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► **To cite this version:**

Philippe Jacquet, Liubov Tupikina. Geometry of Outdoor Virus Avoidance in Cities. GSI 2021 - 5th International Conference on Geometric Science of Information, Jul 2021, Paris, France. pp.869-877, 10.1007/978-3-030-80209-7\_93 . hal-03532645

**HAL Id: hal-03532645**

**<https://hal.science/hal-03532645>**

Submitted on 18 Jan 2022

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# Geometry of Outdoor Virus Avoidance in Cities

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**Abstract.** We present low complexity and efficient path selection algorithms for limiting the exposure to pandemic virus during and between lock-downs. The average outdoor exposure rate reduction is around 3.

**Keywords:** Virus Exposure · Random Walks · City Maps.

## 1 Introduction

The Covid-19 pandemic has caused repetitive lock-downs for most of mankind in order to limit the virus circulation. In many countries outdoor excursion were permitted during lock-downs under strict restrictions; in France, no more than 1 hour within 1km from home. All efforts done in order to reduce the daily exposure time to virus, the lock-down's aim is to "flatten the curve" in order to limit the congestion in hospitals and must be repeated every time the curve becomes critical and before vaccines and herd-immunity take over. But social distancing needs to be applied during inter lock-down periods.

This paper is divided in two parts. In the first part we introduce an application, called Ariadne Covid. It selects outdoor excursion path with limited exposure to virus during lock-downs based on its geometrical properties. The second part is dedicated to inter lock-downs periods, and we introduce a class of algorithms to select commuting paths.

The fundamental tool in Ariadne Covid is the use of random walks which naturally leads to a better load balance between streets while avoiding crowded areas. The application is shown to give a benefit to the first user, although to have an impact on the pandemic, it must be used by a majority of users.

During inter lock-downs periods people are allowed to commute from home to work. The path selection presented in the second part is based on drifted random walks in order to reach the destinations without too much elongations while offering balanced loads on city streets. Both applications take benefit from the geometry of city maps which leads to naturally homogeneous street networks.

## 2 Covid exposure during outdoor excursion in cities

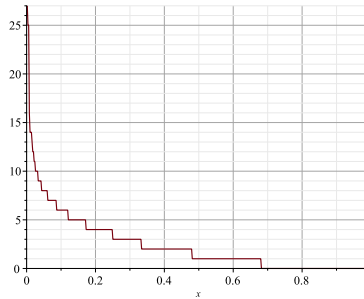
The virus of Covid-19 is supposed to have some probability to be transferred when an infected person is within 1 m of another person during a certain time (10 m when running or biking). The larger is the exposure time the higher is the

contamination probability (the average contamination delay is around 30 min). We call exposure time the cumulated time a given person is within 1m of any other person during an outdoor excursions.

There are 2,900 km of cumulative pedestrian path length in Paris for a population of 2.2 million inhabitants [1]. Therefore there is ample room for all of the inhabitants to walk 24/24h within safe distance. Since only a fraction of the inhabitant are expected to be walking at the same time, it should be easy to get the safe spacing.

The problem is that people show a *social* trend to gather in the same streets, even during lock-downs, rising the exposure to virus. Data sampled via Google Street View show how negative is the impact of the social trends. In the paper we will call *social walkers* (or social joggers) the walkers who follow the social trends during outdoor excursions in the city.

We have sampled 55 km of Paris street via Google Street View. We have made 11 independent random walks starting in different *arrondissements*. A segment (30 m) of pedestrian path is sampled every 100 m. Figure 1 displays the histogram of segment densities sampled in Paris. The Street View pictures date before the lockdown, and mix several periods. The histogram shows a profile close to a Zipf distribution of coefficient close to 0.75. We have no yet records for the period during the lockdown, but we expect a more concentrated distribution with a larger Zipf coefficient, since the most of the commercial areas are closed and therefore attraction streets are reduced.



**Fig. 1.** Histogram of Pedestrian densities in Paris

We represent the city by an abstract graph, with the street intersection (vertices) linked by street segments (edge). We denote  $V$  the set of intersections and  $E$  the set of street segments.

### 3 Path selection during lock-downs

#### 3.1 Ariadne String protocol description

The application consists into delivering on request, a random path for the excursion and a time slot in the day. This path will reduce the exposure to the virus by avoiding the dangerous areas with an abnormal density of social walkers. In

its most basic setting the path will be a pure random walk, since it shows to actually reduce the exposure rate even if the user faces only social walkers. Indeed it is shown that the outdoor exposure to virus is even more reduced compared to the situation where all inhabitants use the application.

During the excursion the user will have his/her own position displayed on her/his mobile phone. At half of the excursion time the walker will receive a notification to reverse the path in order to come back home by keeping path uniformization. The path may contain loops, because they are necessary in order to get the uniform density. If it is the case the walker will proceed with the loops. The walker may skip the loop in the way back but this will shorten the delay of the excursion. The application terminates (at least its embedded mobile part) when the user is back home. There are several options in the randomized path selection.

**Time slot determination** The time slot is selected uniformly on the time schedule proposed by the user in his/her request. In France, the slot has a maximum duration of one hour.

**Path Initialisation** The first step is to randomly select the initial direction: left or right with probability  $1/2$ ,  $1/2$  on the street/edge of the home address.

**Non-backtracking algorithm:** at each intersection  $J$  the algorithm randomly selects the exit road excluding the entry street. If the intersection has four roads, thus three eligible exits, the weight of each exit is  $1/3$ ,  $1/3$ ,  $1/3$ . In case of dead-end, the algorithm selects the U turn. This guarantees the uniformisation of the densities. If the cartesian distance to home address exceeds the authorized distance (1km in France), the path should make a U-turn.

**Non-backtracking algorithm with reduced path diversions:** if the intersection has an even number of streets (thus an odd number of exits) more weight will be given to the exit street facing the entry street, *i.e.* the median exit. For example, the median exit has  $1/2$  probability to be selected, and the other exits are uniformly selected. This minor trick still maintain the uniformization of the densities.

### 3.2 Estimation of exposure reduction during Covid-19 lock-downs

Let  $E$  be the street set and for  $s \in E$ , let  $\ell(s)$  its length and  $\lambda(s, t)$  its linear density at time  $t$ . The total length of the pedestrian network is  $L = \sum_{s \in E} \ell(s)$ . The total pedestrian population at time  $t$  is  $N(t) = \sum_{s \in E} \lambda(s, t)\ell(s)$ . The probability that a pedestrian walk on segment  $s$  at time  $t$  is  $\rho(s, t) = \frac{\ell(s)\lambda(s, t)}{N(t)}$ . If we assume that the distribution on every segment is uniform (and ignoring side effects) the probability that a given pedestrian on segment  $s$  is within distance  $r$

to another given pedestrian on the same segment is  $\frac{2r}{\ell(s)}$  which could be reduced to  $\frac{r}{\ell(s)}$  if the dangerous zone stands forward to the pedestrian.

Thus if a total population of  $N$  pedestrians are present on the streets, each independently following the density pattern of the  $\lambda(s, t)$ , the probability that a pedestrian on segment  $s$  is not on situation of contact at time  $t$  is  $\left(1 - \rho(s, t) \frac{2r}{\ell(s)}\right)^N$  or equivalently  $\left(1 - 2r \frac{\lambda(s, t)}{N(t)}\right)^N$ .

Let  $N_T$  the total population of the city. If the proportion of inhabitants walking outdoor is  $\nu$ , the average exposure rate  $E_G(\nu)$  satisfies

$$E_G(\nu) = 1 - \sum_{s \in E} \rho(s) \left(1 - \rho(s) \frac{2r}{\ell(s)}\right)^{\nu N_T}. \quad (1)$$

assuming that  $\rho(s) := \rho(s, t)$  does not vary (too much) during the permitted hours. If  $\rho(s, t)$  varies during the day and  $\rho(s)$  is its average on segment  $s$ , then by concavity the above expression is an upper bound of the exposure rate.

**Theorem 1.** *The stationary distribution of the non-backtracking random walk is the uniform distribution density.*

By uniform distribution we mean that the average densities in every street are identical.

*Proof.* This is mainly a state of the art result.

But the random walk may be long to converge to its steady state. In our case it is faster because the density of inhabitant in Paris is almost uniform along each street, thanks to Hausmannian construction rules dating from the XIXth century. From now we take the simplified assumption that walkers using the random walk are indeed in uniform density in the streets.

Let  $E_U(\nu)$  be the average exposure rate when all walkers are random walkers. It satisfies:

$$E_U(\nu) = 1 - \left(1 - \frac{2r}{L}\right)^{\nu N_T}. \quad (2)$$

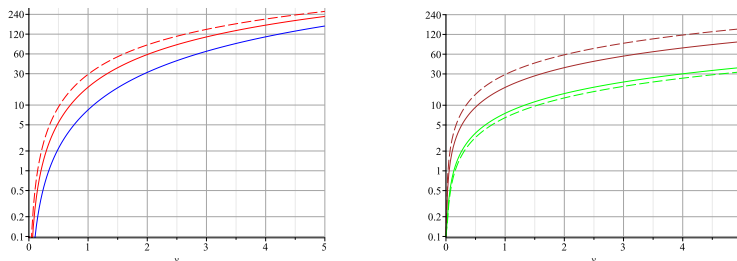
Indeed it suffices to consider the whole pedestrian network as a single segment.

More interestingly we have the case when a single random walker proceeds among  $\nu N_T$  walkers following their social trends. Let's call  $E_{UG}(\nu)$  her/his exposure rate:

$$E_{UG}(\nu) = 1 - \sum_{s \in E} \frac{\ell(s)}{L} \left(1 - \rho(s) \frac{2r}{\ell(s)}\right)^{\nu N_T}. \quad (3)$$

We assume that Parisians have a daily time span (10 hour) when excursions are permitted.

The figure 2 shows the cumulative time spent within unsafe distance to any another individual. Notice that with one hour of excursion time with all social walkers, each walker cumulates nearly half of this time at less than 1m of another individual. This duration drops to 8 minutes with random walking. If the excursions were extended to 5 hours, the social walkers would cumulate 4h30mn within unsafe distance, the random walkers around 2h30mn.



**Fig. 2.** Virus exposure time (min) as function of outdoor excursion duration (in hours). (Left) red: all walkers are social but randomly dispatched on 10 hours (solid) on 5 hours (dashed); blue: when walkers are all random. (Right) red: for a single social walker, when all other walkers are social walkers during 1 hour, green: the single walker is random, dashed: social walkers select slot on an interval of 5 hours instead of 20 hours.

## 4 Path selection during lock-downs periods

Contrary to lock-down outdoor excursions which are mostly aimless (walking, jogging), the excursions during inter lock-downs periods are now aimed (commuting between home and work place). The public transportation will be mostly discarded because of poor social distancing. The city of Paris has increased the number of biking lanes. However the biking lanes will be very busy because people commute around the same hours. Our work is to extend Ariadne Covid application in order to help the user to find biking routes from home to work, which limit the virus exposure.

### 4.1 Path Selection Algorithm

An initial position of a user is on a segment  $(I_U, I_D) \in E$ . The path will take the intersection among  $I_U$  and  $I_D$  which lies ahead of the destination  $z_f$ , *i.e.* if  $\langle z_f - z_0 | I_U - I_D \rangle \geq 0$  then  $I_U$  is selected:  $z_1 = I_U$ , otherwise  $z_1 = I_D$ .

At step number  $k$  of the algorithm, let  $z_k$  be the intersection, where the path is currently ending. Let  $d$  be the degree of the intersection  $z_k$ . If  $d = 1$ , then the path backtracks. If  $d > 1$  then with probability  $\epsilon$  the path proceeds as a non-backtracking random walk, *i.e.* the path takes any other neighbor edge distinct of the entrance street with equal probability. Otherwise, with probability  $1 - \epsilon$ , the path takes one of the two best sectorized streets, defined as follows. Assuming the heading of the segment  $(z_k, z_{k-1})$  is  $\theta_0$ , and we enumerate  $\theta_1, \theta_2, \dots, \theta_{d-1}$  the headings in increasing order of the other exit segments of the intersection  $z_k$ . Let  $\theta$  be the heading of the vector  $(z_k, z_f)$  toward the destination.

**The isotropic walk algorithm** Let  $j$  be such that  $j \frac{2\pi}{d} < \theta \leq (j+1) \frac{2\pi}{d}$ . Note here that  $j+1$  must be considered by *modulo* of  $d$ . Let variables  $\alpha_d(\theta)$  and  $\beta_d(\theta)$  such that  $\alpha_d(\theta) + \beta_d(\theta) = 1$  and  $\alpha_d(\theta)e^{2ij\pi/d} + \beta_d(\theta)e^{2i(j+1)\pi/d}$  be proportional

to  $e^{i\theta}$ . Numerically we have

$$\begin{cases} \alpha_d(\theta) = \frac{\sin(2(j+1)\pi/d-\theta)}{\sin(\theta-2j\pi/d)+\sin(2(j+1)\pi/d-\theta)} \\ \beta_d(\theta) = \frac{\sin(\theta-2j\pi/d)}{\sin(\theta-2j\pi/d)+\sin(2(j+1)\pi/d-\theta)}. \end{cases} \quad (4)$$

The selection of the next intersection  $z_{k+1}$  is done as follows: with probability  $\alpha_d(\theta)$  it selects the edge corresponding to angle  $\theta_j$ , and with probability  $\beta_d(\theta)$  the edge corresponding to the angle  $\theta_{j+1}$ . The algorithm is much less complex than the Dijkstra shortest path algorithm and contrary to the later provides path diversity. Indeed the shortest path algorithm leads to a complexity larger than the number of edges, while the randomized algorithm is at most linear, in the number of vertices, in fact proportional to the diameter of the graph.

**Definition 1 (Isotropic walk condition).** *A walk is isotropic, when reaching any given intersection  $I$ , the difference of an angle  $\theta$  toward a destination with an angle  $\theta_0$  to an arriving edge, satisfies a certain distribution  $P_I(\theta - \theta_0)$ .*

**Theorem 2.** *Under the isotropic walk condition, and assuming the same constant speed  $v$  of all travellers in each street, the aggregation of paths leads to uniform densities of travellers on streets.*

*Proof.* We show that uniform rate situation is stationary. Let's consider that a walker enters an intersection  $I$  of degree  $d$ . In the epsilon mode the walker uniformly exits on the other streets. In the sectorized mode, enumerating the edges in counter clockwise way from the entrance street, the isotropic walk conditions leads to the probability  $p_d(j)$  that the edge  $j$  is selected:

$$p_d(j) = \int_{2j\pi/d}^{2(j+1)\pi/d} P_I(\theta)\alpha_d(\theta) + \int_{2(j-1)\pi/d}^{2j\pi/d} P_I(\theta)\beta_d(\theta). \quad (5)$$

Let us assume that before time  $t$  all streets have same exit rate  $\rho$ . Therefore the entrance flow on any of the streets is  $\epsilon\rho + (1 - \epsilon)\sum_{j=0}^{j=d} p_d(j)\rho = \rho$  and consequently the exit rates are uniform beyond  $t$ . Assuming a uniform speed  $v$  on each street, the density of travellers on each street is  $\rho/v$  per unit length.

## 4.2 The Geo-routing Algorithm

We introduce the geo-routing algorithm inspired from [2], which is similar to the isotropic walk with the difference that we replace the  $2j\pi/d$  by the actual angles of the streets. The consequences are twofold: (i) the average exit path is heading toward the destination, (ii) we lose the pure isotropic property, the densities in the exit streets will now vary with the angles between the streets.

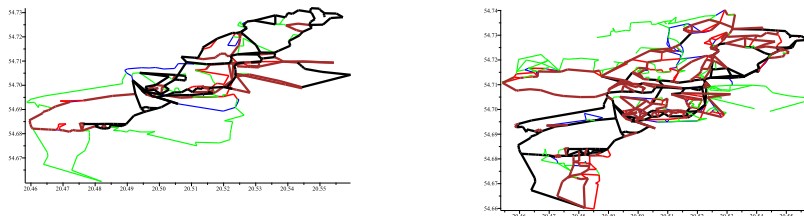
## 4.3 Simulation of the algorithms

We have simulated the algorithms on the map of the city of Kaliningrad, famous for its network of bridges, river and marshes (figure 3). Instead of the classic grid



**Fig. 3.** Left: Illustration of steps of the randomized geo-routing algorithm. The traveller enters the intersection via the large blue arrow. Right: the map of Kaliningrad.

maps of modern cities, the natural obstacles, blocking direct headings, hamper the performances of the algorithms. Indeed we get some blocking loops. In order to cope with looping problems we used the concept of the so-called accessibility graph, see [3]. Let  $G = G(V, E)$  be the street graph. The second degree accessibility graph of the graph  $G * G$  or  $G^{*2}$  is the graph whose vertex set is  $V$  and the edge set is  $E_2$  such that for  $(x, y) \in V^2$ ,  $(xy) \in E_2$  if there exists  $z \in V$  such that  $(xz)$  and  $(zy)$  belong to  $E$ . We have computed  $G^{*2}$ ,  $G^{*4}$  and  $G^{*8}$ . If the algorithm on a path determination fails on  $G$  because of a permanent loop, we run the algorithm on  $G^{*2}$ . Then, if it again fails on  $G^{*2^k}$  we run the algorithm on the accessibility graph of the next degree  $G^{*2^{k+1}}$  until the accessibility graph becomes fully connected graph (we never get to this). When the path is determined one unfolds the path on the original graph  $G$ . We have simulated the application of the isotropic walk and the geo-routing algorithm over 1,000 requests between the same pair initial point and destination point. We set  $\epsilon = 0$ . The figure 4 shows the various path provided by the algorithms.



**Fig. 4.** Path diversity in central Königsberg. Left: geo-routing algorithm. Right: isotropic walks. In green the streets belonging to less than 25 paths, in blue, between 25 and 50, in red, between 50 and 200. In black, above 200.

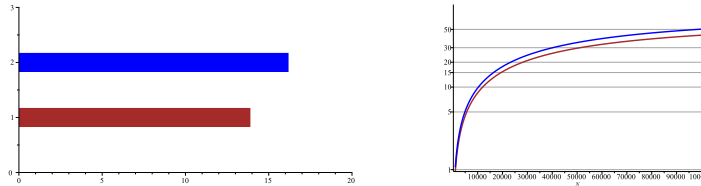
Let  $s \in E$ ,  $\ell(s)$  is the segment length, and let  $\lambda(s)$  be the traffic load after  $N_0$  initial-destination random pairs. The average path length is  $L_G = \frac{1}{N_0} \sum_{s \in E} \lambda(s) \ell(s)$ . The average travel time is  $L_G/v$ . If the simulated time is  $T$ , the segment  $s$  sees an average entrance rate  $\frac{\lambda(s)}{T}$  and the density on the segment length is  $\frac{\lambda(s)}{vT}$ , assumed to be Poisson. If we consider a total number of travellers  $N$ , then the density on a segment is  $\frac{\lambda(s)}{vT} \frac{N}{N_0}$ , where  $N_0$  is a number of travellers



needed to simulate the estimate of  $\lambda(s)$ . Given the Poisson density, a probability that a random traveller at a random time on the segment is not within distance  $R_0$  of another traveller is  $\exp\left(-\lambda(s)\frac{2R_0}{vT}\frac{N}{N_0}\right)$ ,  $R_0$  is the safe distance against the virus (for biking  $10m$ ). We write the expression of an average cumulative exposure time  $E(N)$  for a random walk (traveller):

$$E(N) = \sum_{s \in E} \frac{\lambda(s)\ell(s)}{vN_0} \left(1 - \exp\left(-\lambda(s)\frac{2R_0}{vT}\frac{N}{N_0}\right)\right). \quad (6)$$

Figure 5 shows the simulation of the path length and exposure times with both algorithms after  $N_0 = 5,000$  random initial-destination pairs. The geo-routing algorithm shows a slightly better path length and cumulated exposure.



**Fig. 5.** In blue the isotropic routing, in brown the geo-routing in Kaliningrad (Königsberg). (Left) average path length in km. (Right) the exposure time in minute as function of travellers number  $N$ , peak time duration: 2 hours.

## 5 Conclusions

We have presented the path selection algorithms and efficient path selection, which limits outdoor virus exposure during or between lock-down periods by path diversity and exploiting the geometry of street networks in cities. The proposed solution is based on low complexity algorithms. For practical applications one needs to take into account that success rate of the algorithm implementation strongly depends on users acceptance of those rules, which is left as an outlook.

## References

1. <https://www.pariszigzag.fr/secret/histoire-insolite-paris/combien-y-a-t-il-de-kilometres-de-trottoirs-a-paris>
2. P. Jacquet, B. Mans, P. Muhlethaler, & G. Rodolakis, "Opportunistic routing in wireless ad hoc networks: Upper bounds for the packet propagation speed". IEEE Journal on Selected Areas in Communications, 27(7), 1192-1202 (2009)
3. H.H.K. Lentz, T. Selhorst, I.M. Sokolov "Unfolding accessibility provides a macroscopic approach to temporal networks" (2013) Physical review letters 110 (11).