Identification of Cosserat elastic overall media of quasi-periodic lattices

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Introduction

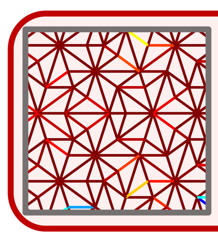
For a few years, the interest in architectured materials has grown since they enable to reach new zones of Ashby diagrams. Quasi-periodic lattices are a class of porous materials that appear locally disorganised but are highly deterministic, as shown by their sharp diffraction diagrams [Shechtman et al. 1984]. Few studies on their mechanical behaviours have been done, but they seem to combine the advantages of foams and periodic lattices: isotropic behaviour and a better toughness [Glacet et al. 2018]. Their widespread use requires the ability to identify an equivalent medium, i.e a fictitious homogeneous medium having the same macroscopic behaviour. An appropriate behaviour law must be chosen. However, it has been shown that classical laws could be insufficient [Poncelet et al. 2018].

Objectives:

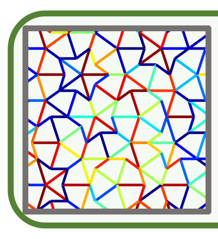
- Identify the apparent Cauchy and Cosserat elastic material parameters for different quasi-periodic patterns and determine which model is the most suitable.

Patterns considered

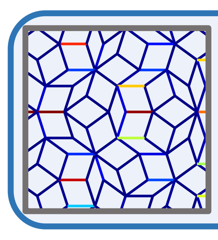
- ❑ 3 quasi-periodic lattices
  - 2D patterns
  - [D_5] symmetry class (diffraction figure)
    - ➢ Homogenized behaviour supposed **isotropic**
  - 3 **different energy classes** [Somera et al. 2022]



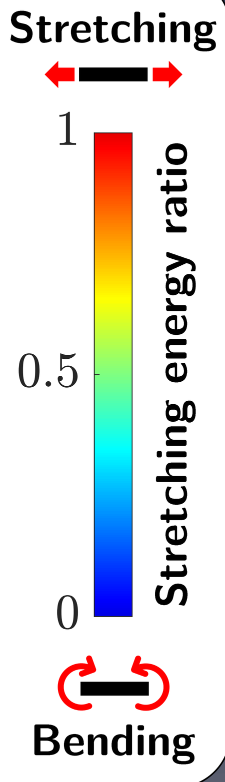
Triangulated Penrose
Always stretching dominated



Penrose P2 (kite & dart)
Varying dominance

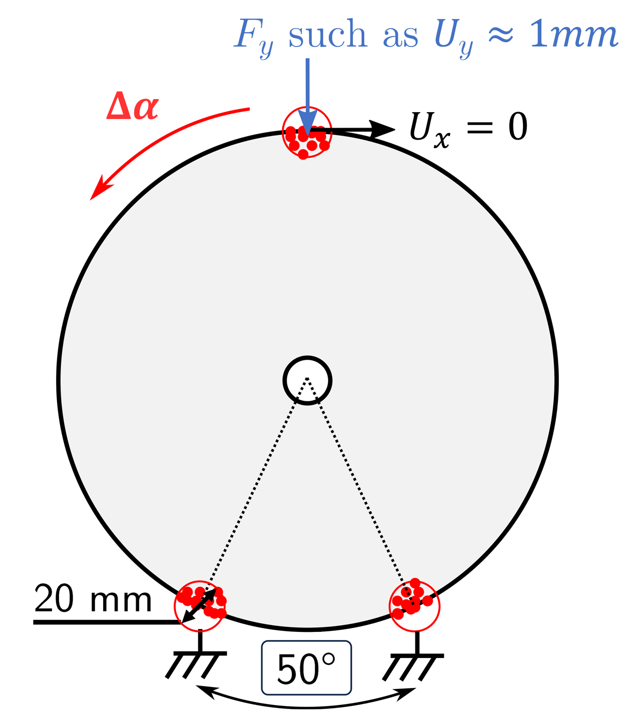


Penrose P3 (rhombus)
Always bending dominated



Identification procedure FEMU

Boundary Conditions



- ❑ 3 points rolling Brazilian test
- ❑ Loading parameters: F_y and $\Delta\alpha$

Initial parameters

Parametrisation :

❑ Cauchy : $\underline{P} = \left(\frac{\nu}{\nu_0}, \frac{G}{G_0}\right)^T$

- ν : Poisson ratio
- G : Shear modulus

❑ Cosserat : $\underline{P} = \left(\frac{\nu}{\nu_0}, \frac{G}{G_0}, \frac{G_c}{G_{c_0}}, \frac{l^2}{l_0^2}\right)^T$

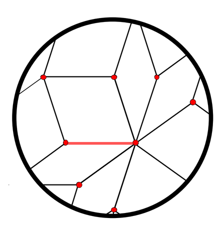
- G_c : Cosserat couple modulus
- l : Cosserat characteristic length

1st iteration — *All iterations*

Beam lattice FEM simulations

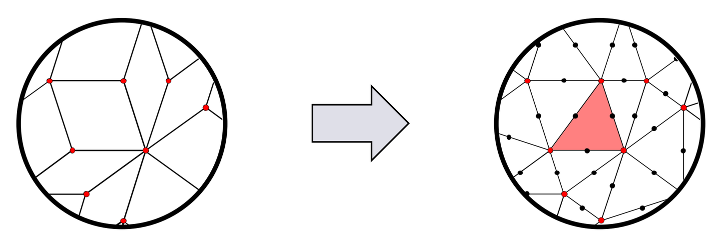
❑ **Mesoscopic model**

- Beam element associated to each strut
- Euler-Bernoulli kinematic
- Elastic behaviour
- Slenderness from 10 to 150



2D continuum FEM simulations

❑ Surface mesh obtained by **Delaunay triangulation** of lattice **physical nodes**



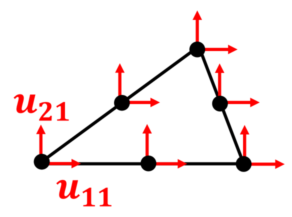
Lattice structure — Surface mesh

❑ Elastic behaviour laws

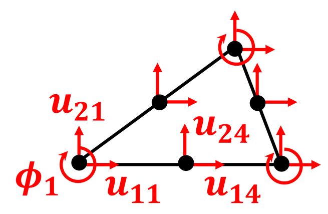
- Plane stress hypothesis
- Cosserat :

$$\underline{\sigma}^{asym} = \begin{bmatrix} \frac{2G}{1-\nu} & \frac{2\nu G}{1-\nu} & 0 & 0 \\ \frac{2\nu G}{1-\nu} & \frac{2G}{1-\nu} & 0 & 0 \\ 0 & 0 & G+G_c & G-G_c \\ 0 & 0 & G-G_c & G+G_c \end{bmatrix} \underline{\varepsilon}^{asym}$$

$$\underline{m} = 2Gl^2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \underline{k}$$

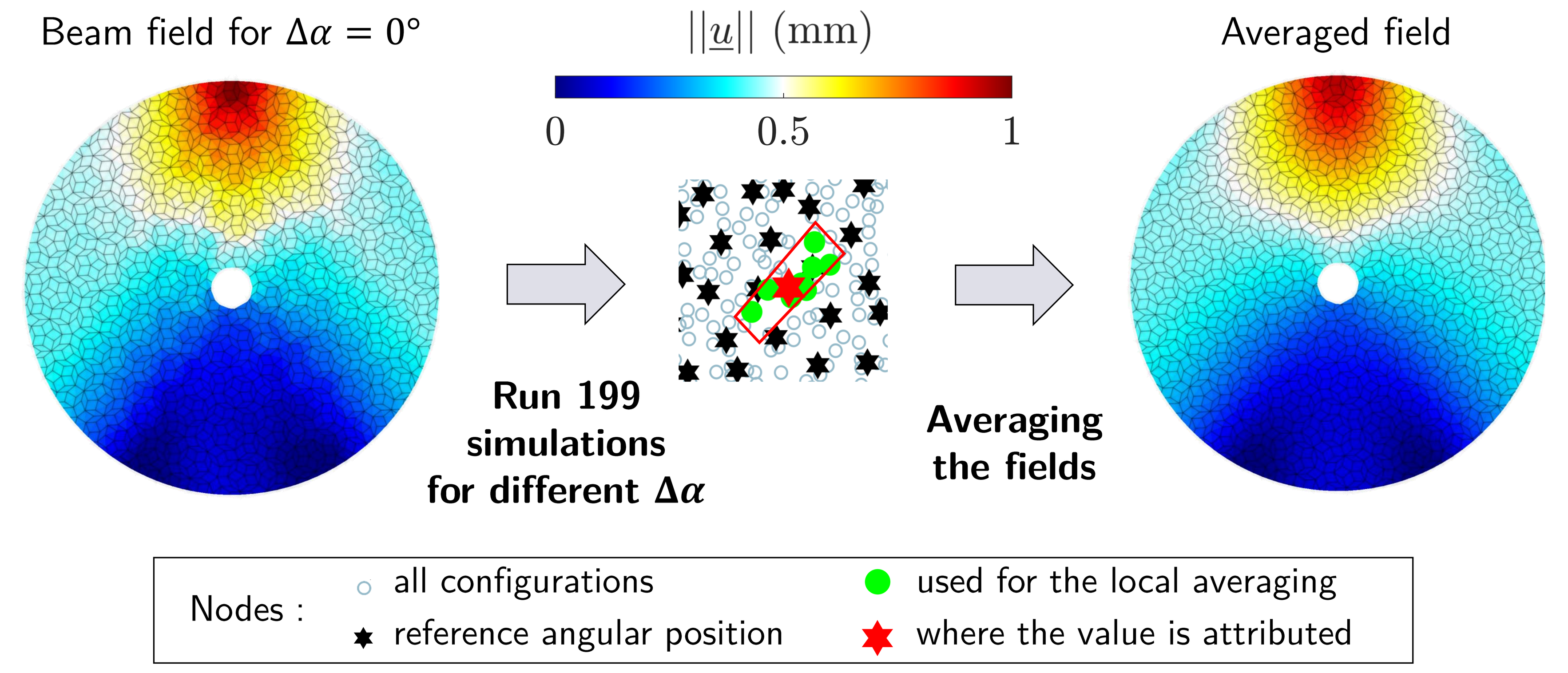


Cauchy mesh



Cosserat mesh

Averaging of the fields



2D continuum fields	Beam lattice fields (Reference)
➢ Prevent the occurrence of biases ➢ Ensuring the same boundary conditions	➢ Prevents disruption of identification due to local variations because of the heterogeneous nature of the lattice

Computation of the cost function $C(\underline{P})$

❑ **Cosserat:**

$$C(\underline{P}) = \begin{pmatrix} \underline{u}_{sim}(\underline{P}) - \underline{u}_{ref} \\ \underline{\phi}_{sim}(\underline{P}) - \underline{\phi}_{ref} \end{pmatrix}^T \begin{bmatrix} [1] & [0] \\ [0] & [L_r^2] \end{bmatrix} \begin{pmatrix} \underline{u}_{sim}(\underline{P}) - \underline{u}_{ref} \\ \underline{\phi}_{sim}(\underline{P}) - \underline{\phi}_{ref} \end{pmatrix}$$

with:

- $\underline{\bullet}_{ref}$: dofs of reference fields
- $\underline{\bullet}_{sim}$: dofs of 2D continuum fields
- L_r: Lever arm used to have homogeneous units, chosen equal to half the average beam length

Updating parameters — NO — $C(\underline{P}) < \varepsilon$ — YES

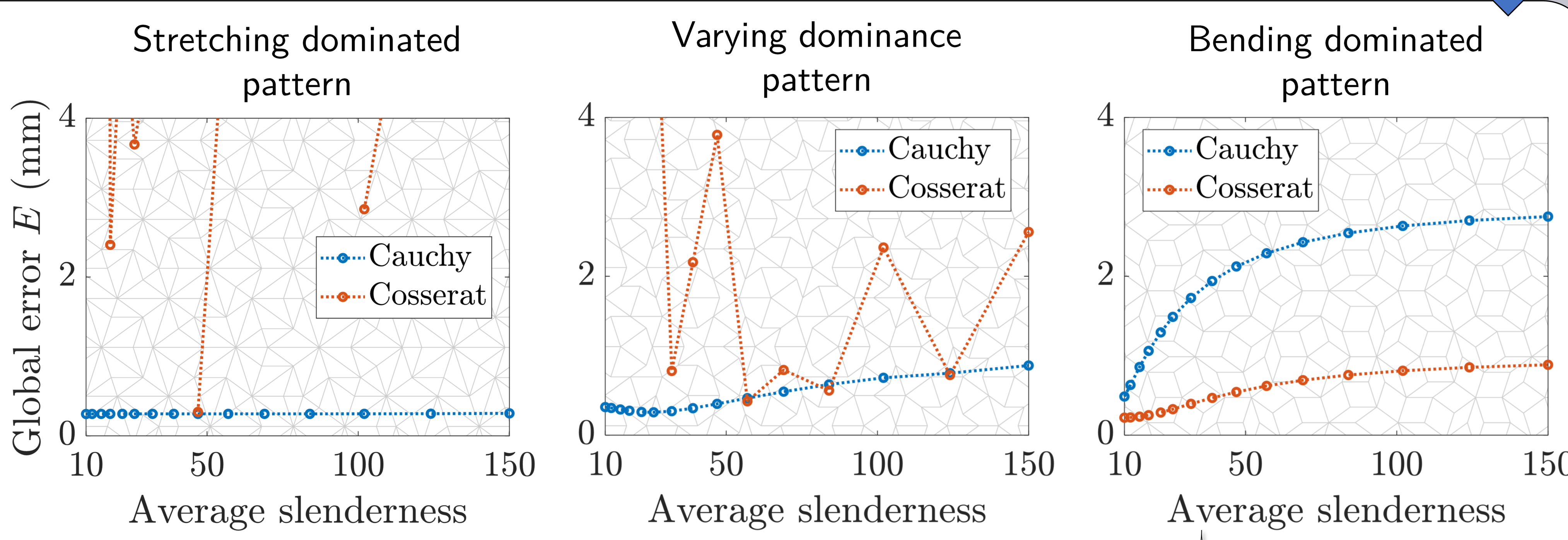
Results

❑ Computation of the **global error** E on the displacement field:

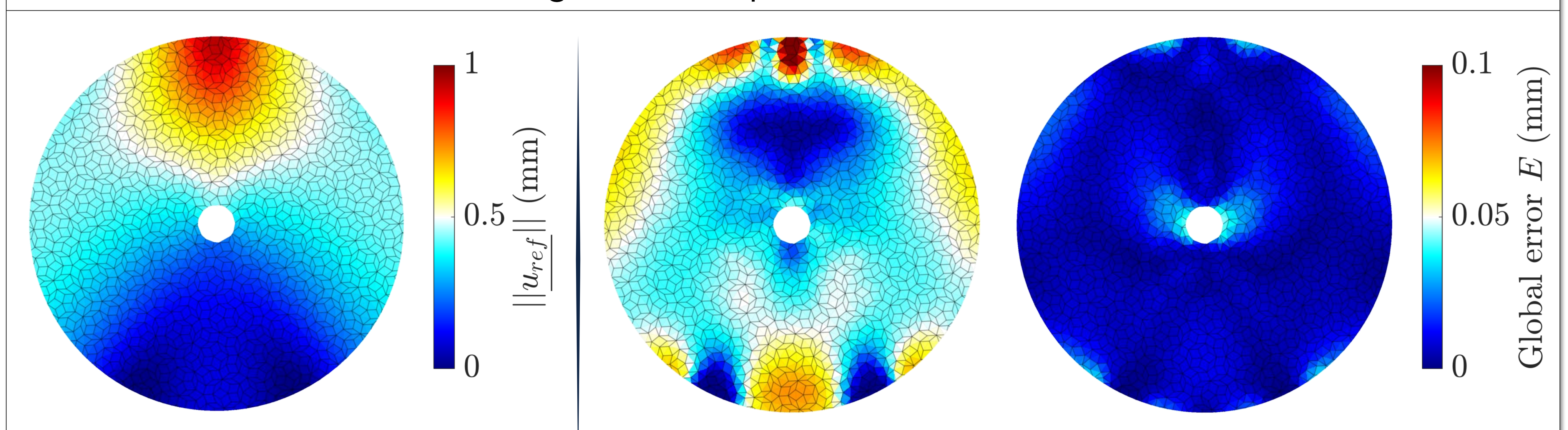
$$E = \sqrt{\sum_{i \in nodes} \left\| \underline{u_{i,sim}} - \underline{u_{i,ref}} \right\|_2^2} \quad \text{with} \quad \begin{cases} \underline{u_{i,sim}}: \textit{Identified displacements at node } i \\ \underline{u_{i,ref}}: \textit{Reference displacements at node } i \end{cases}$$

	Cauchy	Cosserat
Stretching and varying dominance patterns	✓ Convergence of identification regardless of slenderness ✓ Small global errors ⚠ Varying dominance pattern: Unrepresentative averaged reference fields for large slenderness	✗ Difficult convergence ✗ Errors at best equivalent to Cauchy but often much higher ➢ Poor sensitivity as Cosserat parameters are probably close to zero
Bending dominated patterns	✓ Convergence of identification regardless of slenderness ✗ Significant global errors	✓ Convergence of identification regardless of slenderness ✓ Errors 2 to 3 times lower than Cauchy

Most suitable model (shaded cells)



Bending dominated pattern – Slenderness 32



Reference displacement field — Cauchy error field — Cosserat error field

Conclusions

- ➢ **Model of the apparent homogeneous medium dependent on the pattern**
- ➢ **Strong influence of the pattern on the mechanical behaviour**