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Recycling under environmental, climate and resource constraints

Abstract

This paper investigates the recycling opportunities of an industrial sector constrained by resource, climate, and waste capacities. To do this, we model the full lifecycle of a good to consider the waste and greenhouse gas (GHG) emissions coming from both its production — from virgin or recycled materials — and consumption. We identify the optimal trajectories of resources use, mainly depending on the relative scarcity of the resources and on their emissions. Although recycling is usually, and correctly, noted as an opportunity to reduce the impact of consumption on primary resources and waste, we also consider the possible negative environmental consequences of recycling and we discuss the resulting arbitrations. We characterize the optimal recycling strategy and we show that, in some cases, the recycling rate through time is an inverted U-shape, and there can be a catch-up phase of consumption at the end of the social planner program. Finally, we discuss the policy implications of our model by identifying and analyzing the set of optimal tax-subsidy schemes, and we highlight the existence of standard environmental externalities as well as a positive externality linked to the absence of a market for waste.

Keywords— Recycling; Resource extraction; Waste; GHG emissions.

JEL classifications— Q32, Q53, Q54.

1 Introduction

The early economic studies on secondary materials long focused on recycling's ability to save extracted resources (UNEP, 2019). More recently, however, economists have begun to consider recycling's ability to mitigate waste pollution and greenhouse gas (GHG) emissions. With this new view, the concept of a circular economy, in which recycling is one of the cornerstones, appeared as a solution for a more sustainable economical model, as formalized by Braungart and McDonough (2002) in *Cradle to Cradle*. This concept has generated a significant amount of "grey literature" through many non-governmental

organizations like the *Ellen McArthur Foundation*. It can be noted that a large concentration of the grey literature, as well as of the academic literature, occurs in Europe and Asia. Both of these regions have many academic and institutional initiatives in this field (de Jesus and Mendonça, 2018).

In this paper, we contribute to the literature on circular economy by examining the impact of recycling in an industrial sector facing environmental, resource and climate constraints. In this context, we must consider three different balances: a material balance in order to examine the saving of natural resources and the reduction of waste accumulation; a carbon balance for the topic of climate change; and, an economic balance for the evolution of consumption. Hence, we see environmental objectives of recycling going in three different directions: the saving of natural resources while its shortage could lead to economic difficulties; the reduction of waste accumulation that is costly to manage for both public and private entities, and poses a threat to the environment; the fight against climate change and especially the reduction of GHG emissions. A good illustration of these features can be found in the metal sector: scarcity of critical metals raises the question of their sustainable use and end of life management, as well as the emissions rates of their lifecycle (UNEP, 2011).

One intended specificity of this paper is to extend the study of recycling by considering climate change as an additional externality. Recycling can indeed harm the environment, in particular because of the extra energy required by this activity and its resulting potential GHG emissions. This additional externality leads to new arbitrations we describe in the model: in most cases, recycling is a way of reducing the use of resources with a high carbon footprint (ADEME and FEDEREC, 2017), but recycling is still the source of GHG emissions. It has already been highlighted that circularity and environmental issues are connected in an industrial sector, with for instance used tires (Lonca et al., 2018). In France, studies of ADEME and FEDEREC (2017), ADEME and Bio by Deloitte (2017) and ADEME (2019) focused on quantifying different impacts of recycling in terms of GHG emissions, showing that industrial processes are often highly carbon intensive compared to recycling industries (see Table 1). However, in the end, recycling does not appear to be the ideal clean substitute to regular production: recycling produces its own emissions; there is a need in initial production from a regular source; and, recycling comes with a cost (UNEP, 2011). These environmental considerations contribute to the current policy framework, with many countries implementing emissions reduction

targets that affect polluting industries.

HERE INSERT TABLE 1

We do not pretend to calibrate policies to implement, but we try to analytically highlight the cross-effects and eventual synergies between climate change mitigation, preservation of resources, and the limitation of waste disposal. To do that, we develop a dynamic model of resource use where a final manufactured good can be produced from a virgin exhaustible resource or from recycled materials, these two inputs being taken as perfect substitutes. In order to clearly understand their various effects, we successively introduce two negative environmental externalities. First, we examine waste accumulation that harms the economy through a specific damage function, and then we add cumulative GHG emissions that are constrained by an exogenous carbon budget. Consequently, we observe that a third source of (positive) externality must also be considered as long as there is no waste market, as consumption of the final good provided a waste stream which can be reused thanks to an endogenous effort of recycling.

In this framework, we characterize the main properties of the optimal trajectories of the model. In particular, we discuss the merit order in using each type of resource, depending on the relative scarcity of resources and their emissions rates. We analyze the optimized recycling rate curve through time and we show that, under some conditions, it can be an inverted U-shape. We also discuss the respective dynamics of resource use, which sometimes can result in a catch-up phase of consumption at the end of the program. Last, we show that in a competitive market economy, this optimal outcome can be implemented by a set of tax-subsidy schemes and we discuss their policy implications depending on the identity of the tax payer or the subsidy beneficiary. The introduction of a waste market allows to account for the positive externality linked to production that relaxes resources restriction thanks to recycling. This externality comes from the dynamic framework of the model and has not been considered by the static economic literature on recycling.

The rest of this paper is organized as follows. Section 2 provides a brief review of the literature on recycling. Section 3 characterizes the basic model and describes the different possible scenarios of consumption and recycling. Section 4 considers the introduction of GHG emissions through an emissions

ceiling.¹ Section 5 studies the decentralized equilibrium outcome and discusses policy implications. Section 6 concludes.

2 Related literature

Our work can be related to three strands of literature on recycling: resource scarcity, waste management, and environmental policies.

Early studies from the 70s-80s already tackled resource scarcity. For instance, [Smith \(1972\)](#) puts forward social costs linked to waste accumulation and stock diminution. He focuses on the dynamics of waste when recycling is under consideration, and he shows that there is a trade-off between private costs (labor, material) and social costs (waste accumulation, resource depletion). Such dynamic models informed the first economic guidelines motivating recycling. Waste accumulation issues were added to the topic through various models intending to find the optimal level of pollution in an economy (see for instance [Plourde \(1972\)](#), [Forster \(1973\)](#), or [Hoel \(1978\)](#)). Later, the work of [Chakravorty et al. \(2006\)](#) and [Chakravorty et al. \(2008\)](#) focused on the order of resource extraction and gave many insights on situations where resource depletion induces pollution, that changes the extraction order. However, these studies do not include recycling in their model.

An important part of the relevant literature was later developed around the topic of using green policies to promote recycling. [Palmer and Walls \(1997\)](#) use a static micro-economic model to analyze the effects of diverse economic incentives such as subsidies, waste tax and deposit-refunds. This approach gives many policy insights but only takes into account waste and recycling activities. The work started by [Palmer and Walls \(1997\)](#) was then expanded to include environmental effects associated with recycling and resource extraction ([Fullerton and Kinnaman, 1995](#); [Palmer et al., 1997](#)). Going further in this type of analysis, [Walls and Palmer \(2001\)](#) integrate life-cycle aspects of production and consumption and discuss optimal policy instruments. However, they only include an effluent linked to a third, non-material, input in production. Thus they do not differentiate between recycling and extraction. In a later work, [Acuff and Kaffine \(2013\)](#) add carbon emissions to the model with a direct

¹We also propose other three extensions in appendix: accounting for a scrap value for waste, decoupling emissions between collection and transformation in the recycling branch, limiting the capacity of the recycling sector.

link to the input choice and show that the objective of reducing GHG is also a strong incentive to increase recycling, and that green policies can be implemented with this goal.

These articles add a significant contribution regarding public intervention linked to recycling activities. However this kind of static analysis omits the dynamic aspects of resource stocks mentioned above. A further analysis is needed to examine the arbitration between environmental externalities and resource depletion. For instance, extending the [Acuff and Kaffine \(2013\)](#) model (initially being an extension of the [Palmer and Walls \(1997\)](#) model) to a dynamic system.

An example of this can be found in [Huhtala \(1999\)](#). This is one of the few to analyze the optimal use of an exhaustible resource while considering issues of waste accumulation, resource depletion, and pollutant emissions at the same time. She describes the best arbitration of labor between recycling and primary production, and designs a fitting tax-subsidy scheme to achieve it under a balanced budget. This work is complemented by different recent studies on resource economics, with for instance [Pittel et al. \(2010\)](#) who model a decentralized economy with a recycling activity and highlight the market failure resulting from the absence of a market for waste, despite their economical value. They provide an optimum by setting up a market for waste and subsidizing recycling activities.

This dynamic approach is also the perspective of [Di Vita \(2001\)](#) and [Di Vita and Van Long \(2007\)](#) who assess the possibility of an economic and welfare growth under a material constraint, thanks to investments in recycling. [Sorensen \(2017\)](#) also considers a recycling technology in a Ramsey model that alleviates externalities due to resource extraction and consumption. These articles share the use of optimal control theory, but propose different models to represent a circular economy. Additionally, [Di Vita \(2001\)](#) and [Di Vita and Van Long \(2007\)](#) propose models that do not respect a material balance in the economy in contrast to the physical reality of the use of secondary materials. With these propositions, they arrive to the conclusion that recycling allows a stationary growth path. An alternative modeling approach is given by [Boyce \(2012\)](#) who chooses to specify a recycling stock separate from accumulated waste. He examines the dynamic of this stock when there is perfect substitution between virgin and recycled material and he describes economic trade-offs between the two material to manufacture a final good.

The results of all these studies are in part driven by the substitutability characteristics of the

production function. Models often use perfect substitution for virgin and recycled uses (Boyce, 2012; Hollander and Lasserre, 1988; Hoogmartens et al., 2018). While being a strong hypothesis, it remains relevant in our case of metal recycling with efficient processes (Villalba et al., 2002, 2004). While metal recycling has been a motivated topic for a long time (Ayres, 1997; Sigman, 1995), more recent concerns connected to technical changes and the energy transition have begun to be included. The essential place in modern technological applications taken by some materials (such as rare earth or lithium) also raises the question of how to bare the storage costs inherent to the industry as they are not evenly allocated worldwide (Rosendahl and Rubiano, 2019; Ba et al., 2020). As such, one could consider in-use material as a stock of resource (Batteries Europe, 2020).

3 A basic optimal recycling model

3.1 Setup of the model

We consider an industry that is managing a natural resource stock from which a final consumption good can be produced. We set an exogenous time-limit T to the management program, corresponding to the horizon of exploitation of the resource, before a shut down of the mine (Lappi and Ollikainen, 2019).

The industry can produce a quantity q of final good from two different inputs: the virgin resource and a recycled material, of relative quantities v and r . We assume perfect substitution between these two inputs, involving that $q(t) = v(t) + r(t)$ for any time t . We also pose that waste is a one-to-one co-product of the final consumption good. Thus, $q(t)$ also denotes the instantaneous flow of waste, before any potential recycling process.²

Consuming q units of final good provides a gross surplus $u(q)$ to the final user, where $u(\cdot)$ follows the standard hypotheses of utility functions: of class \mathcal{C}^2 , increasing ($u' > 0$), concave ($u'' < 0$) and verifying the Inada conditions, *i.e.* $\lim_{q \rightarrow 0^+} u'(q) = +\infty$ and $\lim_{q \rightarrow \infty} u'(q) = 0$. To account for the previously stated hypothesis of perfect substitution in the production function, we assume the utility drawn from the consumption of a good made out of virgin material is the same as that taken from the

²An alternative would be to consider that a constant share of the final good is destroyed by consumption, and then unrecoverable. In that case, consuming a quantity q of good would release a flow γq of waste, with $\gamma < 1$ being the waste content rate of the manufactured good. This would not change significantly the analysis but does not respect the physical law of material balance.

consumption of a good made out of recycled material. ³

Input flows v and r come from two different primary sources, a virgin stock and a recycled stock. We denote the average delivery costs of these two resources as c_v and c_r , respectively. These average delivery costs include the extraction, production and transportation costs, all of which are assumed to be constant through time.

The industry is initially endowed with a stock $V_0 > 0$ of virgin resource. We suppose that recycling has never happened by the past, meaning that the initial stock of recycled material is zero: $R_0 = 0$. At any time t , the current levels of virgin and recycled stocks $V(t)$ and $R(t)$ are governed, respectively, by the following extraction processes:

$$\dot{V}(t) = -v(t) \quad , \quad V(0) = V_0 \quad (1)$$

$$\dot{R}(t) = -r(t) + \beta(t)q(t) \quad , \quad R(0) = 0, \quad (2)$$

where β , $\beta \in [0, 1]$, represents the endogenous recycling intensity (*i.e.* the share of production which is collected, sorted and incorporated into the recycled stock after use).⁴ However, note that recovering waste from the flow q and using it in production are two different actions that are only correlated by the cost arbitration. As a matter of fact, it can be optimal to use a recycled input in production while deciding not to redirect waste into the recycled stock ($\beta = 0$), as long as this stock is not empty.

Recovering material from waste is costly and this cost is added to the other private costs related to the recycled input. The recycling cost per unit of waste is given by the function $f(\beta)$, such that $f(0) = 0$, $f'(\beta) > 0$ with $\beta \in [0, 1[$, $f'(1) = +\infty$ and $f''(\beta) > 0$. Recycling the share β of the waste flow q thus costs $qf(\beta)$. The convexity of $f(\cdot)$ means decreasing returns in the recycling technology, which reflects the technical difficulties to retrieve all material produced and consumed. This assumption, combined with $\lim_{\beta \rightarrow 1} f'(\beta) = +\infty$, implies that full recycling is not attainable. There should always be a part of the waste that cannot be recovered, at any cost. It can be due to fatal transformation of

³Note that when focusing on the use of different resources, perfect substitution is a common simplifying assumption in the literature (Amigues et al., 2011; Boyce, 2012; Chakravorty et al., 2006, 2008).

⁴While β does not exactly fit the common definition, we will also refer to it as "recycling rate" throughout the rest of the paper. Moreover, as we model recycled resource and waste as two different stocks, β must be a control variable in order to have an endogenous recycling. A similar approach is used by Boyce (2012) as he considers a sorting cost and assigns to the social planner the choice of sorting waste or not (a binary decision). This choice of modeling two stocks allows to represent the material balance constraint in our economy.

the original resource during the manufacturing process or due to a very dispersive use of the resource which renders it technologically impossible to retrieve all of the resource after consumption.⁵

The remaining share of final good which could not be collected and recycled yields a waste stream $(1 - \beta)q$ that accumulates.⁶ We denote $W(t)$ as the resulting stock of waste at time t , with W_0 as the initial stock. This waste stock is reduced by a bio-decomposition, or another natural resorption mechanism, at a constant and exogenous rate α , $\alpha \in (0, 1)$. The dynamics of W are then given by:

$$\dot{W}(t) = (1 - \beta(t))q(t) - \alpha W(t), \quad W(0) = W_0. \quad (3)$$

Finally, waste accumulation generates local environmental degradation that harms the economy through a damage function $D(W)$, with $D'(W) > 0$ and $D''(W) \geq 0$. In order to simplify the analysis, we will consider a linear damage function, with a constant marginal damage denoted by $c_W > 0$.

3.2 Central planner program and optimal conditions

The objective of the central planner is to determine the resource extraction and recycling paths that maximize the discounted sum of net surpluses over the planning horizon.⁷ Formally, denoting the time discount rate as $\rho > 0$, we want to solve the following problem:⁸

$$\max_{\{v, r, \beta\}} \int_0^T [u(v + r) - c_v v - c_r r - qf(\beta) - D(W)] e^{-\rho t} dt, \quad (4)$$

subject to the dynamic constraints (1)-(3) and to the non-negativity constraints on v , r and β . The Hamiltonian can be written as:

$$\begin{aligned} \mathcal{H} = & u(v + r) - c_v v - c_r r - qf(\beta) - D(W) + \lambda_V(-v) + \lambda_R[\beta(v + r) - r] \\ & - \lambda_W[(1 - \beta)(v + r) - \alpha W], \quad (5) \end{aligned}$$

⁵This hypothesis reflects the dispersive use of many materials as well as complex applications at mass production scales. In fact, thermodynamic limitations to recycling can involve important economic costs, or even impracticability, thus forbidding perfect recycling (Ayres, 1999; UNEP, 2011).

⁶Note that the share $(1 - \beta)$ of the waste stream that is not redirected to the recycled stock at instant t cannot be recycled later in the program.

⁷Remind that, here, the ending time T is exogenous and finite. An alternative would be to let T free. However, as the industry evolves in a finite world where none renewable resource is available and because of Inada conditions, it would be optimal to postpone T as much as possible: $T \rightarrow \infty$.

⁸In order to simplify notations, we will hide the time subscript whenever it is convenient and clear from the context.

where λ_V , λ_R and $-\lambda_W$ are the co-state variables attached to the virgin stock, the recycled stock and the waste stock, respectively.⁹

The optimal solution must satisfy the following first-order conditions:

$$u' \leq c_v + f(\beta) + \lambda_V - \beta\lambda_R + (1 - \beta)\lambda_W, \quad (= \text{if } v > 0) \quad (6a)$$

$$u' \leq c_r + f(\beta) + \lambda_R - \beta\lambda_R + (1 - \beta)\lambda_W, \quad (= \text{if } r > 0) \quad (6b)$$

$$qf'(\beta) \geq (\lambda_R + \lambda_W)q, \quad (= \text{if } \beta > 0) \quad (6c)$$

$$\dot{\lambda}_V = \rho\lambda_V \quad (6d)$$

$$\dot{\lambda}_R = \rho\lambda_R \quad (6e)$$

$$\dot{\lambda}_W = (\rho + \alpha)\lambda_W - D'(W), \quad (6f)$$

completed by the transversality conditions:

$$\lambda_V(T)V(T)e^{-\rho T} = \lambda_R(T)R(T)e^{-\rho T} = \lambda_W(T)W(T)e^{-\rho T} = 0. \quad (7)$$

Conditions (6a) and (6b) compare the gross marginal surplus of using each type of resource with its full marginal cost and state that it is optimal to use the resource when both are equal. The full marginal cost (FMC) of each input v or r is composed of: the delivery cost (c_v or c_r) of the input; the unitary cost $f(\beta)$ of recovering material from the waste stream; the scarcity rent (λ_V or λ_R) of the resource stock; the social marginal cost $-\beta\lambda_R$ of replenishing the recycled stock (a negative cost, this stock being a good for society); and, the shadow marginal cost $(1 - \beta)\lambda_W$ of waste accumulation.

We can already determine the motivation of recycling as implemented in this resource model: recycling relaxes the resource constraint by replenishing the recycled stock and reducing waste accumulation. However, the choice of using recycled inputs in production here only relaxes pressure on virgin stock. Damage due to waste accumulation is not impacted, as both inputs have the same impact on waste after consumption. Waste reduction is controlled by the choice of the recycling rate β .

The optimal path for the recycling behavior is obtained from (6c) by comparing the marginal cost $f'(\beta)$ per unit of final good with the marginal social gain $(\lambda_R + \lambda_W)$ of recycling. Indeed, for

⁹As waste accumulation generates a negative externality through a damage function, its shadow value is non-positive. For the purpose of simplifying the notations, we introduced a positive shadow cost by considering formally $-\lambda_W$ as co-state variable.

a given material stream q , increasing the share of recycled material by $\Delta\beta$ allows to increase the stock of recycled resource, whose marginal shadow value is λ_R , by quantity $\Delta\beta q$, and to reduce waste generation by this same unit, thus saving the marginal shadow cost λ_W of the waste stock.

Equations (6d) to (6f) rule the dynamics of the system. (6d) and (6e) illustrate, in particular, the Hotelling rule which state that the scarcity rents of the two resources grow at a rate equal to the discount rate ρ . Hence we can write: $\lambda_V(t) = \lambda_{V0}e^{\rho t}$ and $\lambda_R(t) = \lambda_{R0}e^{\rho t}$, with $\lambda_{V0} \equiv \lambda_V(0)$ and $\lambda_{R0} \equiv \lambda_R(0)$.

In (7), we can replace λ_V and λ_R by the above expressions, and V and R by the solutions of the differential equations (1) and (2), respectively. Assuming that neither resource stock is abundant, *i.e.* $\lambda_{V0} > 0$ and $\lambda_{R0} > 0$, the transversality condition implies that both stocks must be fully depleted at the terminal time T :

$$\int_0^T v(t)dt = V_0 \quad (8)$$

$$\int_0^T [\beta(t)v(t) - (1 - \beta(t))r(t)]dt = 0. \quad (9)$$

Equation (6f), combined with the transversality condition (7), gives us the trajectory of the shadow cost of waste accumulation:¹⁰

$$\lambda_W(t) = \int_t^T D'(W)e^{-(\rho+\alpha)(s-t)} ds. \quad (10)$$

We see here that the shadow cost of waste accumulation is equal to the intertemporal sum of the flows of marginal damages, discounted at rate $(\rho + \alpha)$ since waste is not only a flow, but also a stock that shrinks at rate α . With a constant marginal damage c_W , this expression can be simplified as follows:

$$\lambda_W(t) = \frac{c_W}{\rho + \alpha} \left[1 - e^{-(\rho+\alpha)(T-t)} \right]. \quad (11)$$

This implies that λ_W is decreasing through time: $\dot{\lambda}_W(t) = -c_W e^{-(\rho+\alpha)(T-t)} < 0$. In fact, this social cost accounts for the future damage due to waste. As the program approaches its end, the constraint has less impact.¹¹

¹⁰Solutions of equations (3) and (6f) are, respectively: $W = e^{-\alpha t} [W_0 + \int_0^t (1 - \beta) q e^{\alpha s} ds]$ and $\lambda_W = e^{(\rho+\alpha)t} [\lambda_{W0} - \int_0^t D'(W) e^{-(\rho+\alpha)s} ds]$. Replacing W and λ_W by these expressions into the transversality condition (7), we get: $\lambda_{W0} = \int_0^T D'(W) e^{-(\rho+\alpha)t} dt$. The initial value of the waste social cost is independently given by conditions on waste, only taking into account the dynamic characteristics α of the stock and its cost $D(W)$ for society.

¹¹We discuss in the appendix the case where the damage caused by the left-over waste stock $W(T)$ still harms the economy after the terminal date T .

From first-order conditions (6a) and (6b), we can infer that the optimal solution is one or more consecutive phases of production from one input or the other, with, in parallel, a specific path for rate β ruled by equation (6c). Perfect substitution and fixed private costs do not allow simultaneous use of each input. Following the display of the model, we will be studying the switch from one input to the other, the needed initial conditions, and the optimal choice regarding the redirection of the waste flow to the recycled stock.

3.3 Arbitration on resources use

In order to get the optimal consumption path, we have to compare the full marginal costs of using each specific input (FMC_i , with $i \in \{v; r\}$), given by the right-hand side of (6a) and (6b). We define ΔFMC as their difference:

$$\Delta FMC \equiv FMC_v - FMC_r = c_v - c_r + (\lambda_{V0} - \lambda_{R0})e^{\rho t}. \quad (12)$$

Examining the sign of this expression gives us arbitration at stakes when applying Herfindhal least-cost principle analysis (Herfindahl, 1967). It can be noted that neither recycling nor the waste stock come into play in the trade-off. In fact, products made out of recycled or virgin materials have the same impact on waste accumulation and the activity of waste recovery does not depend on the origin of the input flow. In this basic version of the model, the relative competitiveness of each type of resource is simply driven by the differential of resource extraction costs and by their relative scarcity. The relative magnitude of these two effects can lead to a switch between inputs, mathematically materialized by the change in sign of ΔFMC before the end of the program. Formally, if $\Delta FMC < 0$ then we produce from the virgin input, and if $\Delta FMC > 0$ then we produce from the recycled input.

From this analysis, Proposition 1 below describes the main characteristics of the optimal sequence of input use.

Proposition 1. *Regardless of the recycling rate $\beta(t)$, the optimal path is such that:*

1. *It must start with a phase of production from extraction of virgin resource in all scenarios.*
2. *There can be at most one switch of input during the program. Let us call this time of switch \tilde{T} , whose existence within $[0; T]$ is not guaranteed. Then, if it exists within $[0; T]$, $\exists! \tilde{T} > 0$ such that*

$\Delta FMC(t) < 0$ for any $t < \tilde{T}$ and $\Delta FMC(\tilde{T}) = 0$.

3. When such a switching time \tilde{T} exists, it is defined by:

$$e^{\rho\tilde{T}} = \frac{c_r - c_v}{\lambda_{V0} - \lambda_{R0}}. \quad (13)$$

Proof. (1.) For any $t \in [0, \tilde{T}[$, if $v(t) = 0$ then, from (2), $R(\tilde{T}) = -\int_0^{\tilde{T}} (1 - \beta)rds \leq 0$ which is not possible. (2.) Differentiating (12) with respect to time yields $\Delta \dot{FMC} = \rho(\lambda_{V0} - \lambda_{R0})e^{\rho t}$. As this expression is constant in sign, ΔFMC is monotonic and there can be at most one switch of inputs during the optimal program of the model. (3.) This is simply the solution of $\Delta FMC(t) = 0$. \square

The first result highlights the need for initial conventional production: recycling can only be achieved when there has been enough extracted input consumed, and then collected from the waste stream. Also, by the assumption of the recycled stock being empty at the beginning ($R_0 = 0$), there must always be a first phase of virgin production to fill the recycled stock before being able to use it. Thus, ΔFMC must be initially negative, which requires $\lambda_{V0} - \lambda_{R0} < c_r - c_v$. The second result implies that, if it exists, the time of the switch \tilde{T} is the solution of the following equation $c_v + \lambda_V(\tilde{T}) = c_r + \lambda_R(\tilde{T})$. A single instant solution to $\Delta FMC = 0$ as given by (13) also means that a simultaneous use of virgin and recycled resource cannot occur. Different scenarios can occur, depending on the sign of $(c_r - c_v)$. Proposition 2 below summarizes them.

Proposition 2. *Regardless of the recycling rate path $\beta(t)$, we can summarize the different optimal scenarios to the following ones:*

1. *If extracting input from the recycled stock is costlier than from the virgin stock ($c_r > c_v$):*
 - (a) *We only use the virgin resource if this resource is relatively more abundant than the recycled one ($\lambda_{V0} < \lambda_{R0}$);*
 - (b) *We use the virgin resource up to time \tilde{T} , with $\tilde{T} \leq T$, and then the recycled resource if the later is relatively more abundant than the former ($\lambda_{V0} > \lambda_{R0}$);*

2. If extracting input from the virgin stock is costlier than from the recycled stock ($c_r < c_v$), we use the virgin resource up to time \tilde{T} and then the recycled resource. In this case, the virgin resource must also be relatively more abundant than the recycled one ($\lambda_{V0} < \lambda_{R0}$).

Proof. The different optimal paths can be drawn from the positivity conditions for ΔFMC . Moreover, in case 1(b), as the transversality condition guarantees that the virgin resource must be exhausted at the terminal date T , the switch to the recycled resource occurs no later than T : $\tilde{T} \leq T$. \square

In scenario 1(a), using the recycled resource does not show any interest as the virgin resource is both less expensive and relatively more abundant than the recycled one. In scenario 1(b), the virgin stock exhibits relative scarcity that can be relaxed by using the recycled resource, despite its higher extraction cost. Last, scenario 2 illustrates the case where the recycled resource is cheaper than the virgin one. In this case, the only way to justify an initial phase of virgin resource use, which is required to provide recycled waste, is to assume that the virgin stock is relatively abundant.

As we separate marginal delivery cost c_r and waste recovery cost $f(\beta)$, we do not assume at first any relationship between delivery costs c_i . However, the difference in maturity between these two materials can be observed as the recycling branch is usually not favored when social costs are not internalized, leading to the reasonable hypothesis $c_r + f(\beta) > c_v$ ¹². However we do not assume anything more as differences between sectors could lead to recycling costs relying on delivery ($c_r \gg f(\beta)$ thus $c_r > c_v$) or on collection ($f(\beta) \gg c_r$ thus potentially $c_v > c_r$).

At the end, the optimal path of resource consumption is characterized by the triplet of endogenous elements $\{\lambda_{V0}, \lambda_{R0}, \tilde{T}\}$, which is determined as the solution of the three equations-system (8), (9) and (13). For the remaining of the section, we will consider the most interesting case where both resources are successively used, which means that the switching time \tilde{T} exists within $]0, T[$. Hence, λ_{V0} and λ_{R0} must satisfy the following additional condition:

$$1 < \frac{c_r - c_v}{\lambda_{V0} - \lambda_{R0}} < e^{\rho T}. \quad (14)$$

¹²Di Vita and Van Long (2007) precises that an industry of secondary materials with the same properties as virgin inputs would bear very high cost, not sustainable with current market prices. With the exemple of lithium, Rosendahl and Rubiano (2019) use higher initial costs for extraction than recycling.

3.4 Recovering waste

The optimal path of the recycling rate β is determined from equation (6c). In order to simplify the analysis, we define the function $\Phi_\beta \equiv \lambda_R + \lambda_W$ as the marginal social gain of the recycling effort (both in avoiding accumulation of waste and replenishing the stock of recycled resource). Recycling intensity results from the arbitration between its marginal cost and this marginal gain. Formally, $\beta = 0$ when $\Phi_\beta \leq f'(0)$, and $\beta \in]0;1[$ such that $\Phi_\beta = f'(\beta)$ when $\Phi_\beta > f'(0)$. Remark that there can be recycling $\beta > 0$ while the virgin resource is not used: recycling only alleviates the pressure on waste accumulation in this case.

The optimal recycling program can be described by having three material moments with characteristics detailed in proposition 3. The principle material moment is time \hat{T}_β which represents the time of maximal recycling rate $\beta(t)$. This moment is accompanied by two other material moments \underline{T}_β and \bar{T}_β which represent the start and finish of recycling, respectively. Formally we specify \underline{T}_β and \bar{T}_β such that $\beta(t) = 0$ if $t < \underline{T}_\beta$ or $t > \bar{T}_\beta$. Note that we restrained our analysis to the case where both V and R are extracted, meaning that there must be a phase of recycling during the program.

Proposition 3. *Regardless of the resource extraction regime:*

1. *There can be zero, one or two phases where the recycling rate β saturates and is zero. These phases can either be:*
 - (a) *No saturation phase (we always recycle);*
 - (b) *One phase at the beginning or the end of the program, such that $\beta(t) = 0$ when $t \in [0; \underline{T}_\beta]$ or $t \in [\bar{T}_\beta; T]$, where the values of times \underline{T}_β or \bar{T}_β are the solution in $[0; T]$ of equation $\Phi_\beta(t) = f'(0)$;*
 - (c) *Two phases at the beginning and the end of the program, such that $\beta(t) = 0$ when $t \in [0; \underline{T}_\beta]$ and $t \in [\bar{T}_\beta; T]$, where the values of times \underline{T}_β and \bar{T}_β are the solutions in $[0; T]$ of equation $\Phi_\beta(t) = f'(0)$ with $0 < \underline{T}_\beta < \bar{T}_\beta < T$.*
2. *When the recycling rate $\beta(t)$ is not always zero, it reaches a maximum at the time \hat{T}_β defined as*

follows:

$$\hat{T}_\beta \begin{cases} = 0 & , \text{ if } e^{(\rho+\alpha)T} \leq \frac{c_W}{\rho\lambda_{R0}} \\ = T & , \text{ if } \frac{c_W}{\rho\lambda_{R0}} \leq e^{\rho T} \\ \in]0, T[\text{ and s.t. } e^{\alpha\hat{T}_\beta} = \frac{\rho\lambda_{R0}}{c_W} e^{(\rho+\alpha)T} & , \text{ if } e^{\rho T} < \frac{c_W}{\rho\lambda_{R0}} < e^{(\rho+\alpha)T} . \end{cases} \quad (15)$$

Proof. As the cost function associated to the recycling rate $f(\beta)$ is increasing and convex: from (6c), the proof of proposition 3 comes from the study of the shape of function Φ_β , illustrated by Figure 1(a). We can study the dynamics of β by studying the evolution of Φ_β :¹³

$$\Phi_\beta(t) = \lambda_{R0}e^{\rho t} + \frac{c_W}{\rho + \alpha} \left(1 - e^{-(\rho+\alpha)(T-t)} \right) .$$

Function $\Phi_\beta(t)$ defined on \mathbb{R}^+ is proved to be increasing from $\Phi_\beta(0) = \lambda_{R0} + \frac{c_W}{(\rho+\alpha)}[1 - e^{-(\rho+\alpha)T}]$ to its maximum $\Phi_\beta(\hat{T}_\beta)$, and then declining, with $\lim_{t \rightarrow \infty} \Phi_\beta(t) = -\infty$.

(1.) We can see that the equation $\Phi_\beta(t) = f'(0)$ has zero solution when $\forall t \in \mathbb{R}^+, \Phi_\beta(t) < f'(0)$, one solution when $f'(0) \leq \Phi_\beta(0)$ and two solutions otherwise.¹⁴ We can determine \underline{T}_β and \bar{T}_β as the results of this equation, when they exist in $[0; T]$.

(2.) The existence of \hat{T}_β in the boundaries of the program $[0; T]$, as illustrated by Figure 1(b), directly follows the study of the equation $\Phi'_\beta(t) = 0$ and gives the results of the second item of Proposition 3. □

HERE FIGURE 1

Figure 1: (a) Saturation of β and (b) Maximum time \hat{T}_β

It follows from this analysis that as the optimal path of $\beta(t)$ is defined by the path of function $\Phi_\beta(t)$, the optimal recycling rate is inverted U-shaped in t when positive. Also, the permanent arbitration with the marginal social benefit of recycling can induce phases with $\beta = 0$. When this phase comes first, it means that the resource constraint on stock R is not high enough to start recycling. The social planner only initiates it after \underline{T}_β . On the other hand, there can also be a final phase without recycling

¹³For the purpose of this analysis, we extend the definition of $\Phi_\beta(t)$ to \mathbb{R}^+ , and then discuss the existence of solutions in $[0, T]$.

¹⁴We intentionally omit the specific case when there is a tangential solution for the equation, such as $\max_{\mathbb{R}^+} \Phi_\beta = f'(0)$.

as the shrinking social cost of waste accumulation makes it no more optimal. This is visible from the evolution of maximum time exposed in (15). Given the inverted U-shape of $\beta(t)$, a maximum instant \hat{T}_β is given by the comparison between the sum of discounted marginal waste damage $\frac{c_W}{\rho} = \int_0^\infty c_W e^{-\rho t} dt$, and the initial scarcity of the recycled resource λ_{R0} . For high damage wastes (or a low constraint on the recycled resource), the maximum of recycling occurs at the beginning.

In section 3.5, we will consider the most interesting case where the recycling rate $\beta(t)$ reaches a maximum at instant \hat{T}_β , corresponding to the third case of (15), where $0 < \hat{T}_\beta < T$.

3.5 Dynamics of resource flows

The objective is now to superimpose the respective dynamics of resource use and recycling that have been previously analyzed separately. For that we consider a phase during which $v > 0$ (and $r = 0$) or $r > 0$ (and $v = 0$). We look at the dynamics of $v(t)$ and $r(t)$ by examining the time derivatives of conditions (6a) and (6b). Using (11), for any value of β , we obtain:

$$F\dot{M}C_v = \left[\rho(\lambda_{V0} - \beta\lambda_{R0}) - (1 - \beta)c_W e^{-(\rho+\alpha)T} e^{\alpha t} \right] e^{\rho t}, \quad (16)$$

$$F\dot{M}C_r = (1 - \beta) \left[\rho\lambda_{R0} - c_W e^{-(\rho+\alpha)T} e^{\alpha t} \right] e^{\rho t}. \quad (17)$$

Remark that, defining $\Phi_v(t) \equiv [1 - \beta(t)]e^{\alpha t} + \beta(t)e^{\alpha \hat{T}_\beta}$, with \hat{T}_β such that $e^{\alpha \hat{T}_\beta} = \rho\lambda_{R0}e^{(\rho+\alpha)T}/c_W$, we can rewrite (16) as follows:

$$F\dot{M}C_v = \left[\frac{\rho\lambda_{V0}}{c_W} e^{(\rho+\alpha)T} - \Phi_v \right] c_W e^{\rho t} e^{-(\rho+\alpha)T}. \quad (18)$$

Proposition 4. *The full marginal cost FMC_v (resp. FMC_r) of producing with the virgin resource (resp. recycled) can reach a maximum at time \hat{T}_v (resp. \hat{T}_r).*

1. A maximum for FMC_v exists if $\Phi_v(0) < \frac{\rho\lambda_{V0}}{c_W} e^{(\rho+\alpha)T} < \Phi_v(T)$. It is determined by:

$$\Phi_v(\hat{T}_v) = \frac{\rho\lambda_{V0}}{c_W} e^{(\rho+\alpha)T}.$$

2. If FMC_r is non-monotonous, it reaches a maximum at time $\hat{T}_r = \hat{T}_\beta$, with \hat{T}_β being characterized by (15).

3. If the \hat{T} 's exist, then $\hat{T}_v < \hat{T}_r$ (resp. $>$) if $\lambda_{V0} < \lambda_{R0}$ (resp. $>$).

Proof. (1.) The full marginal cost FMC_v will reach an extremum when \dot{FMC}_v as given by (18) equals 0. In addition, Φ_v is strictly increasing: $\Phi'_v(t) = (e^{\alpha\hat{T}_\beta} - e^{\alpha t})\dot{\beta} + (1 - \beta)e^{\alpha t} > 0$ for any t , as \hat{T}_β is the maximum instant of β . Then, if $\dot{FMC}_v = 0$, the extremum of FMC_v is a maximum. (2.) From (17), and given that $\beta(t) < 1 \forall t$, it comes immediately that $\dot{FMC}_r(t) = 0 \Leftrightarrow t = \hat{T}_\beta$. Since $\rho\lambda_{R0} - c_w e^{-(\rho+\alpha)T} e^{\rho t}$ is decreasing in t , the extremum is a maximum. (3.) From the characterization of \hat{T}_v given by the first part of the proof, we can write: $e^{\alpha\hat{T}_v} - e^{\alpha\hat{T}_\beta} = \left(\frac{1}{1-\beta(\hat{T}_v)}\right) \left[\frac{\rho\lambda_{V0}}{c_w} e^{(\rho+\alpha)T} - \beta(\hat{T}_v)e^{\alpha\hat{T}_\beta}\right] - e^{\alpha\hat{T}_\beta} = \left(\frac{1}{1-\beta(\hat{T}_v)}\right) \frac{\rho(\lambda_{V0}-\lambda_{R0})}{c_w} e^{(\rho+\alpha)T}$. Hence, $\text{sign}(\hat{T}_v - \hat{T}_\beta) = \text{sign}(\lambda_{V0} - \lambda_{R0})$. \square

Analyzing possible outcomes for FMC curves and considering their characteristics described in Propositions 1, 2 and 4 allows us to determine the typical trajectories of the gross marginal surplus $u'(q)$. From (6a) and (6b), we know that $u'(q) = FMC_v(t) \leq FMC_r(t)$ for $t \leq \tilde{T}$ and $u'(q) = FMC_r(t) < FMC_v(t)$ for $t > \tilde{T}$. Given the possible shapes of the FMC 's as described in Proposition 4, the time dependent marginal gross surplus curve can thus be either always increasing, or always declining, or an inverted U-shape. In this last case, the maximum of $u'(q)$ can be attained before the switching time \tilde{T} , after it, or precisely at that time.

Table 2 summarizes all these possible cases depending on whether the recycled material is more expensive (second column) or cheaper (third column) than the virgin resource, and characterizes the conditions on the dates \hat{T}_v and \hat{T}_r for their occurrence. Notice that, as we focus on the scenario of a successive use of both resources, *i.e.* such that $\tilde{T} \in (0, T)$, if $c_r - c_v > 0$ (resp. $<$), then we must have $\lambda_{V0} - \lambda_{R0} > 0$ (resp. $<$) from condition (14), which implies $\hat{T}_v > \hat{T}_r$ (resp. $<$) from Proposition 4.3. This last result allows for reducing the number of possible trajectories.

HERE TABLE 2

By concavity of u , the dynamics of resource extraction q , either from the virgin stock or from the recycled stock, is the inverse of those of the marginal surplus as characterized in Table 2. When the gross marginal surplus curve is increasing, the Hotelling effect prevails due to the scarcity of both

resources (*i.e.* λ_{V0} and/or λ_{R0} large), and the extraction path must be decreasing. However, as previously explained, this scarcity effect on the resource stocks can be counterbalanced by a recycling effect aiming at increasing the resource use in order to provide raw materials to be recycled, which corresponds to phases where $u'(q)$ is decreasing through time.

As the trajectory of u' is possibly inverted U-shaped, the total flow of material can increase at the end of the program (after time \hat{T}_i). This constitutes a catch-up phase: the lower pressure on waste (as the end of the program approaches) and a higher appreciation of the replenishment of the recycled stock counterbalance depletion of the virgin stock. The constraint $\hat{T}_\beta = \hat{T}_r < \hat{T}_v$ follows when \hat{T}_β comes before the end of the program and the scarcity of V is more prominent than R . A catch-up phase with an increasing production comes after the start of a phase when the interest in recovering waste is lowered $\dot{\beta} < 0$, meaning that the social cost of waste accumulation declines more than the social cost of the resource.

4 Accounting for climate change impacts

4.1 Optimal program with greenhouse gas emissions

We now take into account another environmental externality that comes from GHG emissions due to both virgin and recycled resource uses. We assume that input flows v and r respectively contribute at rates δ_v and δ_r to these emissions.¹⁵ Then, the cumulative GHG emissions $E(t)$ at any time t evolves as:

$$\dot{E}(t) = \delta_v v(t) + \delta_r r(t), \quad E(0) = 0. \quad (19)$$

Contrary to waste accumulation which creates local damages, GHG emissions are a global externality which do not directly harm the industry. However, for this last externality to be binding in our model, we define a GHG stabilization cap beyond which damages are supposed to be too high to be supported on a global scale. Such cumulative emission targets are set by international environmental efforts in order to curb global warming. We suppose in our model that this global target has been divided between different countries, leading to national emissions objectives (or budgets) that can be

¹⁵We suppose for the moment that the contribution of the recycling activity represented by rate β to total emissions is included in rate δ_r given for the use of the recycled input. In appendix, we show how relaxing this assumption modify the results.

dispatched between industries. We set this sectoral carbon budget to \bar{E} so that, at any time t , the cumulative emissions cannot be higher:

$$E(t) \leq \bar{E}, \forall t. \quad (20)$$

As we consider neither carbon-free alternative resources nor processes of reducing cumulative emissions, we can no longer use inputs, virgin or recycled, once the cap is reached. With the Inada conditions, it forces $q = 0$ as late as possible, simplifying hypothesis (20) to a final state condition. Hence, the exogenous time-limit T of the model also corresponds to climate objectives of the industrial sector and its associated emissions ceiling, involving the constraint:

$$E(T) \leq \bar{E}. \quad (21)$$

Compared to the set of optimal conditions obtained in the basic model without GHG emissions, only (6a) and (6b) are modified as shown in (22a) and (22b) and an additional condition (22c) on GHG dynamics is introduced. In this condition, $-\lambda_E$ is the co-state variable associated to the dynamics of GHG emissions:

$$u' \leq c_v + f(\beta) + \lambda_V + \delta_v \lambda_E - \beta \lambda_R + (1 - \beta) \lambda_W, \quad (= \text{ if } v > 0) \quad (22a)$$

$$u' \leq c_r + f(\beta) + \lambda_R + \delta_r \lambda_E - \beta \lambda_R + (1 - \beta) \lambda_W, \quad (= \text{ if } r > 0) \quad (22b)$$

$$\dot{\lambda}_E = \rho \lambda_E \Leftrightarrow \lambda_E = \lambda_{E0} e^{\rho t}, \quad (22c)$$

Conditions (6c)-(6f) are unchanged. The transversality condition (7) also remains true but it must be expanded to include:

$$\lambda_E(T)(\bar{E} - E(T))e^{-\rho T} = 0. \quad (23)$$

The new FMC expression for each input now also includes the social marginal cost λ_E of cumulative emissions weighted by the carbon intensity of each input, δ_v and δ_r . We can see that beyond relaxing the resource constraint and reducing waste accumulation, recycling can also be motivated by potentially reducing GHG emissions when the virgin input is more polluting than the recycled one ($\delta_v > \delta_r$).

4.2 Relative scarcity of the stocks and budget

Transversality conditions (7) and (23) state that at the final time T , stock V (resp. R and $\bar{E} - E$) is either a scarce resource, with $V(T) = 0$ (resp. $R(T) = 0$ and $E(T) = \bar{E}$), or it is an abundant resource

and its shadow value is always zero, and thus $\lambda_{V0} = 0$ (resp. $\lambda_{R0} = 0$ and $\lambda_{E0} = 0$). From (1), (2) and (19), final stocks are linked with the following equation:

$$E(T) = \delta_v(V_0 - V(T)) + \delta_r \left(\int_0^T \beta(s)q(s)ds - R(T) \right). \quad (24)$$

Therefore, a full exhaustion of all stocks means that the exogenous decision on a carbon budget perfectly meets the emissions potential of both resources and accounts for the optimal trajectory of recycling. This politically-hard-to-design scenario can then be ruled out.

For the problem to be meaningful, we place our model in a situation where the carbon budget is set to answer a pressing climate constraint, where there should be significant changes of behaviours in the economy. In other words, we assume that the carbon budget \bar{E} to be small enough, as compared with the initial endowment in carbon-emitting resources, so that it will be exhausted no later than time T . As there is no carbon-free option available in this economy, it is then optimal to postpone as much as possible the exhaustion of the budget, $E(T) = \bar{E}$, which implies a strictly positive shadow cost of emissions: $\forall t, \lambda_E(t) = \lambda_{E0}e^{\rho t} > 0$.

However, exhausting the virgin resource would mean $\delta_v V_0 \leq \bar{E}$ from equations (1) and (19). This situation is similar to what we would call a "business as usual" scenario, where imposing a carbon budget has no impact on resource exhaustion on a finite period. For this reason we should consider the virgin resource to be relatively abundant, meaning that $V(T) > 0$ and, from (7), that $\lambda_V(t) = 0 \forall t$. These observations lead us to the following assumption:

Assumption 1. *The terminal state of the model with climate change is such that the carbon budget is saturated $E(T) = \bar{E}$ and the virgin resource is abundant $V(T) > 0$.*

4.3 Arbitration on resources use

FMCs and their difference are now expressed with the social cost of GHG and without the scarcity rent on the virgin resource:

$$FMC_v = c_v - \beta\lambda_R + \delta_v\lambda_E + (1 - \beta)\lambda_W + f(\beta) \quad (25)$$

$$FMC_r = c_r + (1 - \beta)\lambda_R + \delta_r\lambda_E + (1 - \beta)\lambda_W + f(\beta) \quad (26)$$

$$\Delta FMC \equiv FMC_v - FMC_r = -(c_r - c_v) + [(\delta_v - \delta_r)\lambda_{E0} - \lambda_{R0}]e^{\rho t}. \quad (27)$$

The change of inputs now also depends on the GHG emissions difference. Proposition 1 remains overall the same, except for a new eventual time of input switch which is now defined by:

$$e^{\rho \tilde{T}} = \frac{c_r - c_v}{(\delta_v - \delta_r)\lambda_{E0} - \lambda_{R0}} \quad (28)$$

Different scenarios depending on the costs and emissions rates of resources exist and are summarized with Proposition 5:

Proposition 5. *Regardless of the recycling rate path $\beta(t)$, we can summarize the different optimal scenarios under a carbon budget to the following:*

1. *If extracting input from the recycled stock is costlier than from the virgin stock ($c_r > c_v$):*
 - (a) *If the use of the virgin resource emits less than the recycled one ($\delta_v < \delta_r$), or if the scarcity of the recycled resource weighs more than the emissions difference, we use only the virgin resource;*
 - (b) *If the virgin resource is more polluting ($\delta_v > \delta_r$), at a level enabling \tilde{T} to exist as defined in (28) (i.e. $(\delta_v - \delta_r)\lambda_{E0} > \lambda_{R0}$) and provided that $\tilde{T} \leq T$, we use the virgin resource up to time \tilde{T} ;*
2. *If extracting input from the virgin stock is costlier than from the recycled stock ($c_r < c_v$), we use the virgin resource up to time \tilde{T} and then the recycled resource. In this case, the emissions difference must also weigh less than the scarcity of the recycled resource ($(\delta_v - \delta_r)\lambda_{E0} < \lambda_{R0}$).*

Proof. The different optimal paths can be drawn from the positivity conditions for ΔFMC . □

The arbitration on resources use is significantly influenced by the carbon constraint, as the scarcity of the GHG budget also leads to a switch in inputs, while the virgin resource remains abundant. However, the reasoning on recycling rate β developed earlier remains valid, as equation (6c) does not include the social cost of carbon. Last, as previously mentioned, we restrict the analysis to the case of an interior switch of inputs, i.e. $\tilde{T} \in (0; T)$, and when recycling is non-monotonous, i.e. $0 < \hat{T}_\beta < T$.

4.4 Dynamics of resource flows

While no closed-form expression for the consumption trajectories of resources can be determined analytically in this program, first-order conditions can give us some insights on the different phases of production (as seen previously) and the fluctuations of the material flows. For that we consider a phase during which $v > 0$ (and $r = 0$) or $r > 0$ (and $v = 0$). We look at the dynamics of $v(t)$ and $r(t)$ by examining the time derivatives of conditions (6a) and (6b). Using (11), we obtain for any value of $\beta(t)$:

$$F\dot{M}C_v = \left[\rho\delta_v\lambda_{E0} - \rho\beta\lambda_{R0} - (1-\beta)c_W e^{-(\rho+\alpha)T} e^{\alpha t} \right] e^{\rho t}, \quad (29)$$

$$F\dot{M}C_r = \left[\rho\delta_r\lambda_{E0} + \rho(1-\beta)\lambda_{R0} - (1-\beta)c_W e^{-(\rho+\alpha)T} e^{\alpha t} \right] e^{\rho t}. \quad (30)$$

For this we define functions Φ_v and Φ_r as follow:

$$\Phi_v(t) \equiv [1 - \beta(t)]e^{\alpha t} + \beta(t)e^{\alpha\hat{T}_\beta}, \quad (31)$$

$$\Phi_r(t) \equiv [1 - \beta(t)]e^{\alpha t} - [1 - \beta(t)]e^{\alpha\hat{T}_\beta}, \quad (32)$$

where, from (15), \hat{T}_β is such that $e^{\alpha\hat{T}_\beta} = \frac{\rho\lambda_{R0}e^{(\rho+\alpha)T}}{c_W}$ (which is, if it exists in $]0; T[$, the instant when recycling reaches its maximum).

Proposition 6. *The full marginal cost FMC_v (resp. FMC_r) of producing with the virgin resource (resp. recycled) can reach a maximum at time \hat{T}_v (resp. \hat{T}_r).*

1. *A maximum exists if $\Phi_v(0) < \frac{\rho\delta_v\lambda_{E0}}{c_W} e^{(\rho+\alpha)T} < \Phi_v(T)$ (resp. $\Phi_r(0) < \frac{\rho\delta_r\lambda_{E0}}{c_W} e^{(\rho+\alpha)T} < \Phi_r(T)$).*

These instants correspond to maxima of the FMCs and are determined by:

$$\Phi_v(\hat{T}_v) = \frac{\rho\delta_v\lambda_{E0}}{c_W} e^{(\rho+\alpha)T} \text{ and } \Phi_r(\hat{T}_r) = \frac{\rho\delta_r\lambda_{E0}}{c_W} e^{(\rho+\alpha)T}.$$

2. *If true, we have:*

$$\hat{T}_\beta < \hat{T}_r < \hat{T}_v \text{ if } c_v < c_r,$$

$$\hat{T}_\beta < \hat{T}_r \text{ and } \hat{T}_v < \hat{T}_r \text{ if } c_r < c_v.$$

Proof. (1.) The full marginal cost FMC_v will reach an extremum when $\dot{FMC}_v = 0$. Using (29) and (31), this condition becomes: $\dot{FMC}_v = \left[\frac{\rho\delta_v\lambda_{E0}}{c_w} e^{(\rho+\alpha)T} - \Phi_v \right] c_w e^{\rho t} e^{-(\rho+\alpha)T} = 0$. In addition, Φ_v is strictly increasing: $\Phi'_v(t) = \left(e^{\alpha\hat{T}_\beta} - e^{\alpha t} \right) \dot{\beta} + (1 - \beta)e^{\alpha t} > 0$ for any t , as \hat{T}_β is the maximum instant of β . Then, if $\dot{FMC}_v = 0$, the extremum of FMC_v is a maximum. The same reasoning is applied to FMC_r , with $\dot{FMC}_r = \left[\frac{\rho\delta_r\lambda_{E0}}{c_w} e^{(\rho+\alpha)T} - \Phi_r \right] c_w e^{\rho t} e^{-(\rho+\alpha)T}$.

(2.) First, from the expression of Φ_r , condition $\Phi_r(\hat{T}_r) = \frac{\rho\delta_r\lambda_{E0}}{c_w} e^{(\rho+\alpha)T}$ implies that $\hat{T}_\beta < \hat{T}_r$.

Next, when $c_v < c_r$, from (27), we can write $\Delta\dot{FMC} = \rho[\Delta FMC + (c_r - c_v)]$. As \tilde{T} exists, we know that there is a second phase during which the recycled input is used, meaning $\Delta FMC > 0$ for $t > \tilde{T}$, which involves $\Delta\dot{FMC} > 0$. This result implies $[(\delta_v - \delta_r)\lambda_{E0} - \lambda_{R0}] > 0$. As it is independent of time, we can generalize with $\Delta\dot{FMC} > 0, \forall t$. Hence $\dot{FMC}_v > \dot{FMC}_r$. By rewriting expressions of \dot{FMC}_i with Φ_i ($i \in \{v, r\}$), which are proved to be increasing functions of time, thus $\dot{FMC}_i > 0$ before \hat{T}_i and $\dot{FMC}_i < 0$ after. This gives us that $\hat{T}_r < \hat{T}_v$.

Else, when $c_r < c_v$, expression (28) gives us that $[(\delta_v - \delta_r)\lambda_{E0} - \lambda_{R0}] < 0$. When used in the expression of $\Delta\dot{FMC}$, it leads to $\dot{FMC}_r > \dot{FMC}_v$, hence $\hat{T}_v < \hat{T}_r$. \square

From Proposition 6, we can induce the same type of results on the dynamics of the gross marginal surplus and the dynamics of the resource extraction as those contained in Table 2. Whatever the relative cost of each input, we obtain a time trajectory of the resource extraction that is either strictly increasing, or strictly decreasing or U-shaped. Hence, as in the basic model, a catch-up phase can occur at the end of the program, during which the total flow of material increases. However, this phase is not driven anymore by the relative scarcity of each resource stocks (the virgin stock is assumed to be abundant), but rather by the relative carbon footprint of each resource and by the carbon budget constraint on cumulative emissions with respect to the environmental damage due to waste accumulation.

5 Decentralization and policy implications

In order to analyze environmental policy tools used to promote recycling and the reduction of carbon emissions, there is a need to discuss a decentralized version of our model. To decentralize the model, transfer functions for the different actors in the economy are introduced. These functions correct

for the externalities previously developed in this paper. The basis of the decentralized model is the extended model developed in Section 4 that includes GHG emissions, as it offers a richer discussion on combined externalities and allows for the observation of different mixes of policies.

As a basic structure, we assume that the economy is composed of four agents: the final consumer, the producer of the manufactured good, the virgin resource sector, and the recycled resource sector which manages both the recycling activity and the exploitation of the recycled stock.¹⁶ The property rights of each resource stocks are correctly defined so that each extracting sector is owner of its reserves. These agents can take actions on four markets, assumed to be perfectly competitive: the virgin resource market (price p_v), the recycled resource market (price p_r), the manufactured good market (price p_q), and the waste market (price p_w), provided that such a market exists. Initially, we omit the waste market and we introduce it in a second iteration of the decentralized model.¹⁷

The policy-maker can influence private decisions on each market thanks to a set of monetary transfer functions, denoted by $T_C(\cdot)$, $T_P(\cdot)$, $T_V(\cdot)$ and $T_R(\cdot)$, to the consumer, the producer, the virgin resource sector and the recycling sector, respectively.

5.1 Equilibrium in the absence of a waste market

The consumer determines the quantity of final good q they will consume in order to maximize her instantaneous surplus function $S_C \equiv [u(q) - p_q q + T_C(q)]$.¹⁸ The producer of the manufactured good chooses the quantity of inputs v and r in order to maximize instantaneous profits $S_P \equiv [p_q q - p_v v - p_r r + T_P(v, r)]$ subject to the technological constraint $q = v + r$. The virgin resource owner chooses the extraction rate v maximizing profits during an exogenous finite time T , expressed by $S_V \equiv \int_0^T [p_v v - c_v v + T_V(v)] e^{-\rho t} dt$ subject to (1). Last, the objective of the recycling sector is to determine both the share of final consumption good to be recycled β , and the recycled resource extraction r that maximize $S_R \equiv \int_0^T [p_r r - c_r r - q f(\beta) + T_R(r, \beta)] e^{-\rho t} dt$ subject to (2). Note that we directly focus here on interior values of β (the conditions for corner solution has been discussed

¹⁶We can obtain similar results with a less developed model, where the production sector is omitted. In this case, the final consumer directly consumes the two types of resources which are perfect substitutes for each other.

¹⁷Note that by considering this market economy, we only apply a partial decentralization process, in the sense that we do not develop explicitly the financial market. We assume that private agents discount their monetary flows at the rate ρ . Moreover, we do not include transaction and transportation costs here.

¹⁸There is no dynamic budget constraint, as we do not develop the financial market.

previously in the central planner problem).

We introduce notations λ_i^* and $\tilde{\lambda}_i$ ($i \in \{R; E; W\}$) that refer respectively to the co-state variables of the central planner problem and the co-state variables of the decentralized problem.

Proposition 7. *Given λ_i^* $i \in \{E; W\}$, the shadow costs for an optimal trajectory, the set of policy instruments that restores optimality in the absence of a market for waste must satisfy the following conditions:¹⁹*

$$\frac{1}{q} \frac{\partial T_R(r, \beta)}{\partial \beta} = \lambda_W^* \quad (33)$$

$$T'_V(v) + \frac{\partial T_P(v, r)}{\partial v} + T'_C(q) = \pi(\beta) - \delta_v \lambda_E^* - \lambda_W^* \quad (34)$$

$$\frac{\partial T_R(r, \beta)}{\partial r} + \frac{\partial T_P(v, r)}{\partial r} + T'_C(q) = \pi(\beta) - \delta_r \lambda_E^* - \lambda_W^*, \quad (35)$$

with $\pi(\beta) \equiv \beta f'(\beta) - f(\beta) \geq 0$ for any β .

Proof. Maximizing surplus functions of the consumer, the producer, the resource owner and the recycling sector, we get the following first-order conditions:

$$u'(q) \leq p_q - T'_C(q), \quad (= \text{ if } q > 0) \quad (36a)$$

$$p_q \leq p_v - \frac{\partial T_P}{\partial v} \text{ and } p_q \leq p_r - \frac{\partial T_P}{\partial r}, \quad (= \text{ resp. if } v > 0 \text{ or } r > 0) \quad (36b)$$

$$p_v \leq c_v + \tilde{\lambda}_V - T'_V(v), \quad (= \text{ if } v > 0), \quad \text{with } \tilde{\lambda}_V = \tilde{\lambda}_{V0} e^{\rho t} \quad (36c)$$

$$p_r \leq c_r + \tilde{\lambda}_R - \frac{\partial T_R}{\partial r}, \quad (= \text{ if } r > 0), \quad \text{with } \tilde{\lambda}_R = \tilde{\lambda}_{R0} e^{\rho t} \quad (36d)$$

$$f'(\beta) \geq \tilde{\lambda}_R + \frac{1}{q} \frac{\partial T_R}{\partial \beta}, \quad (= \text{ if } \beta > 0). \quad (36e)$$

Given Eq. (36a)-(36d), the market-clearing conditions that characterize an equilibrium are:

$$u'(q) = c_v + \tilde{\lambda}_V - T'_V(v) - \frac{\partial T_P}{\partial v} - T'_C(q) \quad (37a)$$

$$u'(q) = c_r + \tilde{\lambda}_R - \frac{\partial T_R}{\partial r} - \frac{\partial T_P}{\partial r} - T'_C(q), \quad (37b)$$

Additionally, we include condition (36e) to characterize the equilibrium. With these conditions, we can then compare them to the first-order conditions for optimal interior solutions in the central planner model (conditions (22a), (22b) and (6c)) to get equations (33) to (35). \square

¹⁹Here, as we follow a partial equilibrium approach, we do not consider the budget balance equation of the policy-maker. In particular, there is no reason at all to suppose that the net sum of all these monetary transfers must be equal to zero.

Simple economic policies can be illustrated when considering linear and additive separable transfer functions, *i.e.* tax-subsidy schemes: $T_C(q) = T_C \times q$, $T_P(v, r) = T_{Pv} \times v + T_{Pr} \times r$, $T_V(v) = T_V \times v$ and $T_R(r, \beta) = T_{Rr} \times r + T_{R\beta} \times \beta q$. In this case, first-best implementation requires four policy instruments.

The first instrument is a unit subsidy $\pi(\beta)$ (always non-negative given the properties of function $f(\cdot)$) on the flow of produced/consumed goods. For the case of an interior solution ($\beta > 0$), from (6c), this subsidy can be rewritten as follows: $\pi(\beta) = [\beta q \Phi_\beta - q f(\beta)]/q$. This expression reads as the average net social gain of recycling the share β of the flow q of final good (remind that Φ_β is the marginal social benefit of the recycling effort and $f(\beta)$ is the recycling cost per unit of output). The policy-maker must implement such a subsidy in order to correct for the positive externality generated by the waste production for free in the absence of waste market. It stems from the absence of a market for waste and the dynamic framework of the model. Recycled input accumulation is indeed not accounted for in the static modeling literature, thus not finding this externality. To the best of our knowledge, only the dynamic work of [Pittel et al. \(2010\)](#) highlights this market failure.

The second instrument is a unit subsidy of value λ_W^* on the flow of recycled good βq . It is required due to the avoided waste accumulation. This subsidy is always attributed to the recycling industry.

The third and fourth instruments are unit taxes corresponding to the two environmental negative externalities: GHG emissions and waste accumulation.

To integrate the GHG emissions externality in the model, we define the first unit tax using (34) and (35) to write: $T_{Rr} + T_{Pr} - T_V - T_{Pv} = (\delta_v - \delta_r) \lambda_E^*$. In this expression, λ_E^* is a uniform carbon tax that is applied to carbon content rate of each primary material. This tax can be imposed either to the producer of the final good or to the extraction sectors.

To integrate the waste accumulation externality in the model, we define the second unit tax of value λ_W^* that focuses on the potential waste generation from production/consumption of the final good q . This tax can be imposed either to the producer of the final good or to the consumer.

Observing the two monetary transfers associated with waste accumulation (instruments two and four), we can notice that when aggregated together, they represent a single tax of value λ_W^* on the effective waste accumulation $(1 - \beta)q$.

Table 3 illustrates various possibilities of who carries the burden or receives the benefit of these

instruments.²⁰ Note that all these options are revenue-equivalent for the policy-maker.

HERE TABLE 3

These examples show how the carbon tax should be distributed between the producer (T_P) and resource managers (T_V and T_R). The implementation of this tax is the subject of a wide range of literature, and is not the focus of this paper. While existing policies are often developed to favor the recycling industry, our model acknowledges the level of the GHG emissions of the sector, and advocates for an appropriate carbon tax on this industry. The tax does not replace the existing support for the recycling industry. These tax-subsidy support schemes remain as they address other issues unrelated to GHG emissions.

It is interesting to note that whatever the transfer structure, there is a subsidy for the recovered flow of material for the recycling sector. The combination of this subsidy with a taxation on products is often promoted in economic literature as a deposit-refund scheme (Fullerton and Kinnaman, 1995; Choe and Fraser, 1999). Additionally, this combination accounts for the specific characteristics of the industrial sector like emissions rates, private and social costs of waste management (Acuff and Kaffine, 2013). Although the system suggested by our model is still hardly implemented, there are several subsidy schemes for recycling industries and several taxation schemes on waste.

In the examples of Table 3, taxation relies on the global flow of material subject to potential waste damage. Extended Producer Responsibility (EPR) systems are an example where producers have to assume financial responsibility for the potential environmental damage of their products during their whole life-cycle (through an eco-contribution). In example 2 and 3, the cost of potential waste damage is assumed by the final consumer. We can note that even if the tax is applied to the producer, it will eventually be reflected on the price of the product and the burden of the tax will be assumed by the consumer. In existing examples, EPR instruments are implemented by industrial sector, in varying sizes. In France in 2020, Citeo (an agency organizing EPR for packaging industries) fixes a baseline level for eco-contributions of 16,53 ct€/kg for paper and cardboard, and 28,88 ct€/kg for light PET

²⁰As usual, the transfer of the tax burden between producers and consumers may depend on the price elasticity of demand and supply functions. It is not the purpose here to develop this aspect.

(colourless polyethylene terephthalate).²¹ Note that almost 80% of these amounts are redistributed to the recycling sector (collecting, sorting and resource production), thus covering the subsidy part of our monetary transfers (Citeo, 2019).

In the case of example 3, note that subsidy $\pi(\beta)$ is applied to resource producers instead of the final consumer. This reflects the prospective activity of recycling, as consumed resources have the potential to be recycled resources or waste.

5.2 Existence of a waste market

Let us now assume that the final consumer can sell its flow of waste q at price p_w on a specific market. The consumer's surplus must now include the revenue from the waste sale $p_w q$: $S_C = [u(q) + (p_w - p_q)q + T_C(q)]$. The new expression of the intertemporal surplus of the recycling sector is $S_R = \int_0^T [p_r r - c_r r - qf(\beta) - p_w q + T_R(r, \beta, q)]e^{-\rho t} dt$, which must be maximized subject to (2). The private marginal cost of recycling now also includes purchasing waste. Behaviors of the other agents are unchanged.

Proposition 8. *Given λ_i^* $i \in \{E; W\}$, the shadow costs for an optimal trajectory, the set of policy instruments that restores optimality with a market for waste must satisfy the following conditions:*

$$\frac{1}{q} \frac{\partial T_R}{\partial \beta} = \lambda_W^* \quad (38)$$

$$T'_V(v) + \frac{\partial T_R}{\partial q} + \frac{\partial T_P}{\partial v} + T'_C(q) = -\delta_v \lambda_E^* - (1 - \beta) \lambda_W^* \quad (39)$$

$$\frac{\partial T_R}{\partial r} + \frac{\partial T_R}{\partial q} + \frac{\partial T_P}{\partial r} + T'_C(q) = -\delta_r \lambda_E^* - (1 - \beta) \lambda_W^*. \quad (40)$$

Proof. Programs of the final good producer and of the virgin resource extracting sector are not affected by the existence of a waste market. Thus, Eq. (36b) and (36c) still hold. Additionally, the first-order conditions (36d) and (36e) are unchanged for the recycling sector, but another condition appears for this sector (41a) and the condition for the consumer is changed (41b):

$$u'(q) + p_w = p_q - T'_C(q), \quad (41a)$$

$$f(\beta) + p_w = \beta \tilde{\lambda}_R + \frac{\partial T_R}{\partial q}. \quad (41b)$$

²¹This amount is completed by a unit based amount and a bonus-penalty system depending on the eco-design of the product.

When comparing these equations with the first-order conditions for optimal interior solutions in the central planner model given by (22a), (22b) and (6c), we get equations (38) to (40). \square

As expected, the waste market allows to fully internalize the positive externality associated with the waste generation by the final consumer.

6 Conclusion

In this paper, we provided a discussion on optimal use of resources when climate change and waste are both additional constraints for the social planner. Depending on initial characteristics of the industrial sector, the optimal path can be divided into phases of virgin and recycled production, with a potential switch of inputs if recycling becomes more socially profitable. This change in the sector intervenes when the scarcity of the virgin resource prevails and/or private costs also favour the use of recycling. When introducing a climate constraint and a difference between emissions rates, a change of inputs can also occur when this difference is sufficiently large, especially compared to the fixed difference between private delivery costs of inputs. In parallel, the social planner will develop a recycling (or waste recovering) strategy, such as it alleviates the cost of waste accumulation and allows for production from a recycled input. This shows the duality of the activity of sorting waste: a practical goal being the reduction of costs and a speculative goal as the use of recycled input can potentially occur in the future. This leads to inverse U-shaped recycling programs, where it is optimal to increase recycling at first and then decrease it. Moreover, when the marginal cost of recycling is too high, it leads to phases where the recycling rate is zero (although it is still possible to produce from the existing stock of recycled input at the same time). In parallel, under certain conditions, production shows catch-up phases at the end of the program, as full marginal costs start decreasing (involving an increase in production). Our model gives a better understanding of the complementary nature of developing climate change objectives and addressing resource scarcity issues. The model addresses the fact that recycled inputs, while often a cleaner option for production also delaying scarcity, still come with GHG emissions.

While the central planner program gives many insights regarding optimal recycling activities, it

does not give suitable results for policy making. The decentralized model of the sector allows to specify the need for public intervention. This new model highlights a possible tax/subsidy scheme in order to implement a first-best solution. To do this first best solution, a taxation based on GHG emissions must be introduced in both sectors, weighed by the carbon intensity of the branch (the carbon intensity is higher for virgin extraction in most cases). This comes in addition to a tax-subsidy system based on waste and stock replenishment which are already demonstrated in Extended Producer Responsibility programs currently implemented for certain products.

Our model is mainly focused on the case of metal industries, where strong GHG emissions asymmetries and perfect substitution are relevant hypothesis. In the appendix we address other constraints on recycling: the existence of a scrap value for waste, a decoupling of GHG emissions between collection of waste and transformation into an input, and a limited capacity for the stock of recycled input. Extensions toward other industries would require relaxing some hypothesis used here. For instance, loss of quality is in fact an important topic for the academic research on circular economy, especially when the efficiency of a process is studied ([Figge et al., 2018](#)). This is, for instance, the case in the paper and cardboard industry, when each recycling loop lowers the quality of the pulp and only allows a limited amount of cycles depending on the needed quality (usually 7 and 8 cycles are technically possible, but fiber is rather used 3.5 times on average in Europe ([ERPC, 2015](#))). [Lafforgue and Rouge \(2019\)](#) provide the basis for understanding the optimal production path when technological progress in recycling allows for higher quality in recycled inputs.

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A Appendix

Our model can be extended to account more precisely for existing environmental externalities. First, we assume the existence of a scrap value for the waste stock after the program. Second, we decouple the GHG emissions of the recycling process into those related to the collection of waste and to those related to production of the recycled input. To end, we consider the limited capacity of the recycling sector by imposing a cap on the cumulative flows of recycled materials.

A.1 Scrap value for waste

Here, we assume that waste accumulation is no longer damaging from time T onward: $D(W(t)) = 0, \forall t \geq T$. From (3), given that extraction/consumption flows are zero, the remaining stock of wastes evolves as: $W(t) = W(T)e^{-\alpha(t-T)}$, for $t \geq T$. A more realistic approach consists of attaching a scrap value Ω to $W(T)$ in case where the damage caused by the left-over waste stock remains after the end of the exploitation period of time:

$$\Omega(W(T)) = \int_T^\infty D\left(W(T)e^{-\alpha(t-T)}\right) e^{-\rho t} dt. \quad (42)$$

This scrap value must be added to the value function (4) of the social planner program, which slightly modifies the transversality condition relative to stock W :

$$[\lambda_W(T) - \Omega'(W(T))] W(T)e^{-\rho T} = 0. \quad (43)$$

Using (6f), (42) and (43), the shadow cost of the waste stock at any point in time can be expressed as:

$$\begin{aligned} \lambda_W(t) &= e^{(\rho+\alpha)t} \left[\Omega'(W(T))e^{-(\rho+\alpha)T} + \int_t^T D'(W)e^{-(\rho+\alpha)s} ds \right] \\ &= \int_t^T D'(W)e^{-(\rho+\alpha)(s-t)} ds + e^{-\rho T} \int_T^\infty D'(W)e^{-(\rho+\alpha)(s-t)} ds. \end{aligned} \quad (44)$$

At any time t , the shadow marginal cost of waste contains two components: the sum from t to T of the marginal damage, discounted at rate $(\rho + \alpha)$ to reflect the marginal absorption process of the stock of waste by the environment, and the value at time T of the sum from T onward of the discounted marginal damage, i.e. the scrap value term. With constant marginal damage, this shadow cost becomes:

$$\lambda_W(t) = \frac{\theta}{(\rho + \alpha)} \left(1 - (1 - e^{-\rho T})e^{-(\rho+\alpha)(T-t)} \right), \quad (45)$$

and we still have $\dot{\lambda}_W < 0$. The scrap value intervenes by adding an exogenous factor $1 - e^{-\rho T} \in]0, 1[$ to the decreasing term of the shadow cost. The remaining shadow cost for waste at the end of the program is the marginal scrap value $\frac{c_W}{\rho + \alpha} e^{-\rho T} = \Omega'(W(T))$, instead of zero for the model previously discussed.

This additional assumption does not change the change the arbitration in the model, as the first-order conditions (6a)-(6f) remain the same. However, the optimal recycling rate follows a different time path, according to the following proposition.

Proposition 9. *When the recycling rate $\beta(t)$ is not always zero and considering a scrap value for the waste stock damage, it reaches a maximum at time \hat{T}_β . This instant can take the following values:*

$$\hat{T}_\beta = \begin{cases} 0 & , \text{ if } \frac{e^{(\rho+\alpha)T}}{(1-e^{-\rho T})} \leq \frac{c_W}{\rho\lambda_{R0}} \text{ or } \lambda_{R0} = 0 \\ T & , \text{ if } \frac{c_W}{\rho\lambda_{R0}} \leq \frac{e^{\rho T}}{(1-e^{-\rho T})} \\ \frac{1}{\alpha} \left[\ln \left(\frac{\rho\lambda_{R0}}{c_W(1-e^{-\rho T})} \right) + (\rho + \alpha)T \right] \in]0, T[& , \text{ if } \frac{e^{\rho T}}{(1-e^{-\rho T})} < \frac{c_W}{\rho\lambda_{R0}} < \frac{e^{(\rho+\alpha)T}}{(1-e^{-\rho T})}. \end{cases} \quad (46)$$

Proof. Now the marginal benefit of recycling is:

$$\Phi_\beta(t) = \lambda_{R0} e^{\rho t} + \frac{c_W}{\rho + \alpha} \left(1 - (1 - e^{-\rho T}) e^{-(\rho+\alpha)(T-t)} \right)$$

Following the same analysis as the general case, we get Proposition (9). \square

A.2 Decoupling emissions of the recycling branch

Another possible extension of the model is to consider a decoupling of the emissions related to the recycling process. In that case, parameter δ_r only concerns the transformation and use of the recycled input, while we add a constant rate δ_β for the activity of collecting and sorting from the waste flow: $\delta_\beta \beta(t) q(t)$. This leads to the following new dynamic equation for emissions accumulation:

$$\dot{E}(t) = [\delta_v + \beta(t)\delta_\beta]v(t) + [\delta_r + \beta(t)\delta_\beta]r(t), \quad E(0) = 0. \quad (47)$$

Practically speaking, this modification in the model does not change the arbitration between the two resources. This is because the term relative to this type of emissions disappears with the Full Marginal Costs difference, the same way it does for terms relative to waste accumulation, recycled

stock replenishment and the cost of recycling. However, there is an impact on the first-order condition relative to recycling, with a new function Φ_β :

$$\Phi_\beta = \lambda_R + \lambda_W - \delta_\beta \lambda_E \quad (48)$$

This more complex expression of the marginal social gain of recycling can be easily reinterpreted: we now recycle to replenish stock R and alleviate the cost of stock W but it costs emissions to the industrial sector, at rate δ_β . Developing this expression, we obtain: $\Phi_\beta = (\lambda_{R0} - \delta_\beta \lambda_{E0})e^{\rho t} + \frac{c_W}{\rho + \alpha} [1 - e^{-(\rho + \alpha)(T-t)}]$. The same analysis as before can be done for the evolution of the recycling rate. However we can also add the following proposition:

Proposition 10. *It is never optimal to start recycling under the two following conditions:*

- *There is a higher initial social cost for emissions than the depletion of the recycled stock, i.e.*
 $\lambda_{R0} \leq \delta_\beta \lambda_{E0}$;
- *Damage of waste accumulation is such that $\frac{c_W}{\rho + \alpha} \leq f'(0)$.*

In this case, the recycled resource is never used, $\lambda_{R0} = 0$.

Proof. Given the new expression of Φ_β , we see that it is decreasing when $\lambda_{R0} \leq \delta_\beta \lambda_{E0}$. As $\lim_{-\infty} \Phi_\beta = \frac{c_W}{\rho + \alpha}$, equation $\Phi_\beta = f'(\beta)$ does not have a solution when $\frac{c_W}{\rho + \alpha} \leq f'(0)$, proving the necessary condition of the proposition. As there is no recycled stock initially, we never use the recycled resource. Note that a more constraining condition for the absence of recycling is $\Phi_\beta(0) \leq f'(0)$. \square

By adding an emission term for recycling activities, we can highlight a simple condition on the parameters of the model for which recycling is never optimal. If the social cost of GHG emissions relating to the recycling activity is too high, or if the avoided cost of waste accumulation is too low, recycling is never optimal. Finally, note that we also observe the catch-up phase for the use of inputs with this extension of the model.

A.3 Limiting the capacity of the recycling sector

A.3.1 Optimal program with a cap on $R(t)$

In this final iteration of the model, the recycling sector faces an upper limit regarding the size of the recycled stock R . This limit suggests that the speculative strategy for recycling is bounded. One can view this limit as the implicit storage cost of the recycled resource. To analyze this concept more individually, we do not include the previously analyzed GHG ceiling. For this extension, we introduce a cap \bar{R} such as $R(t) \leq \bar{R}$. This new constraint leads to a new first order condition on stock R , introducing the associated co-state variable χ :

$$\dot{\lambda}_R = \rho\lambda_R + \chi \text{ with } (\bar{R} - R(t))\chi(t) = 0 \text{ and } \chi(t) \geq 0. \quad (49)$$

The other optimal conditions refer to the basic case developed in Section 4. The optimal sequence of use has now the following characteristics:

Proposition 11. *Regardless of the recycling rate $\beta(t)$, the optimal path with a cap on the recycled stock is such that:*

1. *While the recycled stock is not full, it follows the optimal program described in Proposition 1, with a sequential order of use.*
2. *When recycled stock is full, two scenarii can occur:*
 - (a) *If there is no recycling, we only use the virgin resource;*
 - (b) *If there is some recycling, there can be a stationary regime for R where the two resources are used simultaneously such as $\beta v = (1 - \beta)r$.*

Proof. While the cap \bar{R} is not reached, equations from the general problem hold, especially $\dot{\lambda}_R = \rho\lambda_R$, thus proving 1. When \bar{R} is reached, meaning a maximum for R , we have $\beta v = (1 - \beta)r$ from equation (2) (it can be only instantaneous and $R(t)$ starts decreasing again). Then this equality for stationarity on R proves 2. □

The cap on the recycling stock offers the possibility of a stationary regime, thus with a simultaneous use of the two resource. At each instant, the recycling rate allows to perfectly compensate the depletion

of R . However, this should not be interpreted as a sustainable scenario, as $\beta < 1$ gives an asymmetry in the material flows: if recycling is low ($\beta < 0,5$) virgin production prevails, the opposite otherwise.

A.3.2 Stationary regime for stock $R(t)$

As seen before, the stationary regime for stock $R(t)$ is defined by a simultaneous production from both inputs, and a ratio v/r governed by the evolution of the recovery rate β . From Proposition 11 and the relationship between flows, we know that this ratio grows when rate β declines: less recovery in recycling means that stock R is less exploited (r diminishes) and eventually consumption is sustained by higher extraction (v increases).

To analyze further, let T^* be the starting point of this stationary regime. From the new FOC on stock R given by equation (49) and its associated transversality condition, we get expression for the shadow value of the recycled stock: $\lambda_R(t) = -\int_t^T \chi(s)e^{-\rho(s-t)} ds$. This cost is negative, as scarcity is not at stake here ($R(T) = \bar{R} > 0$) and the recycling stock is subject to an upper limit (similar to what was done for the GHG emissions stock). The model during the stationary regime is now governed by the following equations, after instant T^* :

$$\forall t \geq T^*, \quad c_v + \lambda_{V0}e^{\rho t} = c_r - \int_t^T \chi(s)e^{-\rho(s-t)} ds \quad (50)$$

$$f'(b) = -\int_t^T \chi(s)e^{-\rho(s-t)} ds + \frac{c_W}{\rho + \alpha} \left(1 - e^{-(\rho + \alpha)(T-t)}\right). \quad (51)$$

Equation (50) comes from the simultaneous use of the resource, meaning that both marginal costs are equal. One necessary condition as seen here is that the cost for the recycled input c_r should be higher than the cost for the virgin input c_v when added to its scarcity rent λ_V . When expressed at the end of the program, it states that both should be equal. In that sense, either the virgin resource is scarce ($\lambda_{V0} > 0$) or both marginal delivery costs are equal.

Another necessary condition for simultaneous use of the resources is that the recycling activity occurs during the whole stationary phase ($\beta > 0$). Equation (51) expresses the equality between cost and marginal benefit of recycling as described earlier. The optimal recycling program can be precised as follow:

Proposition 12. *If the optimal path of the program enters a stationary regime for stock R , the optimal recycling strategy is to always recycle during this phase, and if β reaches a maximum during this phase,*

\hat{T}_β is defined by:

$$T^* < \hat{T}_\beta < T \quad \text{such that} \quad (c_r - c_v) = \frac{cW}{\rho} e^{-\alpha(T - \hat{T}_\beta)}. \quad (52)$$

Proof. From the equality of FMCs (50) expressed at instant T of the program, we get $\lambda_{V0} = (c_r - c_v)e^{\rho T}$. This equality replaces λ_R in the FOC for β , and is used in equality $f'(\beta) = \Phi(\beta)$. The maximum is reached at \hat{T}_β when $\Phi(\hat{T}_\beta) = 0$, thus giving equation (52). \square

During this specific phase, we have an exogenous expression for the maximum of recycling. This expression (52) shows that this instant occurs at a later time when the cost difference $c_r - c_v$ is relatively significant. On the other hand, when the damage of waste accumulation is relatively significant, \hat{T}_β happens earlier. This prompts early recycling as we do not have a scarcity of R and the shadow cost of waste diminishes.