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Dispersion of plasmons in three-dimensional superconductors

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We study the plasma branch of an homogeneous three-dimensional electron gas in an s -wave superconducting state. We focus on the regime where the plasma frequency ω_p is comparable to the gap Δ , which is experimentally realized in cuprates. Although a sum rule guarantees that the departure of the plasma branch always coincides with the plasma frequency, the dispersion and lifetime of the plasmons is strongly affected by the presence of the pair condensate, especially at energies close to the pair-breaking threshold 2Δ . When ω_p is above 1.7Δ , the level repulsion is strong enough to give the plasma branch an anomalous, negative dispersion with a minimum at finite wavelength. At non-zero temperature and at $\omega_p > 2\Delta$, we treat in a non-perturbative way the coupling of plasmons to the fermionic excitations, and show that a broadened plasma resonance inside the pair-breaking continuum coexists with an undamped solution in the band gap. This resonance splitting is associated with the presence of multiple poles in the analytic continuation of the propagator of the Cooper pairs.

Introduction: Despite being a very mature experimental platform, supporting numerous technical applications, superconductors still hold some of the most fundamental open questions of many-body physics. The impressively high critical temperature (T_c) and the unconventional Cooper pairing in cuprates and iron-based superconductors are the most famous of those fascinating questions. However, even some properties of conventional Bardeen-Cooper-Schrieffer (BCS) superconductors are still intensively discussed, such as the existence of an amplitude collective mode [1–3], reminiscent of the Higgs mode in high-energy physics.

In fact, even for such usual behavior as plasma oscillations (the collective modes of the electronic density), superconductors are still not fully understood. In a pioneering work, Anderson [4] has shown that the phononic (Goldstone) branch that exists in a neutral fermionic condensate acquires a gap corresponding to the plasma frequency ω_p in presence of long-range Coulomb interaction. This mechanism later became famous due to its analogy with the phenomenon of mass acquisition in high-energy physics. The work of Anderson has then been revisited in the context of high- T_c superconductivity [5–9], and nuclear/neutronic matter [10]. While Anderson focused on the regime of large ω_p , the frequency of transverse plasmons in layered superconductors such as cuprates often lies below the pair-breaking threshold 2Δ [11–13], such that an undamped plasma branch can be expected. The departure of the plasma branch was shown to always coincide with ω_p [9], and as temperature or excitation momenta were varied, a duplication of the plasma resonance was observed, with a low-energy branch at energies below 2Δ and a high-energy one above [14–16].

The existence of such low-energy plasmons was cited as a possible explanation of the critical temperature increase in cuprates [17]. Besides their importance for the fun-

damental understanding of superconductivity and long-range interactions in many-body physics, those modes can also be used in plasmonics, or to probe and manipulate superconducting materials [18]. Experimental research on Josephson plasmons, which describe transverse plasmonic excitations in superconducting layers is still very active today [19].

At low energy-momentum, plasmons can be described by phenomenological approaches based on London electrodynamics [20], but a microscopic theory is needed when the eigenfrequency of plasmons approaches the pair-breaking threshold. In this regime, the theoretical literature is still hesitant, in particular in the cases where a complex plasma mode describing a damped resonance is expected. Here, we reveal that remarkable phenomena affecting the plasma dispersion in presence of superconducting order have been overlooked. Like Anderson, we consider the reference situation of an isotropic three-dimensional (3D) s -wave superconductor but our study can be readily extended to layered geometries or anisotropic pairing. Due to the repulsion of the pair-breaking threshold, the plasma branch acquires a negative curvature and thus a minimum at non-zero wavenumber when ω_p is between 1.696Δ and 2Δ at zero temperature. At larger wavenumber, nonzero temperature or when $\omega_p > 2\Delta$, the plasma branch enters the pair-breaking continuum but remains observable with a finite lifetime that we calculate using recently developed techniques [21–23] to treat the coupling to the fermionic continuum. We find a rich resonance structure in the density-density response function, with several peaks both above and below the pair-breaking threshold, which we relate to the existence of multiple poles in the analytic continuation [21, 23, 24] of the propagator the density-phase fluctuations. In the quasiphononic regime $\omega_p \ll 2\Delta$ corresponding to the experimental situation in cuprates [13, 15], the

plasma eigenfrequency behaves as $\sqrt{\omega_p^2 + \omega_{q,n}^2}$ where $\omega_{q,n}$ is the phononic dispersion of the neutral fermion condensate [14]. Finally, we show that when temperature increases towards T_c , the plasmons gradually recover their normal (undamped) dispersion [25], except in a region of size Δ^2/T around the pair-breaking threshold. All these unusual behaviors should have important consequences on the electromagnetic and transport properties of superconductors.

Linear response within RPA: We study an homogeneous electron gas evolving in a cubic volume V with a average density ρ , defining the Fermi wavenumber $\rho = k_F^3/3\pi^2$. Electrons interact through both the long-range Coulomb potential $V_C(r) \propto 1/r$ and a short-range part, responsible for s -wave Cooper pairing, and modelled by a contact potential of coupling constant g :

$$V(\mathbf{r}_1, \mathbf{r}_2) = g\delta(\mathbf{r}_1 - \mathbf{r}_2) + V_C(\mathbf{r}_1 - \mathbf{r}_2) \quad (1)$$

We note that a momentum cutoff at the Debye frequency should be used to regularize the divergence caused by the contact potential, although in the following discussion this cutoff can safely be send to infinity. In terms of the electron mass m and wavenumber q , the Fourier transform of the Coulomb potential is $V_C(q) = m\omega_p^2/\rho q^2$ (we use $\hbar = k_B = 1$ throughout the article).

We imagine that the system is driven at fixed frequency ω and wavenumber q by an external field (for example an electromagnetic field) and we study the collective response within linear response theory. The response function can be computed either using a path integral formalism [26] with both a pair and density auxiliary fields, or in the Random Phase Approximation (RPA), neglecting as in [4] the exchange-scattering diagrams (the effect of those diagrams is discussed in Ref. [27]). In a superconductor, since the density response $\delta\rho$ is coupled to the fluctuations of the order parameter, in phase $\delta\theta$ and *a priori* in modulus $\delta|\Delta|$, the linear response function χ is a 3×3 matrix:

$$\begin{pmatrix} 2i\Delta\delta\theta(\mathbf{q}, \omega) \\ 2\delta|\Delta(\mathbf{q}, \omega)| \\ g\delta\rho(\mathbf{q}, \omega) \end{pmatrix} = \chi(\omega, \mathbf{q}) \begin{pmatrix} u_\theta(\mathbf{q}) \\ u_{|\Delta|}(\mathbf{q}) \\ u_\rho(\mathbf{q}) \end{pmatrix}, \quad (2)$$

where u_θ , $u_{|\Delta|}$ and u_ρ are respectively the phase, modulus and density driving fields [28]. The response matrix χ is expressed in terms of the bare propagator Π as $\chi = -M^{-1}\Pi$ with

$$M = \Pi - D \text{ and } D \equiv \begin{pmatrix} V/g & 0 & 0 \\ 0 & V/g & 0 \\ 0 & 0 & V/2V_C(\mathbf{q}) \end{pmatrix} \quad (3)$$

Remark that due to the Coulomb potential M and Π do not commute, such that χ is not a symmetric matrix. The matrix Π was computed for instance in Refs. [28–30].

We give here its generic expression

$$\Pi_{ij}(z, \mathbf{q}) = \sum_{\mathbf{k}} \frac{\pi_{ij}^+(1 - f_+ - f_-)}{z^2 - (\epsilon_+ + \epsilon_-)^2} - \frac{\pi_{ij}^-(f_+ - f_-)}{z^2 - (\epsilon_+ - \epsilon_-)^2} \quad (4)$$

in terms of the Fermi-Dirac occupation numbers $f_\pm = 1/(1 + \exp(\epsilon_\pm/T))$, free-fermion $\xi_\pm = \xi_{\mathbf{q}/2 \pm \mathbf{k}}$ and BCS energies $\epsilon_\pm = \epsilon_{\mathbf{q}/2 \pm \mathbf{k}}$ with $\xi_{\mathbf{k}} = k^2/2m - \mu$ and $\epsilon_{\mathbf{k}} = \sqrt{\xi_{\mathbf{k}}^2 + \Delta^2}$. The coefficients π_{ij}^\pm can be deduced from Eq. (36) in [28]. The first term in Eq. (4) gives rise to the pair-breaking continuum $\{\epsilon_{\mathbf{q}/2 + \mathbf{k}} + \epsilon_{\mathbf{q}/2 - \mathbf{k}}\}_{\mathbf{k}}$, gapped at low q by the pair-breaking threshold 2Δ . The second term exists only at $T \neq 0$ and gives rise to the gapless quasiparticle-quasihole continuum $\{\epsilon_{\mathbf{q}/2 + \mathbf{k}} - \epsilon_{\mathbf{q}/2 - \mathbf{k}}\}_{\mathbf{k}}$

The spectrum of the collective modes is found as the poles of χ , hence as the zeros of M :

$$\det M_\downarrow(z_q, \mathbf{q}) = 0 \quad (5)$$

The \downarrow sign recalls that when the collective mode is coupled to the pair-breaking [21] or quasiparticle-hole continuum [23, 31], it is complex energy is found only after an analytic continuation of M from upper to lower half-complex plane.

The present analysis of the plasmon dispersion focuses on the typical weak-coupling regime of superconductors, with Δ much smaller than the Fermi energy ϵ_F , and the excitation wavelength comparable to the Cooper pair size $\xi = k_F/2m\Delta$. In this regime, the fluctuation of the modulus of the order parameter are decoupled from the phase-density fluctuations:

$$\det M_\downarrow = 0 \iff M_{11,\downarrow}M_{33,\downarrow} - M_{13,\downarrow}^2 = 0 \text{ or } M_{22,\downarrow} = 0 \quad (6)$$

The second condition gives rise the ‘‘pair-breaking’’ or ‘‘Higgs’’ modulus mode which in the weak-coupling regime is insensitive to Coulomb interactions [22, 32]. Here, we study the density-phase modes, fulfilling the first condition.

Anomalous dispersion of long wavelength plasmons: We first study analytically the plasmon dispersion in the limit $q \ll 1/\xi$, where, by analogy with the normal case [25], one can expect the quadratic law [7]:

$$z_q = \omega_0 + \alpha \frac{q^2}{2m} + O(q^4) \quad (7)$$

The expansion in powers of q is more easily performed using the recombined matrix:

$$\widetilde{M} = \begin{pmatrix} M_{11} & zM_{13} + 2\Delta M_{11} \\ zM_{13} + 2\Delta M_{11} & z^2M_{33} + 4\Delta zM_{13} + 4\Delta^2M_{11} \end{pmatrix} \quad (8)$$

In particular the origin ω_0 of the plasma branch is found

simply by solving $\widetilde{M}_{33} = 0$ to lowest order in q . We have

$$\begin{aligned} \widetilde{M}_{33} = & \frac{\rho V q^2}{2m} \left(1 - \frac{z^2}{\omega_p^2} \right) \\ & + \sum_{\mathbf{k}} \frac{\widetilde{m}_{33}^+(1-f_+ - f_-)}{z^2 - (\epsilon_+ + \epsilon_-)^2} - \frac{\widetilde{m}_{33}^-(f_+ - f_-)}{z^2 - (\epsilon_+ - \epsilon_-)^2} \end{aligned} \quad (9)$$

where we have used a sum rule¹ to simplify the first line, and we set $\widetilde{m}_{33}^\pm = [\xi_+ - \xi_-]^2 [\epsilon_+ \pm \epsilon_-] [\epsilon_+ \epsilon_- \mp \xi_+ \xi_- \mp \Delta^2] / 2\epsilon_+ \epsilon_-$. The summation on the second line is of order q^4 and will only affect the expression of the dispersion parameter α . Thus, the origin ω_0 of the plasma branch always coincides with the plasma frequency [9]

$$\omega_0 = \omega_p \quad (10)$$

This expected results shows that superconductivity does not affect the departure of the plasma branch. As we now explain, the situation is quite different for the low- q dispersion and lifetime.

To extract the curvature α of the plasma branch, we expand \widetilde{M}_{33} to subleading order in q , and consider the 2 other matrix elements

$$\begin{aligned} M_{11} &= \sum_{\mathbf{k}} \left[\frac{m_{11}^+(1-f_+ - f_-)}{z^2 - (\epsilon_+ + \epsilon_-)^2} - \frac{m_{11}^-(f_+ - f_-)}{z^2 - (\epsilon_+ - \epsilon_-)^2} \right] - \frac{V}{g} \\ \widetilde{M}_{13} &= \sum_{\mathbf{k}} \left[\frac{\widetilde{m}_{13}^+(1-f_+ - f_-)}{z^2 - (\epsilon_+ + \epsilon_-)^2} - \frac{\widetilde{m}_{13}^-(f_+ - f_-)}{z^2 - (\epsilon_+ - \epsilon_-)^2} \right] \end{aligned} \quad (11)$$

with $m_{11}^\pm = (\epsilon_+ \pm \epsilon_-)(\epsilon_+ \epsilon_- \pm \xi_+ \xi_- \pm \Delta^2) / 2\epsilon_+ \epsilon_-$ and $\widetilde{m}_{13}^\pm = \pm(\epsilon_+ \pm \epsilon_-)(2\xi_+ \xi_- + 2\Delta^2 - \epsilon_+^2 - \epsilon_-^2) \Delta / 2\epsilon_+ \epsilon_-$. At zero temperature, this yields the fully analytic expression of α :

$$\alpha = \frac{6\epsilon_F}{5\omega_p} - \frac{32\epsilon_F}{15\omega_p} \frac{\Delta^2 \text{Arcsin}(\omega_p/2\Delta)}{\omega_p \sqrt{4\Delta^2 - \omega_p^2}} \quad (12)$$

which is shown as a black curve on Fig. 1. This expression remains valid when $\omega_p > 2\Delta$ and the plasma branch is embedded in the pair-breaking continuum. In this case, one should use $\omega_p \rightarrow \omega_p + i0^+$ and $\text{Im } \alpha < 0$ describes the nonzero damping rate of plasmons. We note that the repulsion of the pair-breaking threshold leads to a squareroot divergence of $\text{Re } \alpha$ and $\text{Im } \alpha$ when approaching the pair-breaking threshold respectively from below and above. This opens an interval $\omega_p \in [1.696\Delta, 2\Delta[$ where plasmons have an anomalous *negative* dispersion at the origin ($\text{Re } \alpha < 0$). In the conventional limit

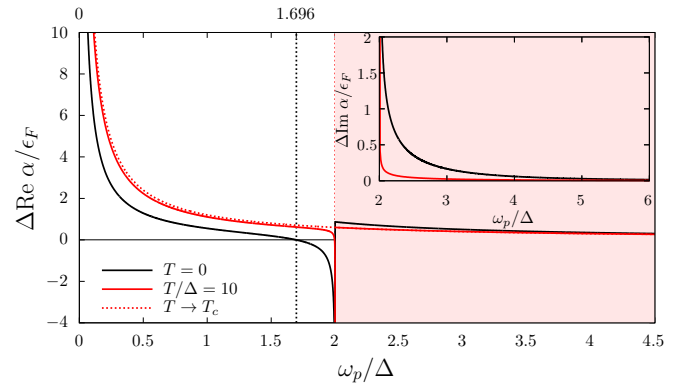


FIG. 1: Dispersion parameter $\text{Re } \alpha$ (multiplied by Δ/ϵ_F to have a finite weak-coupling limit), in function of the plasma frequency at zero (black curve) and high temperature (red curve). The normal dispersion $6\epsilon_F/5\omega_p$ is shown by the red dotted curve. The value where α changes sign at $T = 0$ is indicated by the black dotted line. Inset: the damping parameter $\text{Im } \alpha$, which becomes nonzero inside the pair-breaking continuum $[2\Delta, +\infty[$ (red area).

$\omega_p \gg 2\Delta$, the second term in (12) becomes negligible, such that we recover the normal plasmon dispersion $\alpha \rightarrow 6\epsilon_F/5\omega_p$ [25]. In the opposite « quasiphononic » limit $\omega_p \ll 2\Delta$, which corresponds to the experimental situation of Refs. [13, 15], rather than expanding for fixed z as prescribed by (7), one should expand for $q \rightarrow 0$ while keeping z/v_F comparable to q [14]. This yields²

$$z_{\mathbf{q}} \xrightarrow[q \rightarrow 0]{cq/\omega_p \text{ fixed}} \sqrt{\omega_p^2 + c^2 q^2} \quad (13)$$

where $c = v_F/\sqrt{3}$ is the speed-of-sound of the weakly-interacting condensate of neutral fermions.

At nonzero temperature, α depends on the dimensionless temperature $\bar{T} = T/\Delta$ and plasma frequency $\bar{\omega}_p = \omega_p/\Delta$. We find

$$\alpha = \frac{\epsilon_F}{\Delta} \left[\frac{6\bar{\omega}_p}{5} (I_3 + J_0 - J_2) - \frac{8}{3\bar{\omega}_p} I_1 \right] \quad (14)$$

in terms of the dimensionless integrals $I_n = \int_0^{+\infty} d\xi \frac{\text{th}(\epsilon/2T)}{\epsilon^n (\bar{\omega}_p^2 - 4\epsilon^2)}$ and $J_n = \frac{1}{2T\bar{\omega}_p^2} \int_0^{+\infty} \frac{d\xi}{\epsilon^n \text{ch}^2(\epsilon/2T)}$ with $\epsilon = \sqrt{\xi^2 + 1}$. The red curve in Fig. 1 shows α in the vicinity of the critical temperature $T/T_c = 0.9989$ ($T/\Delta = 10$). We observe that α tends to its normal limit $6\epsilon_F/5\omega_p$ uniformly except in a neighborhood of size $\approx \Delta^2/T$ around the pair-breaking threshold 2Δ . There, the divergence of the real and imaginary part is preserved whenever $T < T_c$, showing that a regime of anomalous

¹ Explicitly, we have used $\sum_{\mathbf{k}} [(1-f_+ - f_-)(\epsilon_+ + \epsilon_-)(\epsilon_+ \epsilon_- - \xi_+ \xi_- - \Delta^2) - (f_+ - f_-)(\epsilon_+ - \epsilon_-)(\epsilon_+ \epsilon_- + \xi_+ \xi_- + \Delta^2)] / 2\epsilon_+ \epsilon_- = \rho V q^2 / 2m$.

² Note that this is consistent with the behavior of α in the limit $\omega_p/\Delta \rightarrow 0$.

plasmon dispersion subsists until the transition to the normal phase. In usual situations, ω_p is fixed in units of the Fermi energy ϵ_F , but the ratio $\omega_p/\Delta(T)$ can still be adjusted by varying the temperature. The negative plasma dispersion will thus eventually occur when increasing the temperature provided $\omega_p < 2\Delta(T=0)$. Near T_c , we note that the plasma branch may also interact with the phononic Carlson-Goldman excitations [7, 29, 33] describing the motion of the superconducting electrons embedded in a majority of normal carriers. A convincing description of this phenomenon requires going beyond the collisionless regime of undamped fermionic quasiparticles [34], which is beyond the scope of this work.

One could be surprised that plasmons remain undamped ($\text{Im}\alpha = 0$) for $\omega_p < 2\Delta$ despite the nonzero temperature, which provides a decay channel through quasiparticle-quasihole excitations. In fact, to absorb a plasmon (i.e. to satisfy the resonance condition $\omega_p = \epsilon_{\mathbf{q}+\mathbf{k}/2} - \epsilon_{\mathbf{q}-\mathbf{k}/2}$) quasiparticles need to have a wavenumber $k > 2m\omega_p/q$. The plasmon lifetime thus follows an activation law $\text{Im}z_q \propto e^{-2m\omega_p^2/q^2T}$ which is exponentially suppressed in the limit $\Delta/\epsilon_F, T/\epsilon_F \rightarrow 0$ with ω_p, q of order $\Delta, 1/\xi$. Intrinsic plasmon damping at $\omega_p < 2\Delta$ is thus essentially a strong-coupling effect.

We conclude this section by computing the matrix residue $Z_q = \lim_{z \rightarrow z_q} (z - z_q)\chi(z, q)$, which quantifies the spectral weight of the plasma resonance. Writing $\chi = -1 - M^{-1}D$ and using $d(\det M)/dz \xrightarrow[z \rightarrow 0]{z \rightarrow \omega_p}$ $M_{11}M_{22}d\tilde{M}_{33}/z^2 dz$ together with $M_{11} = z^2 M_{33}/4\Delta^2 = -zM_{13}/2\Delta$ to leading order in q , we obtain, in the phase-density sector:

$$Z_q = \Delta \begin{pmatrix} \frac{\omega_p}{\rho g} \frac{4m\Delta}{q^2} & 1 \\ \frac{\omega_p}{\rho g} \frac{2m}{q^2} & \omega_p/2\Delta \end{pmatrix} + O(q^2) \quad (15)$$

Note that the phase and density excitation channels (respectively first and second line of Z_q) dominate respectively in the limits $\omega_p \rightarrow 0$ and $\omega_p \rightarrow +\infty$.

Dispersion minimum and resonance splitting at non-vanishing wavenumber: Outside the limit $q\xi \ll 1$, we study the dispersion of plasmons by numerically evaluating the M matrix. We first characterize on Fig. 2 the dispersion minimum of the plasma branch. For $\omega_p > 1.696$, it is reached at a nonzero wavenumber q_{\min} (blue curve in Fig. 2), such that the band gap of the plasma branch (black curve) is strictly lower than ω_p . As visible on Fig. 2, this undamped plasma branch at $\omega_q < 2\Delta$ persists even when $\omega_p > 2\Delta$. This is a first sign of the splitting of the plasma resonance. However, in the normal limit $\omega_p/\Delta \rightarrow +\infty$ the undamped branch tends uniformly to 2Δ with a vanishingly small spectral weight.

Since we expect the plasma branch to eventually enter the pair-breaking continuum, the characterization of the resonance at finite q requires a numerical exploration of

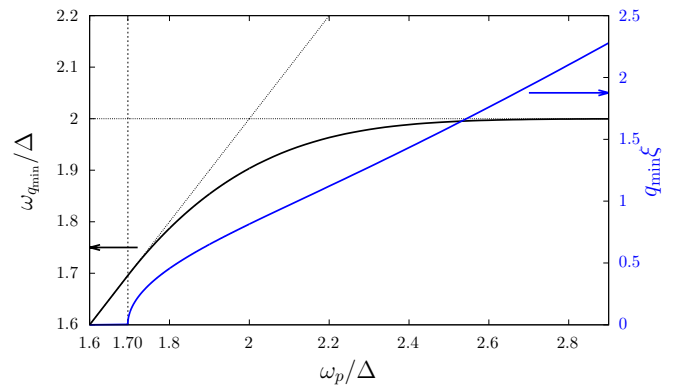


FIG. 2: The dispersion minimum $\omega_{q_{\min}}$ and the wavenumber q_{\min} at which it is reached in function of the plasma frequency. For $\omega_p < 1.696$, q_{\min} is identically 0 and $\omega_{q_{\min}}$ coincides with ω_p (oblique dotted line). In the limit $\omega_p \rightarrow +\infty$, q_{\min} diverges linearly and $\omega_{q_{\min}}$ tends to 2Δ .

the analytically continued matrix M . At $T = 0$, the pair-breaking threshold 2Δ and the second branching point [14]

$$\omega_2 = \sqrt{4\Delta^2 + \epsilon_F \frac{q^2}{2m}} \quad (16)$$

divide the real axis in three analyticity windows (I, II and III, see the inset of Fig. 3), each supporting a separate complex root ($\omega_q^I, z_q^{II}, z_q^{III}$ respectively) of Eq. (5). This suggests a splitting of the plasma resonance into 3 peaks.

While the quadratic Eq. (7) give the low- q dispersion of ω_q^I and z_q^{III} (respectively for $\omega_p < 2\Delta$ and $\omega_p > 2\Delta$), the pole of window II starts from 2Δ and departs following a non-integer power-law:

$$z_q^{II} = 2\Delta - \frac{i \pm 1}{\sqrt{\Delta}} \sqrt{\frac{8}{3\pi^2} \left| 1 - \frac{4\Delta^2}{\omega_p^2} \right|} \left(\frac{k_F q}{2m} \right)^{3/2} + O(q^{7/4}) \quad (17)$$

where the sign of the real part of $z_q^{II} - 2\Delta$ is negative if $\omega_p < 2\Delta$ and positive otherwise (in either case z_q^{II} remains outside the natural interval $[2\Delta, \omega_2]$ of window II). Remarkably, when $\omega_p = 2\Delta$ the quadratic law reemerges $z_q^{II} = 2\Delta - (0.0184 + 0.9953i) \frac{\epsilon_F}{\Delta} \frac{q^2}{2m} + O(q^4)$. Those results are obtained by expanding at low q as prescribed by Eq. (10) in [21].

On Fig. 3, we show the dispersion relation of these solutions for $\omega_p = 1.9\Delta$. In this case ω_q^I supports the main plasma branch departing in ω_p (while for $\omega_p > 2\Delta$ the main branch would be supported by z_q^{III}), z_q^{II} belongs to an indirect region of the analytic continuation (specifically $\text{Re}z_q^{II} < 2\Delta$) and z_q^{III} follows rather closely the angular point ω_2 . This subtle analytic structure is reflected in frequency behavior of the density-density response function shown on Fig. 4. Besides the Dirac peak below 2Δ , one (black curve) then two (grey curves) broadened peaks

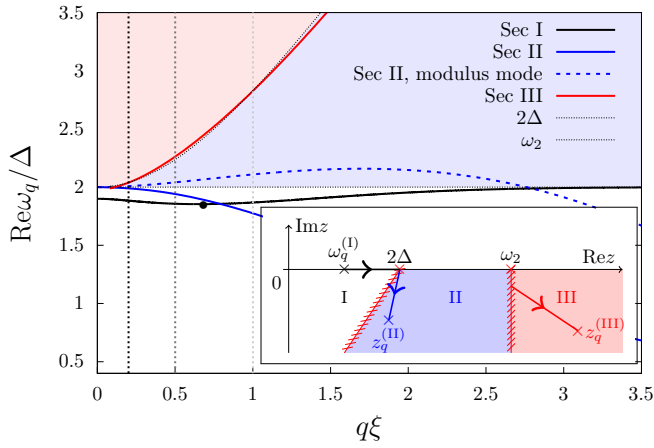


FIG. 3: The eigenfrequency $\text{Re}z_q$ of the plasma branch in function of the wave vector q (in unit of the inverse pair radius $\xi = k_F/2m\Delta$), with $\omega_p = 1.9\Delta$. The angular points 2Δ and ω_2 (Eq. (16)) are shown as dotted lines. The analytic windows are shown in colors: white for $\omega < \omega_1$ (window I), blue for $\omega_1 < \omega < \omega_2$ (window II) and red for $\omega > \omega_2$ (window III). The solution of Eq. (6) in each window is shown as a solid line in the corresponding color. The inset shows their schematic trajectories in the complex plane after analytic continuation. In window II, the pair-breaking mode (solution of $M_{22,\downarrow} = 0$) is shown as a dashed line. The dispersion minimum of the undamped solution below 2Δ is shown by the black dot.

appear inside the pair-breaking continuum [5] as q is increased. In the time domain, this remarkable frequency behavior corresponds to an evolution (after *e.g.* an electromagnetic quench of the electronic density) where the system oscillated at several frequencies: an exponentially decaying high-frequency component and a long-lived low-frequency component.

Conclusion: We have described the low- q quadratic dispersion of superconducting plasmons in 3D, and the resonance splitting which occurs when the eigenenergy nears the pair-breaking threshold. For a more realistic description of plasmons in cuprates, our study should be extended to 2D superconductors [35], with possibly Josephson interlayer couplings [16, 36]. Our work may also be applied to superfluids of ultracold fermions [37] where different kind of long-range interactions can be engineered with dipolar atoms [38].

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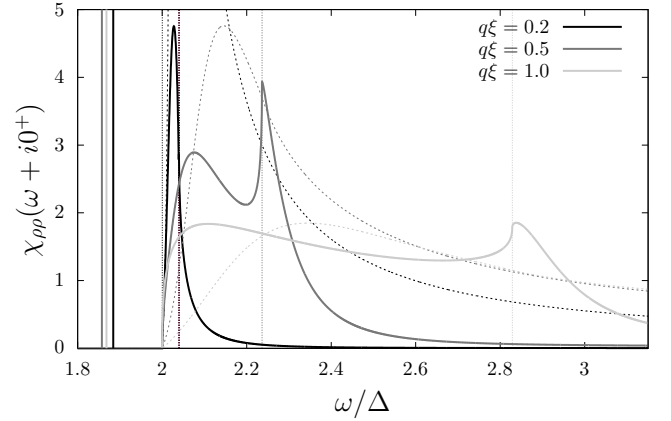


FIG. 4: Density-density response of the superconductor in function of the excitation frequency ω at fixed plasma frequency $\omega_p = 1.9\Delta$ and excitation wave number $q = 0.2/\xi$, 0.5ξ and 1.0ξ (corresponding to the vertical dotted lines in Fig. 3). The angular points 2Δ and $\omega_2(q)$ are marked by vertical dotted lines. Besides the Dirac peaks below 2Δ , broadened peaks are visible inside the pair-breaking continuum. Thin dashed lines show the modulus response $\rho g/\epsilon_F M_{22}(\omega + i0^+)$.

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