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## Phase Synchrony Neurofeedback? BEWARE!

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PRESENTER  
Côme Le Breton

## Background

**Phase synchrony** estimation in **EEG signals** is subject to a number of **pitfalls**, which makes **neurofeedback** protocols **rare**, despite their potential in health care applications (epilepsy syndromes, attention deficit hyperactivity disorder..).

**Phase synchrony** measures the consistency of a relationship between the phase of two signals. It requires in the first place the determination of a phase (or phase difference), and in a second place the quantification of this consistency.

**EEG signals** are the combination at the scalp of the activity of the neurons of the cortex, plus possibly external noises.

A simple, but likely generic assumption (especially in the framework of **phase synchrony**), is to assimilate **EEG signals** to locally stationary weighted sums of sinusoids.

## Methodology

**A]** From the fundamentally equivalent methods to extract the phase from **EEG signals**, we retained the **Morlet Wavelet Transform**.

We derive analytically the convolution of a **Morlet wavelet** with a **sinusoid** and extend the derivation to the **weighted sums of locally stationary sinusoids**. Additionally we propose an **approximation** of the former derivation with **loose constraints**.

The behavior of the **weights** for **varying width** and **frequencies** is analyzed.

We present two cases of **false synchrony**, one of **missed synchrony**, and one of **spurious synchrony**.

**B]** The **correlation coefficients** of two **δt distant Morlet wavelet coefficients** is computed.

**C]** We set up an **experiment** to assess the correlation between common phase synchrony measures. A subject underwent 13 recordings of 275 seconds, alternating between 20s of **Eyes Open Fixation** of the screen (Baseline) and 20s of **Eyes Closed Relaxation**, interleaving 5 seconds of rest between each event.

## Results

**A]** The wavelet transform is an accurate estimation of the argument of a sine wave for  $\Omega > 2$ .

The benefit of increasing the width of the wavelet ( $\Omega$ ) to values higher than 30 is minimal, and at the expense of reducing the temporal accuracy of the transform.

The **higher the frequency** of interest ( $\nu_w$ ) is, the **more mixing between frequency** components ( $\nu_s$ ) around that frequency (respecting the uncertainty principle).

The **amplitude ratio** between components **matters** in the phase estimation, but **lightly** in comparison to the distance ratio between  $\nu_w$  and  $\nu_s$ .

This **phase** between **two pairs of sinusoids** oscillating in the vicinity of the wavelet frequency ( $\nu_w$ ) is almost zero for two different pairs of sinusoids (i.e.  $\nu_{a0}=9.5, \nu_{a1}=11.5; \nu_{b0}=10, \nu_{b1}=11$ ).

It means that the **phase is not unique**: several combinations of different frequencies produce the same phase.

**B]** The derivation of the correlation between two close coefficients show that they are not independent.

**C]** Some synchrony measures are strongly correlated and thus redundant: **PLI and ImCoh** or **PLV and Coh** are strongly **correlated**, on the other hand **PLI and PLV** are **complementary**, and would benefit from being analyzed altogether when assessing synchrony.

## Conclusion

Assessing properly phase synchrony is a real challenge, even for offline analysis.

It has mostly to do with the volume conduction / volume leakage mixing, the uncertainty principle and the redundancy of the wavelet coefficients.

One or more of these issues are often neglected.

We provide a framework for better understanding and interpreting phase extraction with the Morlet Wavelet Transform to show that errors are made when extracting phase information and must be accounted for.

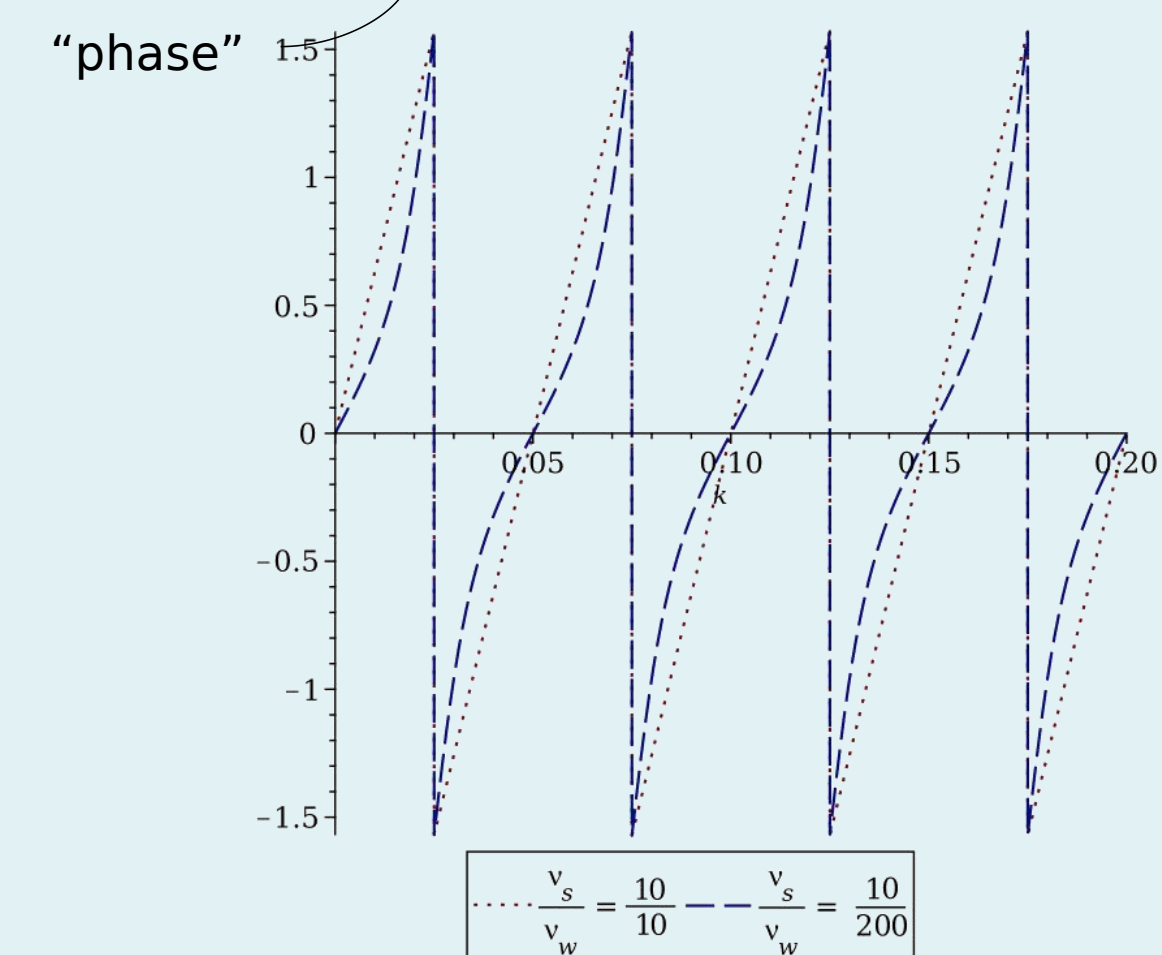
Before attempting to blindly use phase synchrony measures in Neurofeedback, there *a fortiori* a need for a proper and efficient estimation of the statistical significance of the measures calculated on the fly.

**Wavelet Transform :**

$$WT(f(t))(k) = \frac{1}{\sqrt{\frac{\pi}{\omega_w}}} \int_{-\infty}^{\infty} f(k-t) e^{i\omega_w t} e^{-\frac{t^2 \omega_w^2}{2\Omega^2}} dt$$

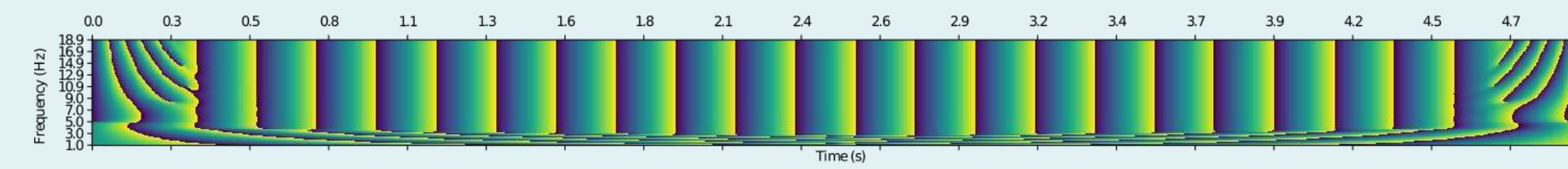
$f(t) = \cos(\omega_c t + \phi_c)$  :

$$\phi(k) = \arctan\left(\frac{\Im(WT(f(k)))}{\Re(WT(f(k)))}\right) = \arctan(\tan(\omega_c k + \phi_c) \tanh\left(\frac{\Omega^2 \omega_c}{\omega_w}\right))$$



Effect on the phase of varying the frequency ratio (signal vs wavelet) up to high values

For  $f(t) = \sin(2\pi\nu_s t + \phi_s)$ , the argument  $2\pi\nu_s t + \phi_s$  is properly recovered for  $\nu_s$  close to  $\nu_w$ . A deviation in the coefficient  $\tanh\left(\frac{\Omega^2 2\pi\nu_s}{2\pi\nu_c}\right)$  causes the phase to warp, nonetheless the jumps occur at the frequency of the signal. The formula shows clearly why one sees on Time-Frequency planes long vertical lines and phase information far from the frequency of the signal.

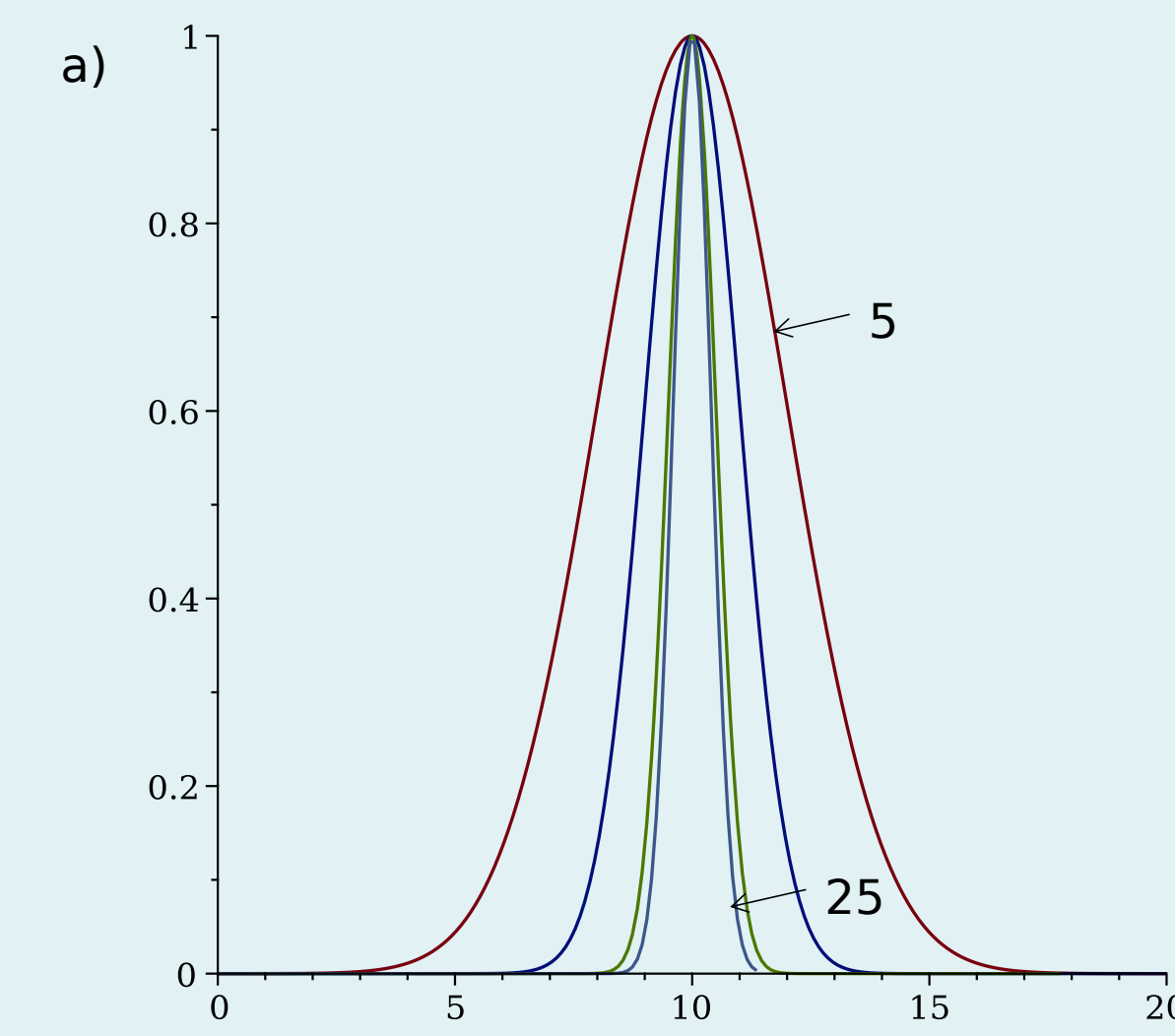


TF plane of  $\sin(10 \cdot 2\pi t)$ ,  $t$  varies from 0 to 5 seconds. The signal is decomposed on frequencies varying from 1 to 19 with wavelets composed of 7 oscillations ( $\Omega$ ). The plot is obtained by numerical convolution (solved with multiplication in the Fourier domain of the Fourier Transform of the signal and the Fourier Transform of the wavelet followed by Fourier Inverse Transform of the product).

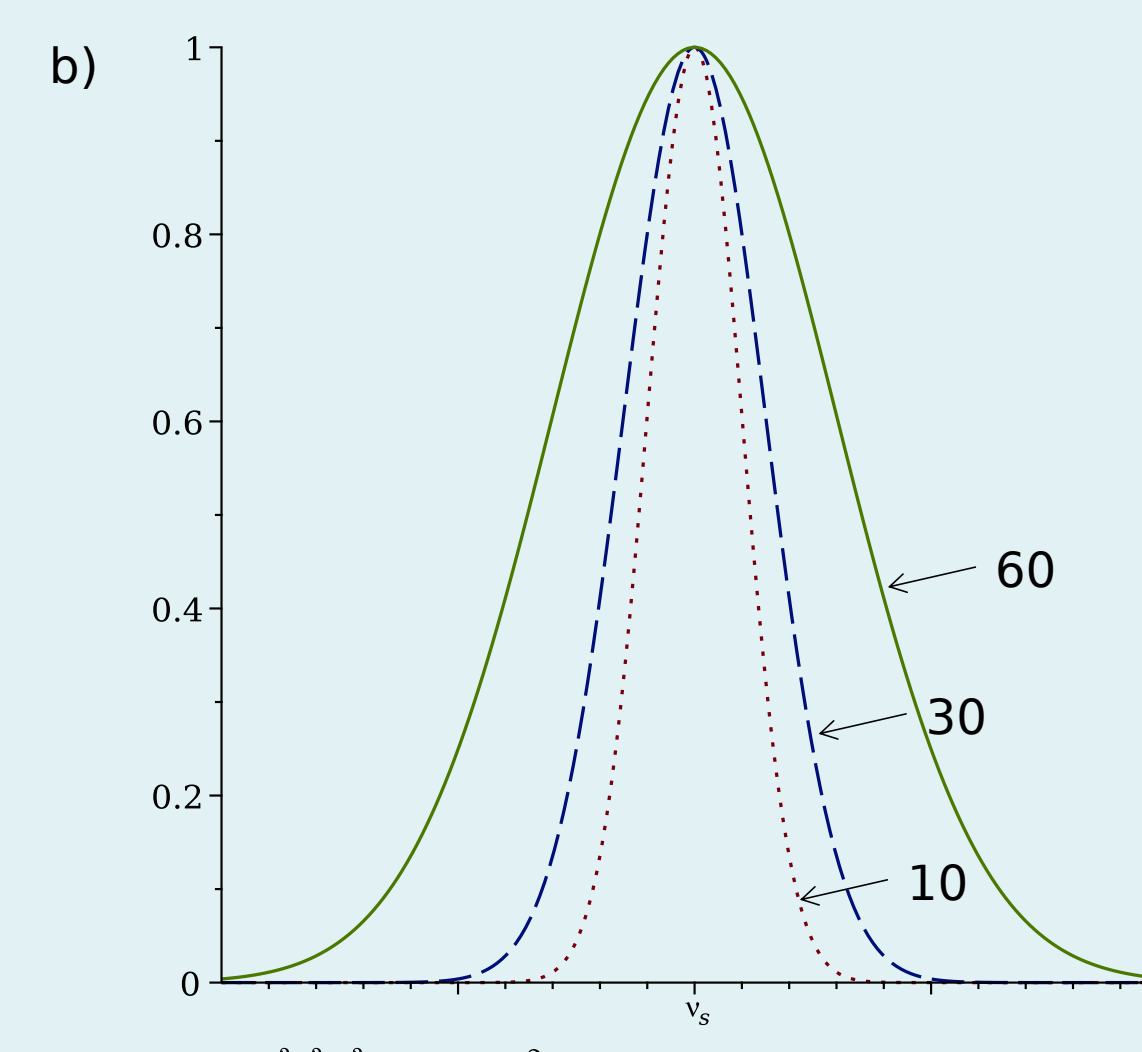
$f(t) = \sum \alpha_i \cos(\omega_i t + \phi_i)$  :

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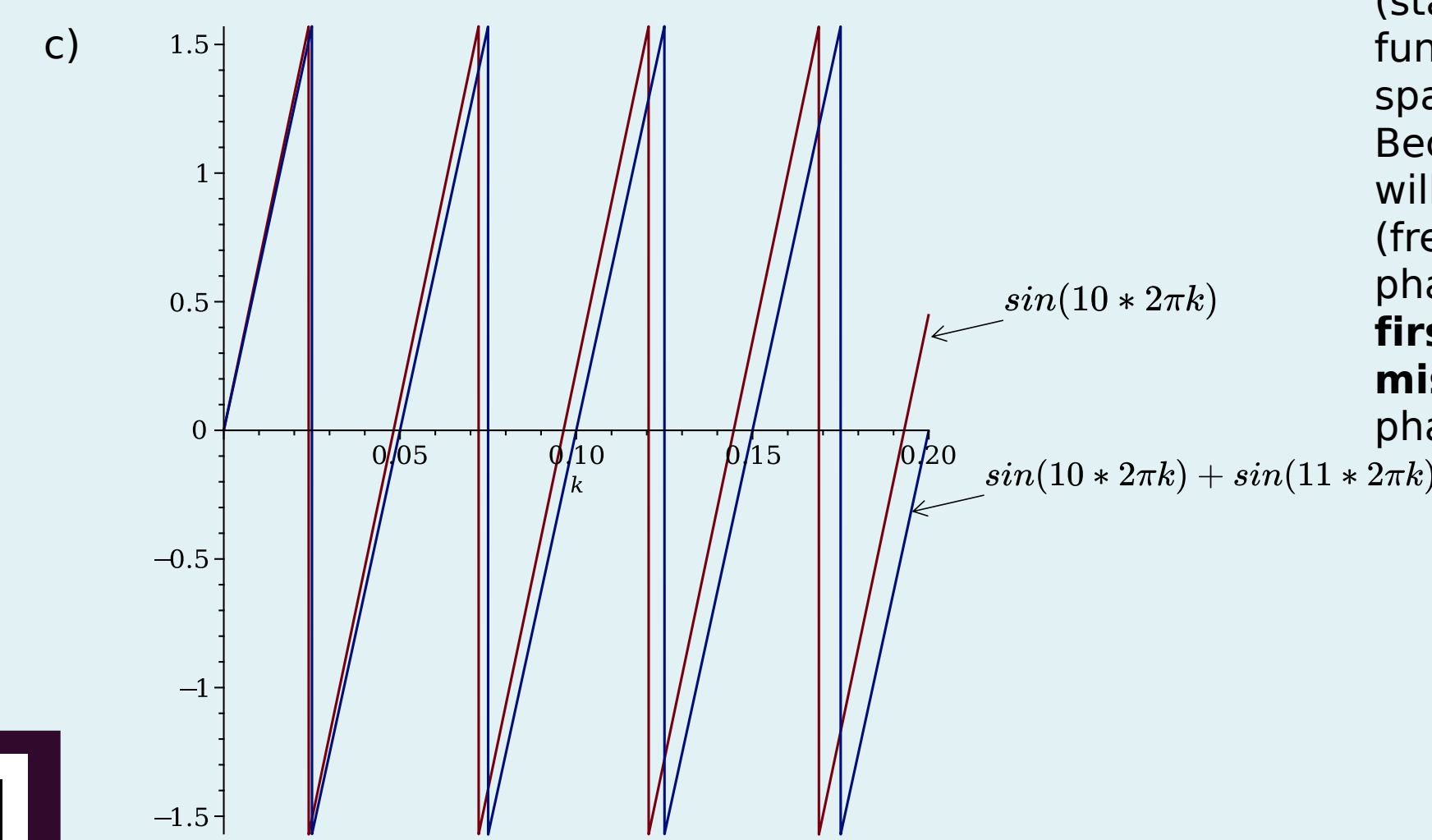
"phase"



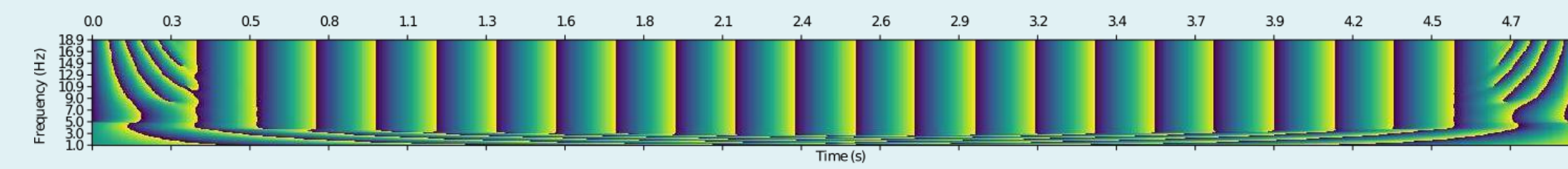
$e^{-\frac{\Omega^2(\omega_i - \omega_w)^2}{2\omega_w^2}} \sinh\left(\frac{\Omega^2 \omega_i}{\omega_w}\right), \nu_w = 10, \Omega = [5, 10, 20, 25]$



$e^{-\frac{\Omega^2(\omega_i - \omega_w)^2}{2\omega_w^2}} \sinh\left(\frac{\Omega^2 \omega_i}{\omega_w}\right), \Omega = 20, \nu_s = \nu_c - 10 : \nu_c + 10$



**c)** This figure illustrates the **importance in phase synchrony estimation of frequency mixing**. We imagine two oscillators (10 Hz) that are in phase (stable  $0^\circ$  difference for illustration purpose) due to functional coupling and a third oscillator oscillating spatially close to the second. Because scalp EEG combination the 11 Hz oscillator will completely perturbate the phase at 10Hz (frequency of the wavelet used to extract the phase), and the **phase synchrony between the first and second oscillators is completely missed**. If the third oscillates at 14 Hz, then the phase synchrony is restored for a wavelet at 10Hz.



TF plane of  $\sin(10 \cdot 2\pi t)$ ,  $t$  varies from 0 to 5 seconds. The signal is decomposed on frequencies varying from 1 to 19 with wavelets composed of 7 oscillations ( $\Omega$ ). The plot is obtained by numerical convolution (solved with multiplication in the Fourier domain of the Fourier Transform of the signal and the Fourier Transform of the wavelet followed by Fourier Inverse Transform of the product).

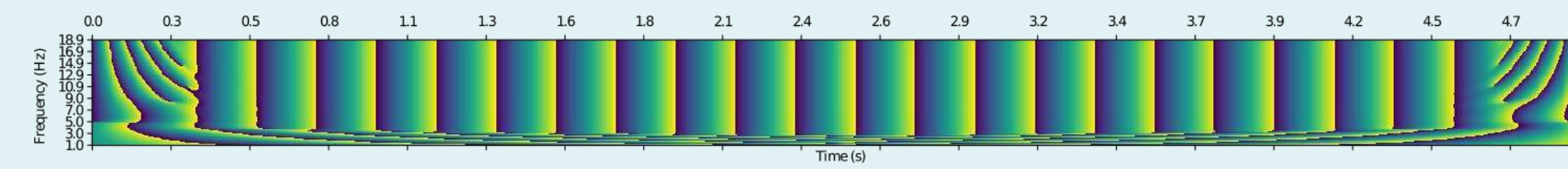
$$\frac{e^{-\frac{\Omega^2(\omega_i - \omega_w)^2}{2\omega_w^2}}}{e^{-\frac{\Omega^2(\omega_i - \omega_w)^2}{2\omega_w^2}}} < 10^{-\theta} \text{ for } \Omega = \frac{\sqrt{-2\bar{\omega} \ln(10^{-\theta})}}{2\bar{\omega}}, \text{ where } \bar{\omega} = \frac{\omega_c}{\omega_w}$$

a) The **larger the wavelet** ( $\Omega: 5 \rightarrow 25$ ), the narrower the peak, the **less mixing between far apart frequencies**.

b) The **higher the frequency** ( $10 \rightarrow 60$ ) the **larger the peak** of the coefficient, hence the **mixing between further apart frequencies**.

Both are in accordance with the uncertainty principle, that compromise should be made on temporal resolution to increase frequency resolution.

**c)** This figure illustrates the **importance in phase synchrony estimation of frequency mixing**. We imagine two oscillators (10 Hz) that are in phase (stable  $0^\circ$  difference for illustration purpose) due to functional coupling and a third oscillator oscillating spatially close to the second. Because scalp EEG combination the 11 Hz oscillator will completely perturbate the phase at 10Hz (frequency of the wavelet used to extract the phase), and the **phase synchrony between the first and second oscillators is completely missed**. If the third oscillates at 14 Hz, then the phase synchrony is restored for a wavelet at 10Hz.



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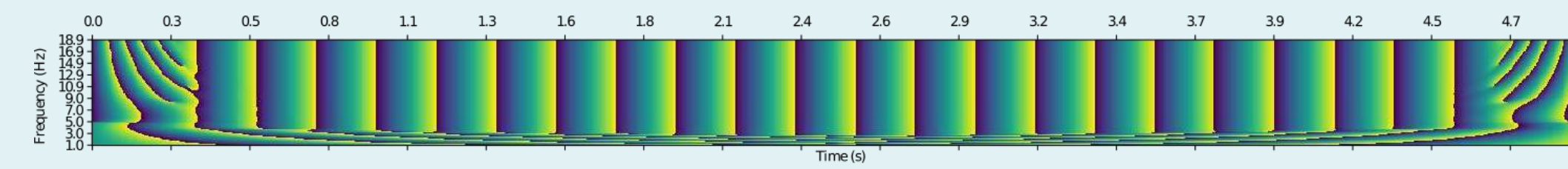
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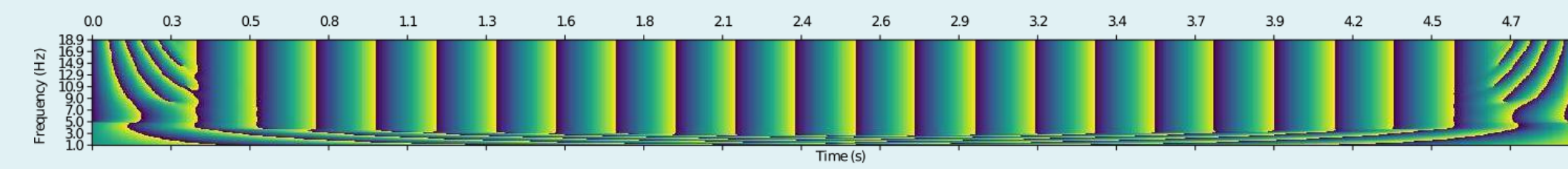
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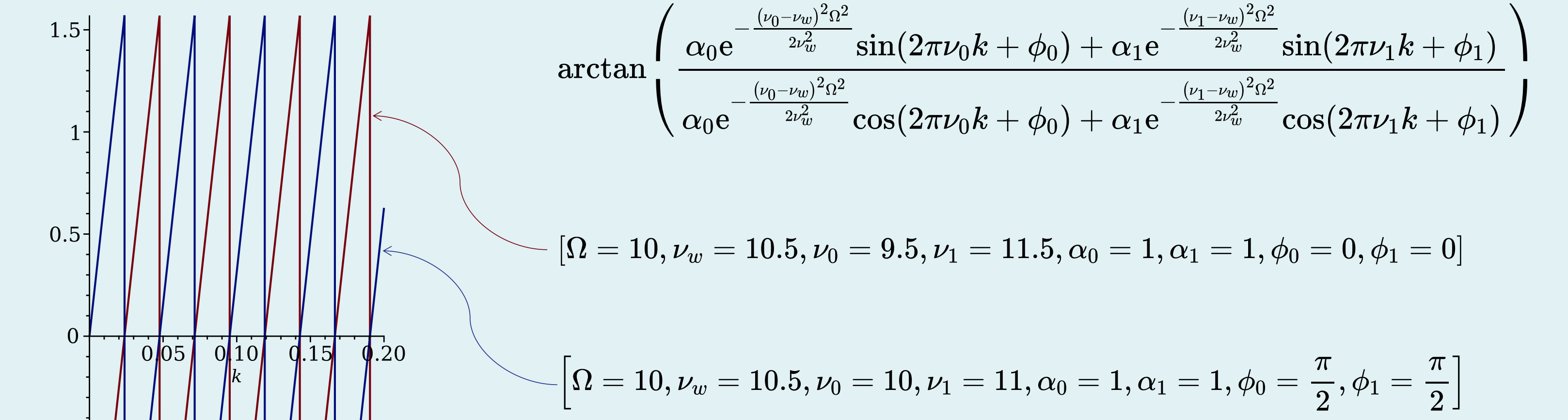
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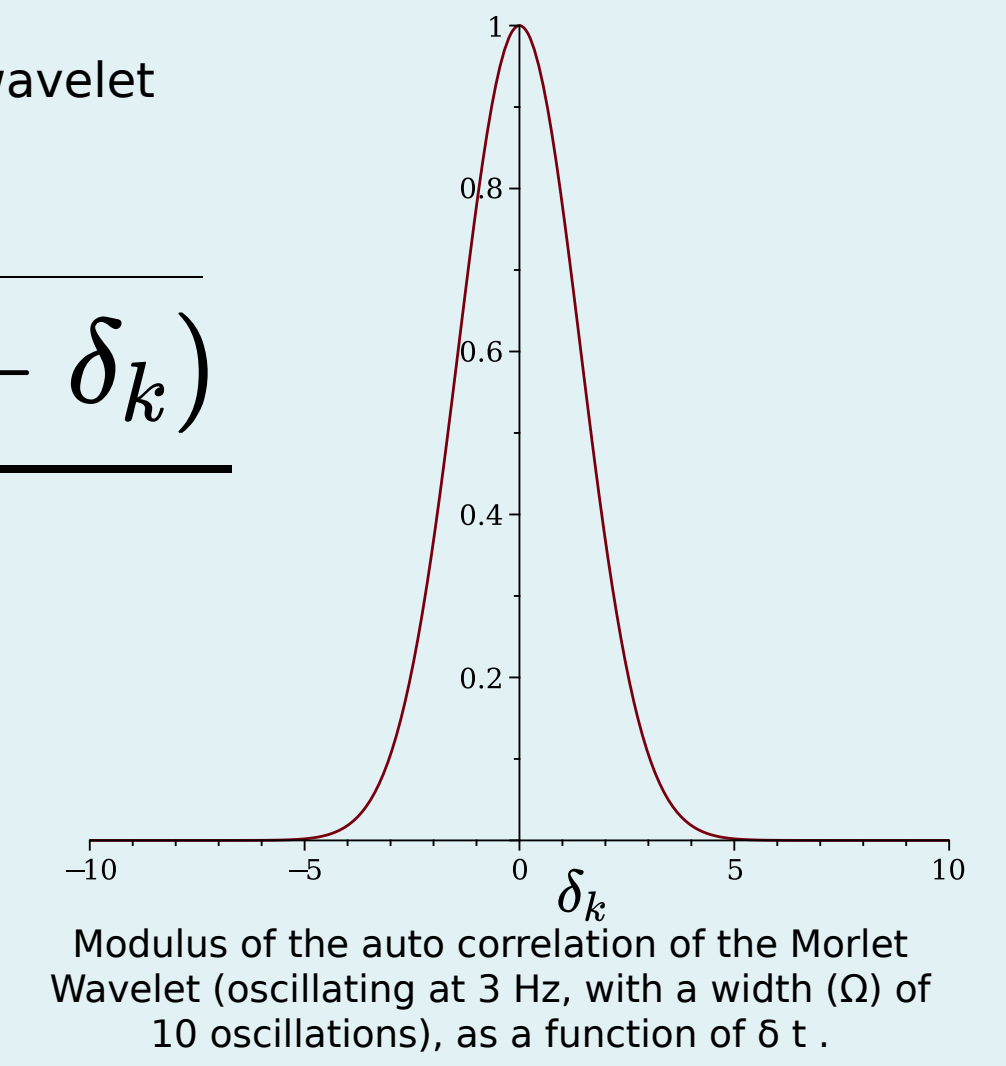
$$\arctan\left(\frac{\alpha_0 e^{-\frac{(\nu_1 - \nu_w)^2 \Omega^2}{2\omega_w^2}} \sin(2\pi\nu_0 k + \phi_0) + \alpha_1 e^{-\frac{(\nu_1 - \nu_w)^2 \Omega^2}{2\omega_w^2}} \sin(2\pi\nu_1 k + \phi_1)}{\alpha_0 e^{-\frac{(\nu_0 - \nu_w)^2 \Omega^2}{2\omega_w^2}} \cos(2\pi\nu_0 k + \phi_0) + \alpha_1 e^{-\frac{(\nu_0 - \nu_w)^2 \Omega^2}{2\omega_w^2}} \cos(2\pi\nu_1 k + \phi_1)}\right)$$

$[\Omega = 10, \nu_w = 10.5, \nu_0 = 9.5, \nu_1 = 11.5, \alpha_0 = 1, \alpha_1 = 1, \phi_0 = 0, \phi_1 = 0]$

$[\Omega = 10, \nu_w = 10.5, \nu_0 = 10, \nu_1 = 11, \alpha_0 = 1, \alpha_1 = 1, \phi_0 = \frac{\pi}{2}, \phi_1 = \frac{\pi}{2}]$

The **phase difference** between **two different combinations** of sinusoids oscillating at different frequencies **can be constant** ( $\frac{\pi}{2}$ ). It means that perfect synchrony can be completely **spurious**. It's nonetheless an edge case, since it requires a symmetry with respect to the wavelet frequency, and be close enough (in the width of the wavelet).

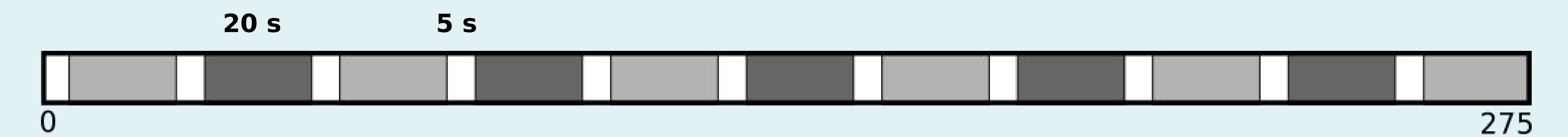
$$\rho(WT(k), WT(k + \delta_k)) = \frac{\sum_{t=-\Gamma}^{\Gamma} W(t)W(t - \delta_k)}{\sum_{t=-\Gamma}^{\Gamma} W(t)^2}$$



Under the hypothesis of an **EEG signal modeled as sequence of random variables with same mean and same variance**, we show that close coefficients are correlated. Hence, wavelet coefficients **independence should not be assumed** in statistical tests. Their correlation equals the auto correlation of the wavelet and thus, depends on the width (and hence the oscillation frequency) of the wavelet.

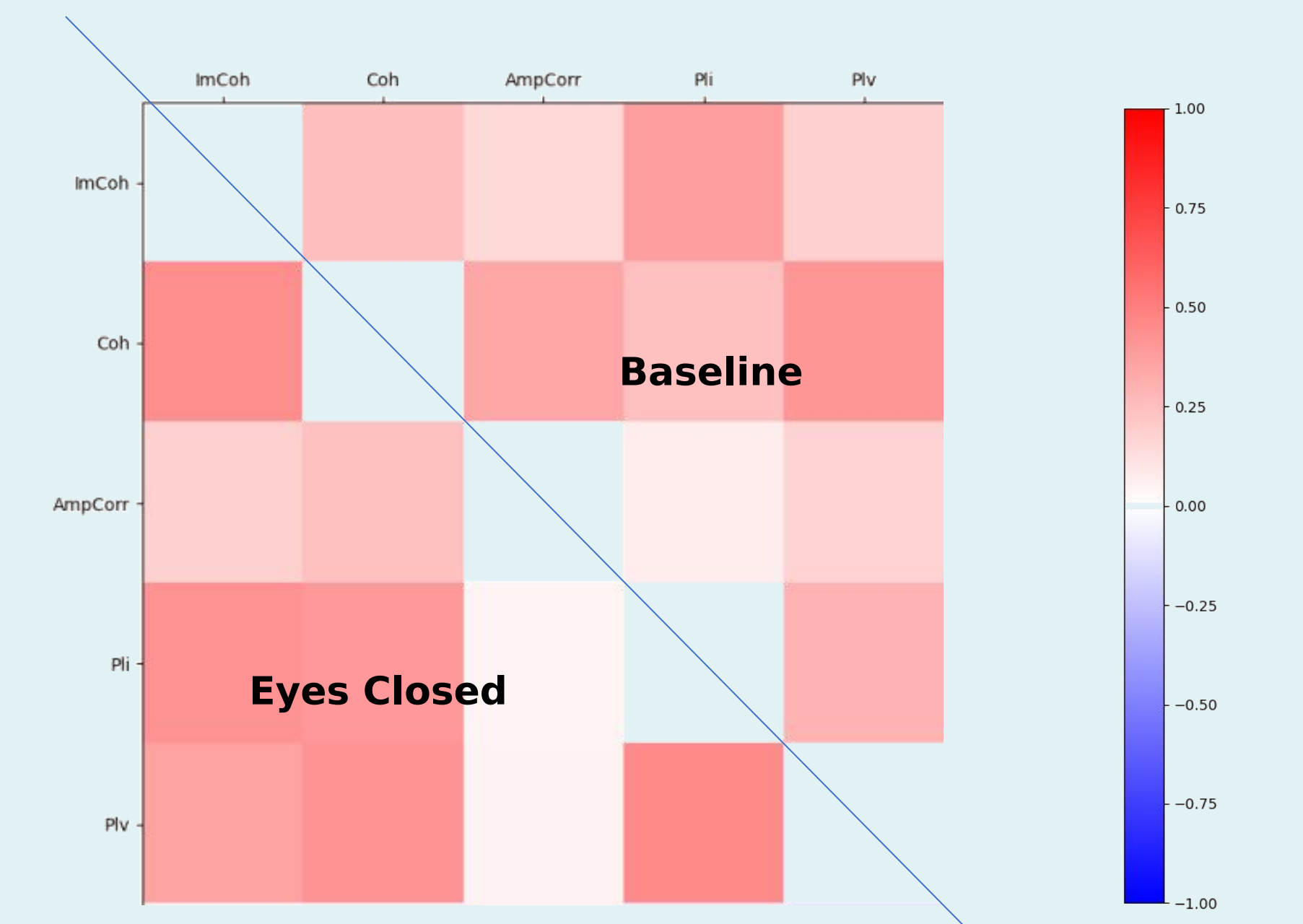
Though, EEG signals **are not strictly non auto-correlated** as we hypothesize, their auto correlation should be integrated in statistical tests.

This correlation coefficient can be used to sub sample the wavelet coefficients in order to minimize their correlation. In turn, this correlation must be introduced in statistical significance testing.



Legend: Baseline (grey), Eyes Closed (black)

Some measures are redundant: **PLI and ImCoh** or **PLV and Coh** are strongly correlated, on the other hand **PLI and PLV** are complementary, and would benefit from being analyzed altogether when assessing synchrony. For instance, **strong PLV and PLI values occurring simultaneously would encourage the assessment of a strong synchrony** between electrodes of a given pair.



Average of 13 recordings x 5 events (for Eyes Closed, 6 for Baseline) correlation matrices computed on 20 seconds event. Vertex referenced. Computed on all available pairs.

ImCoh: Imaginary Coherence; Coh: Coherence; AmpCorr: Correlation between amplitudes envelopes; PLI: Phase Lag Index; PLV: Phase Locking Value



Code



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