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Robust Adaptive PI Control of Wave Energy Converters with Uncertain Dynamics

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Abstract

There exist several approaches to design the optimal control strategy to harvest wave energy with a point absorber. However they are generally based on the assumption that the WEC and the PTO dynamics are well-known. In the practical WEC control implementation, this is generally not the case. The objective of this paper is to design a robust optimal control strategy that can take into account the uncertain WEC and PTO dynamics. Our choice is a robust adaptive PI control law. The proposed controller is validated and compared through simulation for irregular sea states.

Key words: Wave Energy Converter (WEC), Robust Adaptive PI, Uncertain Dynamics, Semidefinite Programming (SDP)

1 Introduction

The ocean is an almost inexhaustible source of energy and represents one of the most reliable and promising forms of renewable energy [4], [5]. According to [18], the ocean holds approximately 8000 – 80 000 TWh/year or 1 – 10 TW. Comparing this to the world's annual energy consumption of approximately 148 000 TWh in 2008, it is shown [9] that wave energy could play an important role in the world's energy portfolio. Currently, several wave energy converter (WEC) concepts are under investigation, but wave energy is still far from being competitive with other forms of renewable energy such as wind or solar. The production cost per unit of power for WEC systems is hampered by two fundamental problems:

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- (1) High cost of maintenance. Under extreme conditions, WEC systems experience large amplitude motions, wave over-topping, wave slamming, etc. These physical phenomena increases the risk of device damage.
- (2) Low energy conversion rate. This concerns virtually all the WEC concepts, regardless of the width of their absorption band. For a broad category of WECs, however, wave-to-grid efficiency strongly depends on the way the machine, and its Power Take-Off (PTO) system in particular, is controlled.

In contrast with the wind energy sector, there is no industrial standard device for wave energy conversion and this diversity is directly related to the PTO system. Many different types of PTO systems have been studied, and the type of PTO system used in a wave energy converter is often correlated with its type [25]. WEC systems are located in very harsh environment, leading to a high wear-rate and are difficult to access due to their location and/or unfavorably weather conditions. As for the rest of the device, the PTO system should be robust, reliable and should require as little maintenance as possible.

When there is a flexible PTO available, capable of both harvesting and drawing power from the grid, respectively in generator and motor modes, an intermittent switching *PI*-controller [28] can be considered as the *standard* solution. The *P* component converts the wave energy into useful energy, while the *I* component modifies the WEC system natural frequency which allows the absorber to be more often in phase with the incoming waves, at the expense of some power being drawn from the grid. Under the assumptions that the PTO system has no dynamics, and that the WEC dynamics are well-known, mathematical tools involved in the computation of the optimal PI gains are remarkably simplified. There exists an elegant analytical expression for the PI gains as a function of wave frequency in the case of regular waves [5]. Unfortunately, a PTO system without dynamics is unrealistic. In addition, due to friction, viscous and many other forces, the WEC and PTO dynamics are often not well-known. In these cases, to the best of the authors knowledge, there is no available numerical method in the literature to calculate the optimal PI gains. The main objective of this paper is to fill this gap.

It is evident that the PI gains should be adapted in real-time as the sea conditions changes continuously. Adaptive PI control has been already studied in [10], [22], [23]. In most literature, only intermittent adaptive PI control laws are considered, where the control gains are constant within a time window of several minutes, e.g., 30 minutes in [10]. Clearly, such intermittent adaptive control laws are sub-optimal in terms of energy recovery. In [22], [23], a new adaptive PI control algorithm is proposed, for which the PI gains are continuously adapted on-line, on a wave-to-wave basis. The most notable feature of the control law in [22], [23] is that the PTO nonlinear efficiency coefficient is taken into account in the controller design phase. However, it is still assumed

that the WEC dynamics are well-known and that the PTO system has no dynamics.

In this paper, we continue the research line of [22], [23]. Using a real-time sinusoidal approximation of the wave excitation force, the PI gains are adapted on-line. The uncertain WEC and PTO dynamics are taken into account in the controller design step. The worst case scenario is considered, i.e., the worst-case harvested average power is maximized. In other words, a min-max optimization problem is solved. We then show how to convert this optimization problem into a convex semidefinite programming (SDP) problem, for which there exists several free available software packages [15], [8].

It is worth stressing that even though the mismatch between model and the actual system is inevitable, the issue of robustness in the WEC field is still rarely to be discussed. Among some attempts in the topic of synthesizing robust WEC controllers, a hierarchical control strategy was considered in [6], [31]. The basic idea is to generate a high-level suboptimal velocity reference, and then a low-level servo control loop. To generate the velocity profile, only a very particular type of model uncertainties can be considered. In addition, the PTO dynamics were not taken into account.

This paper presents a rigorous treatment of the preliminary results in [19]. In this paper:

- (1) Both system and actuator dynamics are uncertain. As a consequence, the nonlinear Morrison force can be addressed. In [19], only the uncertain actuator dynamics case is considered. The system dynamics are assumed to be linear and well-known, see equation (1) in [19].
- (2) A procedure to identify an uncertain transfer function is provided. In [19], there is no such procedure.
- (3) A proof of Theorem 1 is given. In [19], this theorem is presented without proof.
- (4) Theorem 2 and its proof are new.

The paper is organized as follows. Section 2 describes the problem formulation and some preliminaries concerning the SDP optimization problem. A procedure to calculate the real and imaginary parts of the robust optimal control block under regular waves in the presence of a uncertain WEC and PTO dynamics is presented in Section 3. Section 4 explains how the results of Section 3 can be extended to deal with realistic polychromatic sea states. In Section 5, the simulation setup is described used to validate the algorithm, and the results are reported. Finally, some conclusions are drawn in Section 6.

2 Problem Formulation and Preliminaries

2.1 Model of the WEC

We consider point-absorber WECs consisting of a heaving body that moves in one or two degrees of freedom with respect to a fixed anchor or a submerged body. From the relative motion, useful energy can be extracted. In general, the WEC is axis-symmetric so that the energy conversion rate is almost the same for waves coming from all directions, see Fig. 1.

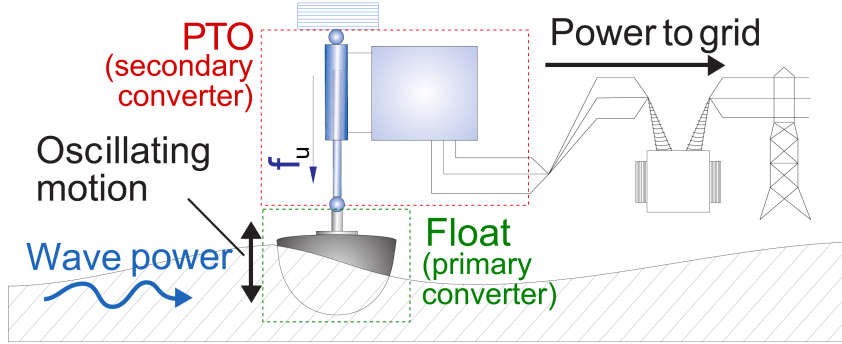


Fig. 1. One-degree-of-freedom floating system for wave energy converter.

The equation of motion for the WEC system can be obtained as,

$$M_s \dot{v}(t) = f_{ex}(t) - f_{pto}(t) + f_r(t) + f_{hd}(t) + f_v(t) \quad (1)$$

where M_s is the WEC mass, $v(t)$ is the heaving velocity, $f_{ex}(t)$ is the wave excitation force, and $f_{pto}(t)$ is the force from the PTO system. The other force terms and their mathematical representation are explained in the following.

$f_r(t)$: Radiation force.

The radiation force due to radiated waves is given as

$$f_r(t) = -M_\infty \dot{v}(t) - f_{rh}(t) \quad (2)$$

where M_∞ is the added mass at infinitely high frequency, and

$$f_{rh}(t) = \int_0^t h(t - \tau)v(\tau)d\tau \quad (3)$$

$h(t)$ is the impulse response function of the radiation force, which can be obtained by boundary element methods [3], [13]. Equation (3) can be considered as a linear system, where the input is $v(t)$ and the output is $f_{rh}(t)$. It can be rewritten as,

$$F_{rh}(jw) = W_r(jw)V(jw) \quad (4)$$

where $F_{rh}(jw), W_r(jw), V(jw)$ are the Fourier transform of $f_{rh}(t), h(t)$ and $v(t)$, respectively.

$f_{hd}(t)$: Hydrostatic force.

The hydrostatic force gives stiffness force as deviation from hydrostatic equilibrium. This force is obtained as

$$f_{hd}(t) = -K_{hd}p(t) \quad (5)$$

where K_{hd} is the hydrostatic coefficient, $p(t) = \int_0^t v(\tau)d\tau$ is the buoy position.

$f_v(t)$: Viscous drag force.

The force due to viscous forces is given as a quadratic function of buoy velocity

$$f_v(t) = K_v|v(t)|v(t) \quad (6)$$

where K_v is the Morison coefficient. Note that this is a simplification of the quadratic term of the Morison equation, which usually considers the relative velocity of buoy and water.

There are other forces such as friction, mooring forces. However they are not considered in the paper. The motion response of the WEC system is given by the following model

$$M\dot{v}(t) + \int_0^t h(t-\tau)v(\tau)d\tau + K_{hd} \int_0^t v(\tau)d\tau + K_v|v(t)|v(t) = f_{ex}(t) - f_{pto}(t) \quad (7)$$

with $M = M_s + M_\infty$.

For convenience of control design, the WEC model is typically expressed in the frequency domain as

$$V(jw) = \frac{1}{Z_i(jw)}(F_{ex}(jw) - F_{pto}(jw)) \quad (8)$$

where $Z_i(jw)$ is the intrinsic impedance of the system. $F_{ex}(jw)$ and $F_{pto}(jw)$ represent, respectively, the Fourier transform of $f_{ex}(t)$, and of $f_{pto}(t)$. Note that $Z_i(jw)$ is uncertain due to the un-modeled forces, due to the presence of the viscous drag force, as well as the variation of the WEC parameters such as K_{hd} .

Define $F_u(jw)$ as the control input. The PTO force $F_{pto}(jw)$ is related to $F_u(jw)$ as, see Fig. 1

$$F_{pto}(jw) = W_{pto}(jw)F_u(jw) \quad (9)$$

where $W_{pto}(jw)$ is the PTO transfer function. $W_{pto}(jw)$ is also uncertain in this study.

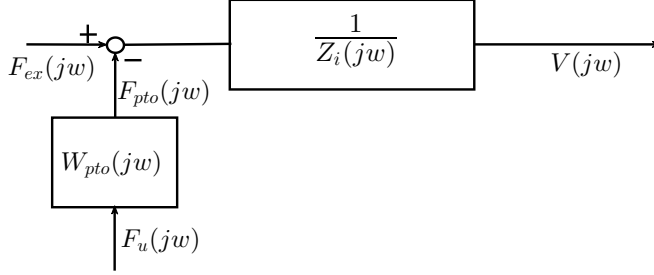


Fig. 2. Structure of the considered mathematical WEC model.

Combining (8), (9), one gets

$$\begin{cases} V(jw) = \frac{1}{Z_i(jw)} (F_{ex}(jw) - W_{pto}(jw)F_u(jw)), \\ F_{pto} = W_{pto}(jw)F_u(jw) \end{cases} \quad (10)$$

Equation (10) will serve as a basic for the results in the next sections.

2.2 Control Objective

The objective for the WEC system is to select the input signal $f_u(t)$ that maximizes the harvested average power P_a

$$P_a = \frac{1}{T} \int_0^T P(t) dt \quad (11)$$

where $P(t) = f_{pto}(t)v(t)$ is the instantaneous power. In the literature, it is generally assumed that $W_{pto}(jw) = 1, \forall w$, and that the WEC dynamics are well known. With these assumptions, the mathematical tool involved to calculate the optimal control signal $f_u(t)$ is greatly simplified, see e.g., [5]. As written in Introduction, many different types of PTO systems exist, and the type of PTO chosen for a particular WEC is often strongly related with the type of converter. For the point absorber type of converter in the paper, a linear electrical generator or a hydraulic system are generally used [25]. Unfortunately, the dynamics of these PTO systems are relatively long with respect to the WEC dynamics, and therefore can not be neglected. In addition, due to viscous, frictions and several other forces, the WEC and PTO dynamics are

not completely known. Therefore, it is important in optimal control of WEC systems to take these uncertainties into account.

Define $R_i(w)$, $X_i(w)$ and $R_{pto}(w)$, $X_{pto}(w)$, respectively, as the real and the imaginary parts of $Z_i(jw)$ and of $W_{pto}(jw)$, i.e.

$$\begin{cases} Z_i(jw) &= R_i(w) + jX_i(w), \\ W_{pto}(jw) &= R_{pto}(w) + jX_{pto}(w) \end{cases} \quad (12)$$

Note that $R_i(w)$, $X_i(w)$ and $R_{pto}(w)$, $X_{pto}(w)$ are not well-known due to the uncertainties of $Z_i(jw)$, $W_{pto}(jw)$. Details of how to obtain a set of possible values of $R_i(w)$, $X_i(w)$ and $R_{pto}(w)$, $X_{pto}(w)$ are described in Section 2.4.

In summary, the control objective for the WEC system is to calculate the control input $f_u(t)$ that maximizes the worst-case harvested average power P_a in the presence of the uncertain WEC and PTO systems (12). In other words, we aim to solve the following max-min problem

$$\max_{f_u} \min_{R_i, X_i, R_{pto}, X_{pto}} \{P_a\} \quad (13)$$

Remark: Fuzzy control [16], [17] might be an alternative solution for the case of uncertain WEC and PTO systems. However, this approach requires an online estimation of the model.

2.3 Notation and Preliminaries

Some standard notations are introduced in this section. For symmetric matrices, the symbol $(*)$ denotes each of its symmetric blocks. We denote $\mathbb{R}^{n \times m}$ the set of real $n \times m$ matrices. For any real symmetric matrix X , the notation $X \succeq 0$ indicates that all eigenvalues of X are nonnegative.

In the following, we recall some basic definitions of semidefinite programming (SDP) that we will use later to design a robust optimal control law.

A linear matrix inequality is a condition of the type

$$F(x) \succeq 0 \quad (14)$$

where $x \in \mathbb{R}^n$ is a vector variable, and the matrix $F(x)$ is affine in x , that is

$$F(x) = F_0 + \sum_{i=1}^n F_i x_i$$

with symmetric matrices $F_i \in \mathbb{R}^{m \times m}$.

LMIs can either be understood as feasibility conditions or constraints for optimization problems. Optimization of a linear function over LMI constraints is called SDP, which is an extension of linear programming [2], [21]. Nowadays, a main benefit in using LMIs is that for solving an LMI problem, several polynomial time algorithms were developed and implemented in free available software packages, such as YALMIP [15], CVX [8], etc.

The Schur complement is a very useful tool for matrix inequalities. It states that the following nonlinear condition

$$\begin{cases} U(x) \succeq 0, \\ P(x) - Q(x)^T U(x)^{-1} Q(x) \succeq 0 \end{cases}$$

can be equivalently written in the LMI form

$$\begin{bmatrix} P(x) & Q(x)^T \\ Q(x) & U(x) \end{bmatrix} \succeq 0$$

Hence the Schur complement allows to convert certain nonlinear matrix inequalities into a higher dimensional LMI condition.

2.4 Identification of Uncertain Transfer Functions

In this section, a transfer function and/or an intrinsic impedance identification problem is considered. Namely, for a given uncertain system $W_s(jw)$, we are looking for a set of all possible values of the real part and of the imaginary part of the transfer function, representing the plant under all relevant operating conditions, on which to base the robust optimal design. It is clear that the value set might be non-convex due to nonlinearities, parameter variations, noise and disturbances, etc.

Denote $U(jw)$, $Y(jw)$, respectively, as the input and output of the considered system, i.e.,

$$Y(jw) = W_s(jw)U(jw) \tag{15}$$

A straightforward way to obtain the value set is to simulate the system (15) with an input $u(t) = A \sin(\omega t)$ for several different amplitudes A , and then to measure the amplitude B and the phase δ of the output $y(t) = B \sin(\omega t + \delta)$. However direct measurements of B and δ are difficult due to nonlinearities, noises, uncertainties [14], [29].

It is worth noticing that the problem of identifying the value set of uncertain systems is well-known [7], [14], [29]. There exist several methods in the literature such as the Lissajou figure method [7], Fourier integral method [14], or correlation method [29]. Due to its simplicity, the Fourier integral approach is chosen in this study. This method is summarized in the following in order to make the paper clearer and more self-contained.

It is assumed that the output measurement $y(t)$ is given as

$$y(t) = B\sin(\omega t + \delta) + e(t) \quad (16)$$

where $e(t)$ is the measurement noise. $e(t)$ is uncorrelated with $u(t)$. Define two new signals

$$y_s = \frac{1}{\mathbf{n}T} \int_0^{\mathbf{n}T} y(t)\sin(\omega t)dt, \quad y_c = \frac{1}{\mathbf{n}T} \int_0^{\mathbf{n}T} y(t)\cos(\omega t)dt$$

where $T = \frac{2\pi}{\omega}$ is the period and \mathbf{n} is any large positive integer. One has

$$\begin{aligned} y_s &= \frac{1}{\mathbf{n}T} \int_0^{\mathbf{n}T} y(t)\sin(\omega t)dt \\ &= \frac{1}{\mathbf{n}T} \int_0^{\mathbf{n}T} B\sin(\omega t + \delta)\sin(\omega t)dt + \frac{1}{\mathbf{n}T} \int_0^{\mathbf{n}T} e(t)\sin(\omega t)dt \end{aligned}$$

Because $e(t)$ is uncorrelated with $u(t)$, one gets $\frac{1}{\mathbf{n}T} \int_0^{\mathbf{n}T} e(t)\sin(\omega t)dt \approx 0$. Hence

$$y_s \approx \frac{B}{2}\cos(\delta) - \frac{B}{2\mathbf{n}T} \int_0^{\mathbf{n}T} \cos(2\omega t + \delta)dt \approx \frac{B}{2}\cos(\delta)$$

Analogously, $y_c \approx \frac{B}{2}\sin(\delta)$. Therefore

$$B = 2\sqrt{y_c^2 + y_s^2}, \quad \delta = \arctan\left(\frac{y_c}{y_s}\right), \quad (17)$$

Once the amplitude and the phase of the output $y(t)$ are obtained, the frequency response of $W_s(j\omega)$ is computed as

$$\begin{cases} |W_s(j\omega)| = \frac{B}{A}, \quad \angle W_s(j\omega) = \delta, \\ R_s(\omega) = |W_s(j\omega)|\cos(\delta), \quad X_s(\omega) = |W_s(j\omega)|\sin(\delta) \end{cases} \quad (18)$$

where $|W_s(j\omega)|, \angle W_s(j\omega), R_s(\omega), X_s(\omega)$ are, respectively, the amplitude, the phase, the real part, and the imaginary part of $W_s(j\omega)$.

In summary, for uncertain system $W_s(j\omega)$, the value set of the real part R_s and of the imaginary part X_s can be obtained using Algorithm 1.

Algorithm 1 : Calculation of the value set of the part R_s and the imaginary part X_s of the uncertain transfer function W_s (or of the intrinsic impedance)

- 1: Select a reasonable finite but large set of frequency samples $0 < w_1 < \dots < w_N$.
 - 2: Select a reasonable finite but large set of amplitude of the input $0 < A_1 < A_2 < \dots < A_L$.
 - 3: For each frequency $w_l, \forall l = \overline{1, N}$, and each amplitude $A_k, \forall k = \overline{1, L}$ in the chosen set
 - 4: Compute the amplitude B and the phase δ of the output as in (17).
 - 5: Compute $R_s^{(l,k)}, X_s^{(l,k)}$ as in (18).
 - 6: The value sets for R_s , and X_s are obtained as the union of $R_s^{(l,k)}$, and $X_s^{(l,k)}, \forall l = \overline{1, N}, \forall k = \overline{1, L}$ respectively.
-

Using Algorithm 1, Fig. 3 presents the value sets of the real and imaginary parts of the WEC intrinsic impedance $Z_i(jw)$. A zoom of the value set of $R_i(w)$ and $X_i(w)$ for one particular frequency $w = 6(rad/s)$ is also presented. The considered frequency interval is $1 \leq w \leq 10$, and the amplitude $1 \leq A \leq 15$. The parameters of the WEC system are given in Section 5.

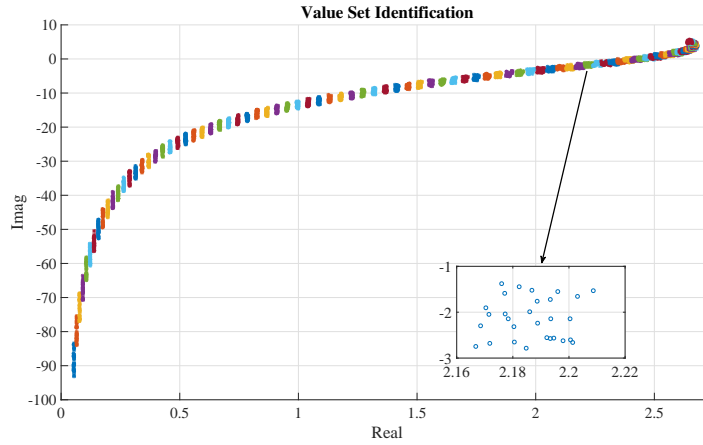


Fig. 3. Value set of the real part $R_i(w)$, and of the imaginary part $X_i(w)$ of the WEC intrinsic impedance $Z_i(jw)$.

3 Robust Optimal Control in Regular Waves

3.1 Average power

For regular waves, the wave excitation force is given as

$$f_{ex}(t) = A \sin(\omega t + \phi) \quad (19)$$

where A, w, ϕ are the amplitude, the angular wave frequency, and the phase angle, respectively. Without loss of generality, ϕ can be set to be zero.

In this work, the control input is assumed to be a linear function of the velocity, i.e.,

$$F_u(jw) = Z_c(jw)V(jw) \quad (20)$$

where $Z_c(jw)$ can be seen as a load impedance, and designed following impedance matching principles [5].

Define $R_c(w), X_c(w)$ as the real and the imaginary parts of $Z_c(jw)$, i.e.,

$$Z_c(jw) = R_c(w) + jX_c(w) \quad (21)$$

For simplicity, the argument w will be omitted from $R_i, X_i, R_{pto}, X_{pto}, R_c, X_c$, whenever possible.

The following theorem holds.

Theorem 1: Given the wave excitation force (19), and the control input (20), the average power is computed as

$$P_a = \frac{A^2(R_{pto}R_c - X_{pto}X_c)}{2((R_i + R_{pto}R_c - X_{pto}X_c)^2 + (X_i + R_{pto}X_c + X_{pto}R_c)^2)} \quad (22)$$

Proof: Using (10), (20), the WEC velocity is calculated as

$$V(jw) = W(jw)F_{ex}(jw) \quad (23)$$

where $W(jw) = \frac{1}{Z_i(jw) + W_{pto}(jw)Z_c(jw)}$ is the transfer function from $F_{ex}(jw)$ to $V(jw)$.

As system (23) is linear, one obtains

$$v(t) = A_w A \sin(\omega t + \theta_w) \quad (24)$$

where $A_w = |W(jw)|$ and $\theta_w = \angle W(jw)$ are the amplitude and the phase of $W(jw)$, evaluated at frequency w , respectively.

Analogously, using (20), one gets

$$f_u(t) = A_c A_w A \sin(\omega t + \theta_w + \theta_c) \quad (25)$$

and, using (10)

$$f_{pto}(t) = A_{pto} A_c A_w A \sin(\omega t + \theta_w + \theta_c + \theta_{pto}) \quad (26)$$

where $A_c = |W_c(jw)|$, $\theta_c = \angle W_c(jw)$, and $A_{pto} = |W_{pto}(jw)|$, $\theta_{pto} = \angle W_{pto}(jw)$ are, respectively, the amplitude and the phase of $W_c(jw)$ and of $W_{pto}(jw)$ at w .

Combining (24), (26), the instantaneous power is computed as

$$\begin{aligned} P(t) &= v(t)f_{pto}(t) \\ &= A_{pto}A_cA_w^2A^2 \sin(\omega t + \theta_w) \sin(\omega t + \theta_w + \theta_c + \theta_{pto}) \end{aligned} \quad (27)$$

and the average power

$$P_a = \frac{1}{T} \int_0^T P(t) dt \quad (28)$$

It is clear that only the relative phase shift of two sines is required for the calculation of P_a . Hence θ_w can be omitted in (27) without loss of generality, i.e.,

$$P(t) = A_{pto}A_cA_w^2A^2 \sin(\omega t) \sin(\omega t + \theta) \quad (29)$$

with

$$\theta = \theta_c + \theta_{pto} \quad (30)$$

It is also clear that it suffices to calculate P_a for one period $T = \frac{2\pi}{\omega}$, since the wave (19) is periodic. Hence

$$P_a = \frac{\omega}{2\pi} A_{pto}A_cA_w^2A^2 \int_0^{\frac{2\pi}{\omega}} \sin(\omega t) \sin(\omega t + \theta) dt$$

and thus, by using the following product-to-sum trigonometric identity

$$\sin(\alpha) \sin(\beta) = \frac{1}{2} (\cos(\alpha - \beta) - \cos(\alpha + \beta))$$

one gets

$$\begin{aligned} P_a &= \frac{\omega}{4\pi} A_{pto}A_cA_w^2A^2 \int_0^{\frac{2\pi}{\omega}} (\cos(\theta) - \cos(2\omega t + \theta)) dt \\ &= \frac{\omega}{4\pi} A_{pto}A_cA_w^2A^2 \left(\cos(\theta)t - \frac{\sin(2\omega t + \theta)}{2\omega} \right) \Big|_0^{\frac{2\pi}{\omega}} \end{aligned}$$

or equivalently

$$P_a = \frac{1}{2} A_{pto}A_cA_w^2A^2 \cos(\theta) \quad (31)$$

Using (30), one has

$$\begin{aligned} \cos(\theta) &= \cos(\theta_c + \theta_{pto}) \\ &= \cos(\theta_c) \cos(\theta_{pto}) - \sin(\theta_c) \sin(\theta_{pto}) \end{aligned}$$

and thus, with (31)

$$P_a = \frac{1}{2} A_{pto}A_cA_w^2A^2 (\cos(\theta_c) \cos(\theta_{pto}) - \sin(\theta_c) \sin(\theta_{pto})) \quad (32)$$

The following equations hold

$$\begin{cases} R_c = A_c \cos(\theta_c), X_c = A_c \sin(\theta_c), \\ R_{pto} = A_{pto} \cos(\theta_{pto}), X_{pto} = A_{pto} \sin(\theta_{pto}) \end{cases}$$

and with

$$A_w^2 = \frac{1}{(R_i + R_{pto}R_c - X_{pto}X_c)^2 + (X_i + R_{pto}X_c + X_{pto}R_c)^2}$$

Equation (32) is rewritten as

$$P_a = \frac{A^2(R_{pto}R_c - X_{pto}X_c)}{2((R_i + R_{pto}R_c - X_{pto}X_c)^2 + (X_i + R_{pto}X_c + X_{pto}R_c)^2)}$$

The proof is complete. \square

Remark: Consider the limiting case when the PTO has no dynamics. This implies that $W_{pto}(jw) = 1, \forall w$ and hence $R_{pto} = 1, X_{pto} = 0$. Equation (22) becomes

$$P_a = \frac{A^2R_c}{2((R_i + R_c)^2 + (X_i + X_c)^2)} \quad (33)$$

Equation (33) is obtained in [5]. Hence the result in [5] is a particular case of Theorem 1 with $W_{pto}(jw) = 1$.

3.2 Optimal Control - Nominal Case

In this section, we consider the case when the WEC and PTO dynamics are perfectly known.

Using (22), the next step is to calculate R_c, X_c to maximize the harvested average power P_a . By taking the partial derivative of P_a with respect to R_c, X_c , and then by solving the following system of equations

$$\begin{cases} \frac{\partial P_a}{\partial R_c} = 0, \\ \frac{\partial P_a}{\partial X_c} = 0 \end{cases} \quad (34)$$

one obtains

$$\begin{cases} R_c^* = \frac{R_{pto}R_i - X_{pto}X_i}{R_{pto}^2 + X_{pto}^2}, \\ X_c^* = -\frac{R_{pto}X_i + X_{pto}R_i}{R_{pto}^2 + X_{pto}^2} \end{cases} \quad (35)$$

with the corresponding average power

$$P_a^* = \frac{A^2}{8R_i} \quad (36)$$

and

$$\begin{cases} R_c^{**} = -\frac{R_{pto}R_i + X_{pto}X_i}{R_{pto}^2 + X_{pto}^2}, \\ X_c^{**} = \frac{-R_{pto}X_i + X_{pto}R_i}{R_{pto}^2 + X_{pto}^2} \end{cases} \quad (37)$$

with $P_a^{**} \rightarrow -\infty$, which is the global minimum, that is not of interest.

Note that P_a^* in (36) is a local maximum. Since the system of equations (34) has only two solutions, for which (37) is the global minimum, it follows that P_a^* is the unique global maximum.

3.3 Optimal Control - Robust Case

In the uncertain WEC and PTO dynamics, the optimization problem becomes

$$\max_{\{R_c, X_c\}} \min_{\{R_i, X_i, R_{pto}, X_{pto}\}} \frac{R_{pto}R_c - X_{pto}X_c}{(R_i + R_{pto}R_c - X_{pto}X_c)^2 + (X_i + R_{pto}X_c + X_{pto}R_c)^2} \quad (38)$$

where the factor $\frac{A^2}{2}$ is omitted, because it has no influence on the optimal solution. This is a difficult max-min optimization problem, since

- The cost function is not convex or concave everywhere.
- The parameters $R_i, X_i, R_{pto}, X_{pto}$ are not exactly known.

Before proceeding further, denote the values sets for (R_i, X_i) and (R_{pto}, X_{pto}) as

$$\begin{cases} (R_i, X_i) \in \{\{R_i\}, \{X_i\}\}, \\ (R_{pto}, X_{pto}) \in \{\{R_{pto}\}, \{X_{pto}\}\} \end{cases} \quad (39)$$

We require that, for each given frequency w

$$R_{pto}R_c - X_{pto}X_c > 0, \quad \forall R_{pto}, \forall X_{pto} \quad (40)$$

since the harvested energy should be positive. Using (39), it is clear that (40) holds if and only if the following two conditions are satisfied

$$\{R_{pto}\}R_c - \{X_{pto}\}X_c > 0 \quad (41)$$

With the constraints (41), the cost function (38) is positive. It is hence possible to rewrite (38) as

$$\min_{R_c, X_c} \max_{R_i, X_i, R_{pto}, X_{pto}} \frac{(R_i + R_{pto}R_c - X_{pto}X_c)^2 + (X_i + R_{pto}X_c + X_{pto}R_c)^2}{R_{pto}R_c - X_{pto}X_c} \quad (42)$$

where instead of solving a max-min optimization problem, we solve the equivalent min-max optimization problem with the inverse cost function.

The following theorem holds.

Theorem 2: The optimal solution to (42) can be obtained by solving the

following convex SDP problem

$$\begin{aligned} & \min_{\gamma, R_c, X_c} \{\gamma\}, \\ \text{s.t.} & \begin{bmatrix} \gamma & \{R_i\} + \{R_{pto}\}R_c - \{X_{pto}\}X_c & \{X_i\} + \{R_{pto}\}X_c + \{X_{pto}\}R_c \\ (*) & \{R_{pto}\}R_c - \{X_{pto}\}X_c & 0 \\ (*) & (*) & \{R_{pto}\}R_c - \{X_{pto}\}X_c \end{bmatrix} \succeq 0 \end{aligned} \quad (43)$$

Proof: It is well known [2] that a quadratic over linear fractional function is convex. Hence (42) can be transformed into the following convex optimization problem

$$\begin{aligned} & \min_{\gamma, R_c, X_c} \{\gamma\}, \\ \text{s.t.} & \begin{cases} \gamma \geq \frac{(R_i + R_{pto}R_c - X_{pto}X_c)^2 + (X_i + R_{pto}X_c + X_{pto}R_c)^2}{R_{pto}R_c - X_{pto}X_c}, \\ R_{pto}R_c - X_{pto}X_c > 0 \end{cases} \end{aligned} \quad (44)$$

where γ is an auxiliary variable, that presents the upper bound of the cost function. Although convex, the optimization problem (44) is still difficult to solve due to the uncertain parameters R_i , X_i , R_{pto} , X_{pto} . Using Schur's complements, rewrite the constraints of (44) as

$$\begin{bmatrix} \gamma & R_i + R_{pto}R_c - X_{pto}X_c & X_i + R_{pto}X_c + X_{pto}R_c \\ (*) & R_{pto}R_c - X_{pto}X_c & 0 \\ (*) & (*) & R_{pto}R_c - X_{pto}X_c \end{bmatrix} \succeq 0 \quad (45)$$

Using (39), it is clear that (45) is satisfied if and only if the following conditions hold

$$\begin{bmatrix} \gamma & \{R_i\} + \{R_{pto}\}R_c - \{X_{pto}\}X_c & \{X_i\} + \{R_{pto}\}X_c + \{X_{pto}\}R_c \\ (*) & \{R_{pto}\}R_c - \{X_{pto}\}X_c & 0 \\ (*) & (*) & \{R_{pto}\}R_c - \{X_{pto}\}X_c \end{bmatrix} \succeq 0 \quad (46)$$

Combining (41), (44), (46), it follows that the min-max problem (38) can be converted into the following SDP optimization problem

$$\begin{aligned} & \min_{\gamma, R_c, X_c} \{\gamma\}, \\ \text{s.t.} & \text{the constraints (46)} \end{aligned} \quad (47)$$

The proof is complete. \square

In summary, in the presence of the uncertain WEC and PTO systems, the real part and the imaginary part of the optimal controller can be optimized using Algorithm 2.

Algorithm 2 : Calculation of the real part R_c^* and the imaginary part X_c^* of the robust optimal control block

- 1: Select a reasonable finite but large set of frequency samples $0 < w_1 < \dots < w_N$.
 - 2: For each frequency in the chosen set
 - 3: Compute the value sets $\{\{R_i\}, \{X_i\}\}$ of the WEC intrinsic impedance.
 - 4: Compute the value sets $\{\{R_{pto}\}, \{X_{pto}\}\}$ of the PTO system.
 - 5: Compute R_c^*, X_c^* by solving the convex SDP problem (47).
-

Remark: The worst case optimal average power is computed as, for a given wave excitation force (19),

$$P_a^* = \frac{A^2}{2\gamma^*} \quad (48)$$

where γ^* is the optimal solution of (47).

Fig. 4 presents the value sets $\{\{R_{pto}\}, \{X_{pto}\}\}$ of a simple first-order uncertain PTO system (49) as a function of frequency. The PTO system is given as

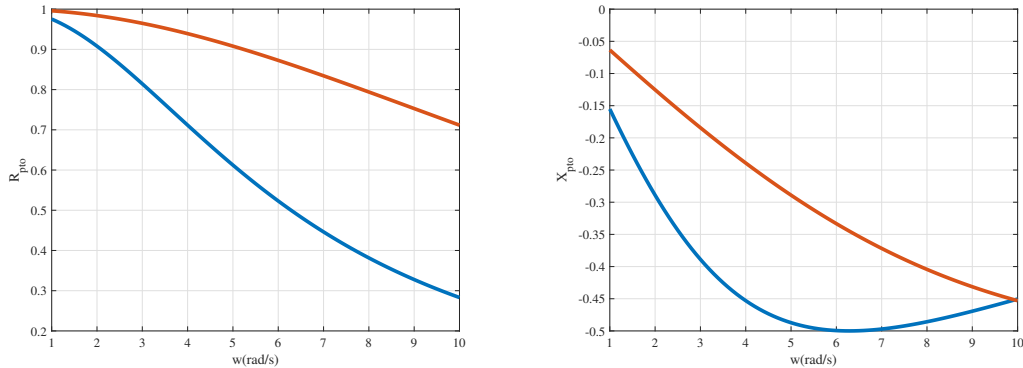


Fig. 4. Value sets $\{\{R_{pto}\}, \{X_{pto}\}\}$ of uncertain PTO system.

$$W_{pto}(jw) = \frac{1}{jw\tau + 1} \quad (49)$$

where τ is a uncertain time constant, $\tau \in [\frac{1}{5\pi}, \frac{1}{2\pi}]$. Note that in this case, the value sets $\{\{R_{pto}\}, \{X_{pto}\}\}$ can be computed analytically as function of frequency w .

Using CVX to solve the SDP optimization problem (47), one obtains the value R_c^*, X_c^* of the optimal controller, see Fig. 5. These two figures also present the value sets of R_i and of $-X_i$ of the WEC system. The parameters of the WEC system are given in the simulation section.

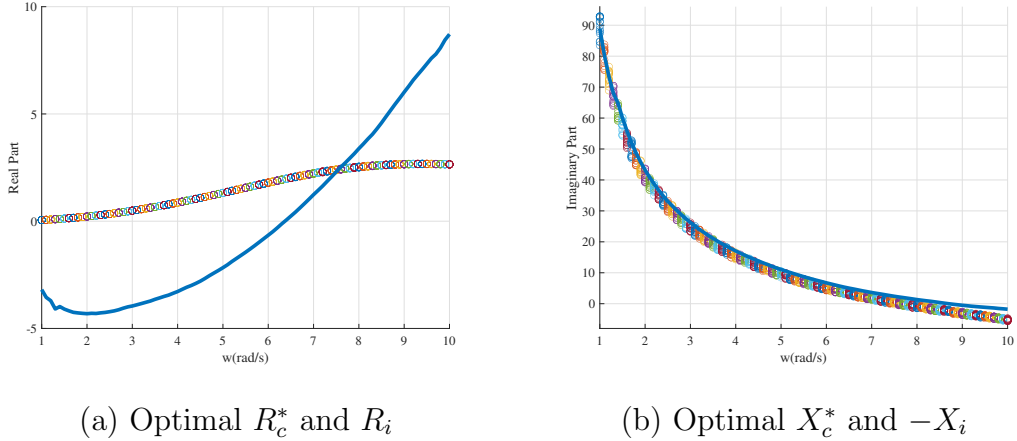


Fig. 5. (a) Optimal R_c^* (solid blue) and R_i (circle marker), (b) X_c^* (solid blue) and $-X_i$ ((circle marker)) as a function of ω .

Remark: It is well known that [5], in the nominal case and without the PTO dynamics, the optimal WEC velocity is in phase with the wave excitation force. Clearly, in the robust case, there is no guarantee that there exists a control law (20) such that the imaginary part of the transfer function $W(j\omega)$ is zero. It implies that the velocity is generally not in phase with the wave excitation force in the robust case.

4 Extension to Polychromatic Waves

We show how to apply the results in Section 3 to polychromatic sea states in this section. For this purpose, the following three main problems need to be addressed.

- (1) The wave excitation force $f_{ex}(t)$ is not directly accessible when the WEC system is running.
- (2) Real sea states are polychromatic, i.e., $f_{ex}(t)$ is not a pure sine.
- (3) A control structure needs to be assigned for $Z_c(j\omega)$.

For the first problem, a few methods have been proposed and tested in real-time to estimate the wave excitation force via measurements of other quantities [12], [1], [20]. In this work, a method in [20] is used. It is based on a linear Kalman filter and a random walk model for the variation of the wave excitation force. Only the float position, the float velocity, and the PTO force are required. The method yields accurate estimates over a large range of sea states.

The second problem is resolved by approximating $f_{ex}(t)$ as a sinusoidal signal

with time-varying amplitude $A(t)$, angular frequency $w(t)$ and phase $\phi(t)$, i.e.

$$f_{ex}(t) = A(t) \sin(w(t)t + \phi(t)) \quad (50)$$

The parameters $A(t)$, $w(t)$ and $\phi(t)$ are estimated on-line. It is clear that this is a nonlinear estimation problem, since $A(t)$, $w(t)$ and $\phi(t)$ enter nonlinearly in (50).

It should be stressed that the problem of estimating the unknown parameters describing a noisy sinusoidal signal is an important one in systems theory with applications in diverse fields [26]. The most of the existing solutions can be listed here are: line enhancers [30], finite impulse response filters [32], infinite impulse response filters or notch filters [27], and frequency locked loop [11].

In this paper an unscented Kalman filter (UKF) [24] is used to estimate $A(t)$, $w(t)$, $\phi(t)$. The main advantages of the UKF filter are: i) the computational complexity is very small, hence it can be implemented in real-time, ii) it requires a modest memory footprint, since only $f_{ex}(t)$ at current time is needed, iii) it is already evaluated experimentally on real-data [23] with a good performance. Note that, like the extended Kalman filter, the UKF consists of the same two steps: state prediction and measurement correction, except they are preceded now by another step for the calculation of sigma points.

For the third problem, the following proportional integral (PI) control structure is chosen

$$Z_c(jw) = K_p + \frac{K_i}{jw} \quad (51)$$

Hence, in the frequency domain

$$F_u(jw) = Z_c(jw)V(jw) = (K_p + \frac{K_i}{jw})V(jw)$$

or equivalently, in the time domain

$$f_u(t) = K_p v(t) + K_i \int_0^t v(\tau) d\tau \quad (52)$$

Hence

$$f_u(t) = K_p v(t) + K_i p(t) \quad (53)$$

where $p(t) = \int_0^t v(\tau) d\tau$ is the float position, which is assumed to be available in real time.

Combining (21) and (51), one obtains

$$\begin{cases} K_p = R_c^* \\ K_i = -wX_c^* \end{cases} \quad (54)$$

The robust optimal PI control strategy is summarized into two stages: offline stage and online stage.

Offline Stage	
1 :	Using Algorithm 2 to calculate the optimal R_c^*, X_c^* .
2 :	Calculate the optimal gains K_p, K_i as in (54).

Online Stage: At each sampling instant t	
1 :	Estimate $f_{ex}(t)$ using $v(t)$, $p(t)$ and $f_{pto}(t)$.
2 :	Estimate the frequency $\hat{\omega}$ using the UKF.
3 :	The optimal gains K_p, K_i are found by a look-up table from the offline stage.
4 :	The control action is computed as: $f_u(t) = K_p v(t) + K_i p(t)$

The robust adaptive PI control algorithm is schematically presented in Fig. 6. It is worth noticing that the algorithm requires only information about $p(t)$, $v(t)$ and $f_{pto}(t)$. No information about $f_{ex}(t)$ is needed.

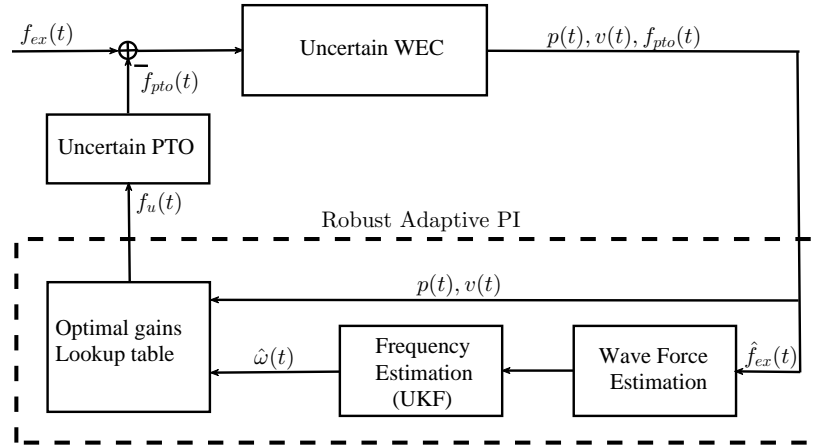


Fig. 6. Robust adaptive PI control structure.

Remark: For simplicity and for a clear physical interpretation, the PI control structure is used in this work. As written in the introduction, the wave energy is converted into useful energy with the P-component, why the I-component modifies the WEC system natural frequency. Other high order control structure such as PID can also be used. The main advantage of these high order control laws is that they offer an additional degree of freedom to harvest more energy.

5 Simulation Results

5.1 WEC Parameters

A laboratory prototype of a point absorber WEC [33] on a 1:20 scale with respect to the well-known Wavestar machine installed near Hanstholm in Denmark from 2009 to 2013, was used to validate the robust adaptive PI control algorithm described in Section 4. The WEC parameters are given as $M = 1.44$, $K_v = 0.01$, $85 \leq K_{hd} \leq 95$, and

$$W_r(jw) = \frac{a_4(jw)^4 + a_3(jw)^3 + a_2(jw)^2 + a_1(jw) + a_0}{b_5(jw)^5 + b_4(jw)^4 + b_3(jw)^3 + b_2(jw)^2 + b_1(jw) + b_0} \quad (55)$$

whose coefficients are given in Table 5.1.

Numerator	Denominator
	$b_5 = 1$
$a_4 = 30.8619$	$b_4 = 208.5959$
$a_3 = 6.4642 \times 10^3$	$b_3 = 8.5834 \times 10^4$
$a_2 = 2.6016 \times 10^6$	$b_2 = 8.8994 \times 10^6$
$a_1 = 2.6550 \times 10^8$	$b_1 = 1.0740 \times 10^8$
$a_0 = -7.0967 \times 10^5$	$b_0 = 7.0311 \times 10^8$

Using Algorithm 1, Fig. 7 presents the Bode plot of the considered WEC system. Note that this Bode plot is very typical for a WEC system of the point-absorber type.

5.2 Simulation Results

In this section, we present the simulation results obtained using the robust adaptive control algorithm described in Section 4. To validate our concept, we choose one sea state, that represents a normal operating condition for the WaveStar WEC. Fig. 8 shows the spectral power density of this wave.

Fig. 9 presents the estimation results of the UKF filter. Recall that the UKF filter is used to estimate the wave frequency as well as the wave amplitude. It can be observed that a very satisfactory performance of the UKF filter is obtained.

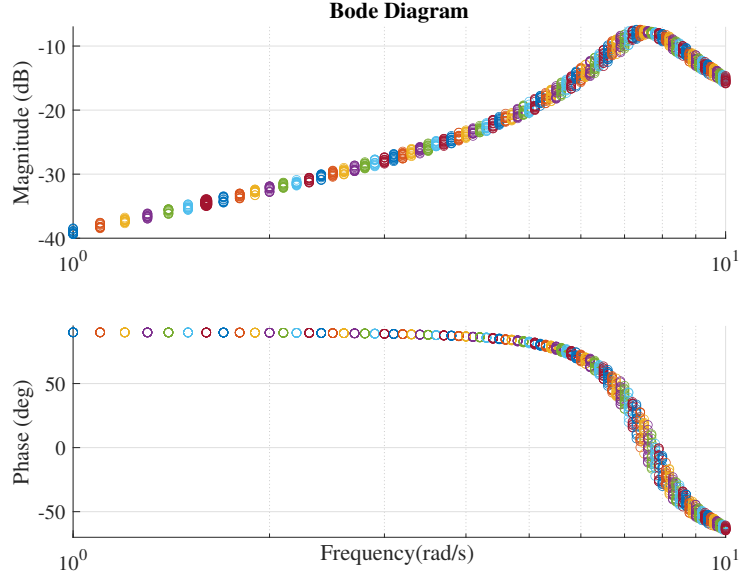


Fig. 7. Bode plot of the considered WEC system.

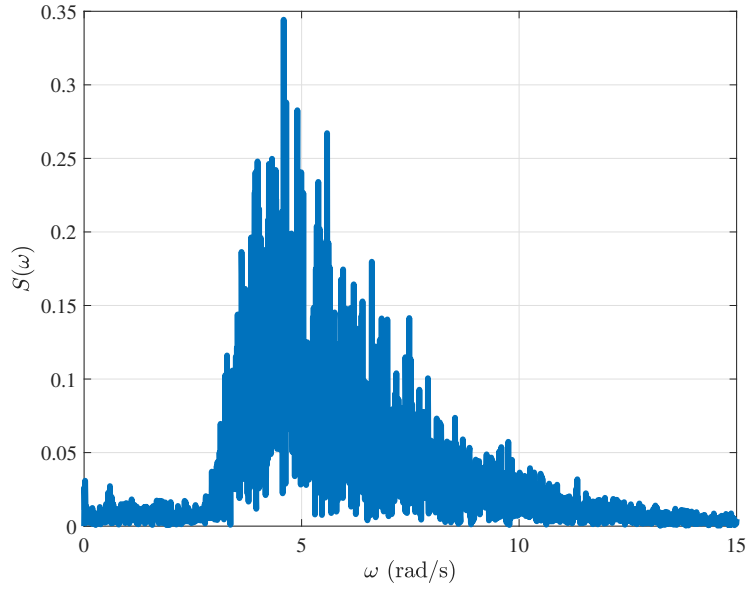
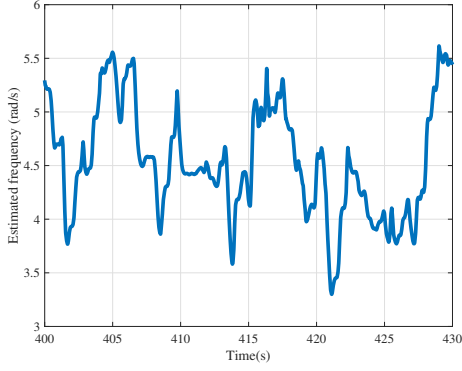


Fig. 8. Wave spectral power density.

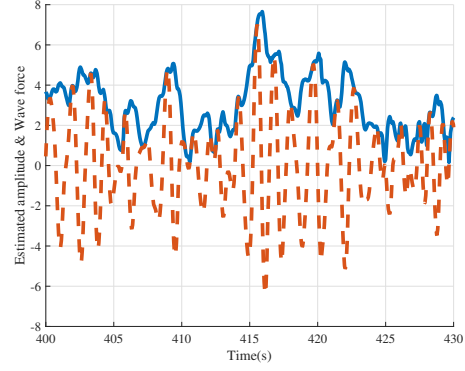
The transfer function of the PTO system with a uncertain time constant is given in (49).

For 20 different realizations of K_{hd} and τ , see Fig. 10, Fig. 11 shows, respectively, different WEC positions and WEC velocities as a function of time using the robust adaptive control law.

Fig. 12(a) presents the harvested energy obtained as the integral of the in-

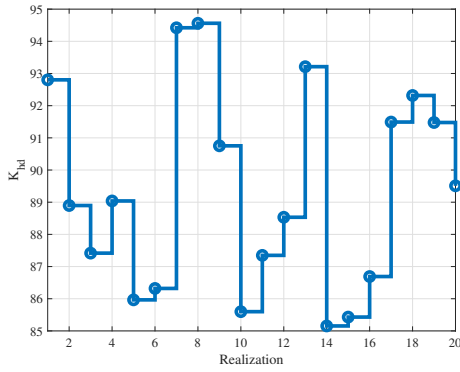


(a) Frequency

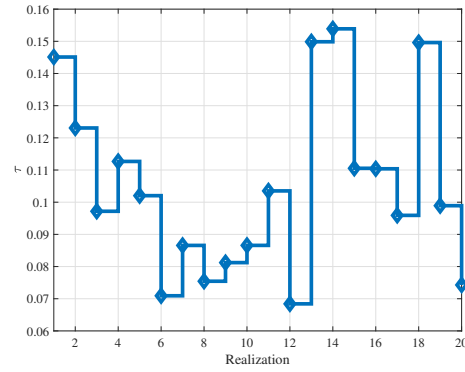


(b) Amplitude & Wave force

Fig. 9. (a) Estimated frequency, (b) Estimated amplitude (solid blue) and wave excitation force (dashed red).



(a) K_{hd}



(b) τ

Fig. 10. 20 different realizations of K_{hd} and τ .

stantaneous power. For comparison, Fig. 12(b) shows the harvested energy obtained applying the nominal adaptive PI control law in Section 3.2. Using 12(b), it is clear that if uncertainties are not properly taken into account in the controller design step, the harvested energy might be negative. For the nominal adaptive controller, R_i and X_i are chosen as the mean of $\{R_i\}$, and of $\{X_i\}$, respectively. The nominal PTO system is selected as

$$W_{pto}(jw) = \frac{1}{3.5(jw) + 1} \quad (56)$$

6 Conclusion

A new control approach to the problem of maximizing the harvested energy with a point absorber in the presence of a uncertain WEC and PTO dynamics

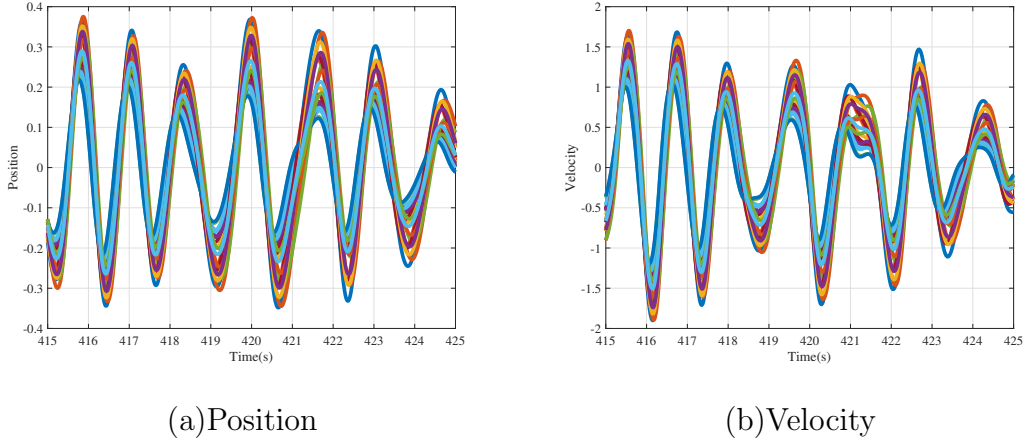


Fig. 11. Buoy position and velocity for 20 different realizations of K_{hd} and τ .

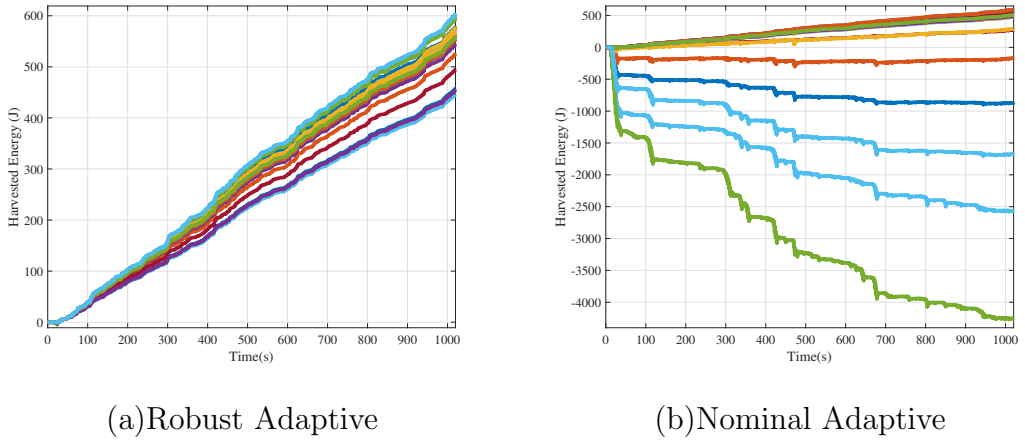


Fig. 12. Harvested energy for 20 different realizations of K_{hd} and τ .

is proposed. The basic idea is to adopt a robust adaptive PI control strategy. The design procedure is decomposed into an offline and online stages. In the offline stage, the optimal PI gains are computed by maximizing the worst-case harvested average power. It is shown that this problem can be converted into a convex semidefinite optimization problem, for which the optimal solution is unique. In the online stage, by approximating the wave excitation force as a time-varying sinusoid, the gains of the PI controller are continuously adapted on-line, on a wave-to-wave basis. A simulation example is given and compared to earlier solutions from literature.

We will in the future, consider the input and output constraints. We are also interested in extending this technique to the multi-float configuration case.

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