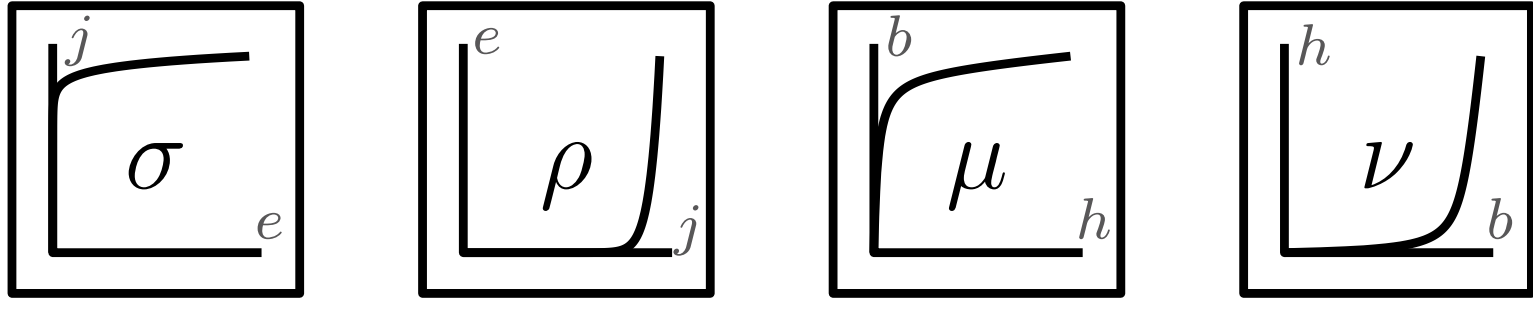


What Formulation Should One Choose for Modeling a 3D HTS Magnet Motor Pole with a Ferromagnetic Material?

Julien Dular¹, Kévin Berger², Benoît Vanderheyden¹, Christophe Geuzaine¹
¹ Liège University, Belgium, ² GREEN, Université de Lorraine

Introduction

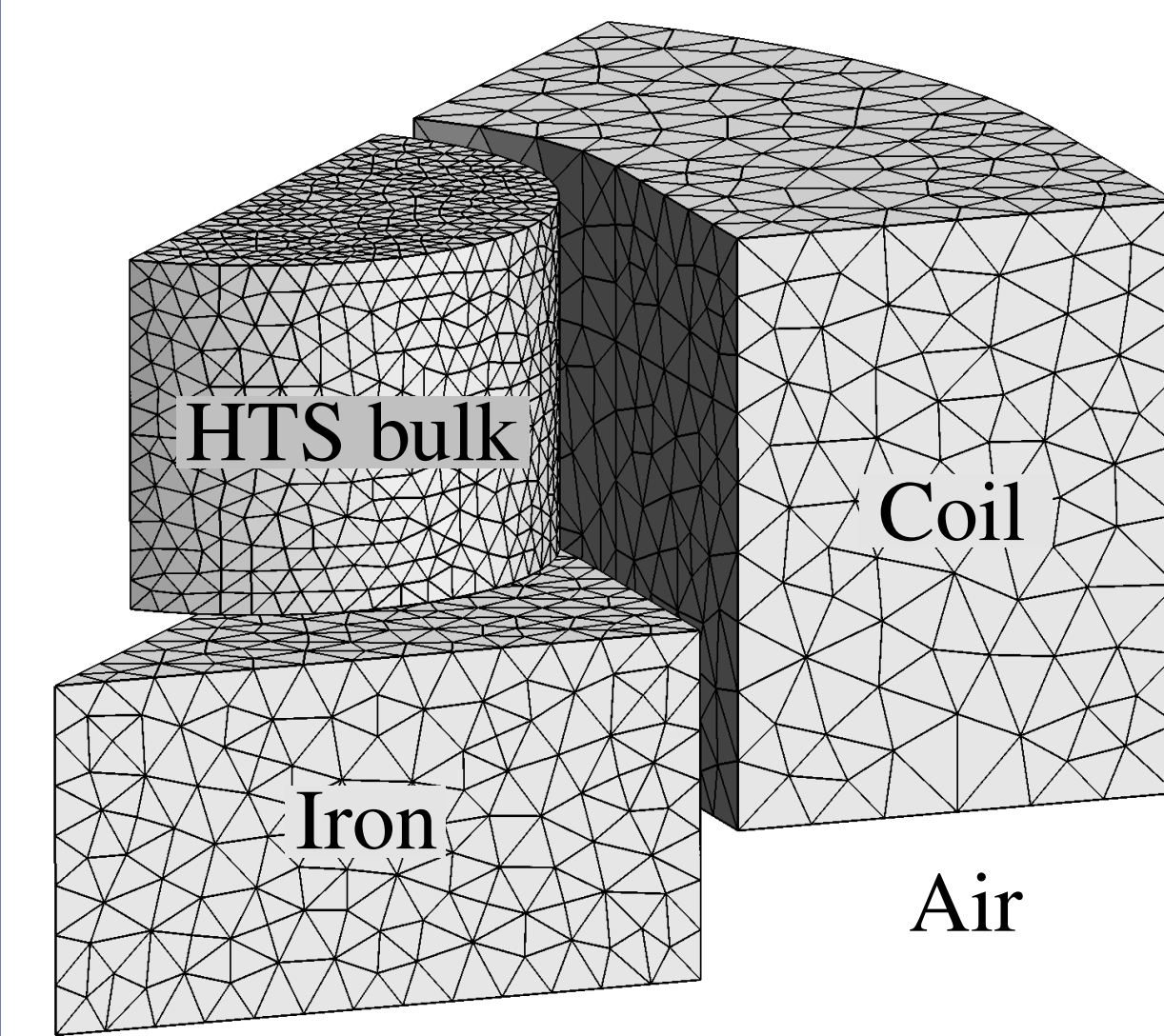
Modeling the magnetic behavior of systems with high-temperature superconductors (HTS) and ferromagnetic materials (FM) is an expensive task due the **strongly nonlinear** behaviors.



- Not all formulations yield the same efficiency. In this work, we compare the performance of different finite element formulations on a 3D problem.
- In particular, we propose a **coupled formulation** as an efficient choice for these kind of systems.

Problem definition

Four HTS bulks are put on an iron substrate, and are magnetized by an inducting coil. **One eighth** is modeled.



HTS bulk Ω_c

- Power law: $\rho = \frac{e_c}{j_c} \left(\frac{\|j\|}{j_c} \right)^{n-1}$ with $e_c = 10^{-4}$ V/m.
- Kim's law: $j_c = j_c(\mathbf{b}) = 5 \times 10^8 / (1 + \|\mathbf{b}\|/b_0)$ A/m², $b_0 = 0.5$ T.
- Power index: $n = n(\mathbf{b}) = n_1 + (n_0 - n_1) / (1 + \|\mathbf{b}\|/b_0)$, $n_0 = 21$, $n_1 = 5$.

Iron substrate

- Saturation law (saturation at ≈ 2.2 T) extracted from measurements.

Magnetizing coil Ω_s

- $I_s(t) = \begin{cases} I_{\max} \sin 2\pi t/T, & t < T, \\ I_{\max} e^{-(t-T)/\tau}, & t > T, \end{cases}$ $I_{\max} = 2$ kA, $T = 2$ ms, $\tau = 10$ ms.
- Number of turns: 55.5. Constant current density $\|j_s\|$, no eddy currents.

h -conform formulations

Classical h -conform formulation, with Ω the full domain, Ω_c the conducting domain, Ω_c^C its complementary:

$$(\partial_t(\mu \mathbf{h}), \mathbf{h}')_{\Omega} + (\rho \operatorname{curl} \mathbf{h}, \operatorname{curl} \mathbf{h}')_{\Omega_c} = 0$$

It involves the **permeability** μ and the **resistivity** ρ .

The source current I_s is **strongly** imposed via a precomputed source field \mathbf{h}_s satisfying $\operatorname{curl} \mathbf{h}_s|_{\Omega_s} = \mathbf{j}_s$.

Two main possibilities for the function space:

- Full **h -formulation**, with edge elements in Ω and a spurious **finite resistivity** in the non-conducting domain. This formulation is widely implemented in commercial softwares (e.g. Comsol) [1].
- Hybrid **h - ϕ -formulation**, where $\mathbf{h} = \mathbf{h}_s + \operatorname{grad} \phi$ in Ω_c^C so that $\operatorname{curl}(\mathbf{h} - \mathbf{h}_s)|_{\Omega_c^C} = \mathbf{0}$ is strongly satisfied. Currents are imposed via cohomology basis functions.

b -conform formulations

Classical b -conform formulation, with Ω_s the coil volume, \mathbf{a} the magnetic vector potential ($\mathbf{b} = \operatorname{curl} \mathbf{a}$, $e = -\partial_t \mathbf{a}$):

$$(\nu \operatorname{curl} \mathbf{a}, \operatorname{curl} \mathbf{a}')_{\Omega} + (\sigma \partial_t \mathbf{a}, \mathbf{a}')_{\Omega_c} = (\mathbf{j}_s, \mathbf{a}')_{\Omega_s}$$

It involves the **reluctivity** $\nu = 1/\mu$ and the **conductivity** $\sigma = 1/\rho$.

The source current I_s is **weakly** imposed via the right-hand side term.

The vector potential \mathbf{a} is not unique in Ω_c^C :

- It can be made unique by introducing a spurious **non-zero conductivity** in the non-conducting domain. This approach is sometimes used in commercial softwares. We denote it as the **\bar{a} -formulation**.
- It can be gauged with a **tree-cotree** technique. We denote this method as the **a -formulation**.

Coupled formulations

The HTS nonlinearity is easier to handle with ρ . By contrast, the saturation law is easier with ν .

\Rightarrow Motivation for a **coupled formulation**.

Domain decomposed in two parts: Ω_a and Ω_h .

HTS domain $\subset \Omega_h$ and FM $\subset \Omega_a$

Depending on where we place the remaining domain, we end up with two different formulations:

- h - a -formulation**: the rest is placed in Ω_a . The vector potential is gauged.
- h - ϕ - a -formulation**: the rest is placed in Ω_h . The ϕ potential is introduced, as well as the source field.

Coupling via the common boundary Γ_m :

$$\begin{aligned} (\partial_t(\mu \mathbf{h}), \mathbf{h}')_{\Omega_h} + (\rho \operatorname{curl} \mathbf{h}, \operatorname{curl} \mathbf{h}')_{\Omega_{h,c}} \\ = \langle \partial_t \mathbf{a} \times \mathbf{n}_{\Omega_a}, \mathbf{h}' \rangle_{\Gamma_m} \\ (\nu \operatorname{curl} \mathbf{a}, \operatorname{curl} \mathbf{a}')_{\Omega_a} - (\mathbf{j}_s, \mathbf{a}')_{\Omega_s} = \langle \mathbf{h} \times \mathbf{n}_{\Omega_a}, \mathbf{a}' \rangle_{\Gamma_m} \end{aligned}$$

The formulation is **mixed**. To avoid instability and satisfy the **inf-sup condition**, one of the fields (\mathbf{h} or \mathbf{a}) has to be enriched on Γ_h [2].

Summary and comparison

Formulation	Function space	Support of DOFs	$\sigma \neq 0$ in Ω_c^C ?
h-formulation	$\mathcal{H}(\Omega) = \{\mathbf{h} \in H(\operatorname{curl}; \Omega)\}$	Edges in Ω	Yes
h-ϕ-formulation	$\mathcal{H}_{\phi}(\Omega) = \{\mathbf{h} \in H(\operatorname{curl}; \Omega) \mid \operatorname{curl} \mathbf{h} = \mathbf{0} \text{ in } \Omega_c^C\}$	Edges in Ω_c , nodes in Ω_c^C	No
\bar{a}-formulation	$\bar{\mathcal{A}}(\Omega) = \{\mathbf{a} \in H(\operatorname{curl}; \Omega)\}$	Edges in Ω	(Yes)
a-formulation	$\mathcal{A}(\Omega) = \{\mathbf{a} \in H(\operatorname{curl}; \Omega) \mid \text{co-tree gauge in } \Omega_c^C\}$	Edges in Ω_c , facets in Ω_c^C	No
h-a-formulation	$\mathbf{h} \in \mathcal{H}(\Omega_c), \mathbf{a} \in \mathcal{A}(\Omega_c^C)$	Edges in Ω_c , facets in Ω_c^C	No
h-ϕ-a-formulation	$\mathbf{h} \in \mathcal{H}_{\phi}(\Omega_a), \mathbf{a} \in \mathcal{A}(\Omega_a)$	Edges in $\Omega_{h,c}$, nodes in $\Omega_{h,c}^C$, facets in Ω_a	No



Results

The problem is solved by the **six formulations** on the **same mesh**. They yield results of **similar accuracy**. The model is implemented in GetDP, and time integrated with an implicit Euler method and adaptive time steps.

Linearization:

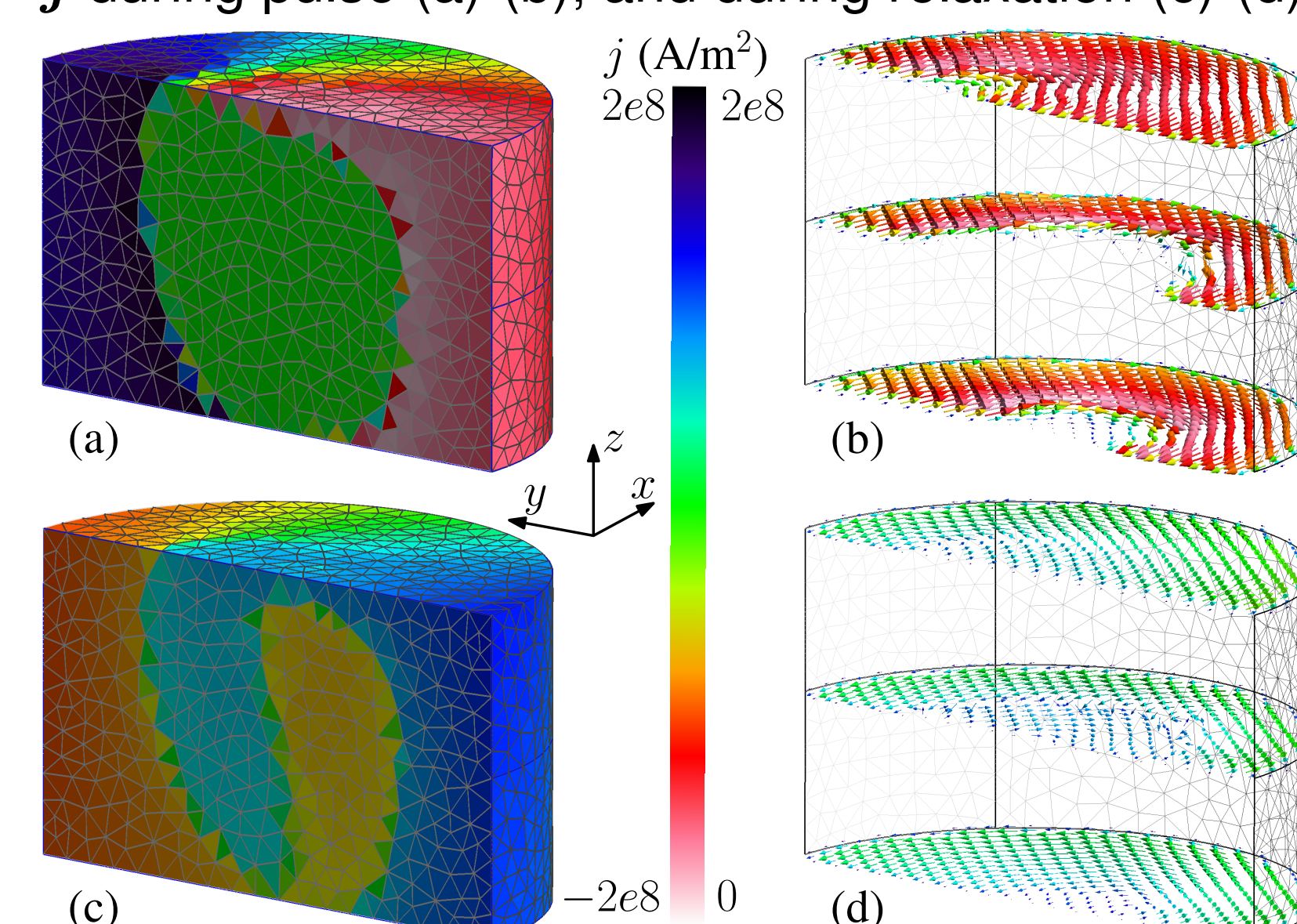
- Newton-Raphson** (N-R) for ρ and ν .
- For σ or μ , N-R is **not robust**, even with relaxation factors.
- Fixed point** method instead \rightarrow slower convergence. This is the main motivation for a **coupled formulation**.

Performance:

Formulation	# DOFs	# iterations	CPU time
h -formulation	35,532	4,057	5h58
h - ϕ -formulation	12,172	3,937	3h38
\bar{a} -formulation	29,010	2,955	4h45
a -formulation	26,964	3,147	3h07
h - a -formulation	32,045	1,124	1h25
h - ϕ - a -formulation	16,070	1,108	1h16

Solution of the h - ϕ - a -formulation:

j during pulse (a)-(b), and during relaxation (c)-(d).



- \Rightarrow When possible, it is beneficial to introduce a scalar potential for \mathbf{h} and to gauge \mathbf{a} .
- \Rightarrow The **h - ϕ - a -formulation** is **robust** and is the most efficient method. Compared to the **h - a -formulation**, it requires many less DOFs.

Conclusions

We presented a **coupled formulation** that is suited for modeling 3D hybrid systems with HTS and FM.

It proved to be **more robust and efficient** than other classical formulations, as was the case in 2D.

Further work: find an alternative to the fixed point method for σ and μ .

References

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