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Soft sensor design for estimation of thermal behavior of encapsulating materials in power electronic module

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A B S T R A C T

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With the emergence of new semi-conductor technologies, an increasing number of high integrated power electronic modules are designed. The increase of reliability of power modules induces the precise knowledge of the local temperature, even if it cannot be measured at any location. In this paper, the design of a soft sensor, more precisely a linear functional observer, is proposed. It enables the estimation of the temperature at any location using measurements provided from thermal sensors located at a number of precise points. The aim is to design a reduced size observer that could be implemented on a real-time embedded target such as Digital Signal Processor. Consequently, it is necessary to obtain a minimal order observer to limit the computation complexity.

1. Introduction

The emergence of Wide Band Gap materials (SiC, GaN, C) and new generation hybrid integration techniques significantly enhances performances of power electronic modules. Such modules should be able to operate under severe environmental conditions and constraints: high temperature and high-power density, fast switching, etc. High temperatures provoke new constraints which become critical for power electronics assemblies. Thus, for power electronic modules, real time condition monitoring is a great subject of interest. It has been shown in [1] that the evolution of local constraints in a power electronic module, which can be thermal or thermo-mechanical, have a negative effect on the lifetime of the module. Consequently, precise knowledge of the temperatures at specific locations in the module, such as the temperature of the semiconductor chips or of the encapsulating silicone gel around these chips, is necessary. However, the use of thermal sensors may be difficult at some locations inside of the power module. For these reasons the objective of the following work is to estimate this physical variable in a specific non-measured location, using measured data by a number of sensors.

Some research deals with the thermal model of power module to estimate the temperature of semiconductor chips. In order to obtain an accurate thermal model of power modules, heat transfers are considered and a spatial discretized thermal model can then be introduced [2,3]. Note that the heat equation is a partial differential equation (PDE) with continuous distribution of parameters leading to infinite order systems.

The main method used to simulate these systems is to discretize PDE leading to finite order systems. This method leads to a high order linear time-invariant state space representation. In order to design an associated observer, many studies propose to reduce the size of the state space representation either of the system [4] or the observer [5]. Reducing the size of the system while discretizing the heat equation leads to additional approximation on temperature. In this paper, a reduced order linear functional observer (LFO) is presented to estimate the state of high order systems. These observers offer the advantage of estimating local temperature without estimating the whole states [6].

From the seminal works of Luenberger [7] solutions of the observation problem for a linear time-invariant system have been developed. Methods have been elaborated for the design of LFO of the full or partial state vector of the system leading to reduced order observers. Thus, in our application, local temperature may be estimated without estimating all states.

As a case study, this paper considers a simple one-dimensional thermal system. Its model is given in Section 2. Section 3 deals with the design of a minimum order linear functional observer. Application of the design methodology is given in Section 4 for the estimation of temperature in the system. The performances of the designed observer on a real thermal system are discussed and results are compared with experimental data to validate the principle of the observer. The noise sensitivity of the observer is discussed. Approximations and simplifications regarding the choice of the observer are also examined in the paper and its real time implementation on embedded computing target such as

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Digital Signal Processor is presented.

2. System modeling

2.1. Description of the studied system

This study deals with the case of a bar of silicone gel (Silgel 616) of length much larger than radius. Thermal phenomena occurring along different directions of the main dimension of the bar will be neglected. Thus, the bar will be considered as thermally insulated outside of its main dimension. Thermal expansion induced by thermal behavior will also be neglected. This means that all thermal parameters will be constant regardless of conditions. The modeling methodology is based on a discretization of the heat equation described in [2].

The material is cast and crosslinked in a glass tube of internal diameter $d = 1 \cdot 10^{-2} m$ over a length $L_0 = 16 \cdot 10^{-2} m$. The tube is then placed vertically on a temperature-controlled heating plate of temperature $T_h(t)$ that is measured by a thermocouple. Ambient temperature $T_a(t)$ is also measured. Thermocouples are inserted into the silicone gel at different positions along the length of the bar to measure the local temperature. All thermocouples are placed at the center of the bar along its radius. Finally, a thermal insulator composed of extruded polystyrene is positioned around the tube. It has a thermal conductivity approximately ten times lower than the silicone gel, avoiding heat transfer along the radius axis of the cylinder. To establish the model, the system is sampled into $n = 16$ elementary volumes of length $\Delta x = 1 \cdot 10^{-2} m$ (see Fig. 1). The centers of each elementary volume are called the nodes of the system. Thermal conduction is considered inside the cylinder along its main dimension. Convection is considered to air at ambient temperature along one base of the cylinder.

2.2. Thermal model

Conductive and convective heat transfers are governed by the heat equation expressed in a single spatial dimension [8]. The heat equation is spatially discretized with the finite centered difference method of order 2 [9].

Using bond graph methodology on spatially discretized heat equation, it is clear that temperature at each node of the system can be considered as state variables of the systems [10]. Associated differential equations of order 1 can then be easily written in a matrix form (see Eqs. (1) to (3)) where $T(t)$ is the state vector of the local temperatures and $T_{mes}(t)$ is the measured temperature located at the 13th node of the system that will be used for thermal estimation. Matrices $A_{th}(n, n)$, $B_{th}(n, p)$ and $C_{th}(m, n)$ are given in Appendix A.

$$\dot{T}(t) = A_{th}T(t) + B_{th}U(t) \quad (1)$$

$$U(t) = \begin{pmatrix} T_h(t) \\ T_a(t) \end{pmatrix} \quad (2)$$

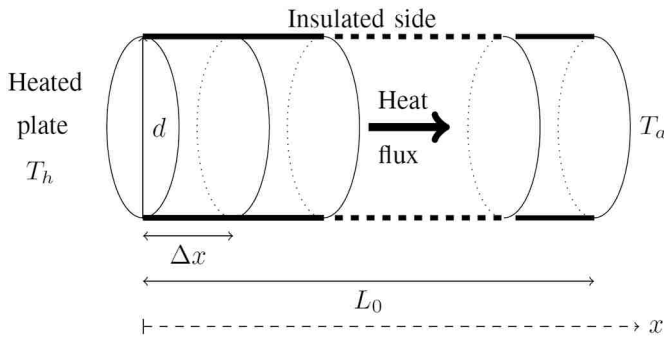


Fig. 1. Studied system.

$$T_{mes}(t) = C_{th}T(t) \quad (3)$$

3. Linear functional observer

3.1. Design of a single LFO

The objective of this section is to describe the design of a single LFO to estimate the vector $v(t) = L.T(t)$ where $L(l, n)$ is a one-row vector i.e. $l = 1$.

Firstly, the triplet (A_{th}, C_{th}, L) has to be functionally observable (Eq. (4)) where $O(A_{th}, C_{th}, n)$ is the observability matrix [11].

$$\text{rank} \left(\begin{bmatrix} O(A_{th}, C_{th}, n) \\ L \end{bmatrix} \right) = \text{rank}(O(A_{th}, C_{th}, n)) \quad (4)$$

Moreover, in order to avoid a trivial algebraic part in the observer, it is supposed without loss of generality that C_{th} and L are linearly independent:

$$\text{rank} \left(\begin{bmatrix} C_{th} \\ L \end{bmatrix} \right) = m + l \quad (5)$$

The observation $\hat{v}(t)$ of $v(t)$ can be carried out by a LFO which is described by the state space equation (Eq. (6)) [3].

$$\begin{cases} \dot{z}(t) = F z(t) + G U(t) + H T_{mes}(t) \\ \hat{v}(t) = P z(t) + V T_{mes}(t) \end{cases} \quad (6)$$

The constant matrices $F(v, v)$, $G(v, p)$, $H(v, m)$, $P(l, v)$, $V(l, m)$ and the order v have to be determined such that $\lim_{t \rightarrow +\infty} v(t) - \hat{v}(t) = 0$. This asymptotic tracking is ensured if F is a Hurwitz matrix, i.e. all the real parts of its eigenvalues are strictly negative (Eq. (7)).

$$\Re(\text{eig}(F)) < 0 \quad (7)$$

The necessary and sufficient condition (Eq. (8)) for the existence of an asymptotic observer (Eq. (6)) are given by the following results [6].

$$\begin{cases} Q A_{th} = H C_{th} + F Q \\ L = P Q + V C_{th} \\ G = Q B_{th} \end{cases} \quad (8)$$

where Q is a constant matrix such that $\lim_{t \rightarrow +\infty} z(t) - Q.T(t) = 0$.

3.2. Dynamic of the observer

As a preliminary, let q be the smallest integer such that:

$$\text{rank}(\Sigma_q) = \text{rank} \begin{pmatrix} \Sigma_q \\ L A_{th}^q \end{pmatrix} \quad (9)$$

with:

$$\Sigma_q = \begin{pmatrix} C_{th} \\ L \\ C_{th} A_{th} \\ L A_{th} \\ \vdots \\ C_{th} A_{th}^{q-1} \\ L A_{th}^{q-1} \\ C_{th} A_{th}^q \end{pmatrix}$$

The design of the observer uses the successive derivations of $v(t) =$

$LT(t)$ that must be expressed only with $\mathbf{v}(t)$, $T_{mes}(t)$, $U(t)$ and their successive derivatives.

Realizing the obtained input-output differential equation [5] leads to a state equation for the observer (Eq. (10)).

$$\begin{cases} \dot{z}(t) = \begin{pmatrix} 0 & & \Lambda_0 \\ 1 & \ddots & \Lambda_1 \\ & \ddots & 0 \\ & & 1 \\ & & & \Lambda_{q-1} \end{pmatrix} z(t) + \begin{pmatrix} \Phi_0 \\ \Phi_1 \\ \vdots \\ \Phi_{q-1} \end{pmatrix} U(t) \\ \begin{pmatrix} \Gamma_0 + \Lambda_0 \Gamma_q \\ \Gamma_1 + \Lambda_1 \Gamma_q \\ \vdots \\ \Gamma_{q-1} + \Lambda_{q-1} \Gamma_q \end{pmatrix} T_{mes}(t) \\ \hat{\mathbf{v}}(t) = \begin{pmatrix} 0 & \cdots & 0 & 1 \end{pmatrix} z(t) + \Gamma_q T_{mes}(t) \end{cases} \quad (10)$$

If Σ_q is a full row rank matrix, the observer is realized through solving (Eq. (11)) where Σ_q^* is the pseudo-inverse of Σ_q .

$$[\Gamma_0 \ \Lambda_0 \ \cdots \ \Gamma_{q-1} \ \Lambda_{q-1} \ \Gamma_q] = L A_{th}^q \Sigma_q^* \quad (11)$$

As a contrary, if Σ_q is not a full row rank i.e. the rank of Σ_q is lower than its number of rows, there exist some degrees of freedom in the design of the observer. The amount of degrees of freedom is denoted r and is defined by (Eq. (12)). It means that r parameters in $[\Gamma_0 \ \Lambda_0 \ \cdots \ \Gamma_{q-1} \ \Lambda_{q-1} \ \Gamma_q]$ may be chosen depending on the desired dynamics of the observer [5]. This case will be detailed in Section 4.

$$r = \dim(\text{Ker}(\Sigma_q^T)) \quad (12)$$

3.3. Procedure for implementing the design of the observer

In a practical way, q is determined through an iterative procedure leading to a possible minimum order for the LFO [5].

The whole procedure is given in Fig. 2.

4. Design of thermal observer

The observer will be designed to estimate the temperature at a distance of 1 cm from the heated plate. The estimation point corresponds to the average temperature of the first and the second node in the model i.e. to the functional matrix L :

$$L = (0.5 \ 0.5 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0)$$

A thermocouple is inserted at the estimation position for the validation of the results of the observer.

4.1. Iterative procedure

Regarding the system modeling in Section 2 and the design procedure given in Section 3, preliminary checks (Eqs. (4), (5)) are carried out. As the system is functionally observable, an iterative procedure may be undertaken to find the smallest integer q such that (Eq. (9)) is verified.

Integer $q = 8$ leads to an observer of size 8 lower than the size of the system ($n = 16$). However, in this case, $r = 1$ meaning that a degree of freedom exists in the design of the observer. This degree of freedom is called Γ in the following. A root locus of the observer (see Fig. 3) demonstrates that a parameter $\Gamma \in [-0.8; 10]$ leads to a stable observer satisfying (Eqs. (7), (8)).

4.2. Performances of the designed observer

Once designed, the performances of the observer are verified in terms of noise sensitivity and decay rate of the estimation error. Note that the noise sensitivity depends on the dynamic of the observer and then on the choice of the parameter Γ . This study is done by using gain behavior of transfer functions from state space representation of the observer regarding its inputs $T_h(t)$, $T_a(t)$ and $T_{mes}(t)$. The transfer

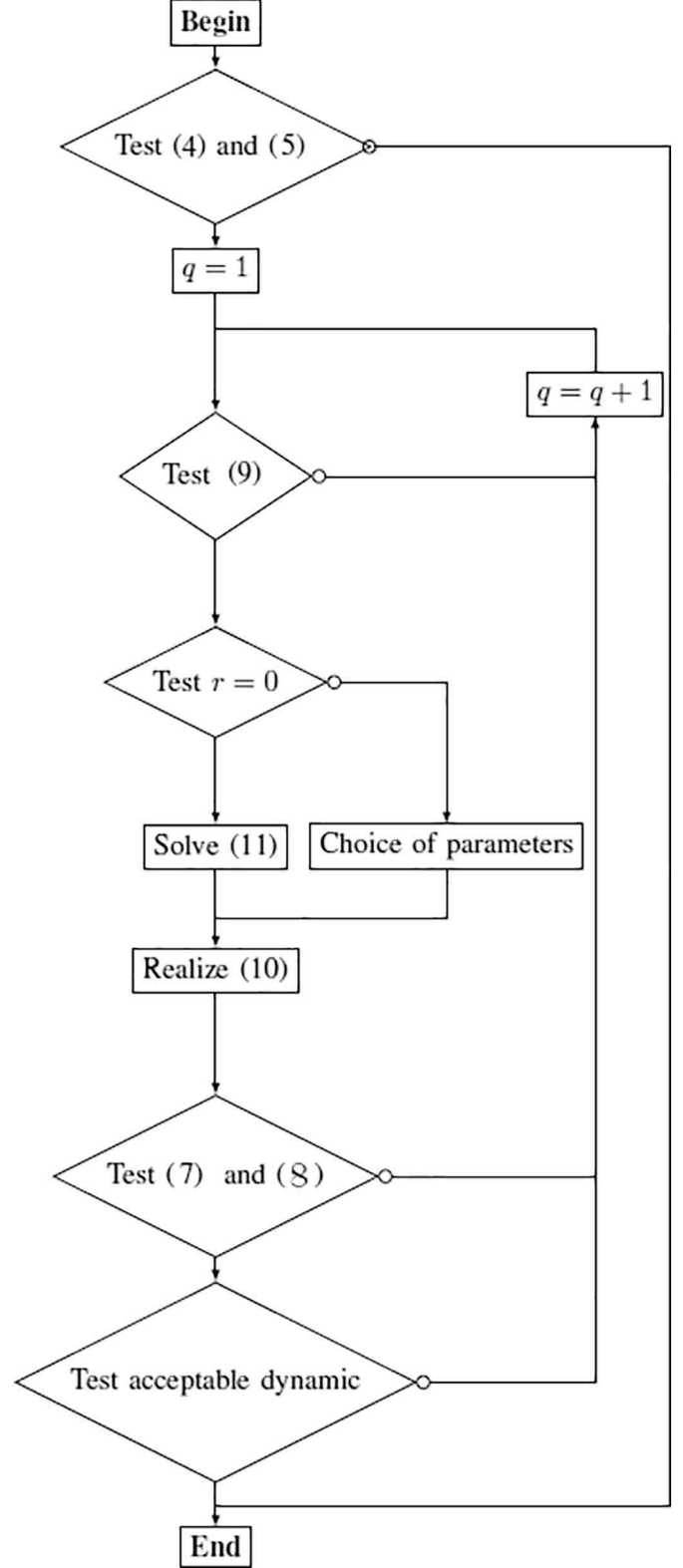


Fig. 2. Algorithm of the procedure for the design of the observer.

functions $H(s)$ of the system are obtained using (Eq. (13)) where I_q is the identity matrix of size q . Associated Bode diagrams, especially gain diagrams are given in Figs. 4 to 6.

$$H(s) = P (s I_q - F)^{-1} [G H] + [0 \ 0 \ V] \quad (13)$$

Regarding $T_h(t)$ and $T_a(t)$, the transfer acts respectively as a first and

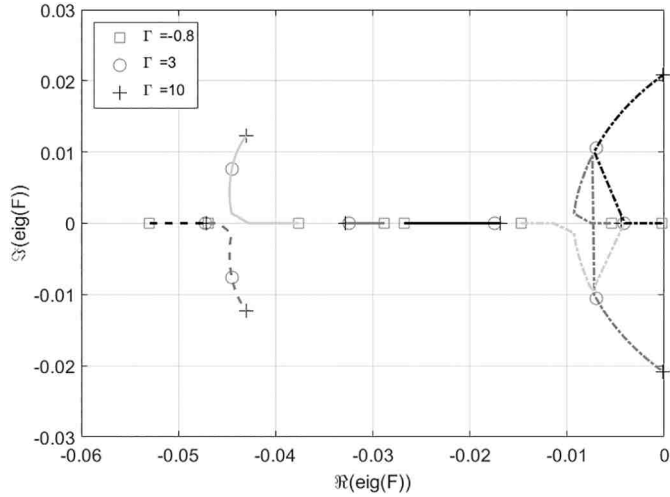


Fig. 3. Root locus of the observer.

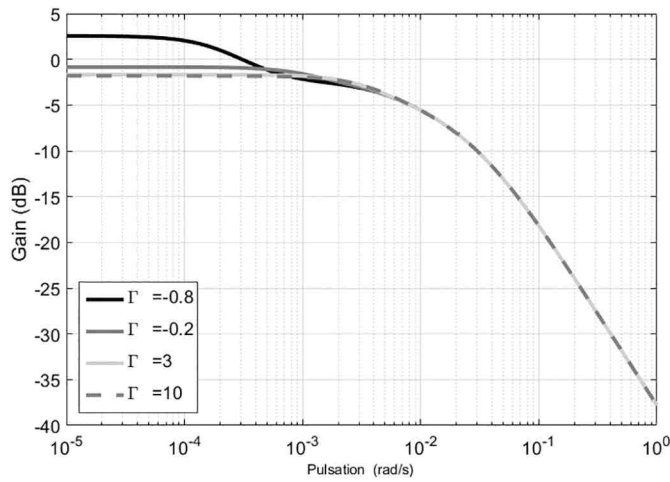


Fig. 4. Gain behavior of $\hat{v}(t)$ regarding $T_h(t)$.

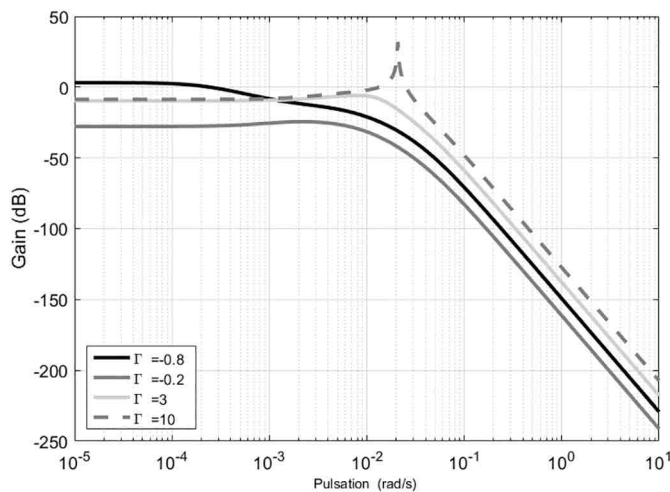


Fig. 5. Gain behavior of $\hat{v}(t)$ regarding $T_a(t)$.

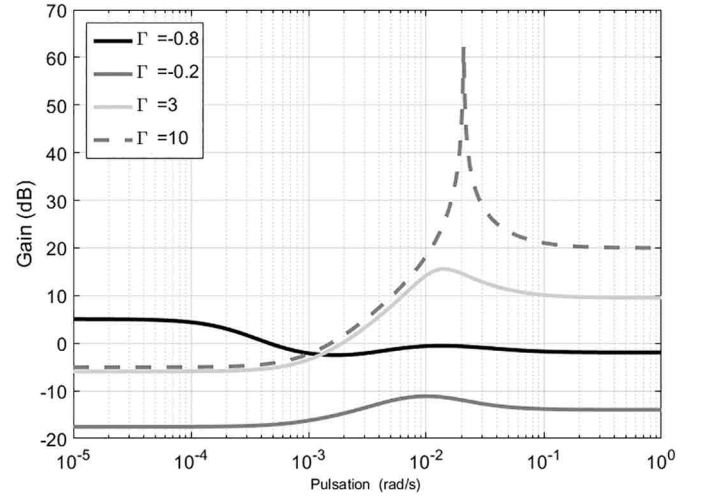


Fig. 6. Gain behavior of $\hat{v}(t)$ regarding $T_{mes}(t)$.

gain at low and high frequencies is obtained. Note that resonant behavior should be avoided. Moreover, it can be remarked that static gains on Figs. 5 and 6 are lower for $\Gamma = -0.2$. This means that the observer becomes less dependent on input values $T_a(t)$ and $T_{mes}(t)$. As a consequence, $\Gamma = -0.2$ could be chosen.

Finally, the observer is realized. Matrices of the observer are given in Appendix B. Eigenvalues of F are then $(-0.0013, -0.0059, -0.0138, -0.0228, -0.031, -0.0393, -0.0468, -0.0512)$. Conditions in (Eqs. (7), (8)) are then verified. The estimated temperature obtained with the observer is compared with that measured at the same position (see Fig. 7). The error rapidly decreases, in accordance with eigenvalues of the observer, resulting in good estimation properties.

However, it seems that a slight estimation error of about 2°C exists even for long duration. This error is null when the observer output is compared to the model one. This may result from some hypothesis used to model the system such as constant parameters whatever the temperature or the absence of mechanical length variations due to temperature increase. Indeed, LFO has been applied to a system that is known to be non-linear. One can conclude that even using this strong simplification, estimation performances are fair enough to estimate potential degradation of the silicone gel.

In addition, it must be remembered that discretizing PDE may lead to

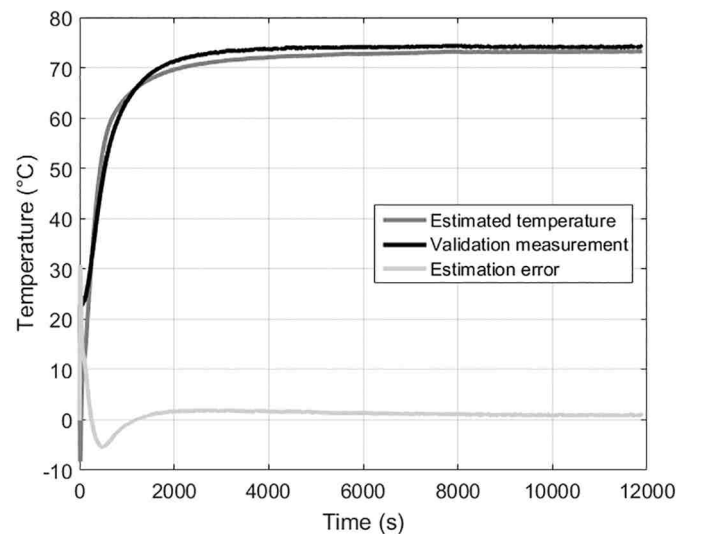


Fig. 7. Observer results.

fourth order low pass filter, thus avoiding the effects of high frequency measurements noise on the estimated variable. Considering Fig. 5, due to direct action of $T_{mes}(t)$ on the estimated variable (Eq. (6)), a static

static error between model and reality. Finally, even with these imprecisions, the estimated temperature could be validated in terms of decay rate of estimation error, given a good approximation of the local temperature in the silicone gel bar. It can be remarked that for $t = 0$, the measured temperature increases whereas the estimated one decreases. This stems from unstable zeros (zeros with positive real part) in the transfer function of the observer regarding two of its inputs $T_h(t)$ and $T_{mes}(t)$. This behavior is due to the fact that the dynamic of the observer may not be fully chosen except by choosing the parameter Γ that has been tuned according to noise sensitivity.

4.3. Real time implementation of the observer

In order to be used in real time embedded computing systems, it is necessary to implement the observer as a recursive equation.

Some methods have been detailed in [12]. It has been demonstrated that the use of a classical numerical integration algorithm such as Euler or Runge-Kutta methods of order 2 and 4 provides the best results for implementing the observer by limiting the number of operations per time sample. For these methods, the maximum number of additions and multiplications to be done at each sample time in order to estimate the temperature can be evaluated and are presented (see Table 1). Moreover, regarding matrices given in Appendix B, it can be remarked that some values may be tuned to 0 without loss of estimation performances. This results in a reduction of the computation complexity of the observer on a real time target.

5. Conclusion

A thermal estimation approach was presented for a number of studied materials used in power electronic modules: the silicone gel used for encapsulation. The proposed approach is based on state space representation. The design of a single LFO has been presented and designed in order to estimate the temperature in any desired location using one measurement and the knowledge of thermal inputs.

This kind of observer results in a relevant reduction in the observer order compared to the initial system dimension. It has been demonstrated, using experimental data, that the observer was able to accurately estimate the temperature evolution of a desired location in the considered system. Moreover, the noise sensitivity of the observer has been detailed depending on its parameters, and real time implementation of the observer on embedded computing target has been discussed.

Several developments have to be investigated regarding the present work. One of our future work axes will be to validate linear functional observers on thermo-mechanical systems to estimate mechanical stress

Appendix A. Expression of thermal matrices

Regarding 1D discretized thermal equations using proposed methodology, matrices of the system have a particular form that is detailed below.

$$A_{th} = \begin{pmatrix} -3\alpha & \alpha & 0 & \dots & 0 \\ \alpha & -2\alpha & \ddots & \ddots & \vdots \\ 0 & \ddots & \ddots & \ddots & 0 \\ \vdots & \ddots & \ddots & -2\alpha & \alpha \\ 0 & \dots & 0 & \alpha & -\alpha - \beta \end{pmatrix} \quad (14)$$

$$B_{th} = \begin{pmatrix} -\alpha & 0 \\ 0 & \vdots \\ \vdots & 0 \\ 0 & \beta \end{pmatrix} \quad (15)$$

$$C_{th} = (0 \quad \dots \quad 0 \quad 1 \quad 0 \quad 0 \quad 0) \quad (16)$$

where:

Table 1
Evaluation of number of operations to calculate $\hat{v}(t)$

	Euler	RK2	RK4
Additions	$\approx 5q$	$\approx 11q$	$\approx 27q$
Multiplications	$\approx 5q$	$\approx 10q$	$\approx 24q$

or elongation depending on thermal behavior. 3D thermo-mechanical behavior will be considered to be more representative of power electronic modules. The proposed method for design minimal order observers may be easily applied to these models. However, such models will be naturally of large dimension, and state space representations will be huge. Thus, it could be supposed that even the use of linear functional observer will not drastically reduce the order of the problem. In this case, the observer designed from experimentally identified transfers could be studied.

Time-varying systems can also be studied and corresponding observers could be designed. This point seems of particularly significant interest in the field of power electronics where parameters depend not only on temperature but also on aging of the material itself.

Finally, another axis of future research is to consider unknown inputs, such as thermal power dissipated by semiconductors or thermal perturbations from passive elements in the power module. The next step will be to design functional observers of unknown inputs to accurately monitor the temperature of semiconductors during the life cycle of the power modules.

CRedit authorship contribution statement

B. Trajin: Methodology, validation, resources, writing original and review, visualization

I. Sakhraoui: Software, investigation

F. Rotella: Conceptualization, formal analysis, supervision, funding acquisition

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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$$\alpha = \frac{\lambda}{\rho \Delta x^2 C_p}$$

$$\beta = \frac{2 \lambda h}{\rho \Delta x C_p (h \Delta x + 2 \lambda)}$$

with:

- $\lambda = 0.1 \text{ W(m K)}^{-1}$ the thermal conductivity,
- $C_p = 80 \text{ W(K kg)}^{-1}$ the heat capacity,
- $\rho = 970 \text{ kg m}^{-3}$ the density,
- $h = 70 \text{ W m}^{-2}$ the convection coefficient.

Appendix B. Expression of matrices of the observer

Matrices of the observer that are obtained through the proposed methodology are detailed below.

$$F = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & -6.86 \cdot 10^{-15} \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & -8.07 \cdot 10^{-12} \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & -2.46 \cdot 10^{-9} \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & -3.23 \cdot 10^{-7} \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & -2.21 \cdot 10^{-5} \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & -8.49 \cdot 10^{-4} \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1.85 \cdot 10^{-2} \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -2.12 \cdot 10^{-1} \end{pmatrix} \quad (17)$$

$$G = \begin{pmatrix} 6.23 \cdot 10^{-15} & -2.81 \cdot 10^{-16} \\ 5.73 \cdot 10^{-12} & 3.66 \cdot 10^{-13} \\ 1.34 \cdot 10^{-9} & 3.2 \cdot 10^{-11} \\ 1.33 \cdot 10^{-7} & 9.22 \cdot 10^{-10} \\ 6.69 \cdot 10^{-6} & 8.58 \cdot 10^{-9} \\ 1.78 \cdot 10^{-4} & 0 \\ 2.4 \cdot 10^{-3} & 0 \\ 1.29 \cdot 10^{-2} & 0 \end{pmatrix} \quad (18)$$

$$H = \begin{pmatrix} 2.28 \cdot 10^{-15} \\ 1.18 \cdot 10^{-12} \\ 7.04 \cdot 10^{-11} \\ -3.05 \cdot 10^{-9} \\ -3.88 \cdot 10^{-7} \\ -1.33 \cdot 10^{-5} \\ -2 \cdot 10^{-4} \\ -1.1 \cdot 10^{-3} \end{pmatrix} \quad (19)$$

$$P = (0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 1), V = -0.2 \quad (20)$$

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