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What Formulation Should One Choose for Modeling a 3D HTS Magnet Motor Pole with a Ferromagnetic Material?

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We discuss the relevance of several finite-element formulations for systems containing high-temperature superconductors (HTS) and ferromagnetic materials (FM) in the context of a 3D motor pole model, in terms of their numerical robustness and efficiency. We propose a coupled h - ϕ - a -formulation as an optimal choice, modeling the problem with an a -formulation in the FM, and an h - ϕ -formulation in the remaining domain.

Index Terms—Finite element analysis, high-temperature superconductors, magnetic materials, nonlinear equations.

I. INTRODUCTION

MODELING the magnetic response of high-temperature superconductors (HTS) is of high importance in many applications. One of the main modeling tools is the finite element (FE) method with the E - J power law in HTS. The resulting strongly nonlinear system of equations requires a carefully chosen formulation to obtain both accurate results and efficient resolutions. Coupling HTS with ferromagnetic materials (FM) introduces additional difficulties. In particular, the power law in HTS and magnetic law in FM are most efficiently solved with distinct formulations.

In this work, we analyse and compare the efficiencies of a number of formulations for a 3D problem with both HTS and FM. Especially in 3D, we have a strong need to optimize the computational cost of the simulations.

II. MAGNET POLE PROBLEM AND FORMULATIONS

The considered problem consists of four HTS bulks placed on top of an iron substrate, and magnetized via an inducting coil. Exploiting symmetry, one eighth of the geometry is modeled and represented in Fig. 1.

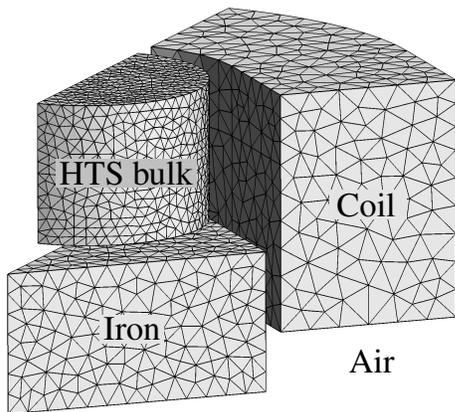


Fig. 1: Mesh on one eighth of the 3D geometry. The mesh in the air domain is not represented.

We model the magnetic response of the system with the magnetodynamic equations. In HTS, we assume a power law for the resistivity:

$$\rho = \frac{e_c}{j_c} \left(\frac{\|j\|}{j_c} \right)^{n-1}, \quad (1)$$

with $e_c = 10^{-4}$ V/m, j_c the critical current density and n a dimensionless number. Both j_c and n depend on the magnetic flux density. The system also contains a non-conducting ferromagnetic material, for which the permeability is a function of h . The excitation is a known current density in the coil; its waveform is a pulse followed by a slow relaxation.

We now briefly present the formulations that enter our comparison. A summary is proposed in Table I. The modeled domain Ω is decomposed into a conducting part, Ω_c , containing the HTS bulk, and a non-conducting part Ω_c^c , containing the air, the iron and the coil (where we do not model eddy currents).

A. h -conform formulations

In classical h -conform formulations, the unknown field is the magnetic field h , sought in a specific function space whose choice has an important influence on the resulting number of degrees of freedom (DOFs). We present here two options.

In the full h -formulation, the magnetic field is discretized with edge functions on the whole domain, and a spurious resistivity is introduced in Ω_c^c [1]. This approach is oftentimes used in commercial softwares, e.g., Comsol.

A second possibility is to strongly impose a zero current density in non-conducting regions by carefully defining the function space, leading to the well-known h - ϕ -formulation [2]. Multiply connected subdomains, as is the case here for the complementary of the coil, are handled with discontinuous or cohomology basis functions.

B. b -conform formulations

In usual b -conform formulations, the magnetic flux density is described by a vector potential a with $b = \text{curl } a$. The vector potential is not unique in Ω_c^c , and can be gauged. For example, a co-tree gauge reduces the number of DOFs, this leads to the a -formulation [2]. As with the h -conform formulations, we can also introduce a spurious conductivity in Ω_c^c , and hence avoid the gauging step. We refer to this choice as the \bar{a} -formulation.

TABLE I: Description of the different formulations in 3D problems

Formulation	Function space	Support of DOFs	$\sigma \neq 0$ in Ω_c^C ?
h -formulation	$\mathcal{H}(\Omega) = \{\mathbf{h} \in H(\mathbf{curl}; \Omega)\}$	Edges in Ω	Yes
h - ϕ -formulation	$\mathcal{H}_\phi(\Omega) = \{\mathbf{h} \in H(\mathbf{curl}; \Omega) \mid \mathbf{curl} \mathbf{h} = \mathbf{0} \text{ in } \Omega_c^C\}$	Edges in Ω_c , nodes in Ω_c^C	No
\bar{a} -formulation	$\bar{\mathcal{A}}(\Omega) = \{\mathbf{a} \in H(\mathbf{curl}; \Omega)\}$	Edges in Ω	(Yes)
a -formulation	$\mathcal{A}(\Omega) = \{\mathbf{a} \in H(\mathbf{curl}; \Omega) \mid \text{co-tree gauge in } \Omega_c^C\}$	Edges in Ω_c , facets in Ω_c^C	No
h - a -formulation	$\mathbf{h} \in \mathcal{H}(\Omega_c)$, $\mathbf{a} \in \mathcal{A}(\Omega_c^C)$	Edges in Ω_c , facets in Ω_c^C	No
h - ϕ - a -formulation	$\mathbf{h} \in \mathcal{H}_\phi(\Omega_a)$, $\mathbf{a} \in \mathcal{A}(\Omega_a)$	Edges in $\Omega_{h,c}$, nodes in $\Omega_{h,c}^C$, facets in Ω_a	No

For the h - ϕ - and h - ϕ - a -formulation with stranded conductors, we add to \mathbf{h} a precomputed source field \mathbf{h}_s reproducing the source current density. For the \bar{a} -formulation, choosing $\sigma = 0$ in Ω_c^C makes the system singular, which is not necessarily an issue, depending on the linear solver algorithm.

C. Coupled formulations

Systems containing both HTS and FM are advantageously solved with a coupled formulation [3], [4]. The domain Ω is decomposed into two parts: Ω_h , to be solved with the h - ϕ -formulation, and Ω_a , to be solved with the a -formulation. We present two choices for distributing the materials among the domains, both without introducing a spurious conductivity.

The HTS is always put in Ω_h and the FM is always put in Ω_a . In the h - a -formulation, we place all non-conducting domains in Ω_a , and gauge \mathbf{a} . In the h - ϕ - a -formulation, only the FM belongs to Ω_a , and an h - ϕ -formulation is used elsewhere.

III. COMPARISON ON THE 3D PROBLEM

We run simulations with GetDP. All model files are available at www.life-hts.uliege.be. The models feature an adaptive time-stepping procedure and several discretization methods are investigated.

All formulations yield results of similar accuracy, but their computational cost is not equivalent. See Table II. With b -conform formulations, involving the conductivity in HTS, we can only obtain a robust iterative technique by using a fixed point method, thus requiring much more iterations to achieve sufficient accuracy than with Newton-Raphson. With h -conform formulations, handling the FM nonlinearity induces the same difficulties. As a consequence, the number of iterations strongly depends on the fine tuning of the iterative parameters. On the other hand, the coupled formulations demonstrate a good robustness in all test cases, with h - ϕ - a -formulation being preferred because of the lower associated number of DOFs.

TABLE II: Efficiency of the different formulations

Formulation	# DOFs	# iterations	CPU time
h -formulation	35,532	4,057	5h58
h - ϕ -formulation	12,172	3,937	3h38
\bar{a} -formulation	29,010	2,955	4h45
a -formulation	26,964	3,147	3h07
h - a -formulation	32,045	1,124	1h25
h - ϕ - a -formulation	16,070	1,108	1h16

Pulse magnetization of the HTS bulk with linear elements (except on the coupling boundary of coupled formulations). The large number of iterations for h -conform formulations is only due to the FM, for which a fixed point method is necessary to obtain a robust method. In some cases, a Newton-Raphson scheme (with or without relaxation factors) works without troubles with efficiency similar to coupled formulations, but this is not guaranteed in general. This behavior is the main motivation for using a coupled formulation.

IV. CONCLUSION

In this work, we compared the relevance of several finite element formulations for modeling 3D systems with high-

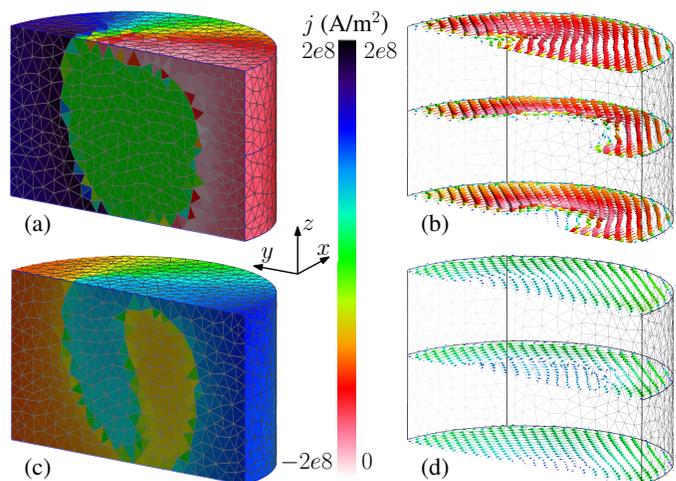


Fig. 2: Current density (h - ϕ - a -formulation) in the bulk during magnetizing pulse (a)-(b), and during relaxation (c)-(d). For (a)-(c) the component along x is represented (labels on the left of the legend). For (b)-(d), the full vector is represented (labels on the right of the legend refer to the norm).

temperature superconductors and ferromagnetic materials. The most efficient choice is a coupled h - ϕ - a -formulation. It combines a good robustness with a low number of degrees of freedom, thus leading to efficient simulations. In the full paper, we will describe the formulations in more details, and compare with results from Comsol.

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