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► **To cite this version:**

Xinyue Wu, Yabin Jin, Abdelkrim Khelif, Xiaoying Zhuang, Timon Rabczuk, et al.. Topological surface wave metamaterials for robust vibration attenuation and energy harvesting. *Mechanics of Advanced Materials and Structures*, 2021, pp.1937758 (9). 10.1080/15376494.2021.1937758 . hal-03549400

**HAL Id: hal-03549400**

**<https://hal.science/hal-03549400v1>**

Submitted on 31 Jan 2022

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# Topological surface wave metamaterials for robust vibration attenuation and energy harvesting

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## Abstract

Metamaterials are extensively utilized to manipulate ground surface waves for vibration isolation within the bandgap frequency ranges whereas topological crystals allow the creation of robust edge states immune to scattering by defects. In this work, we propose a topological surface wave metamaterial working in the Hertz frequency range, constituted of triangle shape concrete pillars arranged in a honeycomb lattice and deposited on the soil ground. Based on the analogue of quantum valley Hall effect, a non-trivial bandgap is formed from the degeneracy lifting of the Dirac cone at the K point of the Brillouin zone by breaking the inversion symmetry of the two pillars in each unit cell. A topological interface is created between two different crystal phases and a topological edge state based on surface acoustic wave propagation is demonstrated. The robustness of the topologically protected edge state is quantitatively analyzed in presence of various defects and disorders. Finally, we take advantage of the robust and compact topological edge state for designing a harvesting energy device. The results demonstrate the functionality of the proposed structure for both robust

34 surface vibration reduction as well as energy harvesting by designing proper topological  
35 waveguides.

36 **Keywords:** Surface wave metamaterial, topological insulator, vibration attenuation,  
37 energy harvesting, robustness.

38

## 39 **1. Introduction**

40 Mechanical vibrations in urban areas resulting from engineering construction and  
41 ground transportation may damage buildings and bridges, impact the operation of  
42 sophisticated instruments in high-tech laboratories and intrude residents' slumber. Apart  
43 from the airborne acoustic wave, the energy of these vibrations is generally  
44 concentrated in the depth of one wavelength underneath the ground surface, that is,  
45 primarily propagating in the form of surface waves, and the frequency range is usually  
46 below 20.0 Hz[1, 2].

47 Mechanical metamaterials[3] whose configurations can be artificially fabricated  
48 and modulated have been proven to manipulate the propagation of mechanical waves  
49 with new effects such as wave isolation[4-9], topological insulator[10-19], Fano  
50 resonance[20-23], chirality[24], focusing and imaging[25], among others. Especially,  
51 surface wave metamaterials[1, 26-36] are designed mainly for transmission suppression  
52 of low-frequency surface wave in certain frequency ranges based on the conception of  
53 hybridized bandgaps. Nonetheless, energy harvesting and signal detection of the  
54 subwavelength ground surface wave can be additional functions for surface wave  
55 metamaterials, remaining a meaningful and challenging issue that has hitherto received  
56 few attentions. It is possible to transform the surface wave into available electric energy  
57 and detect ground dynamics for potential catastrophe early warning.

58 For practical applications, defects and disturbances are ubiquitous in surface wave  
59 metamaterials, hence the surface wave control with high robustness is a crucial  
60 challenge. Recently, the rapid development of metamaterial topological insulator  
61 provides an efficient way to realize robust surface wave metamaterials. Topology  
62 provides a method to describe global wave properties over a band structure which is

63 able to be conserved under certain local perturbations. In recent years, the development  
64 of topological insulators extended from condensed matter physics to classic wave  
65 systems such as acoustic and elastic waves. In general, Dirac cone dispersion results  
66 from the lattice symmetry which is protected by the space inversion and the time-  
67 reversal symmetry. Different mechanisms like the quantum Hall, quantum spin Hall  
68 and valley Hall effects are proposed to open a bandgap from the Dirac cone[12].  
69 According to the elastic analog of the quantum valley Hall effect, topological phases  
70 associated to opened bandgaps from the degeneracy lifting of the Dirac cone can be  
71 achieved via the inversion symmetry breaking [37]. When combining two bulk  
72 metamaterials with the opposite topological phases, topologically protected edge states  
73 can be induced showing strong energy localization at the interface within certain  
74 frequency ranges. The effect has been employed successfully in Lamb wave  
75 manipulation with significant robustness[37-43], revealing a great potential in the  
76 ground surface waves manipulation. For the application to surface acoustic waves,  
77 Wang et al. reported robust guiding valley-dependent edge states for Rayleigh waves  
78 excited by chiral sources at around 30 MHz[44]; Topological chiral edge state is also  
79 realized on a periodically corrugated surface[45].

80 For ground surface waves in macro scale, industrial and traffic activities induced  
81 vibrations are different from chiral excitations. In this work, we study the topological  
82 protected edge modes based on surface acoustic waves within the non-trivial bandgap  
83 of a pillared metamaterial. The edge states are excited by a force point source with  
84 either out-of-plane or in plane polarization. Indeed, these are the main polarizations of  
85 the typical surface waves such as Rayleigh or Love waves. In Sec. 2, we describe the  
86 models and the implemented methods for designing non-trivial bandgaps and  
87 topological edge modes of the pillared metamaterial based on surface acoustic waves.  
88 In Sec. 3, we quantitatively study the robustness of the designed topological edge mode  
89 against three kinds of perturbations. In Sec. 4, we adopt the robust and compact  
90 topological edge mode whose path can be designed, according to the need, to study  
91 energy harvesting properties by using piezoelectric materials. Finally, a summary of the

92 main results is presented in Sec. 5.

93

## 94 **2. Models and Methods**

95 We propose a design constituted by a honeycomb lattice of equilateral triangular  
96 section pillars made of concrete on the surface of soil. We are interested by ground  
97 surface waves with frequency range under 20.0 Hz, so we chose a lattice constant of  $a$   
98 = 2.0 m and the parameters of the pair of identical pillars in the unit cell are fixed as  
99 edge length  $d = 0.8$  m and height  $h = 2.0$  m. The design and the unit cell for the 3D  
100 simulation model are shown in FIG. 1(a). To prevent the physical surface modes being  
101 perturbed by the unphysical Lamb type waves of the thick substrate plate in the  
102 simulation model, the depth of soil in the unit cell is chosen to be  $H = 30.0$  m and a  
103 numerical low-reflecting boundary is applied at the bottom surface to avoid wave  
104 reflection. The soil (Young's modulus  $E_s = 10$  MPa, Poisson's ratio  $\nu_s = 0.3$ , mass  
105 density  $\rho_s = 1800$  kg/m<sup>3</sup>) and concrete pillars (Young's modulus  $E_c = 40$  GPa, Poisson's  
106 ratio  $\nu_c = 0.25$ , mass density  $\rho_c = 2500$  kg/m<sup>3</sup>) are assumed to be homogeneous, isotropic  
107 and linearly elastic materials[1]. Therefore, the equation of motions in solids can be  
108 written with harmonic time dependence assumption as

$$109 \quad \omega^2 \rho u_i = - \sigma_{ij,j} \quad (1)$$

$$110 \quad \sigma_{ij} = c_{ijkl} u_{k,l} \quad (2)$$

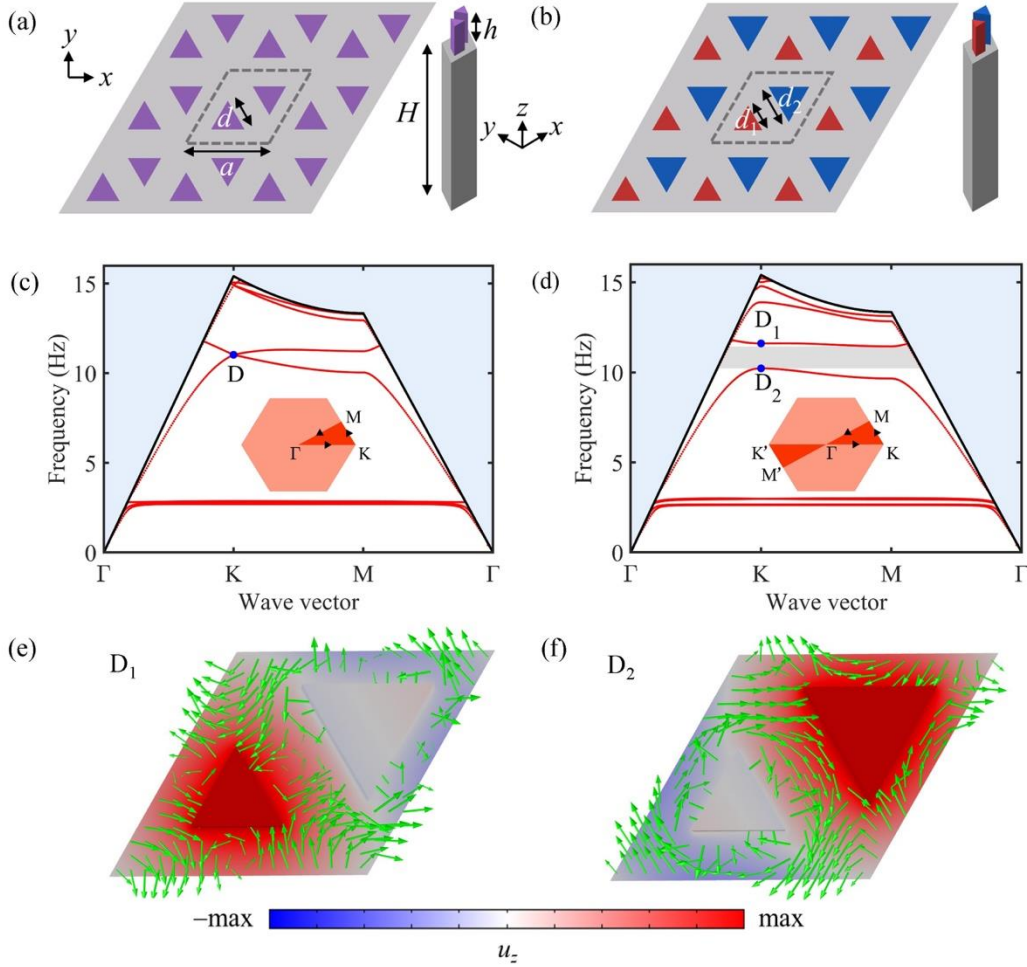
111 where  $\omega$  is the angular frequency,  $u_i$  the displacement component,  $\sigma_{ij}$  the stress  
112 tensor and  $c_{ijkl}$  the stiffness tensors that can be expressed as a function of the Young's  
113 modulus  $E$  and Poisson ratio  $\nu$ . To obtain the dispersion relations, Bloch periodic  
114 conditions are applied on the four vertical surfaces of the unit cells based on Bloch  
115 theorem[46] as

$$116 \quad \mathbf{u}(\mathbf{r} + \mathbf{a}) = \mathbf{u}(\mathbf{r}) e^{i\mathbf{k} \cdot \mathbf{a}} \quad (3)$$

117 where  $\mathbf{a}$  is the lattice vector,  $\mathbf{r}$  the position vector, and  $\mathbf{k} = [k_x, k_y]$  the wave vector. In  
118 addition, stress-free conditions are applied on the other top boundaries of the simulation  
119 model. We sweep the wave vector along the high-symmetry axes  $\Gamma - K - M - \Gamma$  in the

120 first irreducible Brillouin zone of the honeycomb lattice (see FIG. 1(c)) and search the  
121 eigenvalues to find dispersion relations with the help of the finite element method. It is  
122 worth mentioning that this work is dealing with the interaction between surface waves  
123 and pillars, thereby the vibrating modes are represented only under the sound cone of  
124 the substrate which means below the bulk shear wave[4, 47] (shown as the solid black  
125 lines in FIGs. 1(c)-(d)) where the propagation is prohibited towards the bulk. When the  
126 two pillars in the unit cell are identical as in FIG. 1(a), a Dirac cone based on surface  
127 waves dispersion curves emerges at 11.02 Hz at the K point in FIG. 1(c). The frequency  
128 of the Dirac cone is dependent on the pillar's resonant frequency[48]. Thus, the working  
129 frequency of the Dirac cone can be further tuned by designing the proper pillar's  
130 geometry.

131 Then, we decrease the edge length of one equilateral triangular pillar in the unit  
132 cell to  $d_1 = 0.7$  m while we increase that of the other to  $d_2 = 1.0$  m, and keep unchanged  
133 the distance between the two triangle centroids (see FIG. 1(b)). Thereby, the inversion  
134 symmetry of the unit cell is broken while the  $C_{3v}$  symmetry is maintained, which further  
135 induces a non-trivial bandgap in the range [10.22, 11.44] Hz. The degeneracy of the  
136 Dirac point is lifted and the limits of the bandgap at the K point occur at the points,  $D_1$   
137 (11.61 Hz) and  $D_2$  (10.22 Hz) with the inverted symmetry eigenmodes as shown in  
138 FIGs. 1(e)-(f). The elastic energy flux travels anticlockwise and clockwise around the  
139 pillars for  $D_1$  and  $D_2$  valleys, respectively. If  $d_1 = 1.0$  m and  $d_2 = 0.7$  m, the chirality of  
140 the valley states for the lower and higher bands associated to the opened bandgap  
141 becomes inverted. Therefore, the valley Chern number associated with this lower and  
142 upper bands have opposite signs [16, 41], which further supports an elastic analog of  
143 quantum valley Hall effect. As a consequence, the opened bandgap becomes non-trivial.  
144 The wavelength of referenced Rayleigh wave propagating along the free surface  
145 without pillars at the center frequency of the non-trivial bandgap is about 4 m, also  
146 demonstrating a subwavelength property of the pillared metamaterial. The induced non-  
147 trivial bandgap is able to provide robust surface wave attenuation.



148

149 FIG. 1 (a) A honeycomb lattice with two identical triangular shape pillars in the unit cell. The soil  
 150 below the pillars is represented by a thick (gray) substrate. (b) The honeycomb lattice with two  
 151 different pillars in the unit cell breaking the inversion symmetry while preserving the  $C_{3v}$  symmetry.  
 152 (c) Dispersion curves of the surface waves (red lines) for the structure shown in (a) displaying a  
 153 Dirac point (D point, 11.02 Hz). The blue shaded area represents the sound cone. The light red  
 154 hexagon and the crimson right triangle indicate the first Brillouin zone and the first irreducible  
 155 Brillouin zone, respectively. (d) Dispersion curves of the surface waves (red lines) for the structure  
 156 shown in (b) showing the lifting of the Dirac point degeneracy towards the points  $D_1$  (11.61 Hz)  
 157 and  $D_2$  (10.22 Hz). The light blue and gray areas represent the propagating bands of the substrate  
 158 and the bandgap within [10.22, 11.44] Hz, respectively. (e) and (f) The eigenmodes (real part of the  
 159 out-of-plane displacement  $u_z$ ) of  $D_1$  and  $D_2$ . In the color bar, the red and blue colors refer to the  
 160 positive and negative values of the real part of  $u_z$ , respectively. Elastic energy flux is shown as green  
 161 arrows.  
 162

163 We turn now our attention to the design of a topological interface supporting an  
 164 edge mode. Based on the unit cell shown in FIG. 1(b), we consider two types of  
 165 topological interfaces by the juxtaposition of two crystals connected either by two large

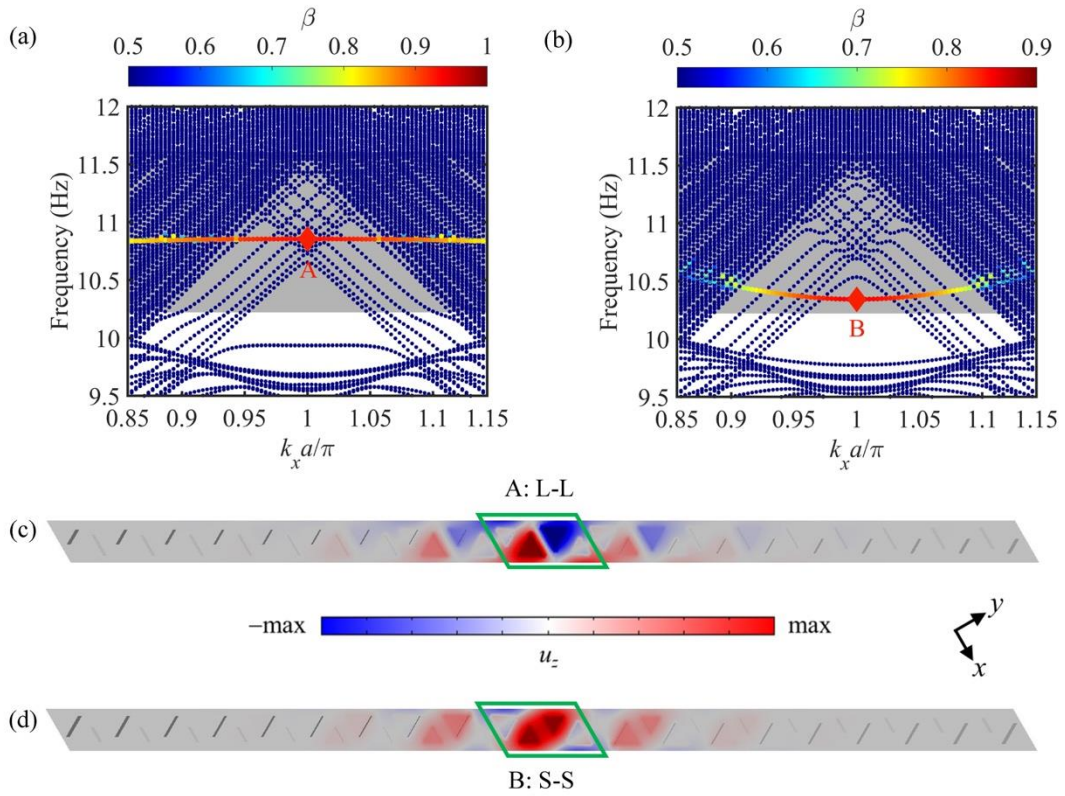
166 pillars (called L-L) or two small pillars (called S-S). We design two strips consisting of  
 167 20-unit cells with either an L-L or an S-S interface at the middle (FIGs. 2(c)-(d),  
 168 respectively). Periodic and stress-free boundary conditions are respectively applied to  
 169 the wider horizontal surfaces and to the narrower vertical surfaces. The detailed  
 170 information can be found in Appendix A. We sweep the wave vector around  $k_x = \pi/a$   
 171 along the  $x$  direction, and calculate the dispersion diagram shown in FIGs. 2(a)-(b) for  
 172 the L-L and the S-S types of interfaces, respectively. To detect the presence of an edge  
 173 mode, localized in the vicinity of each interface, we propose a localization ratio  $\beta$   
 174 defined as the ratio of the integrals of the kinetic energy density ( $\kappa = \rho(\dot{u}_x^2 + \dot{u}_y^2 + \dot{u}_z^2) / 2$ )  
 175 over the two pillars at the interface and over all the pillars. The localization ratio is  
 176 expressed as

$$177 \quad \beta = \frac{\iiint_{\text{interface}} \kappa dV}{\iiint_{\text{all}} \kappa dV} \quad (4)$$

178 A high value of  $\beta$  indicates that the eigenmode is mostly concentrated at the interface,  
 179 revealing an interface mode. In FIGs. 2(a)-(b), the dispersion diagrams display a bright  
 180 edge mode around 10.86 Hz for the L-L type interface and within the interval [10.34,  
 181 10.45] Hz for the S-S type interface. The other branches appearing in the bandgap of  
 182 surface waves result from the band folding of the bulk modes belonging to the soil. We  
 183 also present the dominant out-of-plane displacement fields of the eigenmodes at the  
 184 points A and B in FIGs. 2(c)-(d). One can clearly recognize edge modes localized  
 185 around the interfaces, with a higher confinement for mode B as compared to mode A.  
 186 The two pillars at the interface show a dipolar and a monopolar motion in FIGs. 2(c)-  
 187 (d), respectively. Since the edge modes based on surface acoustic waves are constructed  
 188 within the non-trivial bandgap (shaded grey zone), they are expected to exhibit  
 189 topological protected properties such as showing high robustness against certain  
 190 perturbations and being free of backscattering.

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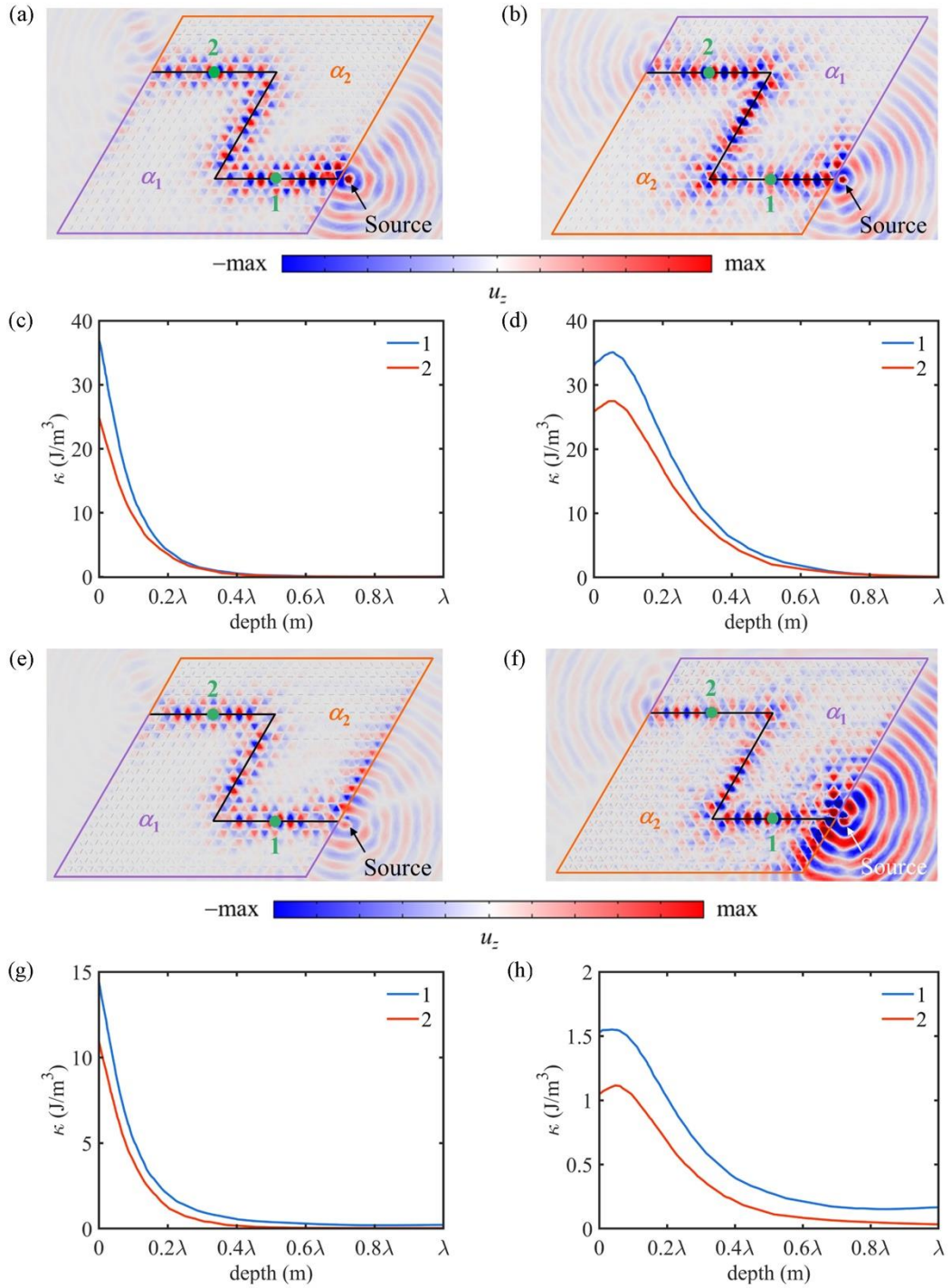
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199

FIG. 2 Two strips are constructed by placing two topologically distinct lattices adjacently, each of them containing 10 unit cells and the interface at the middle being defined by two neighboring large (L-L) or small (S-S) pillars. (a) and (b) Dispersion diagrams for the L-L type and S-S type interfaces. The color represents the value of localization ratio  $\beta$ . The grey zones in (a) and (b) stand for the non-trivial bandgaps. (c) and (d) the out-of-plane displacement of the edge modes A and B in (a) and (b). The unit cells besides the interface are marked out by the green boxes.



200

201 FIG. 3 Out-of-plane displacement fields with the out-of-plane point force excitation for the zig-zag  
 202 waveguide (black curves) for (a) the L-L type and (b) the S-S type at 10.75 Hz and 10.40 Hz,  
 203 respectively.  $\alpha_1$  and  $\alpha_2$  refer to the two distinct topological phases. In (c) and (d), the diagrams of  
 204 the kinetic energy density at the dots 1 and 2 along the soil depth are plotted for the two types.  
 205 Panels (e) to (h) show the same information as (a)-(d), respectively, but for in-plane force excitation  
 206 of the point source.

207

208 To demonstrate the robustness of the topological edge modes, we design a zigzag

209 interface separating the two bulk metamaterials  $\alpha_1$  and  $\alpha_2$  with the L-L and S-S  
210 interfaces respectively. Since typical surface waves such as Rayleigh and Love waves  
211 have different polarizations, we study the surface wave-based topological edge modes  
212 by using both out-of-plane and in-plane force excitations.

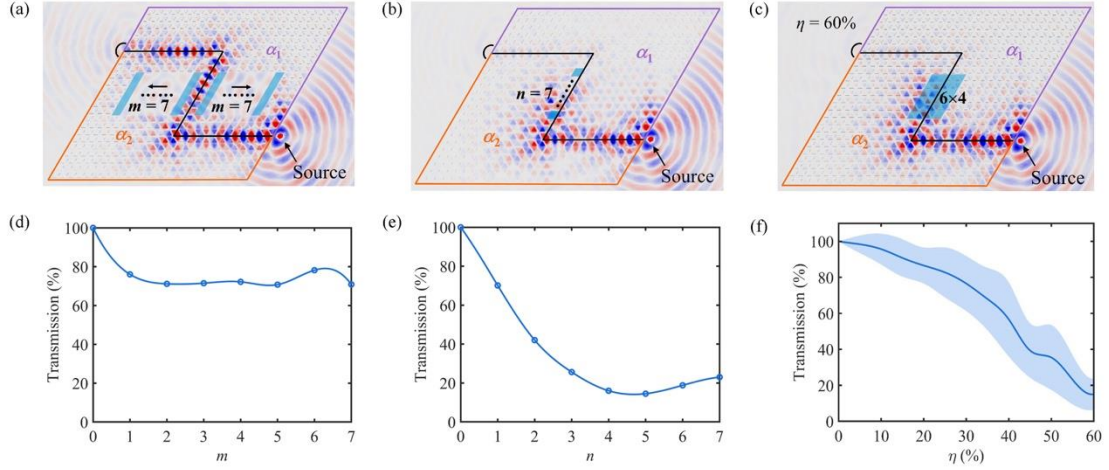
213 A point source represented by an out-of-plane force (marked as the green star in  
214 FIGs. 3(a)-(b)) is excited at the entrance of the zig-zag interface. Low-reflecting  
215 boundaries are applied to the surrounding boundaries to eliminate the reflection effects.  
216 The edge modes are excited at 10.75 Hz and 10.40 Hz and their propagation presented  
217 for the L-L (FIG. 3(a)) and S-S (FIG. 3(b)) types interfaces, respectively. In both  
218 structures, the wave entering the waveguide travels towards the end and passes the  
219 sharp bending corners without a significant backscattering. To further demonstrate that  
220 the edge mode is essentially based on surface acoustic waves, we plot the kinetic energy  
221 density at the green dots 1 and 2 as a function of the depth in the soil in FIGs. 3(c)-(d)  
222 for the two structures. The mode exhibits a high confinement near the surface, with  
223 most of the energy localized within one wavelength depth from the ground surface. This  
224 behavior supports the conclusion that the designed topological edge states are based on  
225 nonleaky surface waves that do not radiate into the bulk. In addition, the energy density  
226 at the dot 2 near the exit is a bit smaller than that at the dot 1 near the entrance, remaining  
227 at relatively the same level, which also supports high transmission and backscattering  
228 immunity of the topological edge mode.

229 We now perform similar calculations for an excitation by an in-plane point source  
230 force with the same amplitude as above. The results are presented in FIGs. 3(e)-(h). The  
231 generated surface wave displays a higher weight of the in-plane displacement over total  
232 displacement than the case of the out-of-plane force excitation. One can see in FIGs.  
233 3(e)-(f) similar high transmission and backscattering immunity of the topological edge  
234 state as above. However, the kinetic energy density (FIGs. 3(g)-(h)) is now smaller than  
235 before.

236 From the dispersion diagram shown in FIG. 2, one can note that the edge mode of  
237 the S-S type interface covers a certain frequency range, which makes it a good candidate

238 for energy harvesting. In the following section, we focus on the edge mode of the S-S  
 239 type excited by the out-of-plane force source for robustness analysis and energy  
 240 harvesting.

### 241 3. Robustness of the topological edge mode



242  
 243 FIG. 4 The robustness of the edge state at the S-S type interface in presence of defects and disorder  
 244 in height of the pillars at 10.40 Hz. The upper panels show the out-of-plane displacement along the  
 245 waveguide for a particular choice of disorder whereas the lower panels give the attenuation in the  
 246 transmission for various degrees of disorder. (a) & (d) The effect of removing  $m$  strips on each side  
 247 of the interface in the middle part of the waveguide. The blue strip supercell constituting the  
 248 interface area contains 6 unit cells. The out-of-plane displacement is detected at the black arcs at the  
 249 exit of the zigzag path. (b) & (e) The effect of introducing  $n$  defects in the unit cells contained in the  
 250 blue area close to the interface. (c) & (f) The effect of height disorder by a factor  $\eta$  for the 24 units  
 251 in the blue area. The light blue domain in (f) demonstrates the standard deviation of the transmission  
 252 for different realizations of the disorder at each  $\eta$ . In the lower panels, the calculations are performed  
 253 for discrete values of the abscissa and the continuous lines are a guide for the eyes.

254

255 We investigate the robustness of the edge state to propagate along the waveguide,  
 256 by calculating the loss in the transmission coefficient in presence of a perturbation.  
 257 Therefore, we define the ratio  $\Delta_1/\Delta_2$ , where  $\Delta_1$  and  $\Delta_2$  represent the integrated  
 258 displacements along the green arcs at the exit of the waveguide with and without  
 259 perturbations, respectively. We consider three kinds of perturbations to quantitatively  
 260 study the robustness of the topological edge state.

261 The first perturbation consists of removing  $m$  strips on each side of the interface  
 262 in the middle part of the waveguide as shown in FIG. 4(a). The area delimited by two  
 263 blue strips on each side of the waveguide is constituted by 6 unit cells. When one strip

264 of defect is introduced ( $m = 1$ ), FIG. 4(d) shows that the transmission decreases  
265 suddenly to about 75%; then, by increasing further  $m$ , it slightly oscillates around the  
266 level (70% -75%), exhibiting a good robustness to stronger disorder. The displacement  
267 field for  $m = 7$  is calculated and presented in FIG. 4(a), showing a preserved zigzag  
268 edge state. Such high robustness is mainly attributed to the strong localization of the  
269 topological edge mode in the vicinity of the interface.

270 The second perturbation (FIGs. 4(b) and (e)) consists of randomly removing  $n$   
271 units at the left side of the middle part of the zigzag waveguide. From FIG. 4(e) one  
272 can observe that when the defect number  $n$  increases from 0 to 4, the transmission  
273 decreases by a large amount to about 20% of the unperturbed situation, due to the  
274 appearance of backscattering effect; then the level of transmission keeps at about 20%  
275 when the defect number continues to increase until 7. The transmission field at  $n = 7$  is  
276 shown in FIG. 4(b) which indicates that although the major part of the wave is reflected,  
277 there still remain weak monopolar edge state after the 7-unit-defect along the last  
278 horizontal part of the zigzag path, showing a good robustness in edge mode shape.

279 The third perturbation is dealing with the effect of disorder in the height of a set  
280 of 24 pillars located in the blue shaded region of FIG. 4(c). These heights are randomly  
281 distributed within the interval  $[h - \Delta h, h + \Delta h]$ , thus defining a disorder degree  $\eta = \Delta h/h$ .  
282 For a given disorder degree, the average transmission coefficient is evaluated over 30  
283 random samplings and the results are shown as the solid line in FIG. 4(f). We also  
284 evaluate the standard deviation of transmissions and plot it as the shaded blue domain  
285 around the solid line in FIG. 4(f). The stable level of the standard deviation supports  
286 that the random sampling number of 30 should be sufficient. Generally, when the  
287 disorder degree  $\eta$  increases from 0 to 60%, the average transmission decreases quasi-  
288 linearly, exhibiting that the backscattering effect also becomes stronger. We show the  
289 transmission field at  $\eta = 60\%$  in FIG. 4(c). Despite the backscattering effect, the relative  
290 weak propagating wave along the last horizontal part of the zigzag path still conserves  
291 the monopolar edge state property, showing a strong robustness of the topological edge  
292 mode.

293

#### 294 **4. Topological edge mode for energy harvesting**

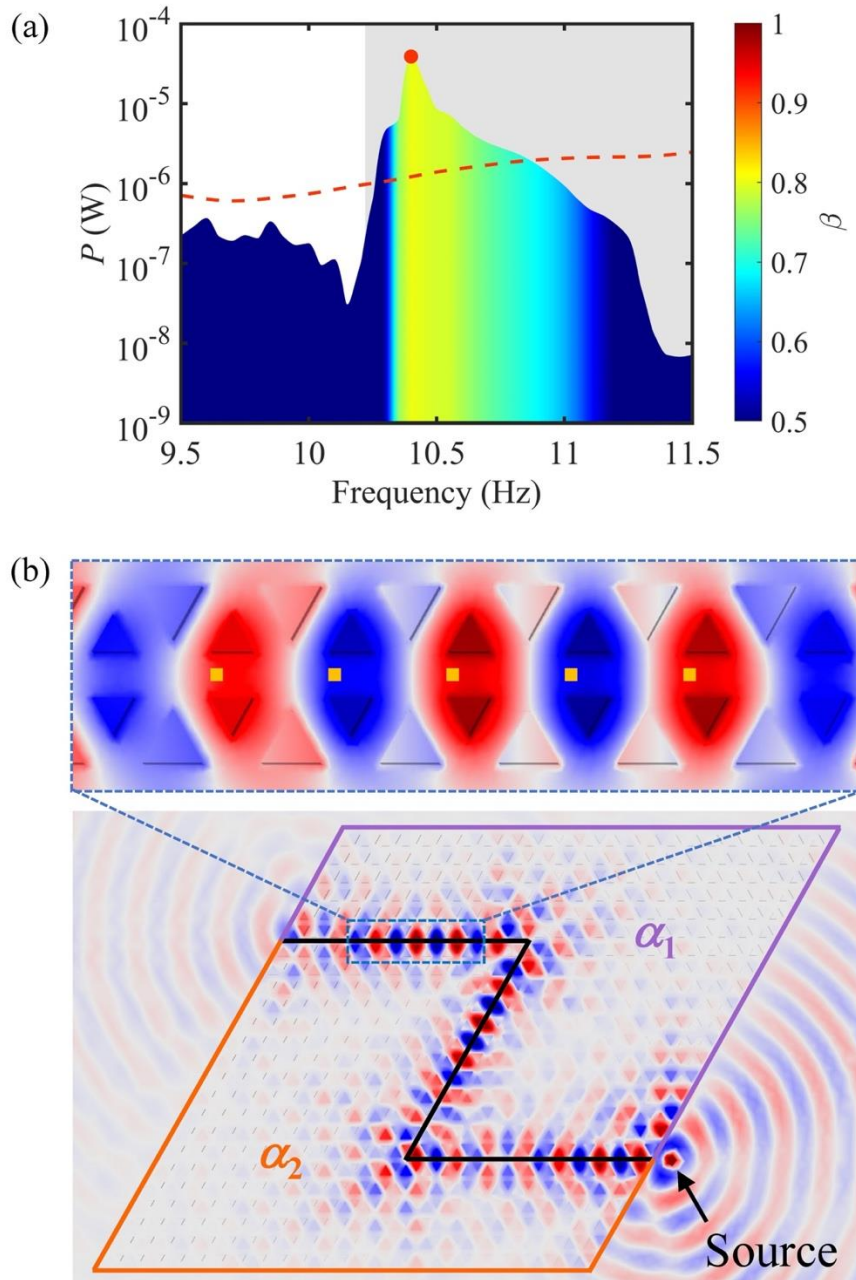
295 The robust and compact topological edge mode has a great potential for energy  
296 harvesting from a practical point of view. We adopt piezoelectric PZT-5H patches and  
297 attach them to the top surface of soil. With the electro-mechanical coupling effect, the  
298 vibrating energy of soil can be transformed to electric power. We choose five identical  
299 square PZT patches with sides  $l = 0.1$  m and thicknesses  $t = 0.001$  m, and arrange them  
300 alternately as shown by the five yellow squares in FIG. 5(b). A simple circuit powered  
301 by each PZT patch with a resistance  $R = 1000 \Omega$  is devised, whose average output power  
302  $P_i$  is calculated by

$$303 \quad P_i = \frac{V_i I_i}{\sqrt{2}\sqrt{2}} = \frac{RI_i^2}{2} \quad (5)$$

304 where  $V_i$  is the output voltage,  $I_i$  is the loop current to be measured. The detailed  
305 information can be found in Appendix B. We sum the total power  $P = \sum_{i=1}^5 P_i$  of all the  
306 six PZT patches, and present the transmission spectra together with the color  
307 information of localization ratio  $\beta$  (Eq. (4)) in FIG. 5(a). From FIG. 5(a), the topological  
308 edge mode with bright color is successfully captured within the frequency range [10.33,  
309 10.97] Hz and the peak is measured at 10.40 Hz which coincides to the design. The  
310 electric power at the peak is about two orders of magnitude higher than that within the  
311 frequency ra [9.50, 10.00] Hz outside the non-trivial bandgap. Besides, it is more than  
312 one order of magnitude higher than that of a bare surface, showing a big advantage of  
313 the energy harvesting by topological edge mode. We also present the displacement field  
314 at 10.40 Hz in FIG. 5(b), almost identical to the field without patches (see FIG. 3(b)).  
315 In fact, all the vibrating pillars at the interface are available for the energy conversion  
316 in practice. Given that the topological edge mode is highly robust to the studied three  
317 kinds of perturbations, the corresponding induced energy harvesting also has an  
318 efficient robustness.

319 At the macro scale, the basic functionality of the structure is to robustly isolate  
320 surface waves with the non-trivial low-frequency bandgap. Energy harvesting is an

321 additional functionality by designing the proper interface path inside the metamaterial.  
 322 With maximum vibrating displacements at the interface of the order of 0.5mm, and the  
 323 small size of the designed PZT patch in FIG. 5, an electric power of about  $10^{-5}$  W is  
 324 achieved. However, if the surface vibrations increase in practice, the PZT patch would  
 325 be able to generate much higher electric power, making the metamaterial as a potential  
 326 ground electric generator.



327  
 328 FIG. 5 (a) Piezoelectric energy harvesting for the edge modes with (b) the displacement fields at  
 329 10.40 Hz. The red dashed curve in (a) represents the electric power measured without pillars. The 5

330 PZT patches are displayed as yellow squares in the zoom-in of (b). The color map in (a) defines the  
331 localization ratio  $\beta$  defined in Eq. (4).

332

## 333 **5. Conclusion**

334 A topological surface wave metamaterial composed of triangular section concrete  
335 pillars arranged in a honeycomb lattice on the soil is proposed for robust vibration  
336 attenuation and energy harvesting at low frequency ranges, below 20 Hz. By breaking  
337 the inversion symmetry of the unit cell while preserving  $C_{3v}$  symmetry, the degeneracy  
338 of the Dirac cone at K point is lifted to open a non-trivial bandgap. By combining two  
339 metamaterials with different topological phases, two types of topological edge modes  
340 (L-L and S-S types) with backscattering suppression and compact properties can be  
341 designed around 10 Hz. We further quantitatively analyzed the robustness of the S-S  
342 type topological edge mode against three kinds of perturbations, namely, (i) stripe  
343 defects in the vicinity of the interface towards the bulk of the crystal, (ii) unit defects at  
344 the interface and (iii) disorders in the height of pillars at the interface. For perturbation  
345 (i), the transmission of the topological edge mode keeps stable at relative high level no  
346 matter how large is the defect in the bulk media, showing a highly compact property of  
347 the topological edge mode. For perturbation (ii), the transmission first decreases very  
348 fast before becoming stable when the unit-defect number reaches 4. For perturbation  
349 (iii), the transmission slowly decreases in quasi-linear shape. For the latter two cases,  
350 the topological edge modes are still preserved at high considered perturbation degree,  
351 exhibiting a good robustness. Finally, piezoelectric patches are designed and attached  
352 on top of the pillars at the interface. The robust and compact topological edge mode  
353 makes it possible for energy harvesting within the frequency range [10.33, 10.97] Hz  
354 of the topological edge mode. The entire size of the pillared metamaterial can be further  
355 minimized according to the potential practical demands. For instance, a thickness such  
356 as 4 or 5 rows of units is sufficient to effectively attenuate surface waves. The unit cells'  
357 geometric parameters can also be optimized for different working frequency range. The  
358 employed concrete and PZT are commercial materials with low price, making the  
359 metamaterial realizable in practical engineering. The proposed new surface wave



360 metamaterial has great potential in robust surface vibration attenuation with the non-  
361 trivial low frequency bandgap and robust energy harvesting with the topological edge  
362 mode.

363

### 364 **Acknowledgment**

365 This work is supported by the National Natural Science Foundation of China  
366 (11902223), the Shanghai Pujiang Program (19PJ1410100), the program for professor  
367 of special appointment (Eastern Scholar) at Shanghai Institutions of Higher Learning,  
368 the Fundamental Research Funds for the Central Universities, high-level of foreign  
369 expert program of Tongji University and Shanghai municipal peak discipline program  
370 (2019010106). This work is also partially supported by the French EIPHI Graduate  
371 School (contract "ANR-17-EURE-0002").

372

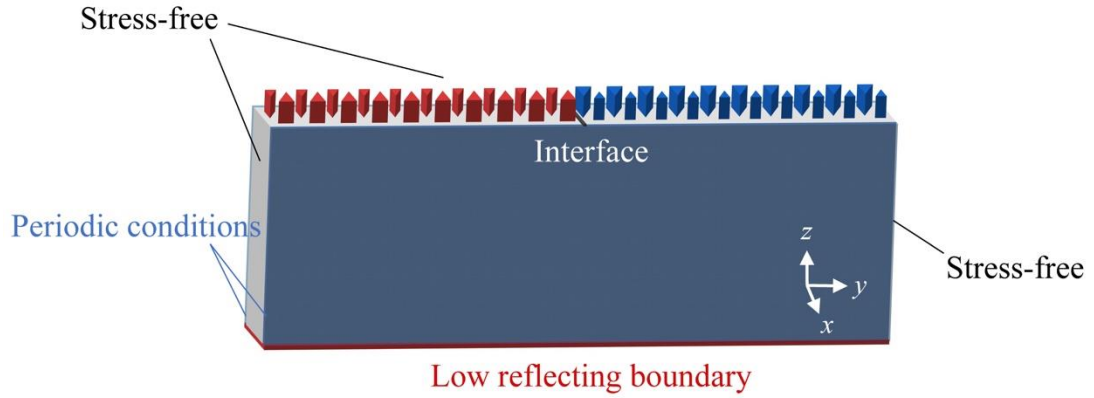
### 373 **Data Availability**

374 The data that support the findings of this study are available from the corresponding  
375 author upon reasonable request.

376

### 377 **Appendix A. The strip's dispersion calculation**

378 The strip's dispersion calculation is modeled via the finite element method. In FIG.  
379 A1, a stripe consisting of lattices red and blue with an interface at the center is deposited  
380 on top of the substrate. The valley Chern number of the two bands associated with the  
381 non-trivial bandgap are opposite for the red and blue lattice bulk media. Low reflection  
382 boundary is applied to the bottom surface. Periodic conditions are applied to the two-  
383 blue side-faces of the substrate along  $x$  direction. Stress free conditions are applied to  
384 the two-grey side-faces of the substrate along  $y$  direction. The built model contains  
385 about 350 thousand domain element meshes with about 2.9 million degrees of freedom.



386

387

FIG. A1 dispersion model of a stripe waveguide.

388

### 389 **Appendix B. Simulating information for the energy harvesting model**

390 In this section, we detail the simulation process of the transmission with  
 391 piezoelectric energy harvesting along the zig-zag interface in the COMSOL  
 392 Multiphysics®. It is calculated by coupling of solid mechanics, electrostatics and  
 393 electrical circuit. The specific process is (I) Geometry model building; (II) Material and  
 394 mechanical boundary conditions settings; (III) Piezoelectric and electrical circuit  
 395 settings; (IV) Meshing; (V) Multiphysics simulation at given frequencies.

396 To enhance the calculation efficiency, the top surface of this model is discretized  
 397 by free triangular elements, and the entire geometry is swept with appropriate element  
 398 size. The meshed model contains 1330471 domain elements and 7879642 degrees of  
 399 freedom, which is sufficient to ensure the convergence of the results. For calculation  
 400 stability, we use the CPU of Intel® Xeon® Gold 6154 Processor with the max turbo  
 401 frequency of 3.70 GHz and the memory of 512 GB. The simulation costs about one and  
 402 a half hours for each frequency.

403

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