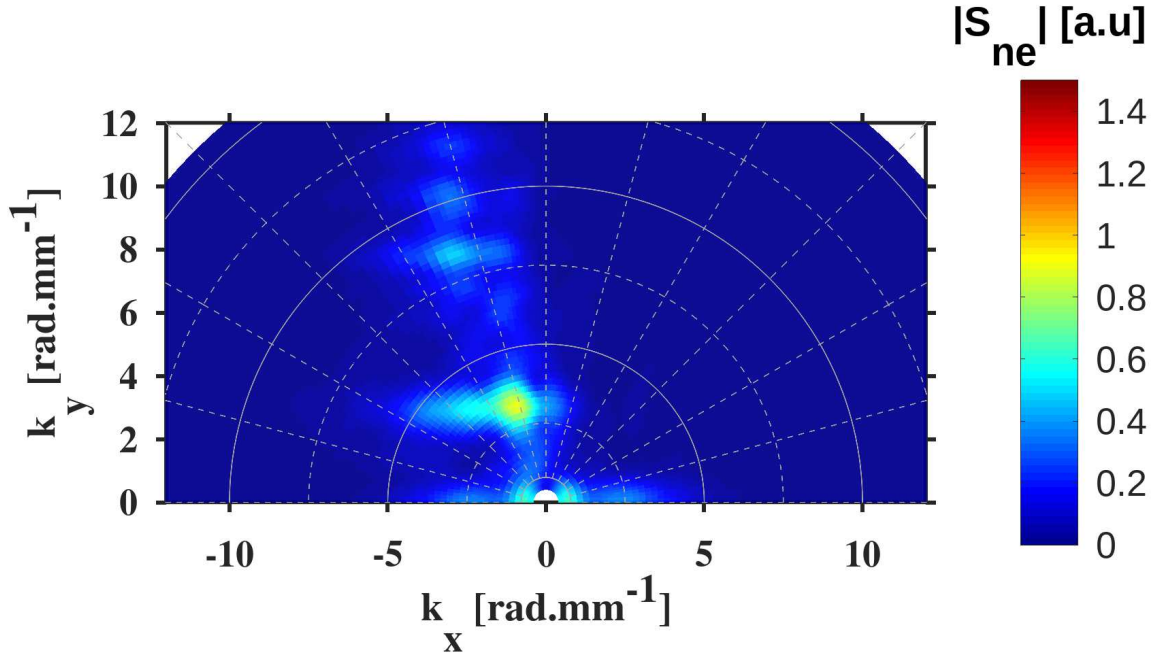
This is the author's peer reviewed, accepted manuscript. However, the online version of record will be different from this version once it has been copyedited and typeset.

PLEASE CITE THIS ARTICLE AS DOI: 10.1063/5.0073215



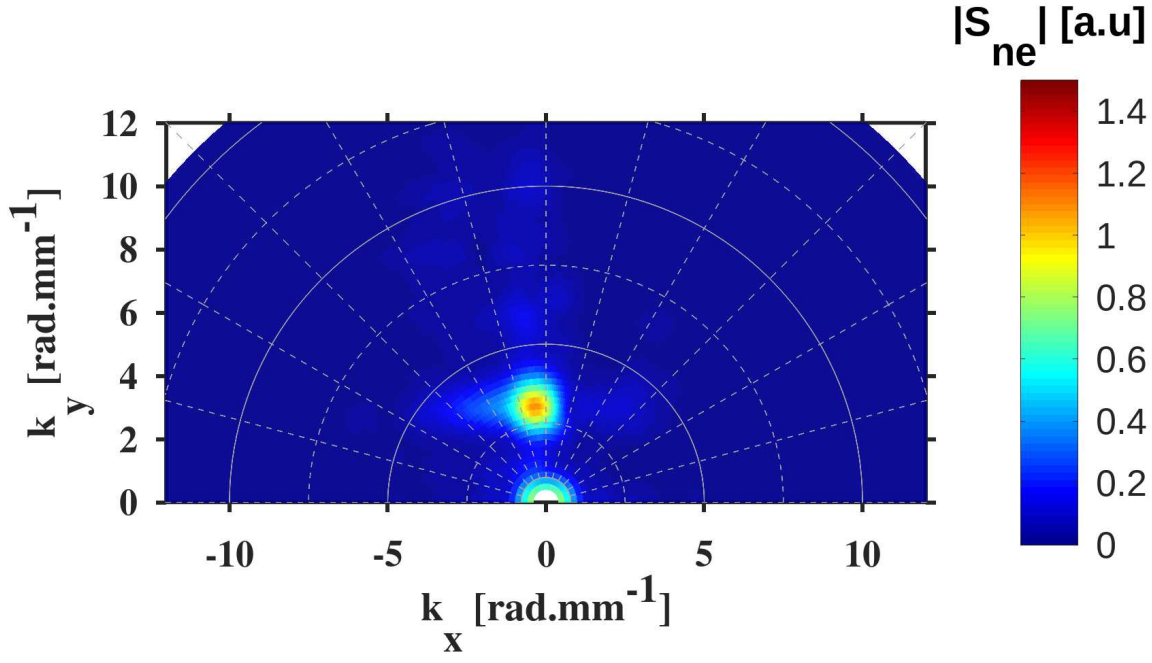
This is the author's peer reviewed, accepted manuscript. However, the online version of record will be different from this version once it has been copyedited and typeset.

PLEASE CITE THIS ARTICLE AS DOI: 10.1063/5.0073215



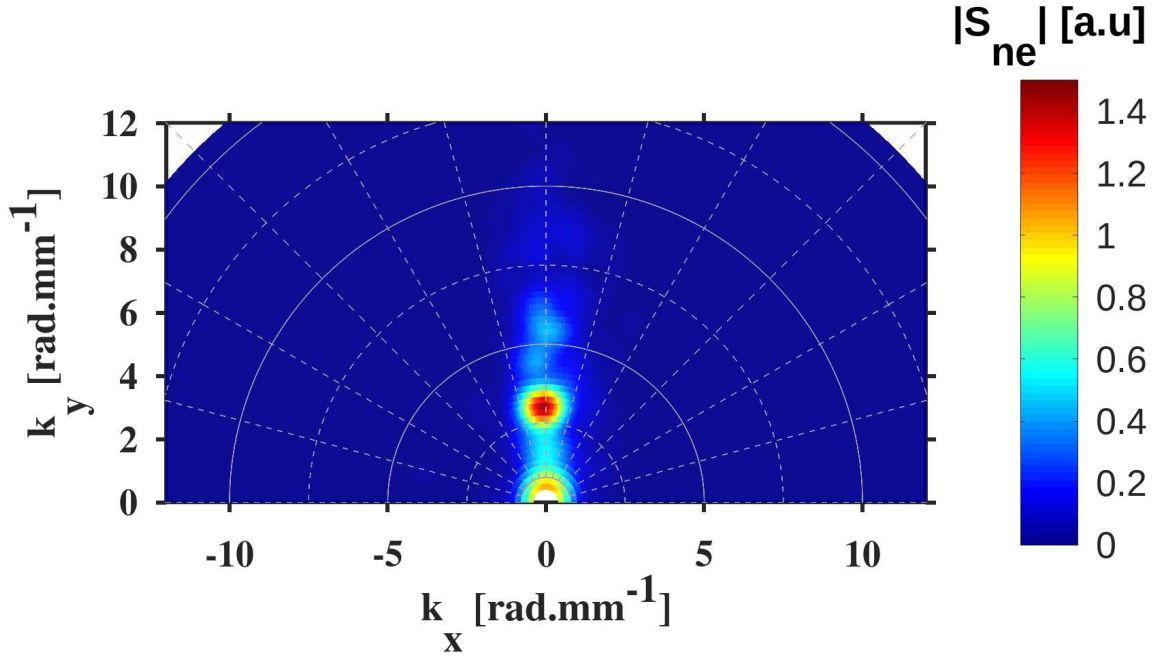
This is the author's peer reviewed, accepted manuscript. However, the online version of record will be different from this version once it has been copyedited and typeset.

PLEASE CITE THIS ARTICLE AS DOI: 10.1063/5.0073215



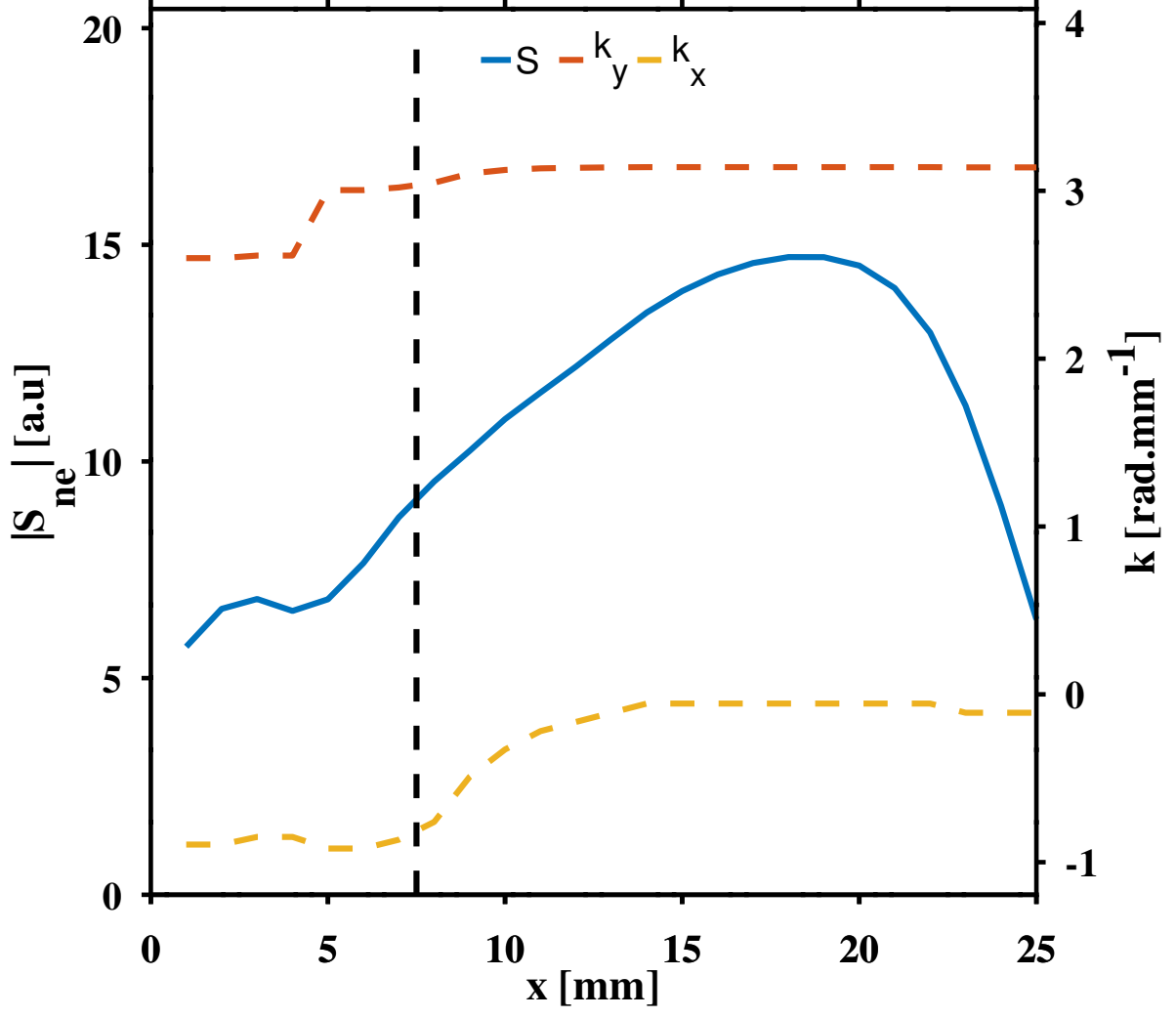
This is the author's peer reviewed, accepted manuscript. However, the online version of record will be different from this version once it has been copyedited and typeset.

PLEASE CITE THIS ARTICLE AS DOI: 10.1063/5.0073215



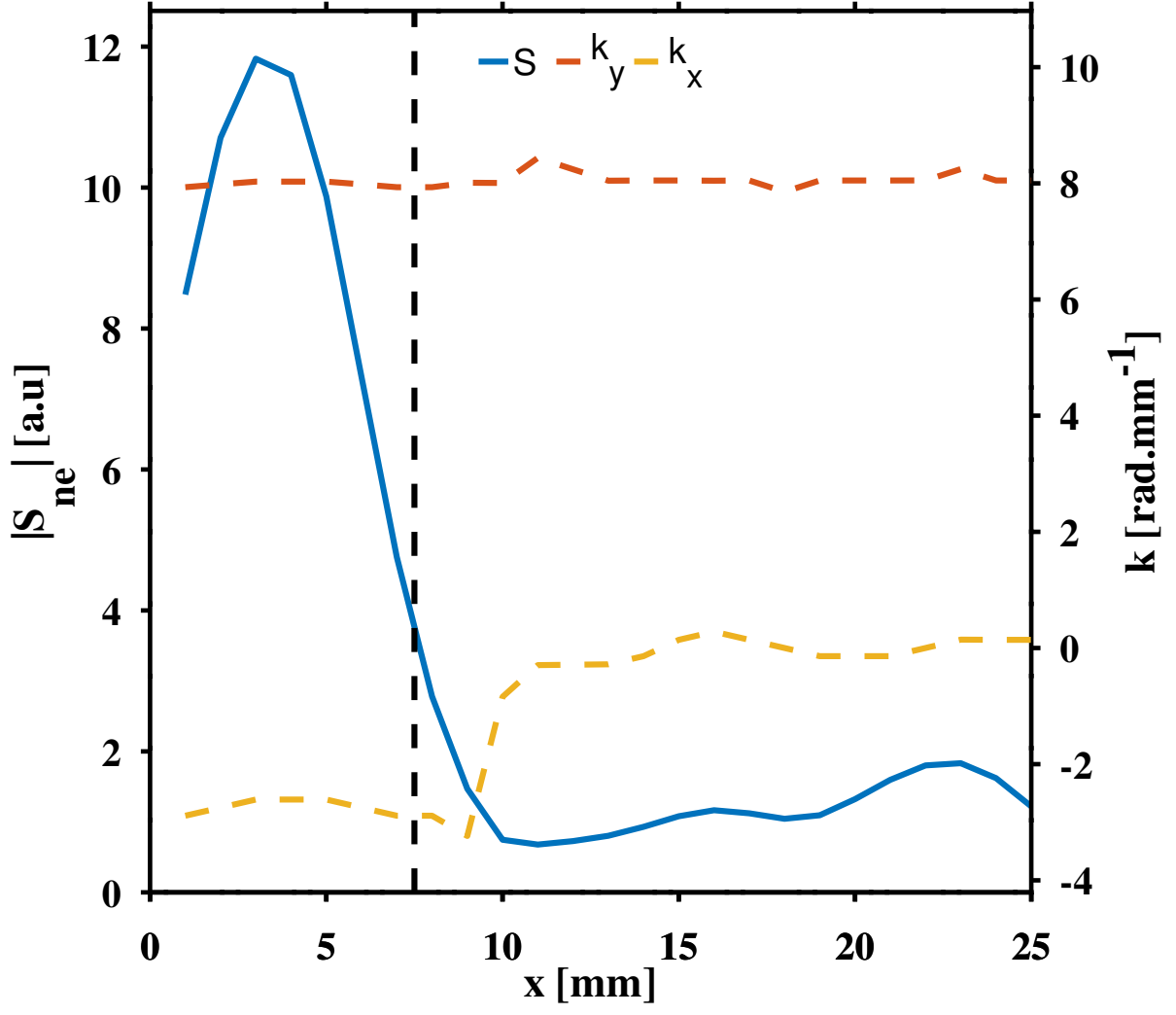
This is the author's peer reviewed, accepted manuscript. However, the online version of record will be different from this version once it has been copyedited and typeset.

PLEASE CITE THIS ARTICLE AS DOI: 10.1063/5.0073215



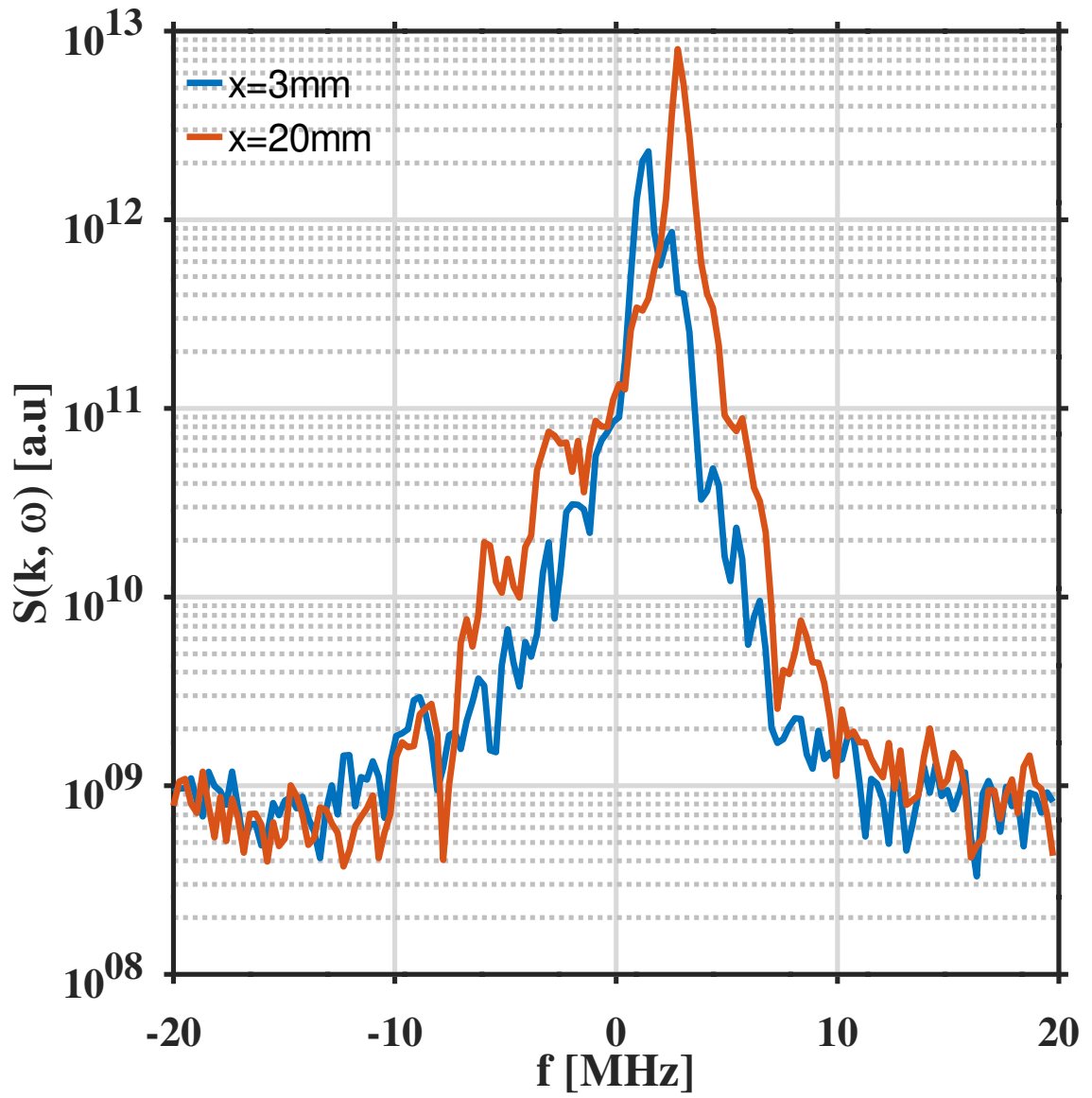
This is the author's peer reviewed, accepted manuscript. However, the online version of record will be different from this version once it has been copyedited and typeset.

PLEASE CITE THIS ARTICLE AS DOI: 10.1063/5.0073215



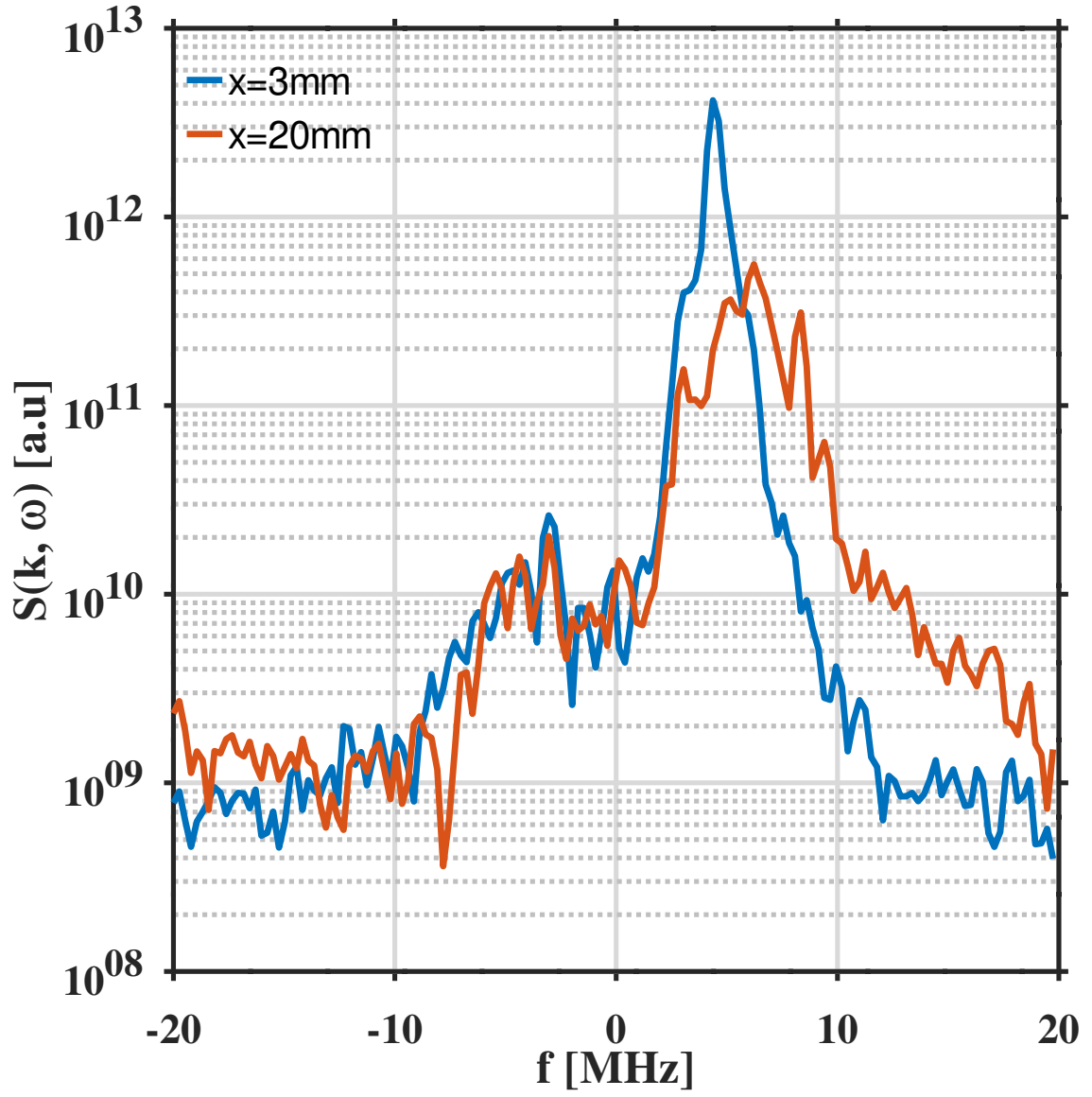
This is the author's peer reviewed, accepted manuscript. However, the online version of record will be different from this version once it has been copyedited and typeset.

PLEASE CITE THIS ARTICLE AS DOI: 10.1063/5.0073215



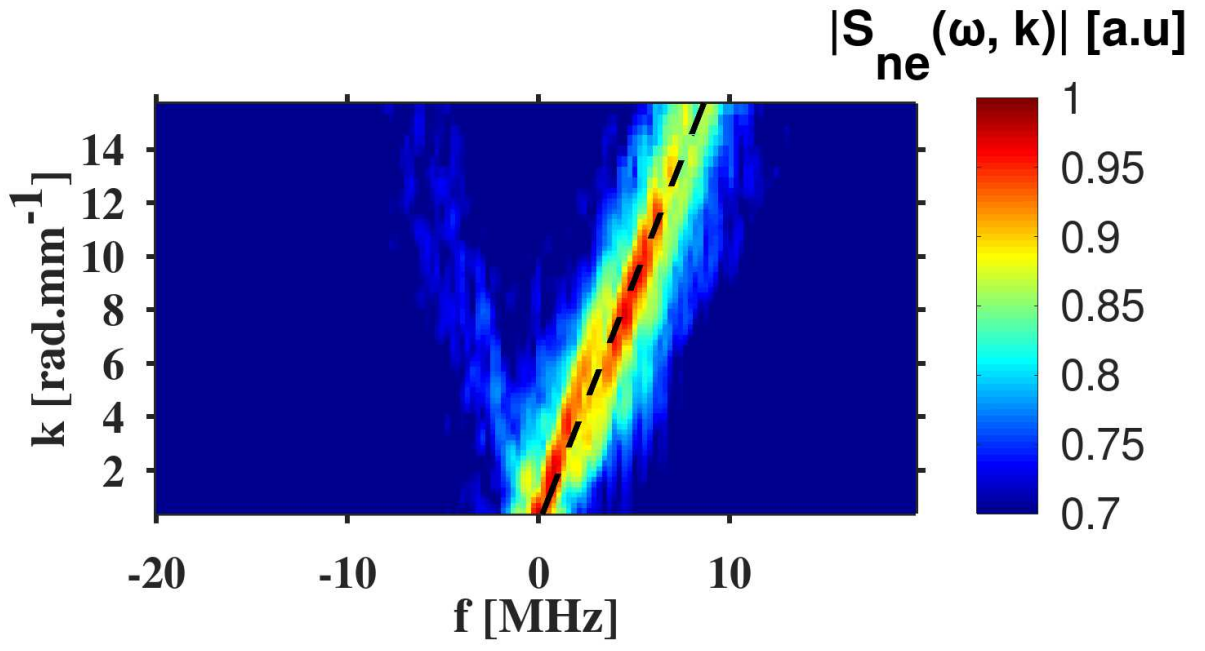
This is the author's peer reviewed, accepted manuscript. However, the online version of record will be different from this version once it has been copyedited and typeset.

PLEASE CITE THIS ARTICLE AS DOI: 10.1063/5.0073215



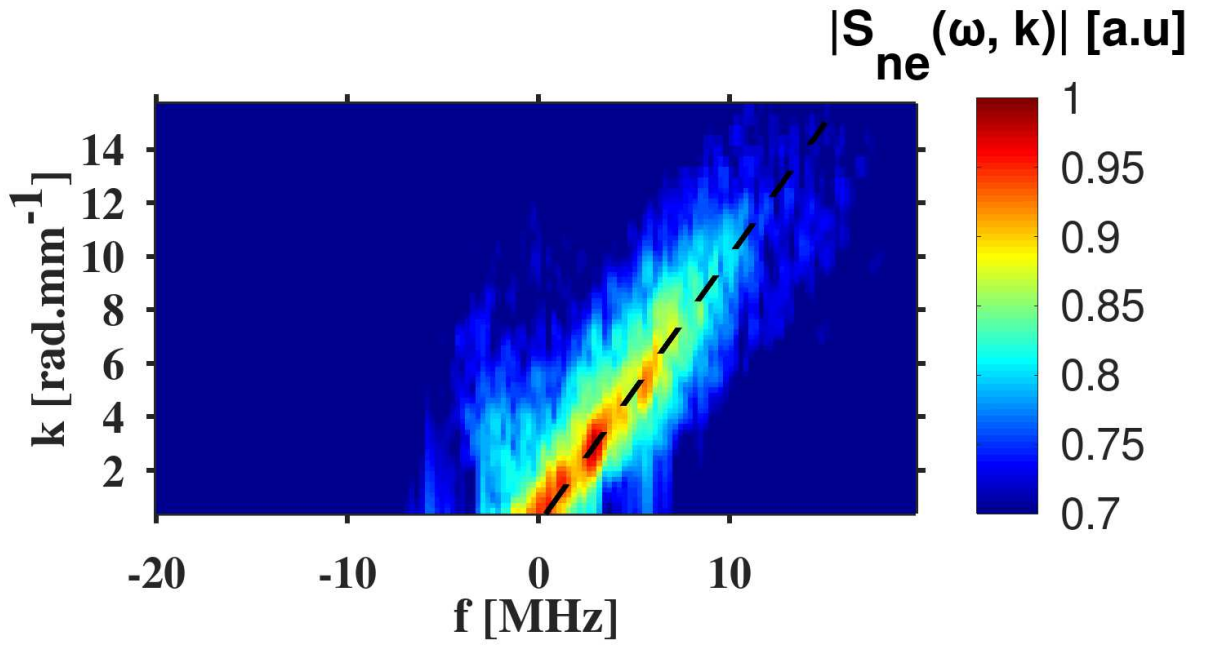
This is the author's peer reviewed, accepted manuscript. However, the online version of record will be different from this version once it has been copyedited and typeset.

PLEASE CITE THIS ARTICLE AS DOI: 10.1063/5.0073215



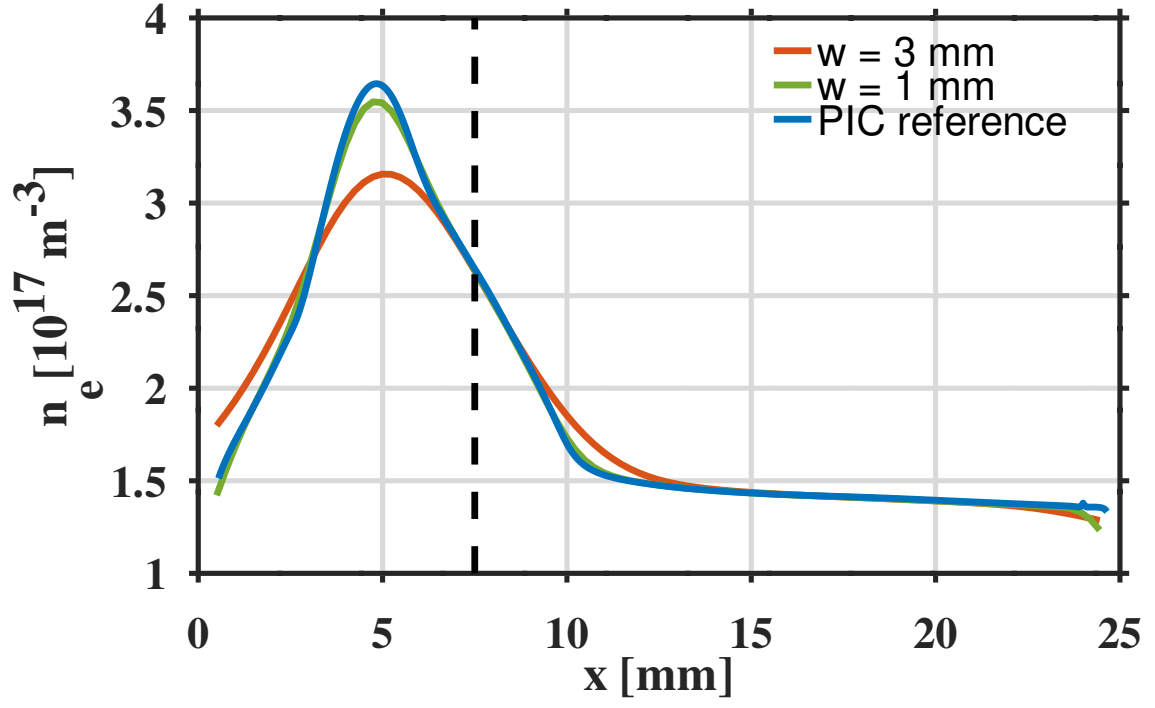
This is the author's peer reviewed, accepted manuscript. However, the online version of record will be different from this version once it has been copyedited and typeset.

PLEASE CITE THIS ARTICLE AS DOI: 10.1063/5.0073215



This is the author's peer reviewed, accepted manuscript. However, the online version of record will be different from this version once it has been copyedited and typeset.

PLEASE CITE THIS ARTICLE AS DOI: 10.1063/5.0073215



This is the author's peer reviewed, accepted manuscript. However, the online version of record will be different from this version once it has been copyedited and typeset.

PLEASE CITE THIS ARTICLE AS DOI: 10.1063/5.0073215

