

# First approach of a mixed domain decomposition method for magnetostatic simulation of rotating machines

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In this work, a mixed domain decomposition method (LATIN) is proposed to improve the efficiency of magnetostatic simulation of rotating machines. This strategy relies on a decomposed formulation of the problem considering both primal and dual fields on the interfaces. From this formulation, the LATIN solver method gives a powerful iterative scheme to build a solution. A first implementation using the potential vector formulation in 2D is developed and is illustrated on two test cases: a ferromagnetic 2D bar and a switched reluctance motor.

*Index Terms*—Domain decomposition, magnetostatics, nonlinearities, parallel machines

## I. INTRODUCTION

THERE is an emerging need to design more compact and efficient machines, which requires the use of numerous numerical simulations. The Finite Element Method (FEM) is commonly used for modeling electromagnetic systems, and particularly rotating machines. It allows through the resolution of Maxwell's equations and the consideration of material behavior laws to simulate, with precision, devices in 2D or 3D [1]. Nevertheless, the 3D simulation of such machines on a complete rotation remains expensive in computation time and is often incompatible with an optimization approach [2].

Thus, the objective is to propose an innovative computational strategy based on a domain decomposition method (DDM), using the possibilities of computers with parallel architectures. Some studies have already been conducted in the context of DDM [3]. Though, unlike the dual and primal methods, the proposed strategy relies on a DDM based on a mixed writing of the equations at the interfaces. Thus, complex behaviors can be described on the interfaces without modification of the problem treated in the sub-domains. This method has been used so far mainly in solid mechanics [5] - [4]. Then, we seek to take advantage of this powerful method in mechanics to simulate efficiently magnetostatic problems.

## II. THE LATIN METHOD

A domain  $\Omega$  of study is considered and sub-divided into a number  $n$  of sub-domains  $\Omega_E$ . All the interfaces around  $\Omega_E$  are defined by one entity  $\Gamma_E$ . The particularity of the method proposed in this work is to introduce both primal and dual fields on the interfaces. In order to simplify the notations, a vector  $\{X\}$  and a matrix  $[K]$  correspond to the concatenation on all sub-domains of  $\{X_E\}$  and  $[K_E]$ . Thus, we denote  $\{W\}$  the restriction on the interfaces of the magnetic vector potential  $\{A\}$  and  $\{T\}$  the tangential component of the magnetic field  $\{H\}$  on the interfaces. These fields are built from the sub-domains using an operator  $[t]$ .

Then, we use the LATIN method which is an iterative solver based on three fundamental principles [5]. The first one consists in separating the equations into two groups: the linear equations and possibly global within the sub-domains (group  $\mathcal{A}_d$ )

$$\mathcal{A}_d : \quad s = (\{W\}, \{T\}) \quad \begin{cases} [K]\{A\} = \{J\} + [t]^T \{T\} \\ \{W\} = [t]\{A\} \end{cases}, \quad (1)$$

where  $\{J\}$  comes from the consideration of a current density, and the local and possibly non-linear equations on the interfaces (group  $\Gamma$ ), which become considering the continuities

$$\Gamma : \quad \hat{s} = (\{\widehat{W}\}, \{\widehat{T}\}) \quad \text{solution of} \quad \begin{cases} [O_d]\{\widehat{W}\} = \{0\} \\ [O_s]\{\widehat{T}\} = \{0\} \end{cases}, \quad (2)$$

where  $[O_s]$  is an operator that applies to the fields on each side of an interface to return the sum. In the same way,  $[O_d]$  returns the difference. One advantage of this method is that the rotation of the rotor can be considered as a behavior on an interface [6]. Thus, only the equations of this interface would be impacted.

The second principle relies on the use of a two stage iterative algorithm, see Algorithm 1. At each stage, partial solutions  $s_i \in \mathcal{A}_d$  or  $\hat{s}_i \in \Gamma$  are successively built according to the search directions  $E^-$  or  $E^+$  that can be written as

$$\begin{aligned} E^+ & : \quad \{\widehat{T}\} - \{T\} - [k^+] \left( \{\widehat{W}\} - \{W\} \right) = \{0\} \\ E^- & : \quad \{T\} - \{\widehat{T}\} + [k^-] \left( \{W\} - \{\widehat{W}\} \right) = \{0\} \end{aligned}. \quad (3)$$

The complete solution is obtained at the intersection of the groups  $\Gamma$  and  $\mathcal{A}_d$ , satisfying all the equations then. A convergence indicator  $\eta$  based on the distance between two successive solutions  $s_i$  and  $\hat{s}_i$  is used to stop the algorithm.

Finally, the third principle advises the use of a reduced order model (ROM) to solve the linear stage [7]. This last aspect is not considered for now.

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**Algorithm 1: LATIN**


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**Input:**  $\hat{s}_0 \in \Gamma$  (generally set to 0)  
**while**  $\eta \geq \text{criteria}$  **do**  
    **Linear stage** for each sub-domain  
    | knowing  $\hat{s}_i \in \Gamma$  search for  $s_i \in \mathcal{A}_d \cap E^-$   
    **Local stage** for each interface  
    | knowing  $s_i \in \mathcal{A}_d$  search for  $\hat{s}_{i+1} \in \Gamma \cap E^+$   
    **Convergence indicator** compute  $\eta$   
**end**

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### III. NUMERICAL EXAMPLE

In this section, the presented method is first validated on a test case with an analytical solution: a ferromagnetic 2D bar with Dirichlet conditions on the left and right ends. For a bar decomposed into two sub-domains, the solution is obtained in 20 iterations with a relative error of  $3 \cdot 10^{-5}$  %. In Fig. 1 we highlight that the number of iterations necessary to reach convergence increases according to the number of sub-domains that are defined for an identical problem to solve. This dependency is a classical issue in mono-scale DDM method.

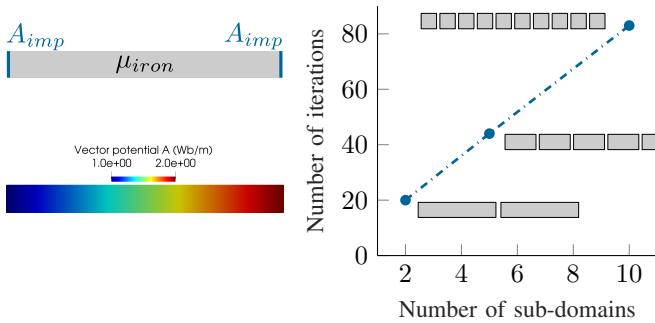


Fig. 1. Definition of the academic problem and its converged solution, and evolution of the number of iterations according to the number of sub-domains

This implemented method is tested on an academic case such as a switched reluctance motor (SRM). The stator consists of a ferromagnetic structure with a coil wound around with a current  $I = 7$  A. A rotor with 4 poles that has a rotational movement is considered. On external boundaries, the magnetic vector potential  $A$  is set to 0 Wb/m as a boundary condition. This problem is decomposed into 15 sub-domains as detailed in Fig. 2.

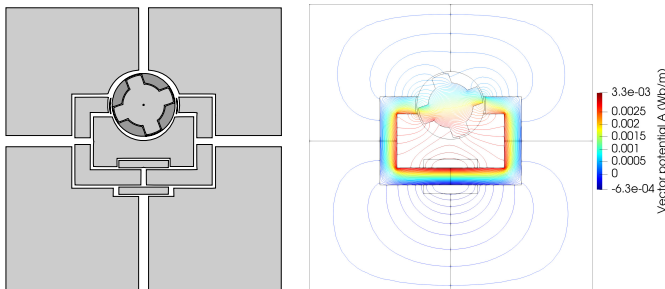


Fig. 2. Decomposition of the problem and equipotential lines of magnetic vector potential  $A$  for a rotor position

In Fig. 2 the equipotential lines of magnetic vector potential  $A$  are shown for a position of the rotor and obtained for a convergence criteria  $\eta \leq 10^{-7}$ . In order to validate this result, it is compared with a solution built with a global simulation. In Fig. 3, the magnetic vector potential is plotted on the circular interface around the rotor for both simulations. This comparison allows to validate the solution computed with the LATIN algorithm.

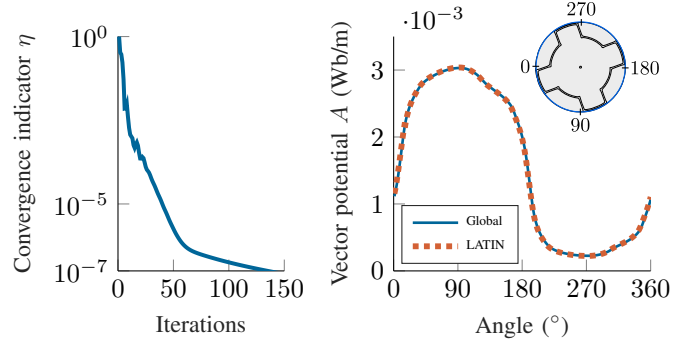


Fig. 3. Evolution of the convergence indicator and comparison of the magnetic vector potential  $A$  around the rotor between a global simulation and a LATIN simulation

### IV. CONCLUSION

We presented a mixed domain decomposition method adapted for magnetostatic simulation. The implementation of the LATIN method have been validated in 2D with the study of a simple rotating machine. This efficient method has the advantage to be able to integrate nonlinearities without impacting the structure of the algorithm. Furthermore, the LATIN method is the perfect ground to add ROM strategy. These areas of improvement will be the subject of upcoming work. In addition, a mono-scale method is highly dependent on the number of sub-domains. A multi-scale computational strategy will be investigated to address this issue.

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