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Wind fractional modeling by spectral analysis

Mohamed Hajjem^{1,2}, Stéphane Victor¹, Pierre Melchior¹, Patrick Lanusse¹ and Lara Thomas²

Abstract

This paper deals with wind turbulence modeling by using spectral analysis, in order to have a real-time wind simulator for dynamic models of parabolic tracking antennas. Indeed, such a realistic wind speed is needed to make dynamic models more complete and to design robust controls. The wind speed can be decomposed into two components: a long-term one representing the wind mean speed and a short-term one corresponding to the wind turbulence. Fast wind turbulence is a stochastic process and is difficult to model in time domain. For this reason, the models presented in this paper are based on the wind spectral characteristics. The most used Von Karman model well approximates the power spectral density of the wind turbulence. However, this model lacks precision in middle range frequency. Indeed, the Von Karman model was originally designed for aircrafts, and consequently for high altitudes and moving systems. In our case, tracking antennas are used under different conditions: low altitude and slow moving systems. Therefore, there is a need in more precisely modeling wind turbulence under these specific conditions. As Von Karman's model expression uses fractional calculus, other models with the same range of parameter number are proposed to better approximate the wind power spectral density by using Cole-Cole and Havriliak-Negami fractional models. The proposed fractional models will enable generating realistic random wind speed turbulence from a random white noise input.

I. INTRODUCTION

Wind turbulence is the main disturbance that acts on many mechanical systems operating outdoors, such as parabolic antennas, and are therefore continuously disturbed. This study was conducted to improve the performance of mobile antennas used for tracking, through a better compensation of wind effects on the system. Indeed, for systems requiring a lot of precision such as antennas, wind turbulence increases the errors and the risks of signal loss. Consequently, it is necessary to take into account the wind effects in the system dynamical model, in order to design a suitable control law capable of effectively rejecting them.

Wind speed measurements are usually stored with a long sampling period. These measurements are then used for statistical and probabilistic modeling generally used for studies on wind power for wind energy conversion systems. However, for the analysis of wind effects on fast-moving systems, the evolution of the wind speed over short periods of time is needed. Access to such measurements is often difficult, hence a more accurate model is needed to well simulate realistic wind speed.

The short-term wind speed component, which corresponds to the turbulence, is a non-stationary stochastic process and thus is difficult to model in time domain. This paper presents an alternative method of wind modeling based on spectral characteristics. These characteristics will be described by models based on the well known Von Karman approximation. The idea is to use these models later to design fractional order filters capable of randomly generating wind speeds.

This paper is organized as follows. Section II presents the data set that will be used to estimate the models. Section III is a bibliographical study of existing models to describe the wind, particularly the Von Karman model. Section IV presents the comparison between the Von Karman approximation with improved existing models such as an auto-regressive model. Then, section V proposes a comparison with fractional models (Cole-Cole and Havriliak-Negami models) and an improved Cole-Cole fractional model. Finally, a conclusion is drawn in section VI.

II. EXPERIMENTAL PROCESS

For the identification of the spectral models, a one-day wind speed data set has been recorded. These measurements were made next to the city of Montpellier, France. The sensor used is an ultrasonic anemometer integrated into a Vaisala WXT520 weather transmitter that measures wind speed and direction, precipitation, air pressure, temperature and relative humidity. The sensor has three identical transducers that generate ultrasonic signals. The wind speed is determined by measuring the time it takes for the ultrasonic signal to travel from one transducer to the others. The anemometer has a measurement range of 0 to 60 m/s and an accuracy of $\pm 3\%$ (0 to 36m/s) and $\pm 5\%$ (36 to 60m/s).

The anemometer is configured to provide data every second with measurements that are averaged over 20s. Such a configuration prevents, first of all, having incoherent measurements and on the other hand, helps detecting long and strong gusts to avoid any antenna damage. Indeed, the Van der Hoven wind power spectrum (see figure 3) shows that above a period of 20s, the turbulence power increases rapidly and therefore when this power exceeds a certain limit, the antenna should no longer be used and should remain in a safety position.

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Such a wind measurement moving averaging can be viewed as a low pass filter because it increases the smoothness of the output and attenuates rapid variations which modifies the high frequency wind behavior. For this reason, this study will focus on spectral characteristics at frequencies below 0.1 Hz, in order to be coherent with the original wind behavior.

This article is about wind turbulence modeling which is the short-term fluctuations of the wind speed, thus the signal will be studied over a period of 2 hours and not a whole day. Figure 1 presents the time signal of the measured wind speed.

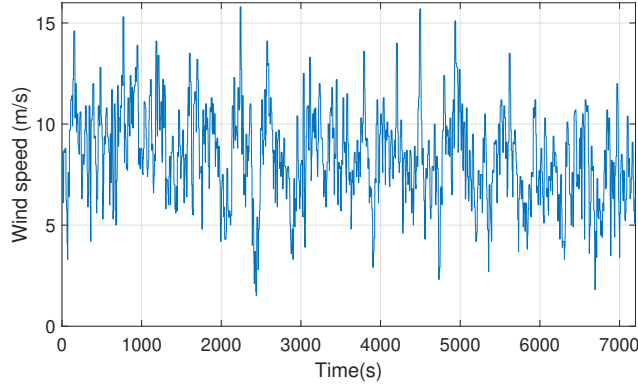


Fig. 1. Measured wind speed during 2 hours

The objective of this paper is to propose simple models that will better approximate the wind turbulence speed in the frequency domain, therefore the reference data will be the power spectral density of the time-series data (figure 2). It represents the spectral distribution of the power of a signal depending on the frequencies that compose it. This calculated power spectral density (PSD) will be used for the parameter estimation of the different models.

III. STATE OF THE ART

A. Wind speed components

The wind energy is distributed over a wide frequency band near the ground. The spectral analysis over a long period of time reveals several scales of fluctuations. Figure 3 shows the results obtained by Van Der Hoven [1] from measurements made at 100m height on the horizontal wind speed. The spectrum shows that the power spectral density can be divided into two parts each with a maximum: one part corresponding to long seasonal variations, and the other part corresponding to turbulence with short period variations. On figure 3, the presence of three peaks can be noticed. The synoptic peak corresponds to a period of about 4 days, it represents the variations due to large-scale atmospheric phenomena. The diurnal peak has a period of about 12 hours and it corresponds to the variations in air mass circulation between day and night. Finally the turbulence peak has a period of about 76s and it is caused by gusts and surface effects.

Between these peaks, there is an energy gap ($15\text{min} < T < 2\text{h}$), where the wind speed can be considered as a stationary process, contrary to turbulence ($T < 10\text{min}$) where the velocity follows a random and non-stationary process. Based on the Reynolds decomposition for fluids, the wind speed can be expressed as a sum of a steady component representing a mean speed and a fluctuating component representing the turbulence:

$$V(t) = \bar{V}(t) + V_{turb}(t) \quad (1)$$

where $\bar{V}(t)$ is the long-term component and $V_{turb}(t)$ the turbulence component.

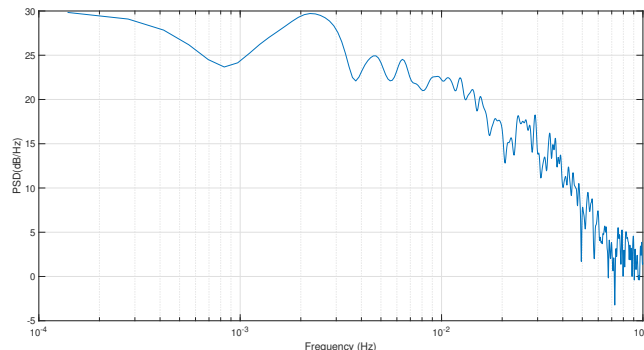


Fig. 2. PSD of the measured wind speed using Welch's method

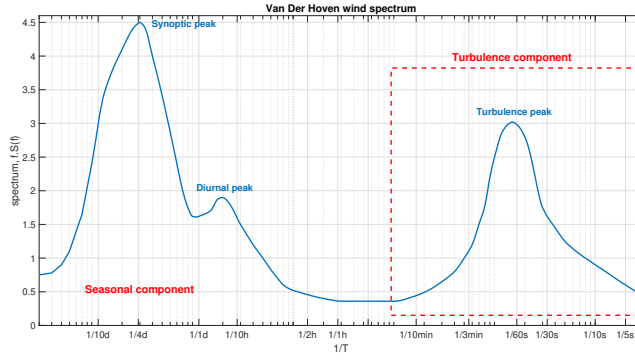


Fig. 3. Van der Hoven's wind spectrum [1]

B. Von Karman model

The Von Karman model [2] [3] [4] is a mathematical model of wind turbulence. The model expresses the power spectral densities of the linear and angular wind speed components. It approximates the PSD by a non-integer order function. Then from this function, filters can be designed, which take white noise as input and return a stochastic signal with the spectral characteristics of the wind turbulence as output. The Von Karman model expresses the spectrum of the longitudinal, lateral and vertical components of the wind speed. The shape being similar for all three components, this study only focuses on Von Karman's longitudinal model.

The wind speed PSD according to the Von Karman's longitudinal model is expressed as:

$$S(f) = \frac{4\sigma^2 \frac{L}{\bar{V}}}{\left(1 + 70.8\left(\frac{L}{\bar{V}}f\right)^2\right)^{5/6}}, \quad (2)$$

where f is the frequency, σ is the turbulence standard deviation, L is the length scale and \bar{V} is the mean speed.

The Von Karman model is tuned by two parameters:

- **the length scale** L , which represents the average size of the fluctuations in terms of distance, depends on the surface roughness, the altitude and the model;
- **the turbulence intensity**, I , expressed by:

$$I = \frac{\sigma}{\bar{V}}, \quad (3)$$

depends on the average wind speed but also on the altitude and the roughness of the surface as the length scale.

The shape of the PSD and thus the quality of the model fitting to the measured wind spectrum is determined by the combination of the length scale and the turbulence intensity. Several possible combinations of these two parameters can produce realistic PSD. There are mathematical models to calculate turbulence intensity and length scale (IEC standards and Danish Standard DS 472). However, these models are sometimes complex and depend on other parameters that are difficult to measure. Therefore they do not always provide the right combination for a good fit.

C. Time simulation of random wind speed signals

The goal of wind models is to have a time signal of the wind speed whose characteristics are as close as possible to reality. In the literature, some methods of time wind simulation have been studied.

Shinozuka and Deadatis [5] proposed a method to reconstruct a random signal like turbulence speed from the PSD in the desired frequency range. The signal is simulated with the following expression:

$$V(t) = \bar{V} + \sum_{i=1}^N A_i \cos(\omega_i t + \phi_i) \quad (4)$$

where ω_i is a frequency from the discretized frequency interval, ϕ_i is a random phase uniformly distributed in $[0, 2\pi[$ and A_i is the amplitude corresponding to each frequency ω_i obtained from the PSD values:

$$A_i = \frac{2}{\pi} \sqrt{\frac{1}{2} (S(\omega_{i+1}) + S(\omega_i)) (\omega_{i+1} - \omega_i)}. \quad (5)$$

For real-time simulation, Diop [6] proposed a method that consists of designing a filter from Von Karman approximated PSD. This filter returns a turbulence speed signal from a random white noise input. The Von Karman model gives filters of

fractional order. Then, in order to be able to simulate these filters, they must be approximated by rational functions using the Oustaloup's poles and zeros approximation [7].

Another proposed method [8] [9] is to use an auto-regressive (AR) model to describe the time series wind speed data. An AR model is identified to fit a given time series data that contains measured output but no measured input. The identification requires only time-domain data. The model is given by:

$$Y(t_k) = \sum_{i=1}^{n_p} a_i Y(t_{k-i}) + e(t_k) \quad (6)$$

where $Y(t_k)$ is the output at instant t_k , $e(t_k)$ is a white noise, a_i are the model parameters and n_p is the parameter number. This equation can be also written through Z transform as:

$$A(z)Y(z) = e(z). \quad (7)$$

The output variable of the AR model linearly depends on its previous values and on a random term.

However, AR models are more often used for the prediction of wind speed or power from measurements than for dynamic simulation.

IV. IDENTIFICATION OF EXISTING MODELS

A. Von Karman model

The nonlinear least squares method is used to estimate the parameters of the Von Karman model allowing a good fit to the PSD of the available data set.

The form of the Von Karman model (2) can be turned into the following expression:

$$\widehat{S}(f) = \frac{K}{(1 + (\tau f)^2)^{5/6}} \quad (8)$$

where $K = 4\sigma^2 \frac{L}{V}$ and $\tau = \sqrt{70.8} \frac{L}{V}$.

The nonlinear least squares problem consists in estimating the parameters K and τ that minimize the normalized sum of the square errors between the measured PSD, S , and the estimated model, \widehat{S} , as:

$$J = \frac{1}{N} \sum_{i=1}^N \left(S(f_i) - \widehat{S}(f_i, K, \tau) \right)^2 \quad (9)$$

where N is the number of samples and f_i are the frequencies defined in the chosen frequency range.

Knowing the constant mean wind speed, the estimated K and τ can be used to find the turbulence intensity and the length scale.

In high frequency, the PSD modulus is tending towards zero, thus leading to suboptimal optimization. In order to maintain a better accuracy in the considered frequency range, the optimization is carried out in dB.

The estimated Von Karman model obtained on @Matlab is presented in figure 4.

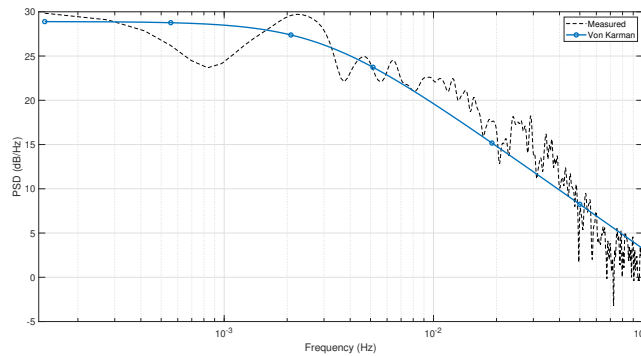


Fig. 4. Comparison between the estimated Von Karman PSD (blue) and the measured wind PSD (dotted)

The Von Karman model provides a good estimate of the low-frequency gain, however the slope is different from that observed for the measured wind PSD.

Overall, the Von Karman model fits the measured wind quite well, especially in low frequencies where a good estimation of the gain is observed. However, for $f > 0.01 Hz$, the slope of the model seems to be different from the one of the measurements. According to the spectrum of Van der Hoven (figure 3), the frequency range of wind turbulence is between $0.01 Hz$ and $0.1 Hz$. Therefore, a good fit is needed over this range, though figure 4 shows that the Von Karman model

does not perform well in this area. Indeed, the Von Karman model was originally designed for aircrafts, and consequently for high altitudes and moving systems. In our case, the data set is used under different conditions: low altitude and slow moving systems. Therefore, there is a need in more precisely modeling wind turbulence under these specific conditions.

B. AR model

The AR model is estimated on @Matlab with least squares approach which consists in minimizing the criterion (9). In this study, the AR model serves as a reference for the performance of other Von Karman-based models, since it is an easily estimable model and does not require knowledge of the wind characteristics in the frequency domain.

The Akaike information criterion (AIC) is used to select the optimal number of parameters for the AR model. Indeed, by increasing the number of parameters to be estimated, a better fit can be achieved but this can cause an overfitting. The AIC criterion estimates the error between the model and the data but it includes a term penalizing the number of parameters. It deals with the trade-off between the goodness of fit of the model and the simplicity of the model. The normalized AIC (nAIC) has the following expression:

$$nAIC = N \log(\det(\frac{1}{N} E^t E)) + \frac{2n_p}{N} \quad (10)$$

where E is the vector of errors expressed as:

$$E_i = S(f_i) - \hat{S}(f_i, K, \tau), \quad \forall i = 1, \dots, N, \quad (11)$$

n_p the number of parameters and N the number of samples.

Figure 5 shows the calculated normalized AIC of the estimated AR wind models with a number of parameters varying from 1 to 10.

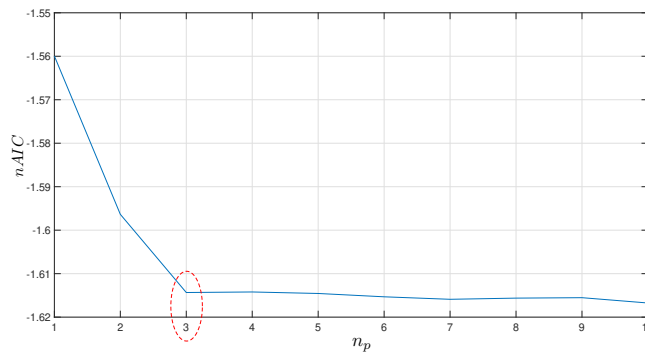


Fig. 5. nAIC criterion of the identified AR models vs parameter number

The nAIC shows that the optimal number of parameters to model the wind turbulence with an AR model is 3.

C. Comparison between Von Karman and AR models

The Von Karman model is now compared with the AR model (see figure 6).

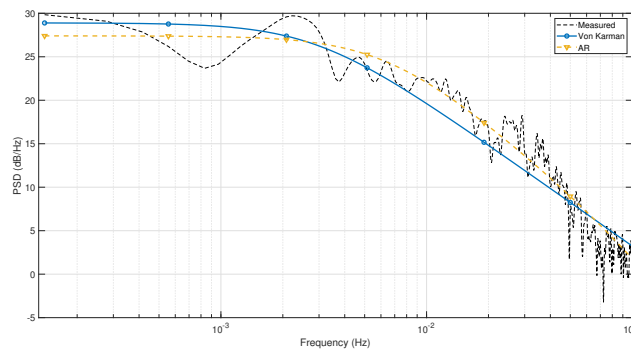


Fig. 6. Comparison between the estimated Von Karman model (blue), AR model (yellow) and the measured wind PSD (dotted)

Better results are obtained with the AR model than with the Von Karman model, especially in the frequency range [0.01 0.1Hz] (see table I for the nAIC criterion comparison). The slope is much closer to that of the measured wind PSD.

However, the disadvantage of the AR model compared to the Von Karman model is that the AR model is a black box model and cannot be adjusted according to the measurement conditions. The only adjustable parameter is the standard deviation of the input white noise which will affect the gain. Therefore, it is better to improve the Von Karman model to get as close as possible to the data and to generate a more realistic wind speed with an adjustable model.

V. FRACTIONAL MODEL APPROXIMATIONS

The Von Karman model does not well fit the measured date in high frequency. Indeed, the slope obtained with Von Karman is fixed by the non integer order $5/6$. Consequently, in order to act on this slope, it is necessary to estimate the correct order of the PSD expression which may not be of $5/6$.

The Von Karman model is in the form of a fractional function, thus a direct improved fractional model can be used such as the two following forms:

- **Havriliak-Negami functions** [10] [11]:

$$F_{hn}(s) = \frac{A}{(s^{\nu_1} + \omega_u)^{\nu_2}} \quad (12)$$

If $\nu_1 = 1$, the Havriliak-Negami function is called the implicit form.

- **Cole-Cole functions** [12] [11]:

$$F_{cc}(s) = \frac{A}{s^\nu + \omega_u}. \quad (13)$$

Therefore, these two forms will be used to have a modified Von Karman model allowing to have a better fit.

A. Havriliak-Negami models

The Von Karman model is in Havriliak-Negami (HN) fractional format with orders ν_1 and ν_2 set at 2 and $5/6$. The fixed order ν_2 will be replaced by an unknown parameter ν which will be estimated by the nonlinear least squares method, along with K and τ . The PSD will therefore be expressed as follows:

$$S(f) = \frac{K}{(1 + (\tau f)^2)^\nu}. \quad (14)$$

The advantage of the HN form is that the structure of the model remains the same as Von Karman's one, so the identified coefficients K and τ can still be used to find the value of the turbulence intensity and the length scale.

B. Cole-Cole models

In order to generate random wind speed, filters will be designed from approximated rational models of the wind PSD fractional ones. Cole-Cole (CC) functions are much easier to approximate than HN models. The CC wind speed model is of the following form:

$$S(f) = \frac{K}{1 + (\tau f)^{2\nu}}. \quad (15)$$

C. Fractional model comparison results

The estimated PSD from the identified HN and CC models are presented in figure 7.

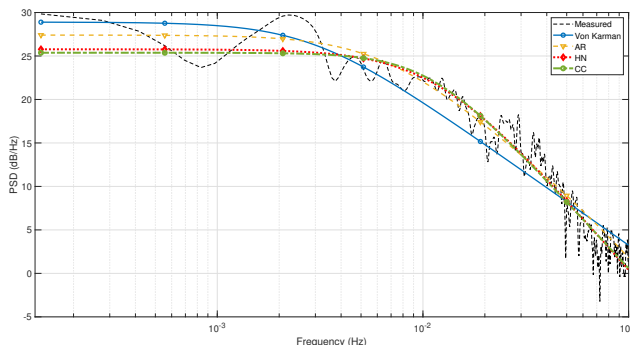


Fig. 7. PSD comparison between the Von Karman model (blue), AR model (yellow), HN model (red), CC model (green) and the measured wind

It can be seen that for both HN and CC models, the high frequency slope well corresponds to the measurements and therefore meets the expectations. However, the gain in low frequencies seems to be a little bit low in comparison with the measured data PSD and the Von Karman model. Thus, both models works well in high frequency ($f > 0.01\text{Hz}$), that is

to say in the range of the turbulence component according to Van der Hoven, but they have difficulty in estimating the low-frequency gain.

In order to better compare the quality of the estimated models, the Akaike information criterion (AIC) will be used (10). Figure 8 shows the normalized AIC of the different models calculated for 10 different wind data sets. They are 2-hour wind speed data measured on the same day but at different times.

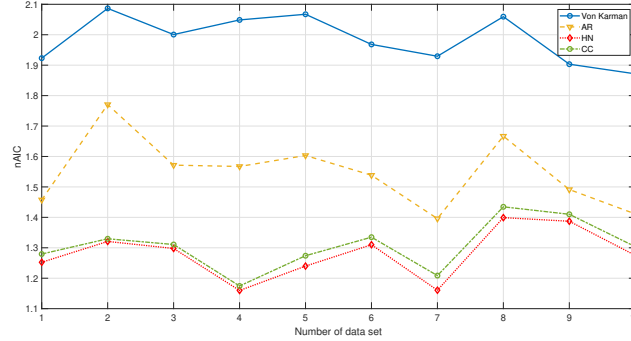


Fig. 8. nAIC comparison between the different identified models: Von Karman (blue), AR (yellow), HN (red), CC (green) for 10 different datasets

The AIC of the Von Karman model is the highest, which confirms the worst fit for the available datasets. The AIC also confirms what has been observed in figure 7. The estimation of the order in the HN and CC models allows to improve the quality of the fit. The goodness of fit of both models is very close, although the HN model seems to be a bit better. Finally, the AR model, as described in section IV-C, is better than the Von Karman model, however for the same number of parameters the proposed fractional models offer better fits.

D. Improved Cole-Cole model

The fractional models well estimate the high frequency slope, however the estimated low frequency gain can cause problems, because it underestimates the amplitude of the turbulence around the mean wind speed and as a result, their impact on the system dynamics. Therefore, there is a need to improve the low-frequency estimation, and thus the fractional model has to be modified. The improvement of the CC model is preferred, to facilitate the approximation to rational functions later, and because it has almost the same quality as the HN model.

To improve the model, a second cell of a fractional order ν_2 is added to the CC fractional function of the PSD, which will be denoted as Cole-Cole $\times 2$:

$$S(f) = \frac{K}{(1 + (\tau_1 f)^{\nu_1})(1 + (\tau_2 f)^{\nu_2})}. \quad (16)$$

The second cell allows the model to have a second slope in low frequency which provides a better fit in low frequency and a more appropriate static gain.

Figure 9 presents the PSD obtained from the CC $\times 2$ model compared to the previous proposed fractional models.

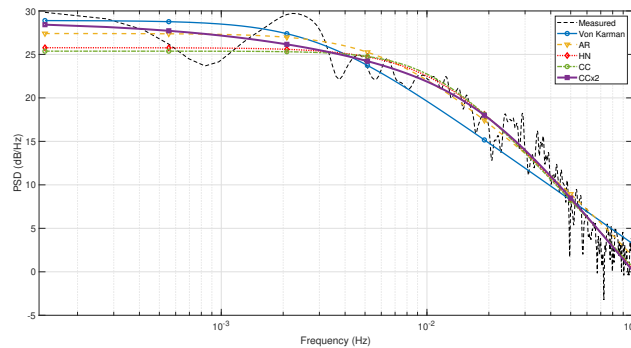


Fig. 9. Comparison between the estimated CC $\times 2$ model (purple), the previous models and the measured wind PSD

The second slope brought by the second cell of the CC model is clearly visible at low frequencies. Indeed, for frequencies $f < 0.001\text{Hz}$, the PSD of the model CC $\times 2$ does not have a constant gain as with the HN and CC models, but a second slope which provides a better fit.

To confirm the quality of the model CC $\times 2$ observed in figure 9, the nAIC of the different models presented in this study will be compared (figure 10).

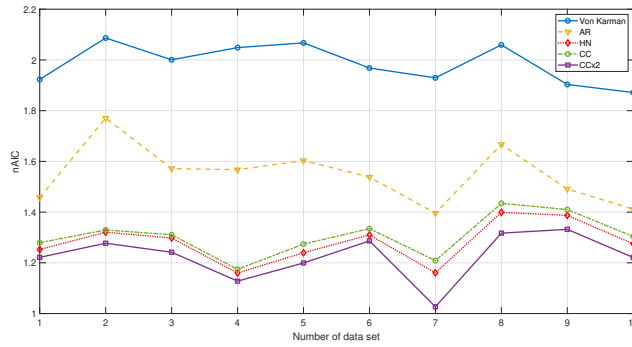


Fig. 10. Comparison between the nAICs of the $CC \times 2$ model (purple) and the other models for 10 different data sets

Figure 10 confirms that the $CC \times 2$ model fits the best the wind data, since its nAIC is the lowest. Table I summarizes the results observed in figures 8 and 10.

TABLE I
COMPARISON BETWEEN THE MEAN nAIC OF THE DIFFERENT MODELS

Model	Number of parameters	Mean nAIC
Von Karman	2	1.985
AR	3	1.556
Havriliak-Negami	3	1.275
Cole-Cole	3	1.301
Cole-Cole $\times 2$	5	1.211

VI. CONCLUSION

Wind turbulence is a non-stationary and stochastic phenomenon, hence the need to rely on spectral characteristics to model wind fluctuations and not on time series data. The reference in terms of spectral modeling of wind turbulence is the Von Karman model, which is widely used and remains very effective, especially for flying vehicles. It is accurate at low frequencies but its fixed order prevents optimal modeling at high frequencies. Consequently, two improved models based on the Von Karman model have been developed and tested. The two models have the Havriliak-Negami and Cole-Cole forms of fractional functions. They allow to have a variable order and thus a better estimation in high frequency which adapts to the different conditions. However, the accuracy of the estimation is reduced at low frequencies, and the estimated gain is lower than the one of the data. As a consequence, another model has been introduced, it is a Cole-Cole two-cell model, allowing the PSD to have a second slope. This model is the one that fits the best but there are two more parameters to identify. A comparison of these models was made with a three parameter auto-regressive model based on time series data. The AIC of the different models showed that the proposed fractional models have a better fit.

These models will be used to synthesize transfer functions of filters capable of generating a random turbulence speed.

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