

# On the convergence test of FFT-based methods

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## ABSTRACT

In the last two decades, FFT-based methods [1] have become a widely used tool for predicting the overall and local behavior of heterogeneous materials. They are all based on the iterative resolution of the Lippmann-Schwinger equation. Unfortunately it was observed that these methods hardly converge in the case of high contrast between the mechanical properties of the constituents, although several acceleration techniques have been proposed in the literature [2,3]. Moreover, the convergence of the methods in the case of infinite contrast (for example with porous materials) is still a matter of debate.

This presentation focuses on the conductivity problem applied to a checkerboard microstructure, for which a solution is known in closed form [4,5]. This explicit solution allows for a comparison between the theoretical convergence of a given iterative scheme to the convergence of its numerical implementation (applied to a discrete image of the microstructure). The present study shows that the theoretical convergence is much better than the one observed in actual implementations. Most noticeably, the method theoretically converges very rapidly in the case of infinite contrast as opposed to what is observed for implemented algorithms. On the other hand, the convergence test based on the equilibrium of the numerical stress fields turns out to estimate inadequately the actual error (see figure).

The reasons of these discrepancies will be analysed in the presentation. Some improvements of the criterion for testing the convergence error will be proposed.

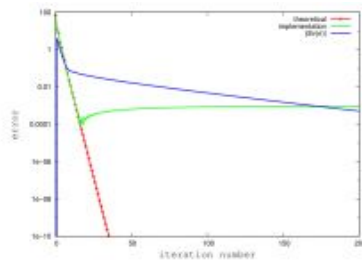


Figure 1: Error on the effective property vs iteration number: algorithm applied on a  $512 \times 512$  pixels image (green), theoretical convergence (red). Equilibrium residue for the algorithm (blue).

## References

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