



HAL
open science

Modeling the Northern eddy-driven jet stream position and wind speed variability with stochastic coupled non-linear lattices

Robin Noyelle, Davide Faranda, Pascal Yiou

► **To cite this version:**

Robin Noyelle, Davide Faranda, Pascal Yiou. Modeling the Northern eddy-driven jet stream position and wind speed variability with stochastic coupled non-linear lattices. 2022. hal-03545111

HAL Id: hal-03545111

<https://hal.science/hal-03545111>

Preprint submitted on 27 Jan 2022

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

1 **Modeling the Northern eddy-driven jet stream position and wind speed**
2 **variability with stochastic coupled non-linear lattices**

3 Robin Noyelle,^a Davide Faranda,^{a b c} Pascal Yiou,^a

4 ^a *Laboratoire des Sciences du Climat et de l'Environnement, UMR 8212 CEA-CNRS-UVSQ,*
5 *Université Paris-Saclay & IPSL, CE Saclay l'Orme des Merisiers, 91191 Gif-sur-Yvette, France*

6 ^b *London Mathematical Laboratory, 8 Margravine Gardens, London, W68RH, UK*

7 ^c *LMD/IPSL, Ecole Normale Supérieure, PSL research University, Paris, France*

8 *Corresponding author:* Robin Noyelle, robin.noyelle@lscce.ipsl.fr

9 ABSTRACT: At the synoptic time scale, the northern mid-latitudes weather is dominated by the
10 influence of the eddy-driven jet stream and its variability. The usually zonal jet can become mostly
11 meridional during so-called blocking events, increasing the persistence of cyclonic and anticyclonic
12 structures and therefore triggering extremes of temperature or precipitation. During those events,
13 the jet shifts northerly or southerly with respect to its mean position. Previous research proposed
14 theoretically derived 1D models of the jet stream to represent the dynamics of such events. Here, we
15 take a data-driven approach using ERA5 reanalysis data over the period 1979-2019 to investigate
16 the variability of the eddy-driven jet latitudinal position and wind speed variability. We show
17 that shifts of the jet latitudinal position occur on a daily time scale and are preceded by a strong
18 decrease of the jet zonal wind speed, 2-3 days prior to the shift. We also show that the dynamics of
19 the zonal wind speed at the jet location can be modelled by a non-linear oscillator with stochastic
20 perturbations. We combine those two results to propose a simple 1D model capable of representing
21 the statistics and dynamics of blocking events of the eddy-driven jet stream. The model is based on
22 two stochastic coupled non-linear lattices representing the jet latitudinal position and zonal wind
23 speed. Our model is able to reproduce temporal and spatial characteristics of the jet. We highlight
24 a potential link between the propagation of solitary waves along the jet and the occurrence of
25 blocking events.

26 SIGNIFICANCE STATEMENT: The meanders of the atmospheric jet stream trigger extreme
27 events in the mid-latitude regions, such as heat waves and cold spells. It is therefore of primary
28 importance to better understand the variability of the jet position. This paper investigates this
29 variability and shows that it can be modelled using coupled non-linear lattices. Our results suggest
30 the possibility of using simple dynamical models to represent the complex dynamics of atmospheric
31 features such as the jet stream.

32 **1. Introduction**

33 The northern mid-latitudes weather and its variability are dominated by the influence of strong
34 zonal westerlies winds located around the tropopause, with a narrow latitudinal extension — a
35 feature known as the jet stream (Charney 1947; Holton 1973; Hurrell and Deser 2010). There are
36 actually two jets arising from two different physical mechanisms: the so-called "thermally-driven"
37 jet (Held and Hou 1980) and the so-called "eddy-driven" jet (Held 1975; Rhines 1975). However,
38 those two jets are not always distinguishable as they can be located around the same latitudes and
39 therefore mix to create what is called a "merged" jet (Lee and Kim 2003; Messori et al. 2021).

40 The jet stream has a typical spatial and temporal variability of a few thousand kilometers and of
41 10 days. This variability is not well represented in most climate models for misunderstood reasons
42 (Davini and d'Andrea 2020) and seems to be sensitive to the model resolution only in some regions
43 (Davini and D'Andrea 2016; Attinger et al. 2019). This shortcoming is crucial as the jet can trigger
44 extreme events in the mid latitudes (Kautz et al. 2021). One strategy to overcome this limitation is
45 to propose an explicit but simplified formulation of the dynamics of the jet.

46 Even though the climatology of winds in the upper troposphere displays a mostly zonal flow,
47 the jets can present large northward and southward meanders (e.g. Koch et al. 2006; Röthlisberger
48 et al. 2016) on the synoptic timescales (≈ 10 days). The flow can even become mostly meridional,
49 or even split or break (Haines and Malanotte-Rizzoli 1991). Those meanders allow air masses
50 coming from the south or the north to persist around mid-latitudes regions, potentially triggering
51 temperature or precipitation extremes (e.g. Dole et al. 2011; Kautz et al. 2021, for a recent review).
52 It is therefore of primary importance to better understand the dynamics of the jet meanders and
53 how it will change with climate change (Woollings et al. 2018a).

54 The seminal paper of Charney and DeVore (1979) showed the existence of multiple flow equilibria
55 in a simple barotropic channel model. This framed the dynamics of meanders and splits in terms of
56 transitions between two situations: zonal, where the jet flows parallel to latitude lines, and blocked,
57 where the jet makes large northward and southward excursions which can last several days and
58 break the zonal symmetry — two situations already identified by Rex (1950).

59 Many explanations have been suggested for the existence of blocked situations in the atmosphere
60 (Lupo 2020), from multiple equilibria in a barotropic flow (Charney and DeVore 1979; Legras
61 and Ghil 1985; Ghil 1987) to resonant or quasi-resonant amplification of Rossby waves (Tung
62 and Lindzen 1979; Mann et al. 2018) or barotropic/baroclinic instability (Simmons et al. 1983;
63 Frederiksen 1982). Other theories have characterized blocking episodes as a manifestation of
64 multiple equilibria in asymmetrically forced flows (Hansen 1986) or soliton-modon structures
65 (McWilliams et al. 1981). Faranda et al. (2016) studied the hypothesis of blocking episodes
66 as unstable fixed points of the atmospheric mid-latitude circulation in a reduced phase space
67 constructed using circulation indices capable to track the symmetry of the mid-latitude flow.

68 Here we propose a data-driven 1D model of blocking events that is calibrated on reanalysis data.
69 In this model, we represent the latitudinal position of the jet as a function of longitude only. We
70 build on the work of Faranda et al. (2019) who also proposed a 1D minimal dynamical system to
71 reproduce the characteristics of breaking events of the jet by an embedding of climate data, and
72 Nakamura and Huang (2018) who proposed a 1D model of blocking events using an analogy with
73 traffic jams. We propose a similar approach but focus on the onset and decay of blocking events.
74 Our model is based on the coupling between the zonal wind speed on the jet and the latitudinal
75 position of the jet itself. We also present analyses of the spatio-temporal behavior of the proposed
76 model and comparisons with reanalysis data.

77 This paper is organized as follows: in section 2, we detail the data used and the methods and
78 tools from dynamical system theory that we will be applying. In section 3 we provide the result of
79 the investigations led on the data and present a point model to represent blocking events. Section
80 4 presents the proposed 1D model and assesses its results with regards to reanalysis data. Finally,
81 the discussion of the results is led in 5 and conclusions are drawn in section 6.

82 2. Data and methods

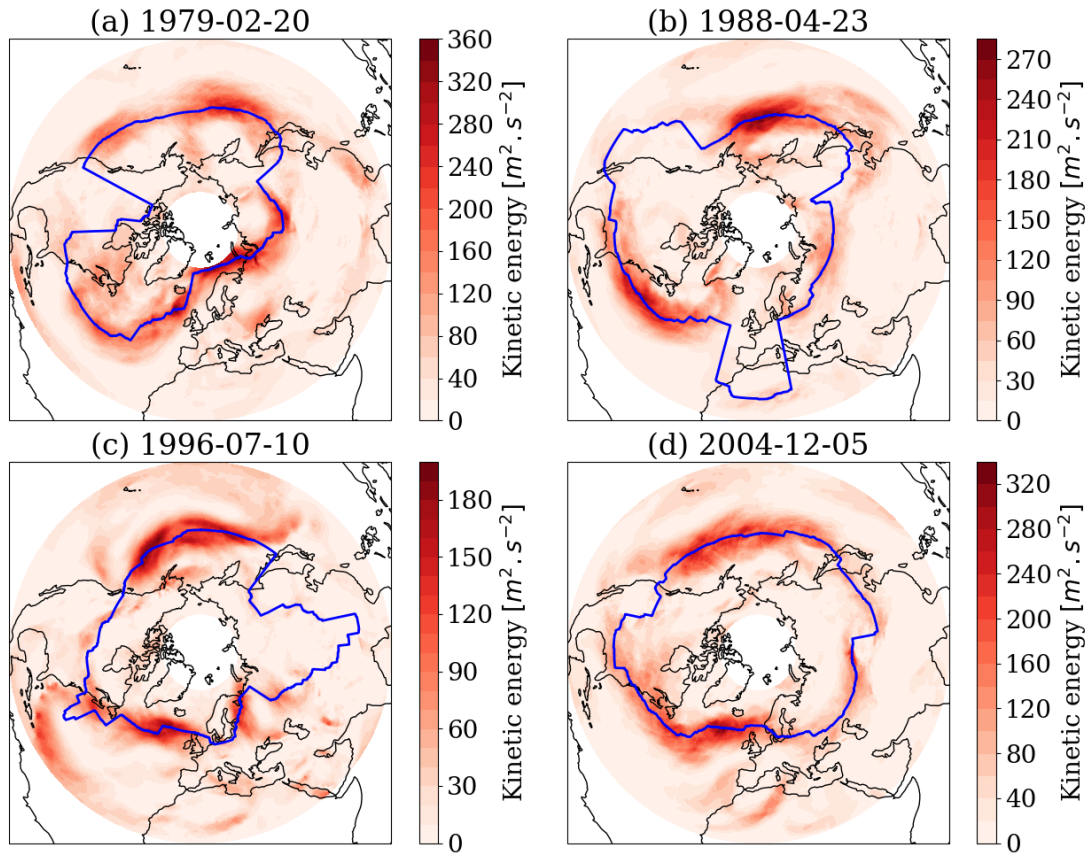
83 *a. Data and jet position algorithm*

84 The analysis and the model proposed here are based on the ERA5 reanalysis data of the European
85 Centre for Medium Range Weather Forecasts (Hersbach et al. 2020). We use daily average data
86 with a 0.25° horizontal resolution over the 1979–2019 period for the Northern Hemisphere between
87 15°N and 75°N . The variables considered are the geopotential height at 500hPa (mid-troposphere)
88 and horizontal wind speed between 850 and 700hPa.

89 The method used to diagnose the jet position is close to the ones proposed by Woollings et al.
90 (2010) and Faranda et al. (2019). The former employed wind speed averaged over 925–700hPa
91 pressure levels (low-level jet), whereas the latter considered the average over 200–400hPa (high-
92 level jet).

93 The two jets (thermally-driven (Held and Hou 1980) and eddy-driven (Held 1975; Rhines 1975))
94 behave differently across the troposphere: the subtropical jet is located on the upper troposphere
95 and displays a strong vertical shear, whereas the eddy-driven jet has a barotropic structure extending
96 to almost all the troposphere (Woollings et al. 2010). To avoid mixing the two jets in the diagnosis
97 of the jet position, we chose to take daily mean wind speed averaged over 850–700hPa pressure
98 levels as the eddy-driven jet is only detectable within these pressure heights. We therefore assume
99 that the barotropic nature of this eddy-driven jet for the position of the jet is well diagnosed. We
100 also apply a 10 days low-pass Lanczos filter to remove the influence of transient eddies (Duchon
101 1979) as in Woollings et al. (2010).

102 To find the position of the jet at each day, we use a two-step algorithm. The first step consists
103 in finding, for each longitude, the latitude at which the wind horizontal kinetic energy $E = \frac{1}{2}\vec{u}_H^2$
104 is maximum. The second step is to apply a 25° rolling median to the previous positions found.
105 This rolling median is applied to avoid a nonphysical detection of breaks in the jet, as with a 0.25°
106 horizontal resolution and considering the low-level jet, the algorithm sometimes detects high-wind
107 speeds in the lee of mountains. This is especially the case in the Atlantic region around Greenland.
108 25° approximately corresponds to 2000km at 45°N , which is also the typical size of mid-latitudes
109 baroclinic disturbances (Hoskins and James 2014) so that this rolling median has a physical basis.
110 Finally, once we have the position of the jet diagnosed by this algorithm, we also consider the zonal



119 **FIG. 1. Horizontal wind kinetic energy and jet position.** Snapshots of horizontal wind kinetic energies E
 120 (colors) and jet positions determined by the algorithm (blue line) from ERA5 data for four dates in the 1979-2019
 121 period: (a) 20th February 1979, (b) 23rd April 1988, (c) 10th July 1997 and (d) 5th December 2004).

111 and meridional wind speed on the jet as being the values of zonal and meridional wind speed at
 112 the latitudes found for the jet position by the algorithm.

113 Figure 1 shows snapshots of the horizontal wind kinetic energies and jet positions determined by
 114 the algorithm for four arbitrary dates in the period studied (20th February 1979, 23rd April 1988,
 115 10th July 1997 and 5th December 2004). The four dates illustrate the behavior of the jet for the
 116 four seasons and as one can notice, the jet is much stronger and well defined during the winter
 117 season, which is consistent with Woollings et al. (2010). As a consequence, in the following —
 118 especially for the model design — we will focus on the winter behavior of the eddy-driven jet.

122 *b. Local dynamical systems metrics*

123 The dynamical relevance of the proposed model will be assessed matching some important
124 dynamical quantities of the model with those extracted from the data. As the atmospheric dynamics
125 can be formulated in terms of a complex dynamical system (Ghil et al. 2008; Dijkstra 2016; Ghil
126 and Lucarini 2020), it is natural to investigate the properties of its attractor.

127 We will use dynamical systems metrics: the local dimension of the attractor d and the persistence
128 of phase-space trajectories θ^{-1} , which both characterise instantaneous state of a system in the phase-
129 space (Lucarini et al. 2016; Faranda et al. 2017). Both of these metrics are computed using the
130 fact that the probability for a recurrence of a system configuration (a state) can be linked to the
131 generalized Pareto distribution. To compute this probability from data, we compute the series of
132 distances $\text{dist}(x(t), \zeta)$ between a state of the system ζ and all other points $x(t)$ on the trajectory of
133 the system. This time series of distances is then transformed into: $g(t) = -\log(\text{dist}(x(t), \zeta))$ so
134 that being close to state ζ is equivalent to exceeding a threshold $s(q)$ where q is a percentile of
135 the series $g(x(t))$. We will use the 97.5% percentile of all values of $g(t)$, which ensures to have
136 enough data while keeping only the extremes. It can be shown that the probability distribution
137 of $g(t)$ when it exceeds $s(q)$ converges to a Pareto distribution (Lucarini et al. 2016) with scale
138 parameter σ , and a shape parameter $\xi = 0$.

139 The local dimension d is defined as the inverse of the scale parameter of the generalized Pareto
140 distribution fitted on the data which satisfies $g(x(t)) > s(q)$. d is a proxy for the system's active
141 number of degrees of freedom when reaching a region of phase space, so that even when considering
142 a system with a large number — possibly infinite — of degrees of freedom, d provides the local
143 number of dimensions that the system can be summarized to. Therefore a state 1 with a local
144 dimension d_1 greater than the local dimension $d_2 < d_1$ of another state 2 means that the behavior of
145 the system around state 1 has more dimensions on which to evolve and is therefore less predictable
146 than around state 2. Additionally, Pons et al. (2020) showed that d can be used as a measure of
147 synchronization: a low value of d is associated with a high degree of synchronization between the
148 variables defining the system.

149 The second dynamical system metrics that we will be using is the persistence θ^{-1} of a given state
150 ζ on the attractor, which is equivalent to the mean residence time of the trajectories when they enter
151 the neighborhood of ζ . This metric corresponds to a well defined statistical quantity introduced

152 in extreme value statistics, namely the extremal index θ . The latter is here estimated using the
 153 Siveges (2007) estimator. Note that in the framework of dynamical systems, we find $\theta = 0$ at stable
 154 fixed points of the dynamics (the trajectory resides an infinite amount of time in the neighborhood
 155 of this state), with an infinite number of infinitely time resolved trajectories. Instead, $\theta = 1$ is found
 156 at non persistent states of the dynamics (see Moloney et al. (2019) for more details). In general,
 157 for time-continuous systems sampled at a given resolution dt , $\theta^{-1} > 1$. For daily sea-level pressure
 158 fields over the North Atlantic, Faranda et al. (2017) found θ^{-1} values varying between 2 and 3 days
 159 ($0.3 < \theta < 0.5$).

160 *c. Blocking metrics*

161 In order to objectively diagnose blocking events we use the classical blocking index of Tibaldi
 162 and Molteni (1990) defined as follows. For each longitude, two geopotential height gradients are
 163 computed:

$$\begin{cases} GHGS = \frac{Z(\phi_0) - Z(\phi_s)}{\phi_0 - \phi_s}, \\ GHGN = \frac{Z(\phi_n) - Z(\phi_0)}{\phi_n - \phi_0} \end{cases} \quad (1)$$

164 where:

$$\begin{cases} \phi_n = 80^\circ N + \Delta, \\ \phi_0 = 60^\circ N + \Delta, \\ \phi_s = 40^\circ N + \Delta, \\ \Delta = -4^\circ, 0^\circ \text{ or } 4^\circ. \end{cases}$$

165 A given longitude is said to be "blocked" at a specific instant in time if for at least one value of
 166 Δ :

167 1. $GHGS > 0$,

168 2. $GHGN < -10m/^\circ lat$.

169 Using this index, "blocking" is an instantaneous and local property of each longitude. However,
170 blocking events usually have a large spatial and temporal extension. Therefore, we follow Tibaldi
171 and Molteni (1990) by defining for each longitude (i) large scale blocking if longitudes are blocked
172 within a range of 15° around this longitude and (ii) episodes blocking if longitudes are blocked
173 within a range of 10° for at least 4 days. The smaller spatial extent used for episodes blocking
174 allow blocked longitudes to slightly move.

175 There is no unique way to define blocking, and therefore many diagnostics tools are available in
176 the literature (Schwierz et al. 2004; Pelly and Hoskins 2003; Barriopedro et al. 2010) — especially
177 2D blocking indices (Scherrer et al. 2006; Masato et al. 2013). Here we choose to use the Tibaldi
178 and Molteni (1990) index because it is a widely used index in the literature, its definition is simple
179 and more importantly it is a 1D index. In the following we use this index for illustration and
180 checking purposes and our analysis is not based on the relevance of this index. Therefore, the
181 choice of one index over another should not have a great impact.

182 **3. Jet behavior and construction of the model**

183 In this section, we display results on the dynamical characteristics of the eddy-driven jet stream
184 and we propose a stochastic point model of the onset and decay of blocking events on the jet. As
185 mentioned earlier, the eddy-driven jet stream is much stronger — and hence well-defined — in
186 winter than during the other seasons. Therefore, even though we will show empirical behavior of
187 the jet during each season, the model will be built by solely restricting the data set to winter months
188 (December to February).

189 *a. Climatology of jet positions, blocking events and dynamic metrics*

190 First, Figure 2 displays the climatology of jet positions found with our algorithm, namely the
191 probability density of latitudes where the jet is located for each longitude, splits into the four
192 seasons. The densities presented in the figure are consistent with previous climatologies of the jet
193 position in terms of mean latitudes (Woollings et al. 2018b), except for summer where for some
194 longitudes mainly in Asia the algorithm seems to track the subtropical jet at the southern part of the
195 spatial domain under study for some longitudes. If we focus on the winter season, we see that the
196 jet is located in a very narrow band of latitude at the west of the ocean basins (North Atlantic and

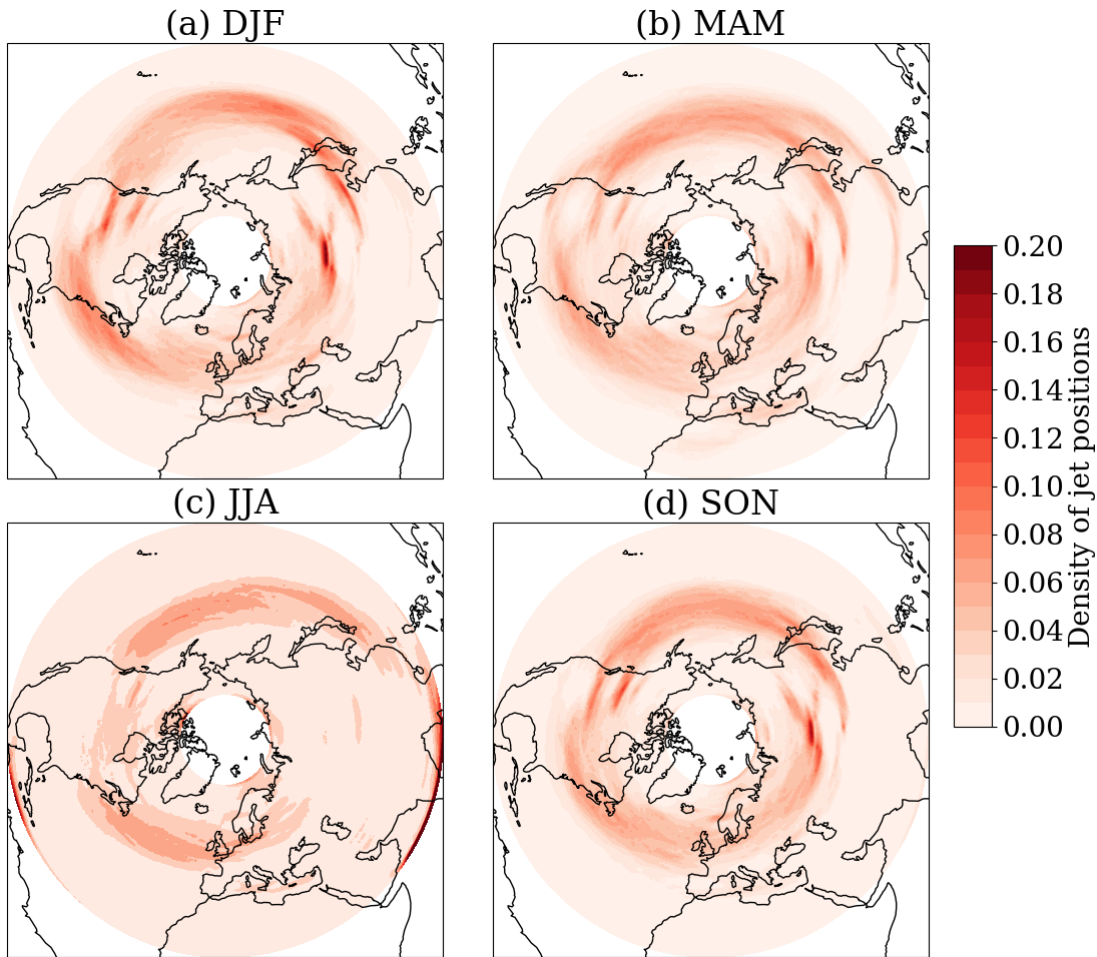
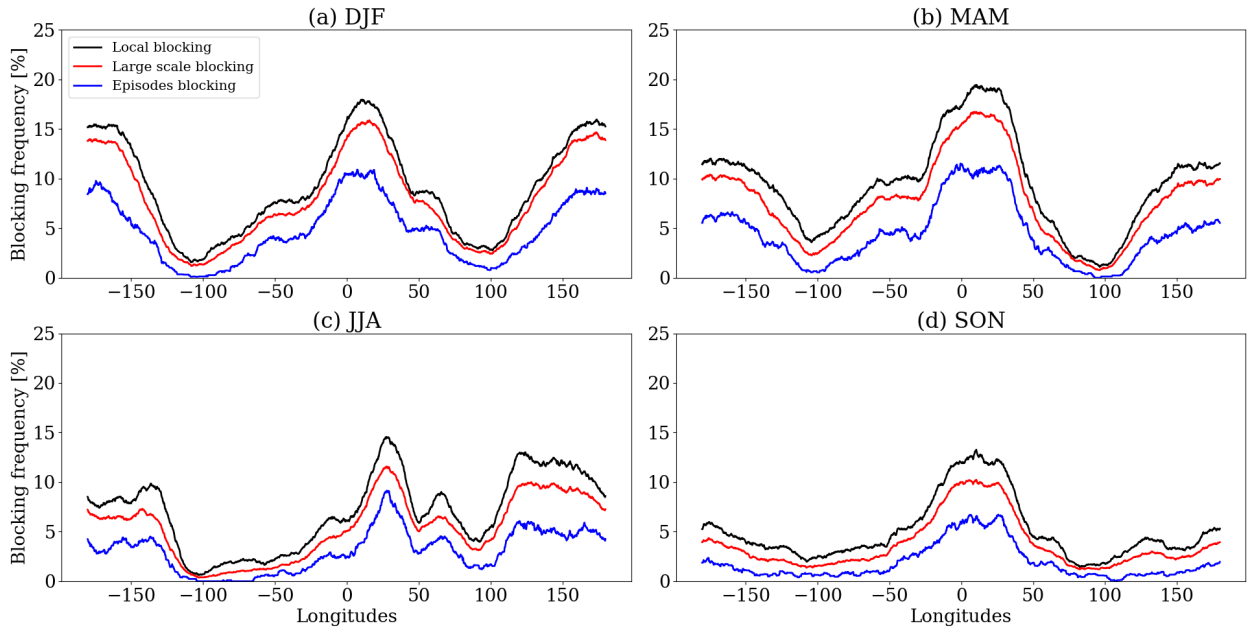


FIG. 2. Probability density of the jet positions for the four seasons over the period 1979-2019.

197 North Pacific) whereas the spread is much more important at the east of those basins. As we will
 198 show (see in particular Figure 3), those locations are the preferential places where blocking events
 199 occur. One drawback of using low level jet is that its position is not well detected over mountain
 200 ranges. Indeed, the jet over the Rocky mountains is located in a very narrow latitudinal band in
 201 Figure 2.

202 Figure 3 displays the frequency of blocking for each longitude for each season. We use the
 203 three Tibaldi and Molteni (1990) indicators defined above: local blocking (plain lines), large scale
 204 blocking (dotted line) and episodes blocking (dashed lines). As one could expect, local blocking
 205 is more frequent than large scale blocking, the latter being more frequent than episodes blocking.
 206 Blocking frequencies vary strongly from season to season and from longitudes to longitudes,
 207 ranging from almost 0% at 100°W in summer to almost 20% at 0° in spring, which is consistent



211 **FIG. 3. Blocking frequencies for each longitudes for the four seasons (1979-2019).** We display the three
 212 indexes defined in c: local blocking (black), large scale blocking (red) and episodes blocking (blue).

208 with the literature (Barriopedro et al. 2006; Tyrlis and Hoskins 2008; Lupo 2020). For winter,
 209 the longitudes which are the most often blocked are located near 0° and near 180°W , which are
 210 longitudes closed to the one with the maximum spread of jet positions.

213 Figure 4 shows the analysis made on the series of the jet positions using the dynamic metrics
 214 defined in section 2b. Figure 4 panel (a) displays the histograms of local dimension and local
 215 persistence for the jet. It should be emphasized that those metrics are computed on the vectors
 216 of the latitudinal positions of the jet, which, as the horizontal resolution is 0.25° , cover all the
 217 Northern hemisphere and have a length of 1440 grid points. Therefore, finding local dimensions,
 218 i.e. approximately the number of degrees of freedom, between 5 and 20 suggests that this dynamics
 219 is rather low dimensional. This is consistent with the results found in Faranda et al. (2017) who
 220 used SLP data for a similar analysis. Figure 4 panel (b) shows the evolution of the two metrics
 221 over the entire period with a one year rolling median (red and blue curves) and a 10 year rolling
 222 median (corresponding black curves). As one can see, there are strong inter-annual variations, and
 223 even inter-decadal variations as for example there is a clear decreasing trend of the local dimension

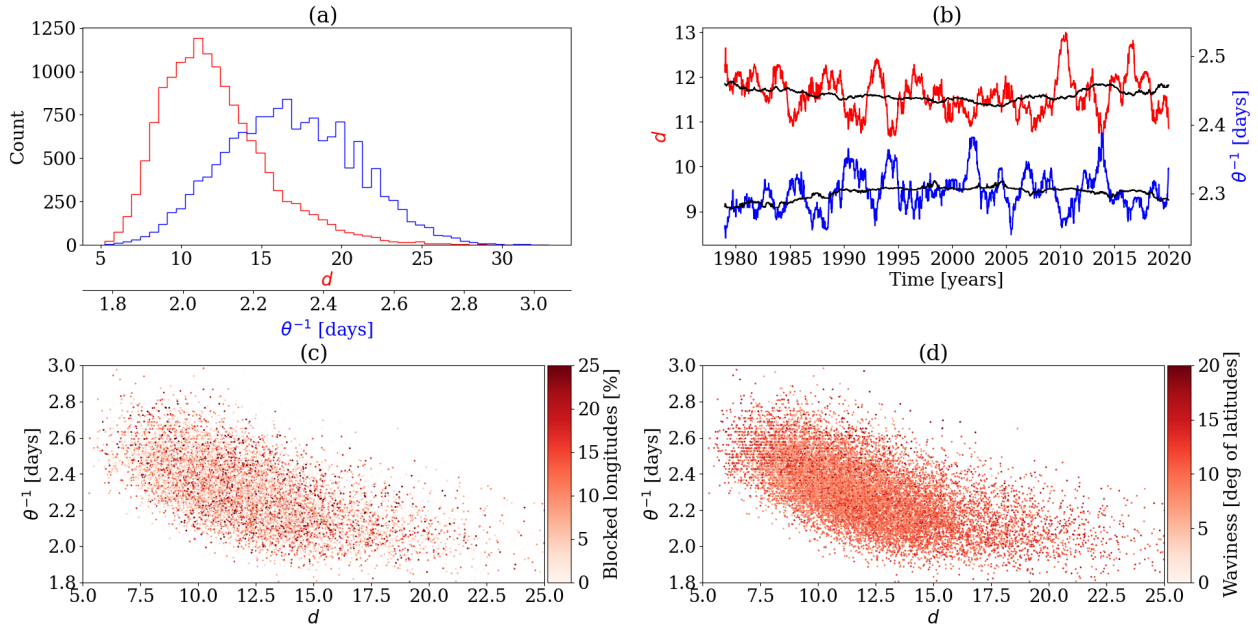
224 between 1980 and 2005 (from $d \approx 12$ to $d \approx 11.5$) which is consistent with the variations in the jet
225 variability identified by Woollings et al. (2018b).

226 Figure 4 panels (c) and (d) displays d - θ^{-1} plots of two measures of the jet non zonality: the
227 percentage of latitudes that are blocked (using the large scale blocked index) and the waviness,
228 which we define as the horizontal standard deviation. Apart from the overall negative correlation
229 between d and θ^{-1} (states with a lot of degrees of freedom are less persistent), which has already
230 been identified (e.g. Faranda et al. 2017), there does not seem to be any link between our two
231 measures of non zonality and the two metrics. The reason could be that we are computing mutual
232 distances between the positions of the jet over the entire Northern Hemisphere, therefore taking
233 into account different disconnected weather regimes. In Figure 5 we provide the same analysis as
234 in Figure 4 but we computed the mutual distances only over the Euro-Atlantic sector (45°W - 45°E).
235 Even though the percentage of large scale blocked longitudes does not show any structure, there is a
236 clear structure in the waviness indicator: high waviness is correlated with a higher local dimension
237 and a lower persistence, which is consistent with the hypothesis of Faranda et al. (2016) that
238 blocking events, i.e. states with a high waviness, are fixed points of the dynamics (low persistence
239 and high dimension).

253 *b. Jet position and blocking events dynamics*

254 We now investigate whether a 1D stochastic model of the jet can be used to represent the dynamics
255 and statistics of blocking events. We wish to represent the jet latitudinal position as a function
256 of longitude only and therefore we will investigate dynamical constraints for such a model. The
257 first step is to investigate whether the jet position can be used as an effective tool for diagnosing
258 blocking events. Figure 6 panel (a) shows the probability density of jet positions at longitude 0° in
259 the case of blocked and non blocked situations for winter (1979-2019) using Tibaldi and Molteni
260 (1990) large scale blocking index and Figure 6 panel (b) displays probability density difference of
261 jet positions in the case of blocked and non blocked situations for all longitudes. In Figure 6 panel
262 (b), red (resp. blue) regions indicate that the jet has a higher (resp. lower) probability to be at that
263 latitude when it is blocked.

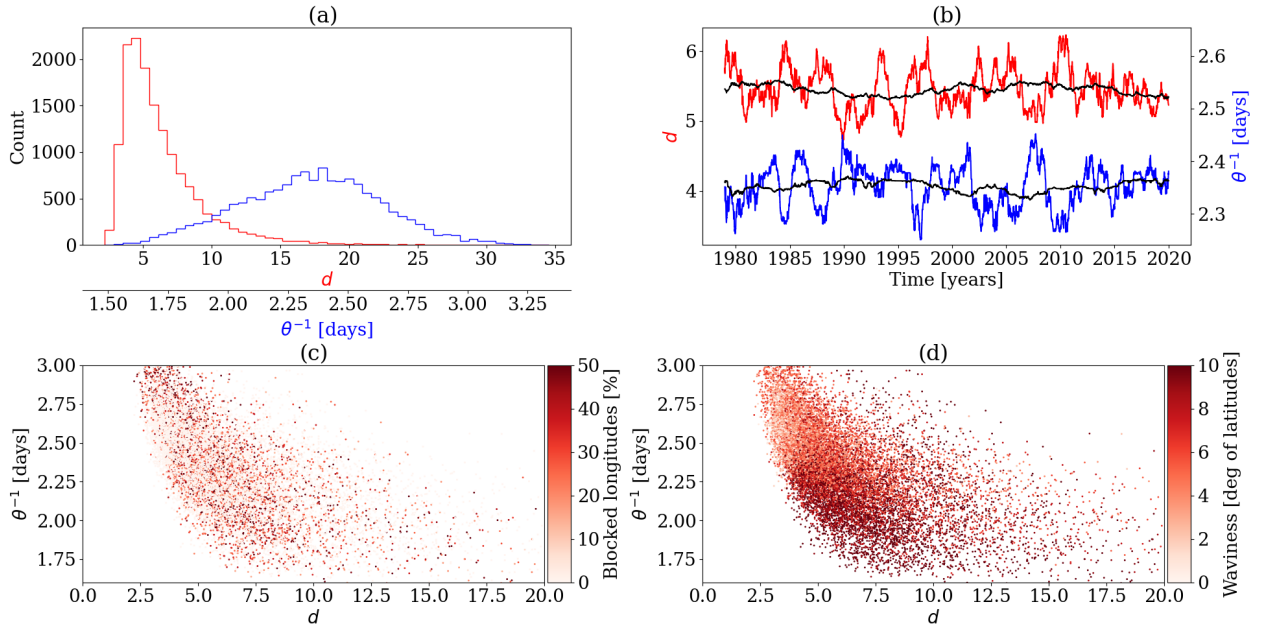
264 The behavior of the blocked jet in Fig. 6 is consistent with the switching between a zonal jet
265 (mainly in blue) with a meandering jet (in red), the latter displaying large northward and southward



240 **FIG. 4. Analysis of the jet positions using local dynamical system metrics.** (a) Histograms [day counts] of
 241 local dimension d (red) and local persistence θ^{-1} (blue) of the jet positions over the entire Northern hemisphere.
 242 (b) one year rolling median for 1979-2019 time series of d (red) and θ^{-1} [days] (blue). Black curves indicate a
 243 ten years rolling median. (c) scatter-plot of $d-\theta^{-1}$ [days] computed on the Northern hemispheric jet positions.
 244 Colorscale indicates the percentage of large-scale blocked longitudes. (d) Same as in (c) with waviness (computed
 245 as the horizontal jet-position standard deviation) in colors.

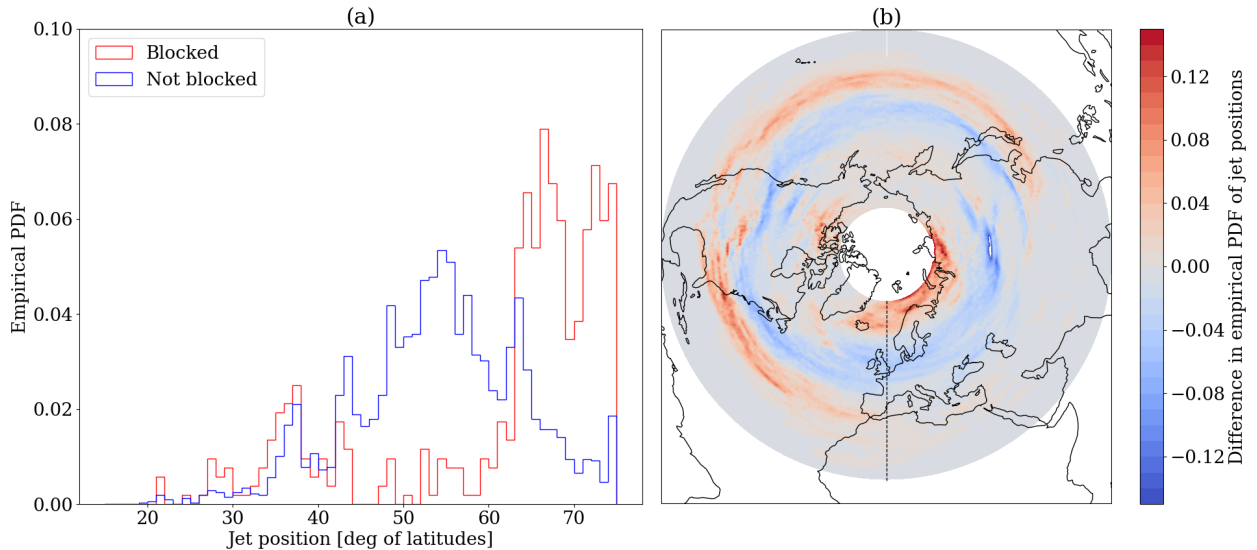
266 excursions when it is diagnosed as blocked. One should note that, when using the blocking index
 267 of Tibaldi and Molteni (1990), the preferred jet position is different from one longitude to another:
 268 northward (resp. southward) in the east (resp. west) of the Atlantic basin. Figure 6 shows that
 269 blocking situations are associated with anomalous jet positions, either northward or southward.
 270 Thus the jet position can be used as an alternative index of a blocked situation, a point on which
 271 we will build our 1D model.

277 The analysis reported in Figure 6 is performed with 0.25° increments in longitude. If we consider
 278 the differences between jet position densities conditional on blocking at longitude 0° only and not
 279 blocked at each longitude, the structure close to 0° is very similar, i.e. with a large northward
 280 meander (not shown).



246 **FIG. 5. Analysis of the jet positions over the Euro-Atlantic sector using local dynamical system metrics.**
 247 (a) Histograms [day counts] of local dimension d (red) and local persistence θ^{-1} (blue) of the jet positions over
 248 the Euro-Atlantic sector (45°W-45°E). (b) one year rolling median for 1979-2019 time series of d (red) and θ^{-1}
 249 [days] (blue). Black curves indicate a ten years rolling median. (c) scatter-plot of d - θ^{-1} [days] computed on
 250 the Euro-Atlantic sector (45°W-45°E) jet positions. Colorscale indicates the percentage of large-scale blocked
 251 longitudes. (d) Same as in (c) with waviness (computed as the horizontal jet-position standard deviation) in
 252 colors.

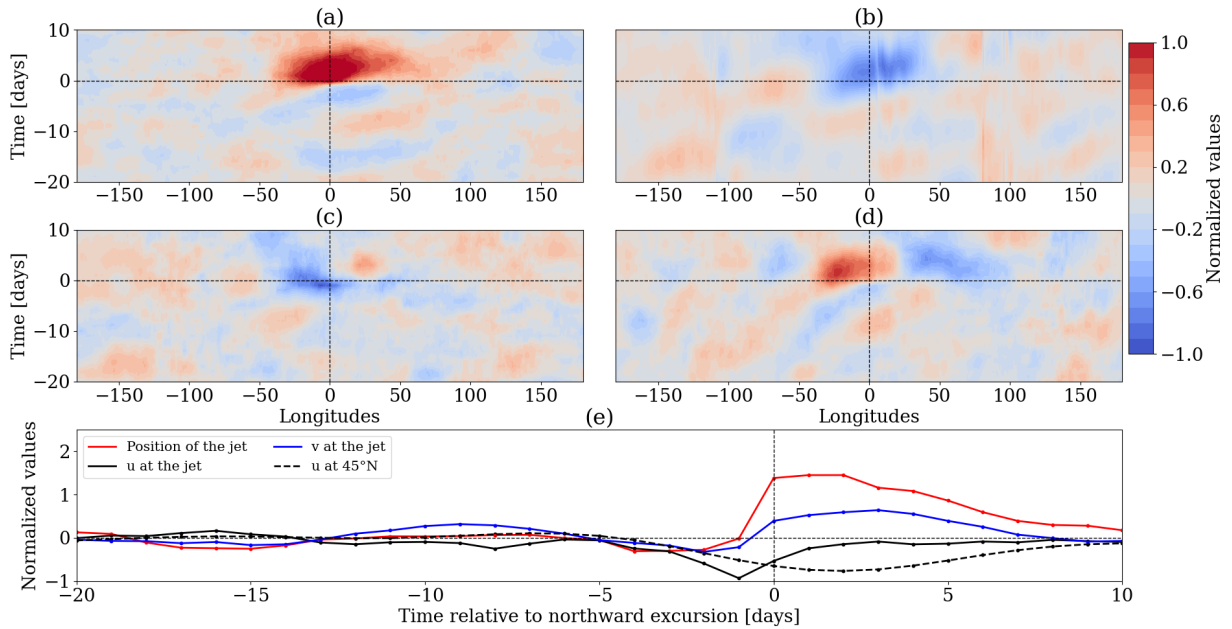
281 Since we can use the latitudinal position of the jet as a relevant index of a blocked situation, we
 282 study the behavior of the jet before and after large northward and southward excursions. We focus
 283 on the behavior at lon=0°, but the results are similar when we take another longitude that is often
 284 blocked, such as 180°W (not shown). We look at events of northward (southward) excursions of
 285 the jet at lon=0° by imposing that the jet position is above (below) the 85% (15%) quantile of the
 286 jet position at lon=0° for at least 3 consecutive days. These quantiles are chosen to have a sufficient
 287 pool of data. The results are not sensitive to a change to the 80% or 90% quantiles (and 20% and
 288 10% quantiles, respectively). We then look at the behavior from 20 days before to 10 days after
 289 the northward (southward) excursion.



272 **FIG. 6. Jet position and blocking events.** (a) Empirical probability density of jet positions at lon=0° in the
 273 case of blocked and non blocked situations for winter (1979-2019) using Tibaldi and Molteni (1990) large scale
 274 blocking index and (b) probability density difference of jet positions in the case of blocked and non blocked
 275 situations (red indicates a greater probability of jet position at this latitude in the case of large scale blocking) for
 276 all longitudes. The dashed line in panel (b) indicates the cross section at which panel (a) is taken.

296 Figures 7 and 8 show the composites of northward and southward excursions, containing respec-
 297 tively $n = 73$ and $n = 79$ events. We display the results for four variables: jet latitudinal position,
 298 zonal wind speed at 45°N, zonal and meridional wind speed at the jet. All variables are normalized
 299 by subtracting their averages and dividing by their standard deviations over all winters longitude
 300 by longitude.

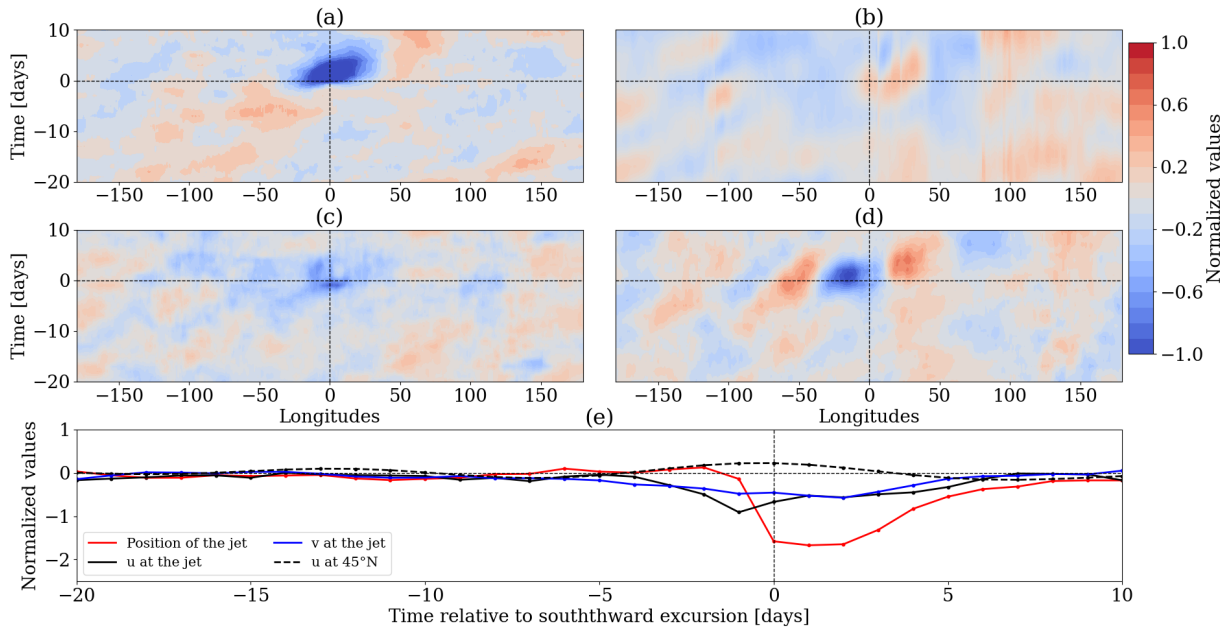
307 Figure 7 panel (a) shows that the northward excursion that we diagnosed has temporal and spatial
 308 extensions that are typical of blocking events (respectively 8–9 days and 40° of longitudes, see
 309 Lupu (2020)). Panel (d) also shows the positive anomaly east of 0° and negative anomaly west
 310 of 0° of meridional wind, so that the structure of the jet looks like so-called omega blocks. We
 311 also note on the meridional plot a teleconnection 20 days before the blocking events at 100°W.
 312 This longitude corresponds roughly to the location of the Rocky mountains over North America
 313 and it is well known that the presence of these mountains intercepting the jet stream can trigger
 314 Rossby waves which then travels across the North Atlantic and break near lon= 0° (Kalnay-Rivas



290 **FIG. 7. Composite behavior during northward excursions of the jet.** Composite mean of $n=73$ events of
 291 northward jet excursions at $\text{lon}=0^\circ$ during the winter season: (a) jet position, (b) zonal wind speed at 45°N , (c)
 292 zonal wind speed at the jet u_J , (d) meridional wind speed at the jet v_J and (e) cross section of the previous figures
 293 at $\text{lon}=0^\circ$ (dashed vertical line). All variables are normalized by subtracting their time averages and dividing by
 294 their standard deviations. The dashed horizontal lines on panel (a)-(e) show the moment when the northward
 295 excursion occurs.

315 and Merkin 1981). However, the anomaly is weakly positive so that it is not really possible to
 316 validate this explanation with our data.

317 The zonal wind speed at the jet decreases 2–3 days before the northward excursions, whereas
 318 the jet position displays no sign of change even the day before its large increase (which means that
 319 the "jump" of the jet to a northern position happens at a time scale smaller than a day). The cross
 320 section displayed in Figure 7 panel (e) further shows that the zonal wind speed at the jet has a
 321 fairly different behavior than the zonal wind speed at 45°N . The former decreases by one standard
 322 deviation before the northward jet excursion before reaching the average within 2 days after the
 323 jump, whereas the latter decreases when the jet jumps and remains negative 5–10 days after that,
 324 which is coherent with a northern position of the strong zonal winds of the jet.



301 **FIG. 8. Composite behavior during southward excursions of the jet.** Composite mean of $n = 79$ events of
 302 southward jet excursions at $\text{lon}=0^\circ$ during the winter season: (a) jet position, (b) zonal wind speed at 45°N , (c)
 303 zonal wind speed at the jet u_J , (d) meridional wind speed at the jet v_J and (e) cross section of the previous figures
 304 at $\text{lon}=0^\circ$ (dashed vertical line). All variables are normalized by subtracting their time averages and dividing by
 305 their standard deviations. The dashed horizontal lines on panel (a)-(e) show the moment when the southward
 306 excursion occurs.

325 To ensure that this excursion corresponds to a blocking event, we display in Figure 9 the composite
 326 anomaly of zonal wind, jet position and geopotential height at 500 hPa (Z500) from 4 days before
 327 to 4 days after the northward excursion of the jet over the entire northern hemisphere. The strong
 328 negative anomaly of zonal wind over Europe, the positive anomaly of zonal wind between Iceland
 329 and Norway and the characteristic wave breaking footprint of Z500 isolines are all elements that
 330 show a switch from a mostly zonal to a mostly meridional jet, therefore a blocked state. This
 331 mostly meridional state of the jet is confirmed by Figure 10 which is similar to Figure 9 with the
 332 meridional wind speed in colors.

333 Figure 8 displays different results from Figure 7 for southward excursions of the jet at 0° .
 334 First, the spatial and temporal extensions of the southward excursions are a bit smaller than for
 335 the northward excursions. Secondly, the structure of the meridional wind after the excursion is

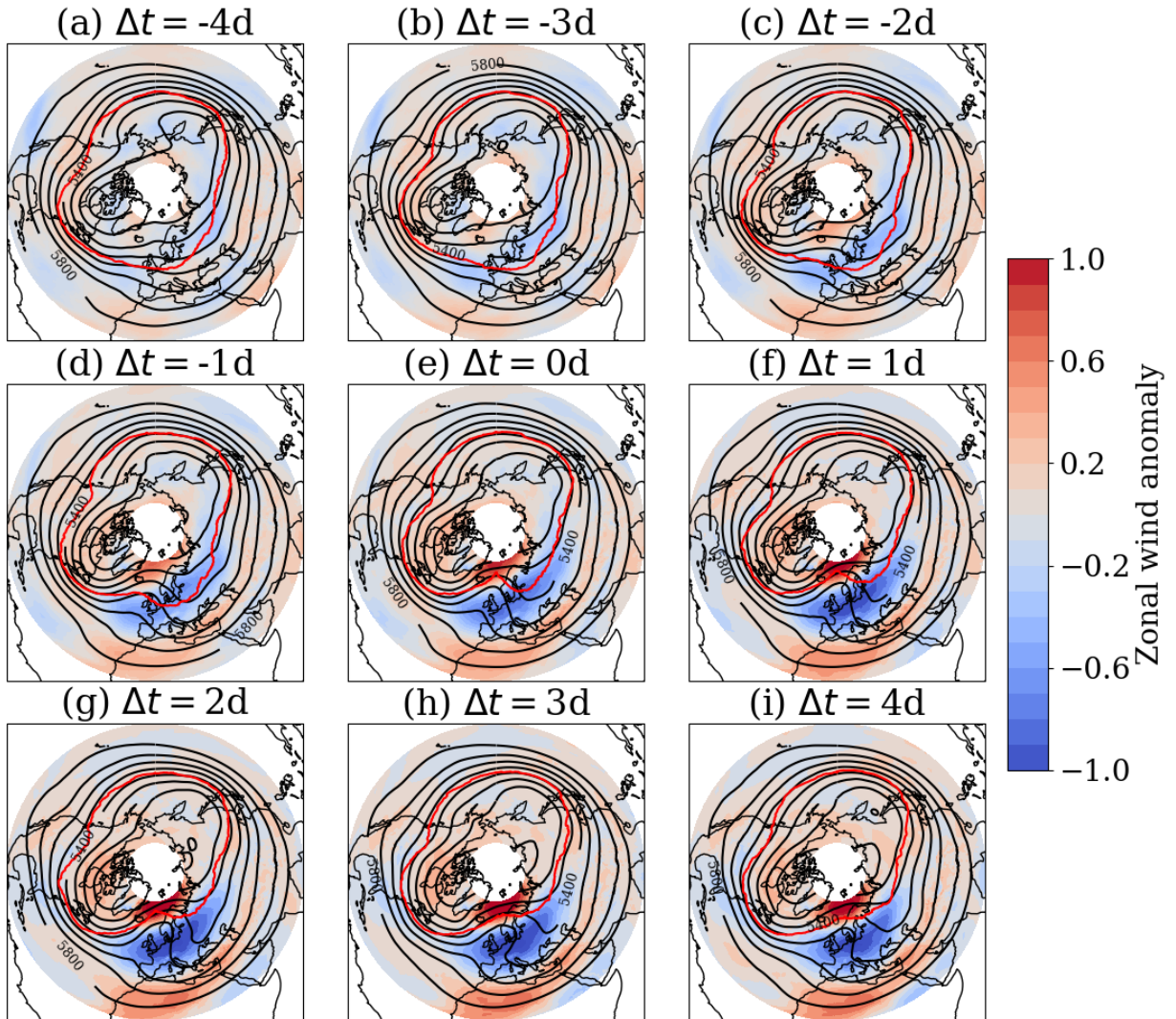
336 not dipolar (negative anomaly – positive anomaly) but tripolar: with a positive meridional wind
 337 anomaly approximately 70° of longitudes west of 0° , then a negative anomaly west of the excursion
 338 and finally a positive anomaly east of the excursion. Finally, zonal wind speed at 45°N does not
 339 decrease but rather increases during the excursion, and the amplitude of the anomaly is not as
 340 big. The common point is that there is also a weakening of the zonal wind speed at the jet 2–3
 341 days before the excursion. Figures 11 and 12 confirm this analysis for the behavior of the jet over
 342 all the northern hemisphere during large southward excursions of the jet. The structures displayed
 343 in these figures are rather different from the traditional omega blocking shape, as if the blocking
 344 event was occurring 15° west of the central longitude 0° and the southward excursion of the jet at
 345 0° was a consequence of this more western blocking. The structure seems to be more similar with
 346 the so-called Atlantic blocking pattern (Vautard 1990).

363 To check that the observed slowdown in the zonal wind speed before large northward and
 364 southward excursions of the jet is statistically significant, we compute the 95% confidence interval
 365 using 1000-iterations bootstrap on the sample of events. The results are displayed in Figure 13 and
 366 show the significance of this precursory slowdown.

370 *c. Point stochastic model*

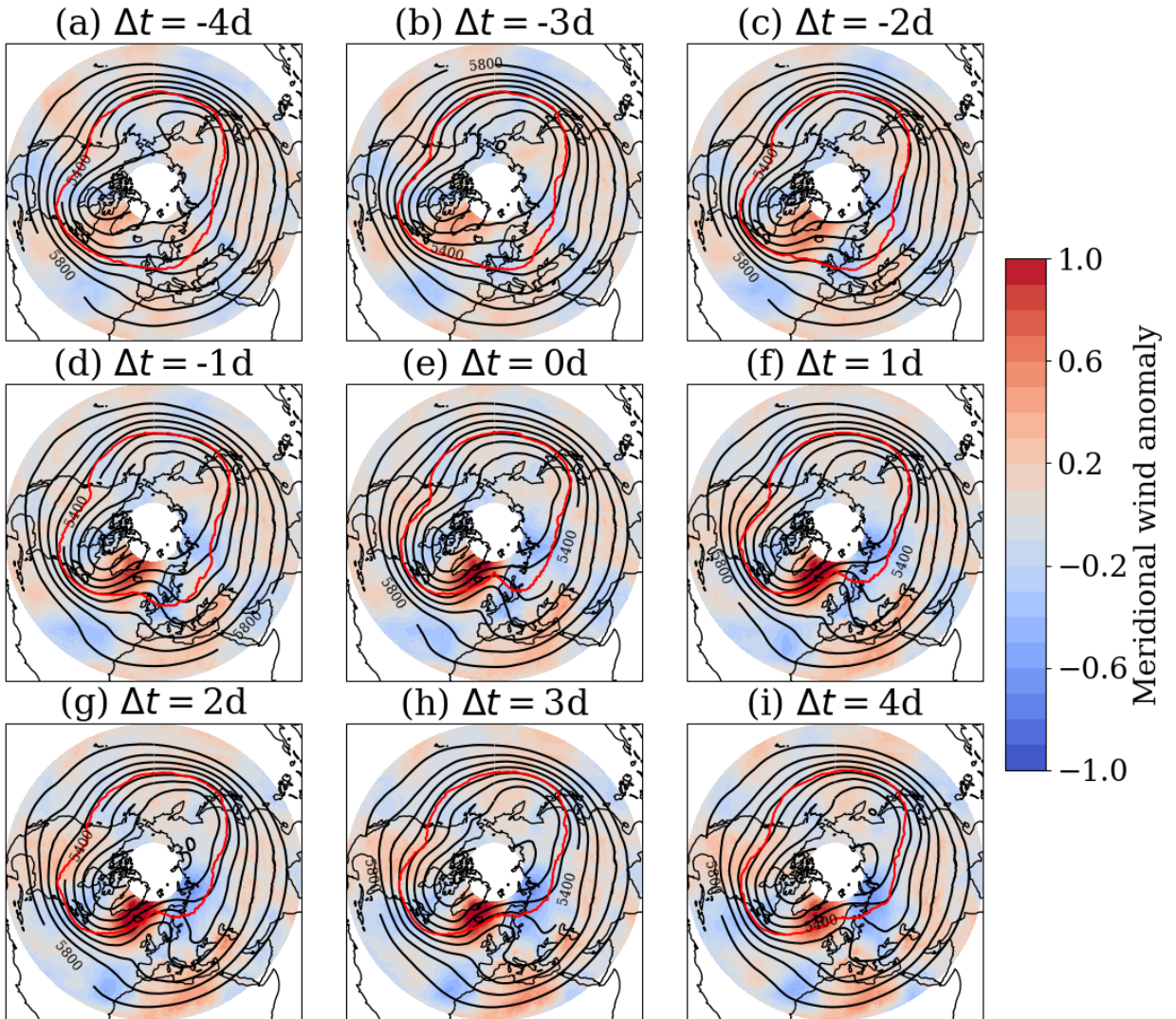
371 We have shown that strong excursions of the jet, either in the northward or southward direction,
 372 can be characterized as blocking events. Moreover, we have shown that large "jumps" of the jet
 373 from a central position to either the north or south, leading to blocking events, are closely linked
 374 to a decrease in the zonal wind speed measured at the jet position 2 to 3 days before the jump,
 375 as is found by the literature: Woollings et al. (2018b) showed that a decrease in the zonal wind
 376 is associated with a higher variability of the jet, and Nakamura and Huang (2018) showed that
 377 blocking events are characterized by high values of local wave activity (Huang and Nakamura
 378 2016) which is itself negatively correlated with zonal wind speed. As we want to derive a 1D
 379 model for the onset and decay of blocking events on the jet, we focus on the behavior of the zonal
 380 wind speed at the jet u_J . In the following, apart when specified so, all analyses are made with
 381 winter data.

386 To do so, we plot in Figure 14 panel (a) the phase portrait of the zonal wind speed u_J at the
 387 jet. The temporal derivative $\frac{du_J}{dt}$ is computed as $\frac{du_J}{dt} = u_J(t+1) - u_J(t)$, as dt is set to be equal



347 FIG. 9. **Composite behavior of zonal wind (normalized) and geopotential height (m) during northward**
 348 **excursions over the Northern Hemisphere.** Composite anomaly of zonal wind (colors), jet position (red line)
 349 and Z500 (black lines) from 4 days before to 4 days after the northward excursion of the jet (n=73 events). Z500
 350 lines are traced every 100m from 5000 m to 6000 m.

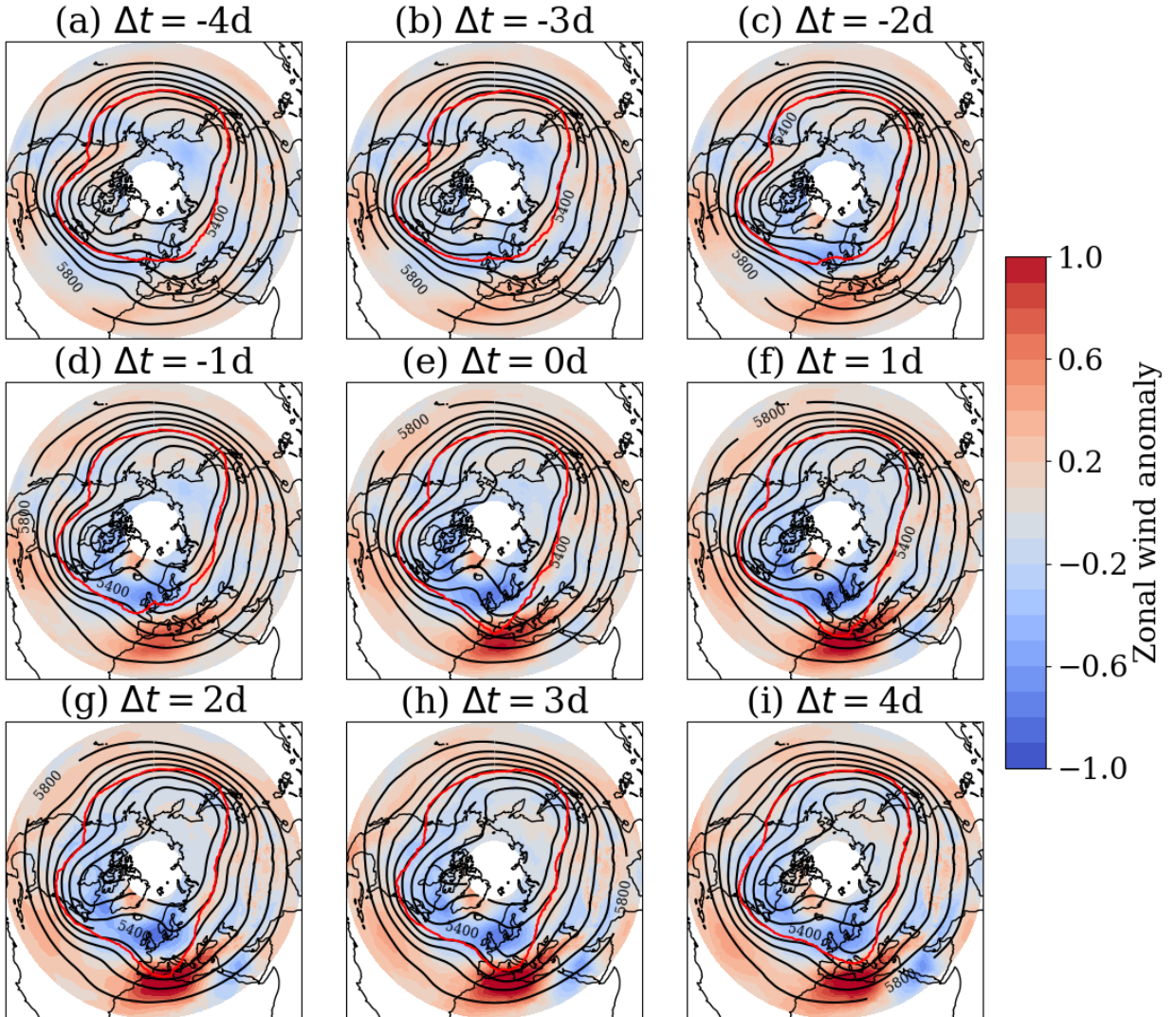
388 to 1 day. Apart from outliers, the bulk of points is located within the interval $[-2, 2]$ and has an
 389 elliptic shape, so that u_J seems to display an oscillator-like structure. Figure 14 panel (b) shows
 390 the relation between $\frac{d^2 u_J}{dt^2}$ and u_J . For an oscillator, either linear or non-linear, we would have
 391 $\frac{d^2 u_J}{dt^2} = F(u_J)$ with $x F(x) < 0$. In this analysis the double temporal derivative is computed as:



351 FIG. 10. Composite behavior of meridional wind (normalized) and geopotential height (m) during
 352 northward excursions over the Northern Hemisphere. Composite anomaly of meridional wind (colors), jet
 353 position (red line) and Z500 (black lines) from 4 days before to 4 days after the northward excursion of the jet
 354 ($n = 73$ events). Z500 lines are traced every 100m from 5km to 6km.

$$\frac{d^2 u_J}{dt^2} = u_J(t+1) - 2u_J(t) + u_J(t-1).$$

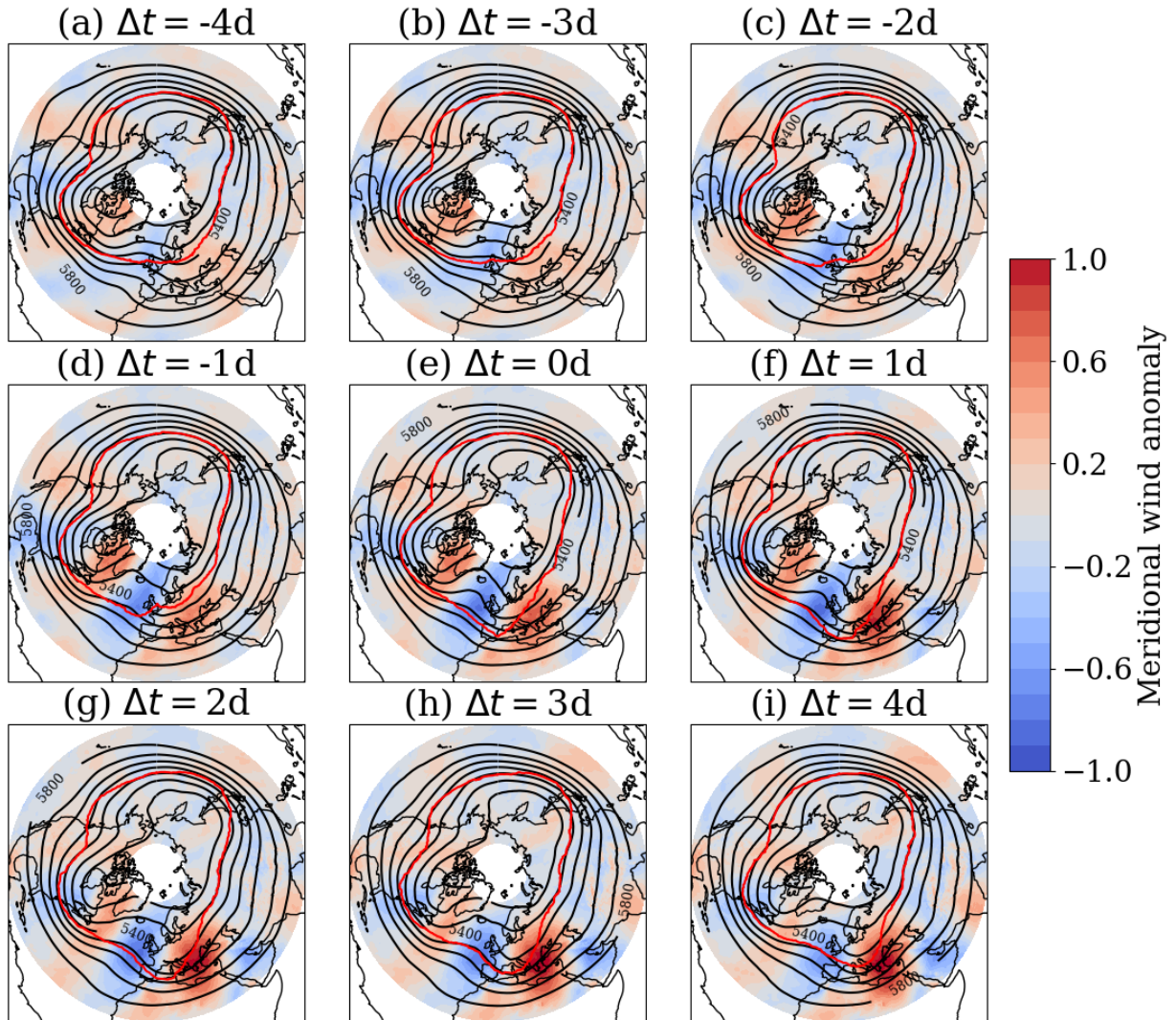
392 Again, there are several outliers in the data points, but we recall that approximately 3500 points
 393 are drawn in this figure, so that the vast majority of them is located within the interval $[-2, 2]$. It



355 FIG. 11. Composite behavior of zonal wind (normalized) and geopotential height (m) during southward
 356 excursions over the Northern Hemisphere. Composite anomaly of zonal wind (colors), jet position (red line)
 357 and Z500 (black lines) from 4 days before to 4 days after the southward excursion of the jet ($n = 79$ events). Z500
 358 lines are traced every 100m from 5000 m to 6000 m.

394 is also clear that there is a strong asymmetry relative to the y -axis. While many functions would
 395 fit those data, we choose an exponential-like model

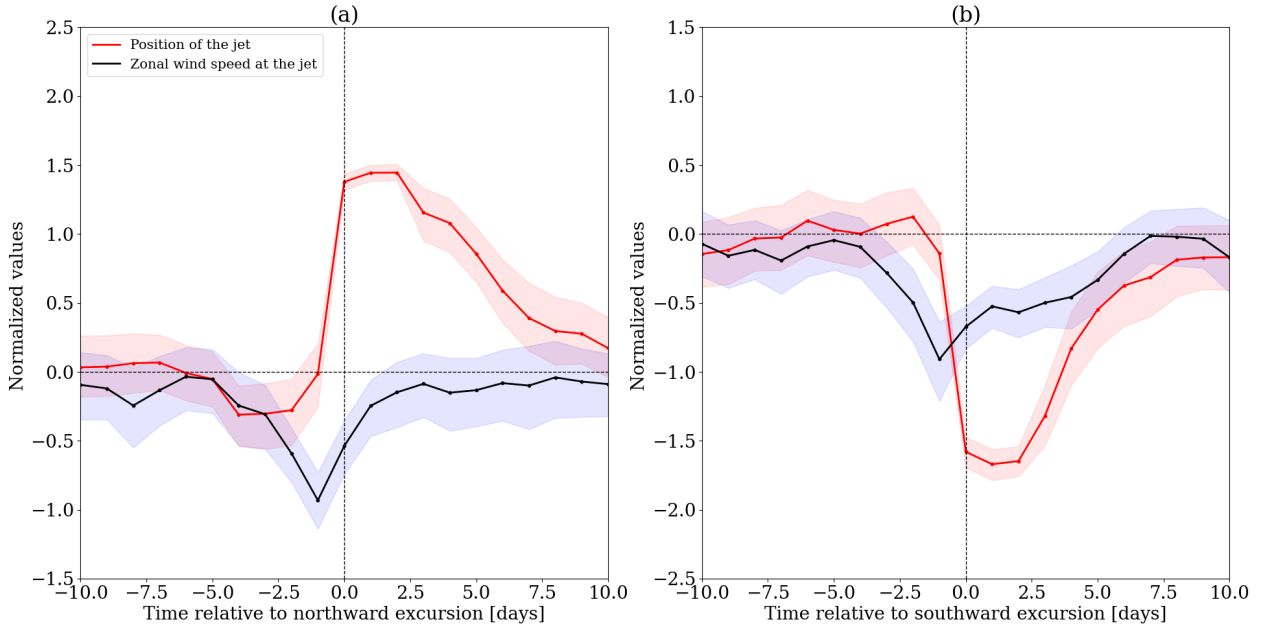
$$F_{a,b}(x) = a(e^{-bx} - 1).$$



359 FIG. 12. **Composite behavior of meridional wind (normalized) and geopotential height (m) during**
 360 **southward excursions over the Northern Hemisphere.** Composite anomaly of meridional wind (colors), jet
 361 position (red line) and Z500 (black lines) from 4 days before to 4 days after the southward excursion of the jet
 362 ($n = 79$ events). Z500 lines are traced every 100m from 5000 m to 6000 m.

396 We fit the parameters by minimizing the Euclidean distance between the model and the data. We
 397 find parameter value $a = 0.27$ and $b = 0.77$. To alleviate notations, we will write simply u (rather
 398 than u_J) the zonal wind speed at the jet in the remainder of the article.

399 The proposed model corresponds to a non-linear oscillator evolving in a potential well $V_{a,b}(x) =$
 400 $a\left(\frac{1}{b}e^{-bx} - x\right)$, so that its total energy $E = \frac{1}{2}\left(\frac{du}{dt}\right)^2 + V(u)$ is preserved. However, a standard

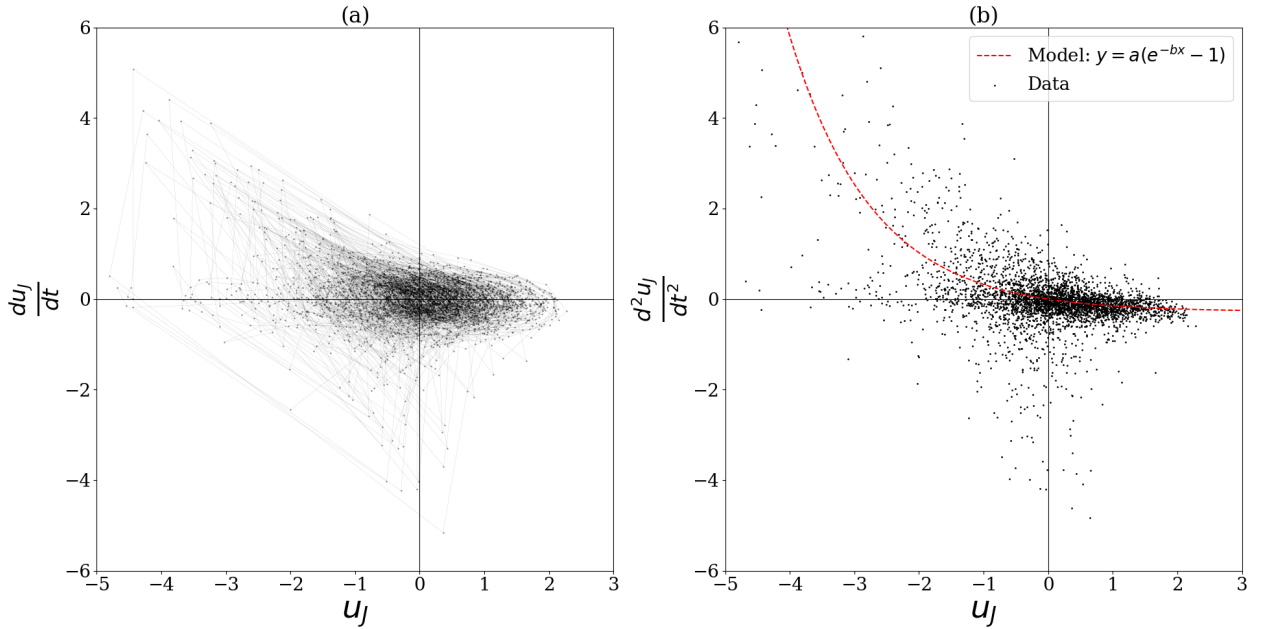


367 FIG. 13. Composite of jet position (red) and zonal wind speed at the jet (blue) for large (a) northward and
 368 (b) southward jet excursions (respectively $n = 73$ and $n = 79$). The shaded areas represent the 95% confidence
 369 interval computed using 1000-iterations bootstrap on the sample of events.

401 oscillator should display regular oscillations, which is not the case in Figure 14. Therefore, we add
 402 a stochastic term to represent perturbations. In the end, we propose the following model for the
 403 behavior of jet zonal wind speed u at the jet at lon= 0° :

$$\frac{d^2u}{dt^2} = F_{a,b}(u) + \eta(t) - \alpha \frac{du}{dt}, \quad (2)$$

404 where $F_{a,b}$ is the function defined above, η is a white noise with standard deviation σ and $-\alpha \frac{du}{dt}$
 405 is a damping term added to avoid instability (η brings too much energy in the system). Figure 15
 406 panel (a) displays 3 winters example (2000-2003), comparing the normalized zonal wind speed
 407 (black) and the proposed stochastic model (red). The model is integrated using a fourth-order
 408 Runge-Kutta scheme with a time step of 0.1 day and the forcing is applied at every time step.
 409 We found $\sigma = 0.35$ and $\alpha = 0.1$ to fit reasonably well the data by a semi-objective inspection of
 410 the results without performing an objective parameter optimisation, that would be left for a future
 411 study. Figure 15 panel (c) displays the comparison between the two histograms. The model is
 412 very close to the data in the range $[-2, 2]$, but overestimates the proportion of high winds ($u > 2$)

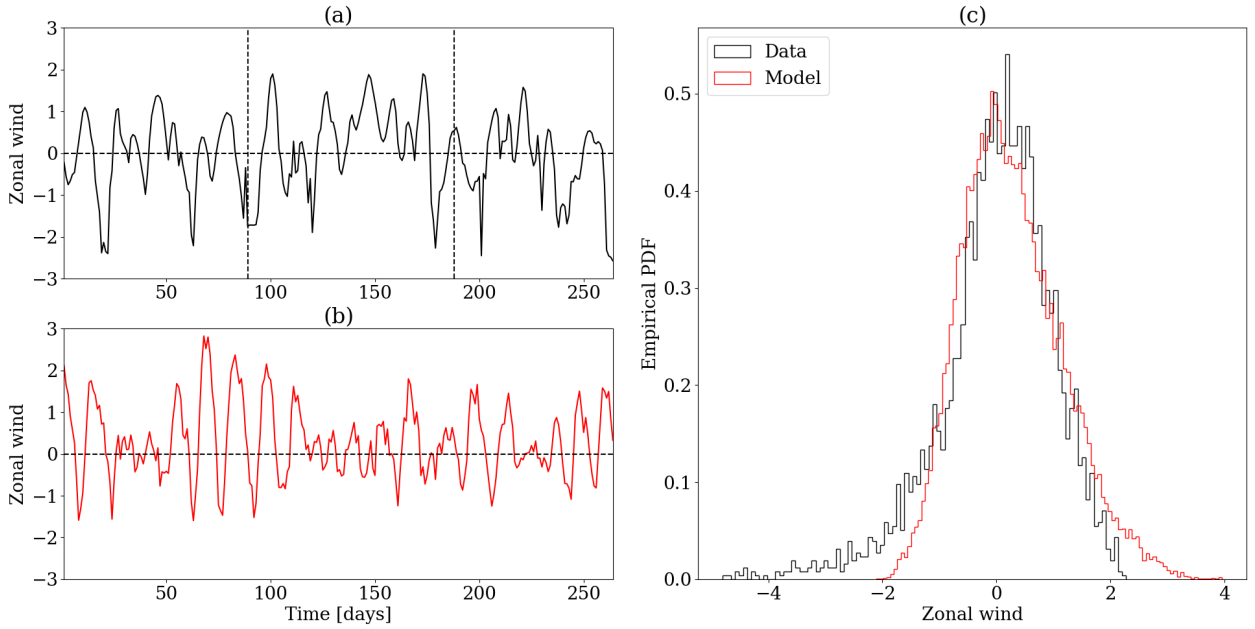


382 **FIG. 14. Dynamical behavior of zonal wind speed at the jet.** (a) Phase portrait of zonal wind speed at the jet
 383 u_J at $\text{lon}=0^\circ$ and (b) relation between $\frac{d^2 u_J}{dt^2}$ and u_J at $\text{lon}=0^\circ$ (points) and proposed model (red). The red model is
 384 fitted using points such that $|u_J| < 2.5$ and $\frac{d^2 u_J}{dt^2} > -0.8$ to remove outliers. By minimizing the euclidean distance
 385 between the model and the data we find $a = 0.278$ and $b = 0.771$.

413 and underestimates the proportion of low winds ($u < -2$). This is not an issue because (i) for low
 414 winds, as we have seen above, a slight slowdown of zonal wind is enough to trigger transition to a
 415 blocked state, so that there is no need to go below $u = -2$ — moreover one has to take into account
 416 that those values may be outliers and not a physical feature as we don't look at temporal and spatial
 417 means — and (ii) for high winds, $u > 1$ is already associated with a zonal jet so that when u takes
 418 bigger values there is no further influence on the position of the jet.

424 Since our main purpose is to build a model for blocked states, we propose the following forced
 425 Langevin equation for the jet position X at $\text{lon}=0^\circ$:

$$\frac{dX}{dt} = -\beta X + F(X, u) + \eta'(t), \quad (3)$$

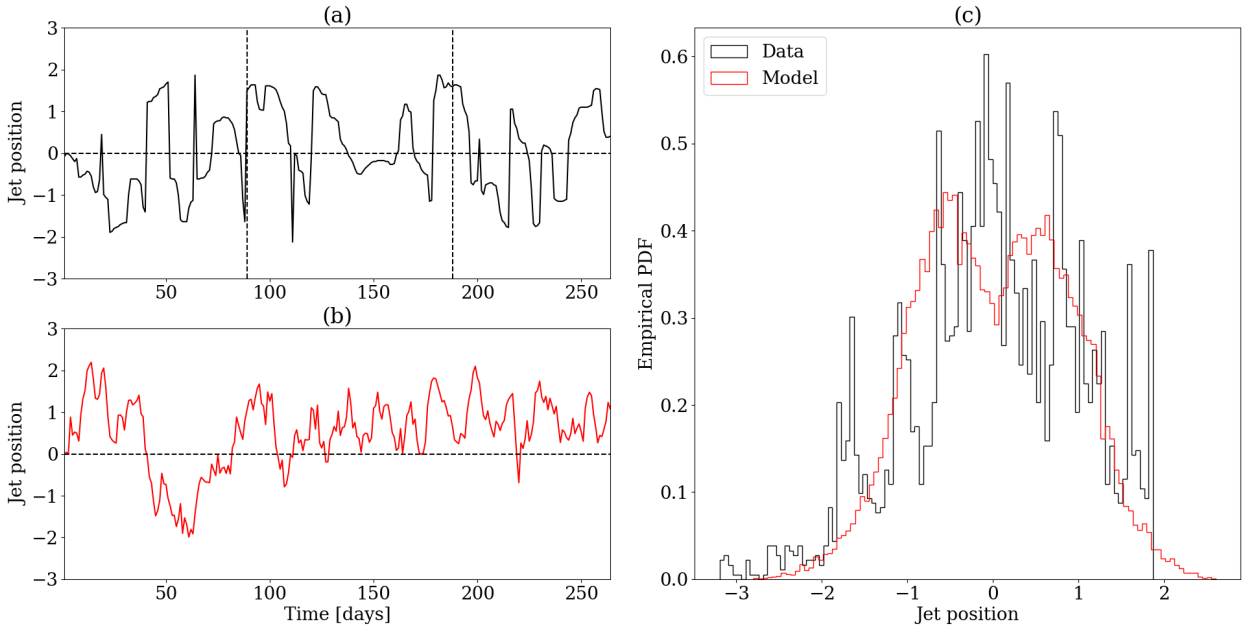


419 **FIG. 15. Point model for the zonal wind speed at the jet.** (a) Normalized zonal wind speed at $\text{lon}=0^\circ$ over
 420 three winters (2000-2003) (black) and proposed stochastic model (red) and (b) comparison of the histograms for
 421 the data (black) and the model (red). Vertical black dashed lines in panel (a) represent the limits between the
 422 three winters for the data, so that there is no continuity in the black lines at those times. The model is computed
 423 at a 0.1 day time step over 10 years but is plotted every 10 time steps (one day) to be comparable to the data.

426 where β is fitted to Figure 13 so that we find $\beta^{-1} = 10$ days, $F(X, u)$ is a forcing term defined as
 427 follows:

$$F(X, u) = \begin{cases} C(|u| - |X|)\text{sign}(X) & \text{if } u < 0 \text{ and } |X| < |u|, \\ 0 & \text{otherwise,} \end{cases} \quad (4)$$

428 so that the forcing is null when the jet zonal wind speed is strong ($u > 0$) and is linearly forced
 429 when the jet zonal wind speed is weak ($u < 0$), which is consistent with the negative correlation
 430 between the jet zonal wind speed and the jet position variability investigated by Woollings et al.
 431 (2018b) (see in particular their Figure 1). Multiplying by the sign of X , combined with the random
 432 perturbation, allows the model to display northward and southward excursions of the jet. The
 433 rationale of this operation is that if the jet is already anomalously high/low, it tends to stay there.
 434 The condition $|X| < |u|$ allows to take into account the fact that the forcing is asymmetric, as can



445 **FIG. 16. Point model for the jet position.** (a) Normalized jet position at $\text{lon}=0^\circ$ over three winters (2000-2003)
 446 (black) and proposed stochastic model (red) and (b) comparison of the histograms for the data (black) and the
 447 model (red). Vertical black dashed lines in panel (a) represent the limits between the three winters for the data,
 448 so that there is no continuity in the black lines at those times. The model is computed at a 0.1 day time step over
 449 10 years but is plotted every 10 time steps (one day) to be comparable to the data.

435 be seen in Figure 13: when u is decreasing the jet latitude drifts away from the central position,
 436 but the relaxation period when u increases again does not see immediately a come back of the jet
 437 to its central position. We choose $C^{-1} = 1$ day to be the time scale of increase for the forcing, as
 438 in figure 13. Finally η' is a white noise with standard deviation $\sigma' = 0.35$ as for u . The proposed
 439 model is close to a modified red noise model: $X_{t+1} = \mu_t + \phi X_t + \epsilon_t$, where μ_t is the moving average,
 440 the equivalent of our forcing. In our case, with a constant forcing $u < 0$, we would have two
 441 equilibrium positions: $X_{eq}^1 = \frac{C}{\beta+C} |u| \simeq |u|$ and $X_{eq}^2 = -\frac{C}{\beta+C} |u| \simeq -|u|$.

442 The results for the stochastic model are displayed in Figure 16. Even though the jet has a tendency
 443 not to stay at anomalously high or low positions as in the data, the model is able to reproduce the
 444 large and sudden shifts of the jet.

450 To conclude this section, we note that the analysis provided here using the zonal wind "sitting"
 451 on the jet is different from that one obtained at a fixed latitude, for example at 45°N as illustrated in

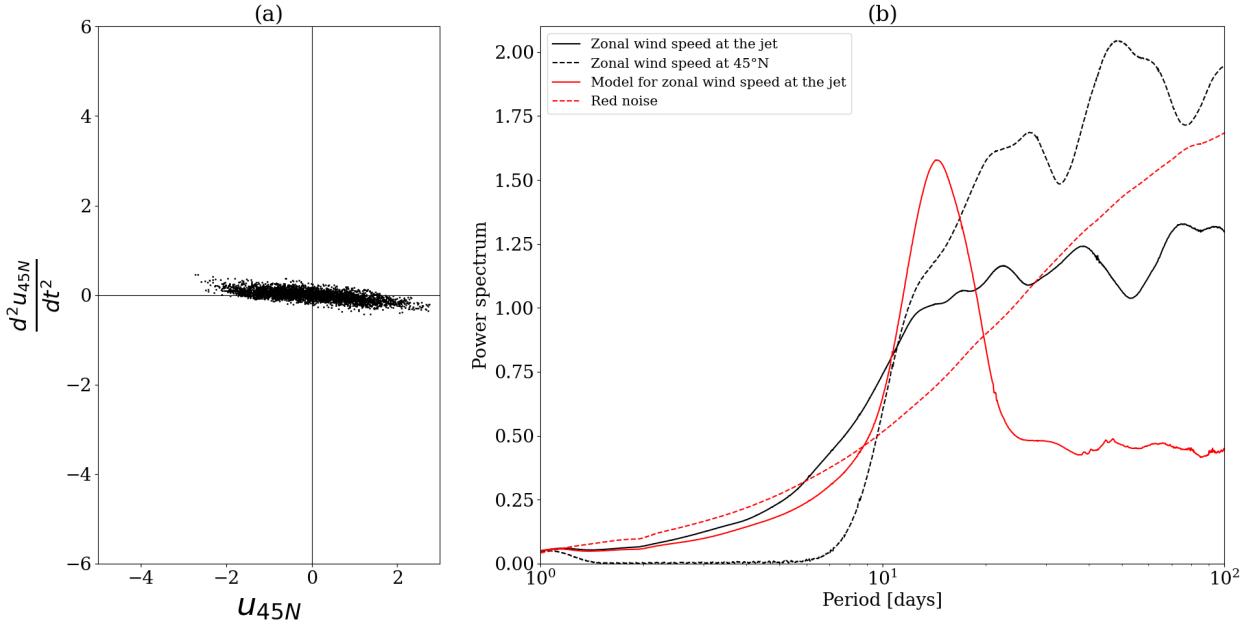
452 Figure 17. Panel (a) shows the relation between $\frac{d^2 u_{45N}}{dt^2}$ and u_{45N} and Figure 17 panel (b) displays
453 the wavelet spectrum of the zonal wind at the jet u , at 45°N u_{45N} , for the model and for a red noise
454 computed using parameters that best fit the data for each winter. For the wavelet spectrum of the
455 data, either at the jet or at 45°N , we used the full data set over the 40 years and then we aggregated
456 the amplitudes for winter times. We show the spectrum only up to a period of 100 days but the
457 main peak of the spectrum is of course located at 365 days. A red noise is fitted for each of the 40
458 winters and the spectrum is computed as the aggregation of the 40 spectra. One striking feature of
459 this spectrum is that, contrary to the zonal wind speed at 45°N which displays two local maxima
460 (at 27 and 49 days), zonal wind speed at the jet displays four local maxima (at 17, 22.5, 38 and 74.5
461 days), which correspond to temporal properties of the series as is shown with the difference with
462 a corresponding red noise spectrum. Those maxima are typical of the time scales associated with
463 the Rossby waves propagating in the atmosphere at these latitudes (Hoskins and James 2014). The
464 spectrum associated with the model does not fit with the one of the data. The main reason for that
465 is that the model is a simple 0D model without spatial interactions. The goal of the next section is
466 therefore to propose a model with spatial interactions that could explain those features.

471 **4. Spatially extended model: coupled non-linear oscillators and Toda lattice**

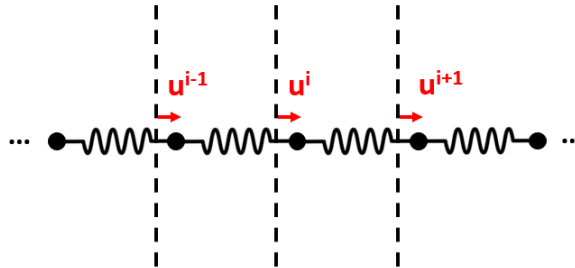
472 In the previous section we proposed a 0D model for the zonal wind speed at the jet u and for
473 the jet latitudinal position X at $\text{lon}=0^\circ$. In this section, we extend this model along all longitudes
474 i . The basic idea is to assume that the jet zonal wind speed at each longitude can be represented
475 as a material point in the potential well $V_{a,b}(x) = a(\frac{1}{b}e^{-bx} - x)$ defined above. Our idea is then
476 to introduce a coupling between those oscillators (see Figure 18). For any oscillator, the force
477 applied by the oscillator on its right is: $-F_{a,b}(u^{i+1} - u^i)$, whereas the force applied on its left is:
478 $F_{a,b}(u^i - u^{i-1})$. So that the dynamics of the oscillator is:

$$\frac{d^2 u^i}{dt^2} = F_{a,b}(u^i - u^{i-1}) - F_{a,b}(u^{i+1} - u^i) = a \left(e^{-b(u^i - u^{i-1})} - e^{-b(u^{i+1} - u^i)} \right) \quad (5)$$

479 and we have a periodic string of oscillators so that: $u^{i+N} = u^i$. This coupled dynamics is known
480 as the Toda lattice and has been used to model interaction between electrons in a solid (Toda
481 1967). The advantage of using a function which gives the Toda lattice is that this system can be



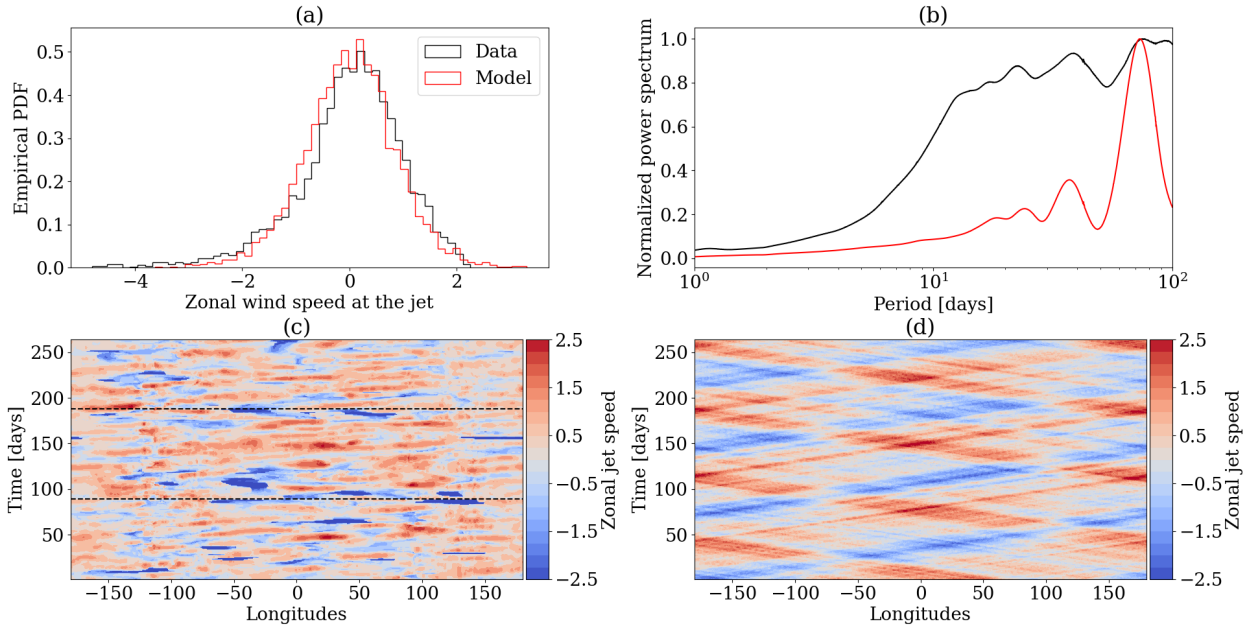
467 FIG. 17. **Comparison of the analysis at the jet and at a fixed latitude.** (a) Relation between $\frac{d^2 u_{45N}}{dt^2}$ and u_{45N}
 468 at $\text{lon}=0^\circ$ and (b) wavelet power spectrum of zonal wind speed at the jet (black plain line), zonal wind speed
 469 at 45°N (black dashed line), model for the zonal wind speed at the jet (red plain line) and a red noise whose
 470 parameters are fitted to the series of zonal wind speed at the jet data (red dashed line).



484 FIG. 18. **Schematic of the spatial model as coupled oscillators.** u^i represents the position of the oscillator i
 485 with respect to its equilibrium position.

482 derived from a Hamiltonian and therefore its total energy is conserved, which makes it very stable,
 483 especially for numerical simulations (Toda 1975).

486 Figure 19 presents the result of this model with 1440 oscillators (the number of longitude grid
 487 points in the data) for $a = 200$ and $b = 2$, initiated with random values taken from a uniform
 488 distribution over $[-c, c]$ with $c = 0.4$ and integrated using a fourth-order Runge-Kutta scheme with



498 **FIG. 19. Unperturbed spatial model for zonal wind speed at the jet.** (a) Histograms at $\text{lon}=0^\circ$ of zonal
 499 wind speed at the jet for the data (black) and the model (red). (b) Normalized wavelet power spectrum for the
 500 data (black) and the model (red). The power spectrum is normalized by dividing by the greatest value of the
 501 spectrum over the range of periods 1-100 days. (c) Hovmöller diagram of zonal wind speed at the jet for the
 502 data, over three winters (2000–2003). Horizontal dashed lines represent the limits between the three winters for the
 503 data, so that there is no continuity at those times. (d) Hovmöller diagram of zonal wind speed at the jet for the
 504 model over three winters. The model is computed at a 0.01 day time step over 10 years but is plotted every 100
 505 time steps (one day) to be comparable to the data.

489 a time step of 0.01 day over 10 years (3.65×10^5 time steps). The values of a , b and c were
 490 found using a trial-and-error procedure to fit the data on the histogram, the wavelet spectrum and
 491 the horizontal wavelength on the Hovmöller diagram. We find that the model succeeds in fitting
 492 those diagrams, especially for the wavelet spectrum, which shows a series of peaks similar to the
 493 one observed in the data (even though the periods corresponding to the peak are not perfectly
 494 identical). Similarly, the model is able to represent propagating waves seen in the data, albeit with
 495 too much emphasis on eastward propagating waves. The values for a and b are different from the
 496 ones proposed in the point model (respectively $a = 0.278$ and $b = 0.771$). This is justified by the
 497 fact that they are used here in a very different settings due to the coupling between the oscillators.

506 As in the model proposed by Faranda et al. (2019), we need to take into account the stochastic
507 disturbances applied by baroclinic and barotropic motions, wave breaking events, convective
508 processes and other sub-grid processes on the jet. As proposed by Barnes and Hartmann (2011)
509 and supported by Woollings et al. (2018b), a purely barotropic model of the atmosphere can
510 represent such a variability of the jet due to barotropic Rossby wave breaking on each of its
511 sides. Therefore the proposed stochastic disturbances could be considered to represent only this
512 mechanism on the jet zonal wind speed variability. For that, similar to Vallis et al. (2004) (see also
513 Paradise et al. (2019) who took a similar approach with the 1D model proposed by Nakamura and
514 Huang (2018)), we apply the following stochastic noise:

$$S^i(t) = \frac{\gamma}{N} \sum_{n=1}^N w_n(t) \cos\left(\frac{2\pi k_n}{L} i + \phi_n(t)\right), \quad (6)$$

515 where γ controls the intensity of the noise, N is the number of horizontal wave numbers on which
516 the system is perturbed, $L = 1440$ is the number of oscillators, k_n are the horizontal wave numbers
517 on which the system is forced and we take $k_n = 20$ to 30 to stick with the typical horizontal scale of
518 weather disturbances (as in Faranda et al. (2019)). Finally, to ensure time consistency, amplitudes
519 $w_n(t)$ and phases $\phi_n(t)$ follow an Ornstein-Uhlenbeck stochastic process that we approximate in
520 discrete time by:

$$\begin{cases} w_n(t+dt) = (1 - e^{-2dt/\tau})^{\frac{1}{2}} Q_n + e^{-dt/\tau} w_n(t), \\ \phi_n(t+dt) = (1 - e^{-2dt/\tau})^{\frac{1}{2}} Q'_n + e^{-dt/\tau} \phi_n(t), \end{cases} \quad (7)$$

521 where τ is the decorrelation time that we choose to be equal to 2 days, $dt = 0.01$ the time step
522 of integration of the model and Q_n and Q'_n random numbers chosen at each time step from a
523 uniform distribution over respectively $[-\delta, \delta]$ with $\delta = 0.1$ and $[-\pi, \pi]$. $w_n(0)$ and $\phi_n(0)$ are taken
524 randomly from a uniform distribution over respectively $[-\delta, \delta]$ with $\delta = 0.1$ and $[-\pi, \pi]$.

525 To ensure that the model is numerically stable, we also add a damping term $-\alpha \frac{du^i}{dt}$, so that in the
526 end, we have:

$$\frac{d^2 u^i}{dt^2} = a(e^{-b(u^i - u^{i-1})} - e^{-b(u^{i+1} - u^i)}) + S^i - \alpha \frac{du^i}{dt}. \quad (8)$$

527 Figure 20 presents the results for the perturbed model with $\gamma = 0.3$ and $\alpha = 0.05$, which we found
528 to be a good compromise to fit the data by a trial-and-error procedure. The model is integrated
529 using a fourth-order Runge-Kutta scheme with a time step $dt = 0.01$ day and the forcing is applied
530 at each time step over a period of 10 years. Apart from the local maximum for periods below 10
531 days in the wavelet spectrum in panel (b), the model reproduces the spectrum observed in the data
532 at $\text{lon}=0^\circ$, especially the shape and the position of the three local maxima for periods greater than
533 10 days. The temporal spectrum for longitudes close to 0° (20°W - 20°E) is similar to the displayed
534 spectrum — with peaks at 23 and 40 days and a trough around 52 days — so that this result is
535 not sensitive to the particular choice of longitudes. Therefore, the temporal waves present in the
536 model are closed to the ones of the data as is also confirmed when comparing panel (c) and panel
537 (d). There are two main differences in those plots: (i) the propagating speed either in the westward
538 or eastward direction is larger in the model than in the data and (ii) the temporal spectrum displays
539 local maxima for periods below 10 days, which correspond to the added perturbation S^i .

548 Finally, we propose to adapt the 0D forced Langevin equation of the previous section to model
549 the latitudinal position of the jet:

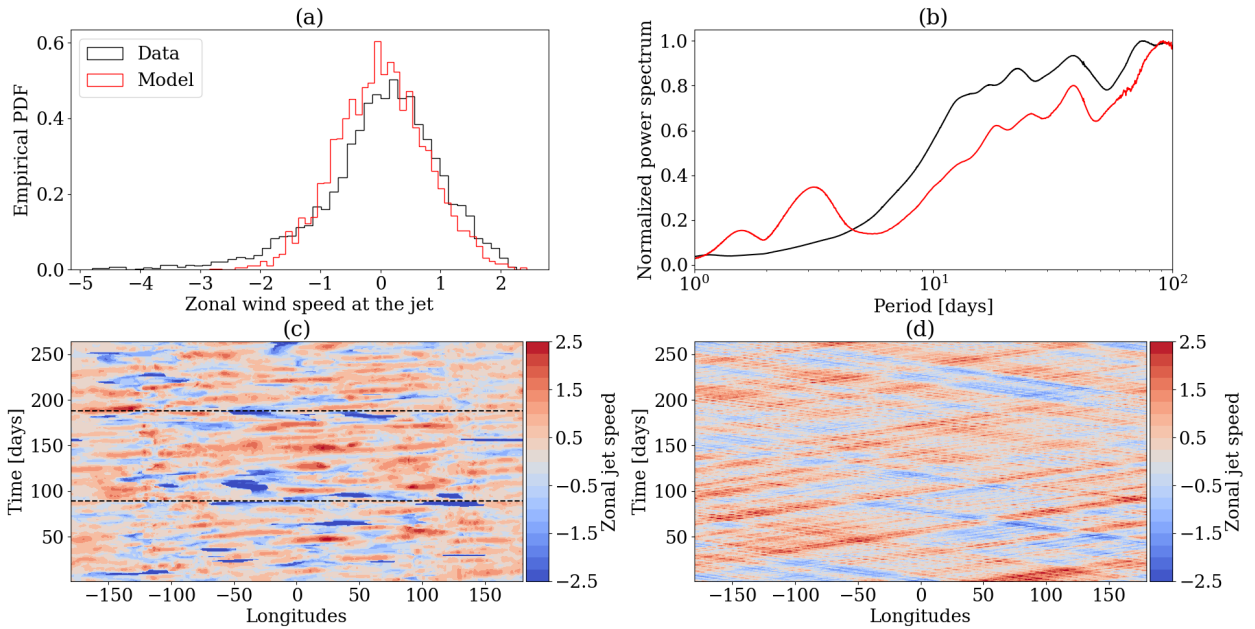
$$\frac{dX^i}{dt} = -\beta X^i + F(X^i, u^i) + \hat{S}^i + D(X^{i+1} - 2X^i + X^{i-1}), \quad (9)$$

550 where $\beta^{-1} = 10$ days, $F(X^i, u^i)$ is the same forcing as previously:

$$F(X^i, u^i) = \begin{cases} C(|u^i| - |X^i|)\text{sign}(X^i) & \text{if } u^i < 0 \text{ and } |X^i| < |u^i|, \\ 0 & \text{otherwise,} \end{cases} \quad (10)$$

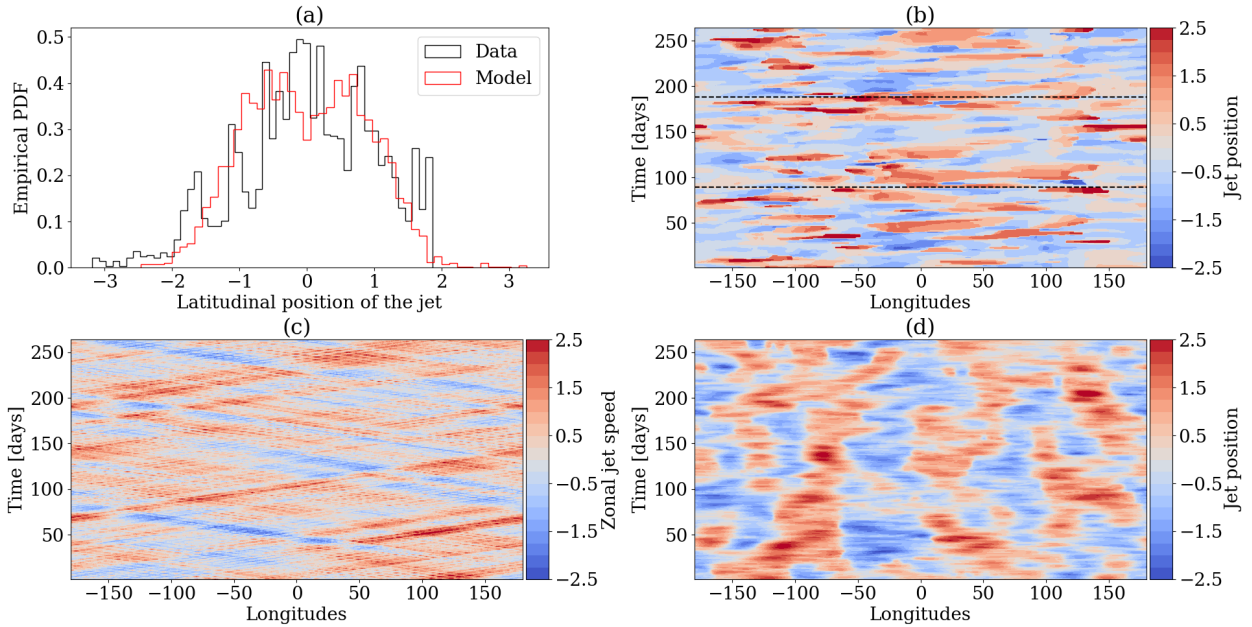
551 where $C = 1$, \hat{S}^i is a source term defined the same way as S^i above but we take $\gamma = 0.6$ and $k_n = 2$
552 to 8 to take into account the greater spatial coherence of the jet position than the zonal wind speed
553 and the last term on the right hand side is a diffusion term, to smooth the position of the jet, with
554 $D = 20$. Again, the model is integrated using a fourth-order Runge-Kutta scheme with a time step
555 $dt = 0.01$ day and the forcing is applied at each time step over a period of 10 years.

563 Figure 21 presents the results for the spatial model of the jet position. On Figure 21 panel (a) we
564 present the comparison between the histograms of the model and the data at $\text{lon}=0^\circ$, whereas panels
565 (b), (c) and (d) present Hovmöller diagrams of jet position for the data, for jet zonal wind speed



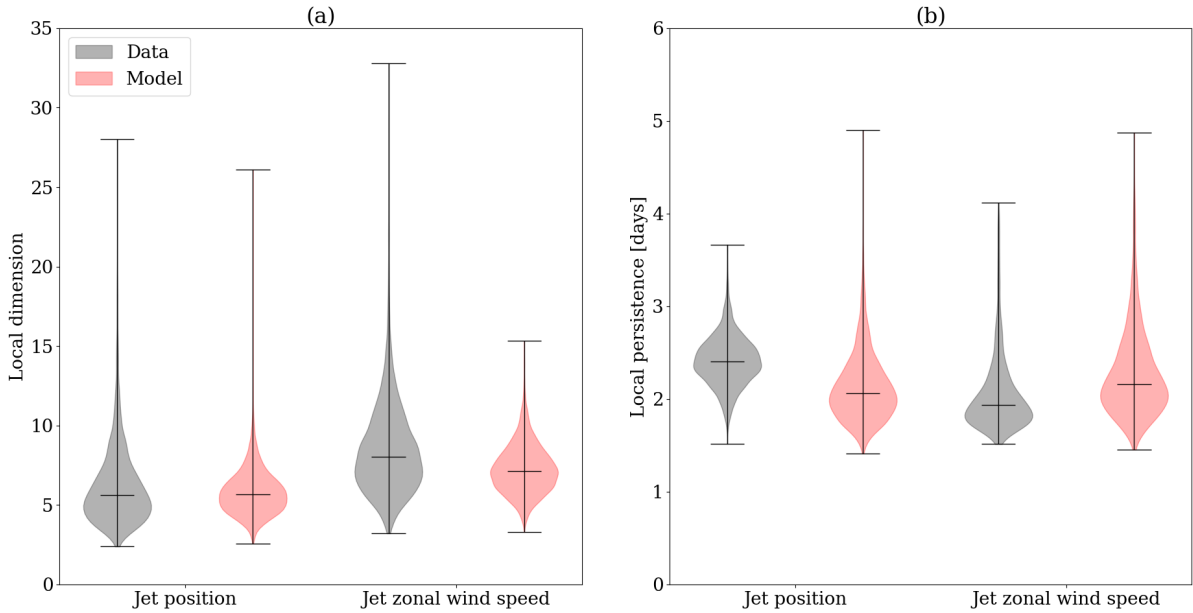
540 **FIG. 20. Perturbed spatial model for zonal wind speed at the jet.** (a) Histograms at $\text{lon}=0^\circ$ of zonal wind
 541 speed at the jet for the data (black) and the model (red). (b) Normalized wavelet power spectrum for the data
 542 (black) and the model (red). The power spectrum is normalized by dividing by the greatest value of the spectrum
 543 over the range of periods 1-100 days. (c) Hovmöller diagram of zonal wind speed at the jet for the data over three
 544 winters (2000-2003). Horizontal dashed lines represent the limits between the three winters for the data, so that
 545 there is no continuity at those times. (d) Hovmöller diagram of zonal wind speed at the jet for the model over
 546 three winters. The model is computed at a 0.01 day time step over 10 years but is plotted every 100 time steps
 547 (one day) to be comparable to the data.

566 of the model and corresponding jet latitudinal position in the model. The model for the latitudinal
 567 jet position reproduces the large northward and southward excursions corresponding to low values
 568 of the jet zonal wind speed as can be seen comparing panels (c) and (d). Even though the jet
 569 latitudinal position in the model displays spatial extensions coherent with the data, the temporal
 570 coherence of the model seems too large compared to ERA5 data, where there is more sudden shifts
 571 in the jet position that we were not able to obtain. Given that the variability of our model is assured
 572 by the combination of the random noise and the sign of X in front of the forcing term in equation
 573 9, the reason could be that the value of γ taken to model the strength of disturbances is not large
 574 enough.



556 **FIG. 21. Perturbed spatial model for jet position.** (a) Histograms at lon=0° of jet latitudinal position for the
 557 data (black) and the model (red). (b) Hovmöller diagram of latitudinal jet position for the data over three winters
 558 (2000-2003). Horizontal dashed lines represent the limits between the three winters for the data, so that there
 559 is no continuity at those times. (c) Hovmöller diagram of zonal wind speed at the jet for the model over three
 560 winters and (d) corresponding Hovmöller diagram of latitudinal jet position for the model over three winters.
 561 The model is computed at a 0.01 day time step over 10 years but is plotted every 100 time steps (one day) to be
 562 comparable to the data.

575 Finally we present the "violin" plots of local dynamical metrics computed on the data and on the
 576 model in Figure 22. The violin plots display the distributions of local metrics in a manner similar
 577 to a vertical histogram. Comparing the local dimension d and local persistence θ^{-1} between the
 578 model and the reanalysis data allows us to ensure similarity between their dynamical properties.
 579 More precisely, on the first hand, d corresponds to the active number of degrees of freedom in
 580 the system and therefore a low value of d means that the coupled oscillators behave as a lower
 581 order system: they are partly synchronized (see Pons et al. (2020) for the link between a low local
 582 dimension and synchronization). On the other hand, θ^{-1} compares the time scale of the persistence
 583 between the different states, that is the mean time spend around each particular state in the phase
 584 space. As in Figure 5, both indicators are computed using only data over the Euro-Atlantic sector



595 **FIG. 22. Local dynamical metrics: model and data.** Violin plots of (a) local dimension and (b) local
 596 persistence for the jet position (left) and jet zonal wind speed (right) for the data (grey) and the model (red).
 597 The model is computed at a 0.01 day time step over 10 years but local dynamical metrics are computed with the
 598 series of the model output taken every 100 time steps (one day) to be comparable to the data.

595 (45°W–45°E), and therefore 360 longitude grid points for the model, because the inferred dynamics
 596 is supposed to be valid only around this region.

597 For the results of the model to be comparable with the data, the local dynamical metrics are
 598 computed with a sample of the model output taken every 100 time steps (one day). For the model
 599 of zonal wind, we also apply a 4-day low-pass Lanczos filter to remove the effect of baroclinic
 600 instabilities represented here by the perturbation term S^i . The results are not sensitive to the cutoff
 601 period of the filter, as long as it is between 4 and 10 days. This is particularly important for local
 602 persistence as we explained previously. The distributions of local dimension and persistence, both
 603 in the jet position model and jet zonal wind speed model are similar to the one observed on the
 604 data, which comforts us in the dynamical relevance of our model.

599 **5. Discussion**

600 In the previous sections we investigated the behavior of a model of the eddy-driven jet. We have
601 shown that the northward and southward excursions of the jet can be used as a diagnostic tool for
602 blocking events. The latter occur on a time scale smaller than a day and are associated with a strong
603 decrease of zonal wind speed on the jet 2–3 days prior to the excursion. By looking at Northern
604 Hemisphere maps of the geopotential height and horizontal wind speeds, we have shown that those
605 excursions are strongly linked to blocking-like events.

606 By investigating the recurrent dynamics of zonal wind speed at the jet, we proposed a non-linear
607 and stochastically perturbed oscillator point model to represent it. The jet dynamics was then
608 represented as a forced Langevin equation. Even though those 0D coupled stochastic models
609 exhibit some of the characteristics of the behavior of zonal wind speed at the jet and jet position
610 variability, they do not take into account the spatial variability and their temporal characteristics
611 are not consistent with those observed on the jet.

612 We therefore proposed two stochastic coupled dynamical models. The first one is for the zonal
613 wind speed on the jet based on the coupling between the oscillator previously identified, leading
614 to the Toda lattice, and a stochastic but spatially meaningful disturbance, modeling in particular
615 the spatial characteristics of Rossby wave breaking. The second one is for the latitudinal jet
616 position based on coupled forced Langevin equations. The model for the zonal wind speed on
617 the jet has the remarkable property of having temporal and spatial characteristics very close to
618 the ERA5 data, especially with the temporal spectrum. It therefore naturally comes to mind to
619 ask why this model seems suitable to represent the dynamics of zonal wind speed on the jet. The
620 answer could lie in the following two points: (i) the Toda lattice can be shown to be a discretized
621 version of the Korteweg-de Vries equation (Toda 1975), which is known to sustain solutions in
622 the form of solitons, i.e. solitary waves that propagate without deformation ; (ii) blocking events,
623 or large northward/southward excursions of the jet, can be linked to soliton-like structures in the
624 atmosphere (Malguzzi and Malanotte-Rizzoli 1984). Therefore the series of peaks observed in the
625 spectra presented previously (Fig. 20 panel (b)), and well reproduced by the model, could represent
626 a train of soliton waves that propagate in both directions along the jet and that cause northward
627 and southward shifts in the jet latitudinal position when the amplitude of the soliton is negative.

628 This interpretation is consistent with the Hovmöller diagrams presented above, for the data or for
629 the model.

630 When analyzing the variability of the jet, we have shown that coupled forced Langevin equations
631 represent reasonably well the variability behavior of the latitudinal position of the jet. To explain
632 physical mechanisms, we conjecture that the oscillatory nature of zonal wind speed in our model
633 does not come from a feedback such as those proposed by Barnes and Hartmann (2011) and
634 Woollings et al. (2018b) between jet speed, jet variability and wave-mean flow interaction. The
635 feedback mechanism, creating an oscillatory behavior is intrinsic of the model behavior via the
636 recurrence function applied on u . This intrinsic oscillatory behavior may be related to the mech-
637 anism proposed by Ambaum and Novak (2014) to model storm track variability with a non-linear
638 oscillator evolving in a potential well very similar to the one we proposed: the interaction between
639 local baroclinicity and eddy heat flux. The physical law explaining the increasing variability of
640 the jet position when its speed is low is however still not clear, even if it is strongly supported by
641 either empirical (Woollings et al. 2018b) and numerical (Vallis et al. 2004) studies. Those are the
642 reasons why a physics-informed stochastic model such as the one we proposed here is relevant and
643 consistent with what has been proposed in the literature (e.g. Masato et al. 2009; Paradise et al.
644 2019).

645 Overall, we demonstrated that by mixing ideas of dynamical systems theory, especially embed-
646 ding of climate data, and knowledge of the physical phenomena at stake, in particular the fact that
647 the variability of the jet latitudinal position increases as the jet zonal wind speed decreases, we can
648 propose reasonable coupled models of the eddy-driven jet stream latitudinal position and its zonal
649 wind speed, representing in particular the northward and southward excursions events, therefore
650 extending the model proposed by Faranda et al. (2019). The proposed model is obviously a very
651 crude representation of the real jet and therefore our model has three main caveats. First, the model
652 was build to represent blocking events in the Euro-Atlantic sector in winter, and the jet behavior
653 could be different in other sectors and during other seasons, especially over land and at the western
654 side of ocean basins as revealed by the blocked frequency at these locations (fig. 3). Second, the
655 model does not include any geographical inhomogeneities, contrary to the real jet as seen in section
656 3. It is likely that such inhomogeneties could be taken into account in the model, for example by
657 introducing constant forcing which would differ over land and over sea as in Faranda et al. (2019).

658 Last, the temporal and spatial spectrum of jet position and zonal wind speed do not perfectly fit
659 the real data as shown previously, which reveals the presence of dynamical features that were not
660 taken into account.

661 **6. Conclusions**

662 We investigated the variability of the northern eddy-driven jet stream latitudinal position and
663 zonal wind speed with ERA5 reanalysis data over the period 1979–2019. We showed that northward
664 and southward shifts of the jet latitudinal position are preceded by a strong decrease in jet zonal
665 wind speed 2-3 days before the excursion, which is not the case for zonal wind speed at a fixed
666 latitude, e.g. at 45°N. We also showed that the dynamics of the jet zonal wind speed can be
667 represented by a non-linear randomly perturbed oscillator.

668 Those two results are used to construct a point stochastic model of jet latitudinal position and jet
669 zonal wind speed, which is then extended to all longitudes using two coupled non-linear lattices
670 modelling those two variables. The jet zonal wind speed is modelled as a perturbed Toda lattice,
671 whereas the jet latitudinal position is modelled as coupled forced Langevin equations. Those
672 coupled models compare surprisingly good to empirical data when it comes to their temporal
673 and spatial characteristics, and are therefore able to grasp the main features of the jet behavior
674 during the onset and the decay of blocking events. We also proposed that the success of the Toda
675 lattice to model the propagation of disturbances along the jet can be a result of the capacity of this
676 lattice to propagate solitons, which could themselves be related to blocking events. In the end,
677 we extended the minimal dynamical model proposed by Faranda et al. (2019), which strengthens
678 their conclusion regarding the possibility of reducing complex mid-latitude circulation dynamics
679 to low-dimensional representations given by conceptual models.

680 The results are consistent with previous results available in the literature, either on the link
681 between jet variability and jet wind speed (Woollings et al. 2018b), or the use of non-linear
682 oscillators to represent storm track variability (Ambaum and Novak 2014). Future analysis could
683 investigate whether a similar model can be made in the Southern hemisphere, where the role of
684 topography is much less important. It could also be relevant to investigate how this model can
685 be used in a context of global warming, i.e. the sensitivity of key parameters to the reduction
686 of the tropical–polar temperature gradient (Petoukhov et al. 2013). Intense research has indeed

687 been carried out to better understand how the dynamics of the jet will change with climate change
688 (Woollings et al. 2018a), especially in relation with Arctic amplification and the decrease of the
689 meridional temperature gradient.

690 The performance of coupled climate models to represent blocking statistics has been deemed
691 to be low (Davini and d'Andrea 2020). We hence do believe that our simple models of the jet
692 behavior, can help improve the next generation of climate models in representing key atmospheric
693 features in the midlatitudes.

694 *Acknowledgments.* This work was part of RN’s Master’s thesis at the University Paris-Sorbonne
695 (Master MOCIS). We thank Francis Codron, Gabriele Messori, Sandro Vaienti and Vladimir Zeitlin
696 for their comments on the behavior of the jet stream and the use of simplified dynamical systems to
697 model it. This work was funded by French ANR project No. ANR-20-CE01-0008 (SAMPRACE),
698 the ANR-19-ERC7-0003 project (BOREAS) and the LEFE-MANU (DYNCLIC) CRNS grant.

699 *Data availability statement.* The ERA5 data are available on the Copernicus web site
700 (<https://cds.climate.copernicus.eu/cdsapp!/home>). The results of the detection of the jet position
701 are available on demand.

702 **References**

703 Ambaum, M. H., and L. Novak, 2014: A nonlinear oscillator describing storm track variability.
704 *Quarterly Journal of the Royal Meteorological Society*, **140 (685)**, 2680–2684.

705 Attinger, R., J. H. Keller, M. Köhler, J. Riboldi, and C. M. Grams, 2019: Representation of
706 atmospheric blocking in the new global non-hydrostatic weather prediction model icon. *Meteo-*
707 *rologische Zeitschrift*, **28 (5)**, 429–446.

708 Barnes, E. A., and D. L. Hartmann, 2011: Rossby wave scales, propagation, and the variability of
709 eddy-driven jets. *Journal of the atmospheric sciences*, **68 (12)**, 2893–2908.

710 Barriopedro, D., R. García-Herrera, A. R. Lupo, and E. Hernández, 2006: A climatology of
711 northern hemisphere blocking. *Journal of Climate*, **19 (6)**, 1042–1063.

712 Barriopedro, D., R. García-Herrera, and R. M. Trigo, 2010: Application of blocking diagnosis
713 methods to general circulation models. part i: A novel detection scheme. *Climate dynamics*,
714 **35 (7)**, 1373–1391.

715 Charney, J. G., 1947: The dynamics of long waves in a baroclinic westerly current. *Journal of*
716 *Atmospheric Sciences*, **4 (5)**, 136–162.

717 Charney, J. G., and J. G. DeVore, 1979: Multiple flow equilibria in the atmosphere and blocking.
718 *Journal of Atmospheric Sciences*, **36 (7)**, 1205–1216.

719 Davini, P., and F. D’Andrea, 2016: Northern hemisphere atmospheric blocking representation in
720 global climate models: Twenty years of improvements? *Journal of Climate*, **29 (24)**, 8823–8840.

- 721 Davini, P., and F. d'Andrea, 2020: From cmip3 to cmip6: Northern hemisphere atmospheric
722 blocking simulation in present and future climate. *Journal of Climate*, **33** (23), 10 021–10 038.
- 723 Dijkstra, H. A., 2016: Understanding climate variability using dynamical systems theory. *The*
724 *Fluid Dynamics of Climate*, Springer, 1–38.
- 725 Dole, R., and Coauthors, 2011: Was there a basis for anticipating the 2010 russian heat wave?
726 *Geophysical Research Letters*, **38** (6).
- 727 Duchon, C. E., 1979: Lanczos filtering in one and two dimensions. *Journal of Applied Meteorology*
728 *and Climatology*, **18** (8), 1016–1022.
- 729 Faranda, D., G. Masato, N. Moloney, Y. Sato, F. Daviaud, B. Dubrulle, and P. Yiou, 2016: The
730 switching between zonal and blocked mid-latitude atmospheric circulation: a dynamical system
731 perspective. *Climate Dynamics*, **47** (5), 1587–1599.
- 732 Faranda, D., G. Messori, and P. Yiou, 2017: Dynamical proxies of north atlantic predictability and
733 extremes. *Scientific reports*, **7** (1), 1–10.
- 734 Faranda, D., Y. Sato, G. Messori, N. R. Moloney, and P. Yiou, 2019: Minimal dynamical systems
735 model of the northern hemisphere jet stream via embedding of climate data. *Earth System*
736 *Dynamics*, **10** (3), 555–567.
- 737 Frederiksen, J., 1982: A unified three-dimensional instability theory of the onset of blocking and
738 cyclogenesis. *Journal of Atmospheric Sciences*, **39** (5), 969–982.
- 739 Ghil, M., 1987: Dynamics, statistics and predictability of planetary flow regimes. *Irreversible*
740 *Phenomena and Dynamical Systems Analysis in Geosciences*, Springer, 241–283.
- 741 Ghil, M., M. D. Chekroun, and E. Simonnet, 2008: Climate dynamics and fluid mechanics:
742 Natural variability and related uncertainties. *Physica D: Nonlinear Phenomena*, **237** (14-17),
743 2111–2126.
- 744 Ghil, M., and V. Lucarini, 2020: The physics of climate variability and climate change. *Reviews of*
745 *Modern Physics*, **92** (3), 035 002.
- 746 Haines, K., and P. Malanotte-Rizzoli, 1991: Isolated anomalies in westerly jet streams: A unified
747 approach. *Journal of Atmospheric Sciences*, **48** (4), 510–526.

- 748 Hansen, A. R., 1986: Observational characteristics of atmospheric planetary waves with bimodal
749 amplitude distributions. *Advances in Geophysics*, Vol. 29, Elsevier, 101–133.
- 750 Held, I. M., 1975: Momentum transport by quasi-geostrophic eddies. *J. Atmos. Sci.*, **32** (7), 1494–
751 1497.
- 752 Held, I. M., and A. Y. Hou, 1980: Nonlinear axially symmetric circulations in a nearly inviscid
753 atmosphere. *Journal of the Atmospheric Sciences*, **37** (3), 515–533.
- 754 Hersbach, H., and Coauthors, 2020: The era5 global reanalysis. *Quarterly Journal of the Royal*
755 *Meteorological Society*, **146** (730), 1999–2049.
- 756 Holton, J. R., 1973: An introduction to dynamic meteorology. *American Journal of Physics*, **41** (5),
757 752–754.
- 758 Hoskins, B. J., and I. N. James, 2014: *Fluid dynamics of the mid-latitude atmosphere*. John Wiley
759 & Sons.
- 760 Huang, C. S., and N. Nakamura, 2016: Local finite-amplitude wave activity as a diagnostic of
761 anomalous weather events. *Journal of the Atmospheric Sciences*, **73** (1), 211–229.
- 762 Hurrell, J. W., and C. Deser, 2010: North atlantic climate variability: the role of the north atlantic
763 oscillation. *Journal of marine systems*, **79** (3-4), 231–244.
- 764 Kalnay-Rivas, E., and L.-O. Merkin, 1981: A simple mechanism for blocking. *Journal of Atmo-*
765 *spheric Sciences*, **38** (10), 2077–2091.
- 766 Kautz, L.-A., O. Martius, S. Pfahl, J. G. Pinto, A. M. Ramos, P. M. Sousa, and T. Woollings, 2021:
767 Atmospheric blocking and weather extremes over the euro-atlantic sector—a review. *Weather and*
768 *Climate Dynamics Discussions*, 1–43.
- 769 Koch, P., H. Wernli, and H. C. Davies, 2006: An event-based jet-stream climatology and typology.
770 *International Journal of Climatology: A Journal of the Royal Meteorological Society*, **26** (3),
771 283–301.
- 772 Lee, S., and H.-k. Kim, 2003: The dynamical relationship between subtropical and eddy-driven
773 jets. *Journal of the atmospheric sciences*, **60** (12), 1490–1503.

- 774 Legras, B., and M. Ghil, 1985: Persistent anomalies, blocking and variations in atmospheric
775 predictability. *Journal of Atmospheric Sciences*, **42** (5), 433–471.
- 776 Lucarini, V., and Coauthors, 2016: *Extremes and recurrence in dynamical systems*. John Wiley &
777 Sons.
- 778 Lupo, A. R., 2020: Atmospheric blocking events: a review. *Annals of the New York Academy of*
779 *Sciences*.
- 780 Malguzzi, P., and P. Malanotte-Rizzoli, 1984: Nonlinear stationary rossby waves on nonuniform
781 zonal winds and atmospheric blocking. part i: The analytical theory. *Journal of Atmospheric*
782 *Sciences*, **41** (17), 2620–2628.
- 783 Mann, M. E., S. Rahmstorf, K. Kornhuber, B. A. Steinman, S. K. Miller, S. Petri, and D. Coumou,
784 2018: Projected changes in persistent extreme summer weather events: The role of quasi-
785 resonant amplification. *Science advances*, **4** (10), eaat3272.
- 786 Masato, G., B. J. Hoskins, and T. Woollings, 2013: Wave-breaking characteristics of northern
787 hemisphere winter blocking: A two-dimensional approach. *Journal of Climate*, **26** (13), 4535–
788 4549.
- 789 Masato, G., B. J. Hoskins, and T. J. Woollings, 2009: Can the frequency of blocking be described
790 by a red noise process? *Journal of the atmospheric sciences*, **66** (7), 2143–2149.
- 791 McWilliams, J. C., G. R. Flierl, V. D. Larichev, and G. M. Reznik, 1981: Numerical studies of
792 barotropic modons. *Dynamics of Atmospheres and Oceans*, **5** (4), 219–238.
- 793 Messori, G., N. Harnik, E. Madonna, O. Lachmy, and D. Faranda, 2021: A dynamical systems
794 characterization of atmospheric jet regimes. *Earth System Dynamics*, **12** (1), 233–251.
- 795 Moloney, N. R., D. Faranda, and Y. Sato, 2019: An overview of the extremal index. *Chaos: An*
796 *Interdisciplinary Journal of Nonlinear Science*, **29** (2), 022 101.
- 797 Nakamura, N., and C. S. Huang, 2018: Atmospheric blocking as a traffic jam in the jet stream.
798 *Science*, **361** (6397), 42–47.

- 799 Paradise, A., C. B. Rocha, P. Barpanda, and N. Nakamura, 2019: Blocking statistics in a varying
800 climate: Lessons from a “traffic jam” model with pseudostochastic forcing. *Journal of the*
801 *Atmospheric Sciences*, **76 (10)**, 3013–3027.
- 802 Pelly, J. L., and B. J. Hoskins, 2003: A new perspective on blocking. *Journal of the atmospheric*
803 *sciences*, **60 (5)**, 743–755.
- 804 Petoukhov, V., S. Rahmstorf, S. Petri, and H. J. Schellnhuber, 2013: Quasiresonant amplification of
805 planetary waves and recent northern hemisphere weather extremes. *Proceedings of the National*
806 *Academy of Sciences*, **110 (14)**, 5336–5341.
- 807 Pons, F. M. E., G. Messori, M. C. Alvarez-Castro, and D. Faranda, 2020: Sampling hyperspheres
808 via extreme value theory: implications for measuring attractor dimensions. *Journal of statistical*
809 *physics*, **179 (5)**, 1698–1717.
- 810 Rex, D. F., 1950: Blocking action in the middle troposphere and its effect upon regional climate.
811 *Tellus*, **2 (4)**, 275–301.
- 812 Rhines, P. B., 1975: Waves and turbulence on a beta-plane. *Journal of Fluid Mechanics*, **69 (3)**,
813 417–443.
- 814 Röthlisberger, M., S. Pfahl, and O. Martius, 2016: Regional-scale jet waviness modulates the
815 occurrence of midlatitude weather extremes. *Geophysical Research Letters*, **43 (20)**, 10–989.
- 816 Scherrer, S. C., M. Croci-Maspoli, C. Schwierz, and C. Appenzeller, 2006: Two-dimensional
817 indices of atmospheric blocking and their statistical relationship with winter climate patterns in
818 the euro-atlantic region. *International Journal of Climatology: A Journal of the Royal Meteorological*
819 *Society*, **26 (2)**, 233–249.
- 820 Schwierz, C., M. Croci-Maspoli, and H. Davies, 2004: Perspicacious indicators of atmospheric
821 blocking. *Geophysical research letters*, **31 (6)**.
- 822 Simmons, A., J. Wallace, and G. Branstator, 1983: Barotropic wave propagation and instability,
823 and atmospheric teleconnection patterns. *Journal of Atmospheric Sciences*, **40 (6)**, 1363–1392.
- 824 Süveges, M., 2007: Likelihood estimation of the extremal index. *Extremes*, **10 (1-2)**, 41–55.

- 825 Tibaldi, S., and F. Molteni, 1990: On the operational predictability of blocking. *Tellus A*, **42 (3)**,
826 343–365.
- 827 Toda, M., 1967: Vibration of a chain with nonlinear interaction. *Journal of the Physical Society of*
828 *Japan*, **22 (2)**, 431–436.
- 829 Toda, M., 1975: Studies of a non-linear lattice. *Physics Reports*, **18 (1)**, 1–123.
- 830 Tung, K., and R. Lindzen, 1979: A theory of stationary long waves. part i: A simple theory of
831 blocking. *Monthly Weather Review*, **107 (6)**, 714–734.
- 832 Tyrllis, E., and B. Hoskins, 2008: Aspects of a northern hemisphere atmospheric blocking clima-
833 tology. *Journal of the Atmospheric Sciences*, **65 (5)**, 1638–1652.
- 834 Vallis, G. K., E. P. Gerber, P. J. Kushner, and B. A. Cash, 2004: A mechanism and simple dynamical
835 model of the north atlantic oscillation and annular modes. *Journal of the atmospheric sciences*,
836 **61 (3)**, 264–280.
- 837 Vautard, R., 1990: Multiple weather regimes over the north atlantic: Analysis of precursors and
838 successors. *Monthly weather review*, **118 (10)**, 2056–2081.
- 839 Woollings, T., A. Hannachi, and B. Hoskins, 2010: Variability of the north atlantic eddy-driven
840 jet stream. *Quarterly Journal of the Royal Meteorological Society*, **136 (649)**, 856–868.
- 841 Woollings, T., and Coauthors, 2018a: Blocking and its response to climate change. *Current climate*
842 *change reports*, **4 (3)**, 287–300.
- 843 Woollings, T., and Coauthors, 2018b: Daily to decadal modulation of jet variability. *Journal of*
844 *Climate*, **31 (4)**, 1297–1314.