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# Investigation of suitability of the method of volume averaging for the study of heat transfer in superconducting accelerator magnet cooled by superfluid helium

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## A B S T R A C T

In the field of applied superconductivity, there is a growing need to better understand heat transfers in superconducting accelerator magnets. Depending on the engineering point of view looked at, either 0-D, 1-D, 2D or 3D modeling may be needed. Because of the size of these magnets, alone or coupled together, it is yet, impossible to study this numerically for computational reasons alone without simplification in the description of the geometry and the physics. The main idea of this study is to consider the interior of a superconducting accelerator magnet as a porous medium and to apply methods used in the field of porous media physics to obtain the equations that model heat transfers of a superconducting accelerator magnet in different configurations (steady-state, beam losses, quench, etc.) with minimal compromises to the physics and geometry. Since the interior of a superconducting magnet is made of coils, collars and yoke filled with liquid helium, creating channels that interconnect the helium inside the magnet, an upscaling method provides models that describe heat transfer at the magnet scale and are suitable for numerical studies. This paper presents concisely the method and an example of application for superconducting accelerator magnet cooled by superfluid helium in the steady-state regime in considering the thermal point of view.

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### Keywords:

Superconducting accelerator magnet

Liquid helium

Porous media

Method of volume averaging

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## 1. Introduction

CERN, with the support from the European community and the international high energy particle accelerator community, currently researches and develops next generation (very) high field magnets based on Nb<sub>3</sub>Sn and/or HTS superconductors, necessary for the LHC collimation upgrades and HL-LHC (High Luminosity LHC). These magnets will be at the forefront of technology and shall operate in a thermally very challenging environment where they will be subject to high doses of high-energy particle-showers. Analysis of steady state and transient heat flow and temperature and pressure distributions in these new magnet designs are a necessary ingredient for the success of these R&D efforts. These magnets will function either in sub-cooled, pressurized superfluid helium, in supercritical helium or in saturated normal helium. Due to the size of these magnets, it is yet, impossible to study this directly numerically for computational reasons alone with reasonable computing time and without simplification in the description of the geometry and the physics, especially if we consider several

coupled magnets. For that reason a mathematical description, suitable for numerical modeling, which preserves as much as possible the geometrical information and the physics, is needed. In this framework, the development, and later-on application, of a generic method to numerically simulate the behavior of aforementioned complex systems is required.

In the field of porous media, it is also often impossible to simulate directly all the physical phenomena that arise in the microstructure and researchers currently use upscaling methods to model the physical behavior of porous media in some average sense. These techniques (method of volume averaging [1], homogenization [2] for instance) consider the physical problem at the pore scale and apply mathematical techniques to derive the equations that model the porous medium behavior at the porous media scale, for some Representative Elementary Volume (REV). The main idea of this study is to consider the interior of a superconducting magnet as a porous medium and to apply the method of volume averaging in order to get the equations that model the behavior of the magnet at the magnet scale; i.e. suitable for numerical simulation. Since the interior of a superconducting accelerator magnet is made of coils, collars and yoke filled with liquid helium creating channels that interconnect the helium inside the magnet, the magnet can be seen as a particular

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porous medium. This method can be applied to any type of system of equations which implies that this method can be used for magnets cooled by pressurized superfluid helium, supercritical helium or saturated normal helium. We apply here the method to superconducting accelerator magnet cooled by superfluid helium on the thermal point of view in the steady-state regime and we present the first results in considering a simplified structure of these magnets.

## 2. The method of volume averaging

### 2.1. Principle

The method of volume averaging (see [3] for a review and [1] for a textbook introduction) is a technique that can be used to derive continuum equations for multiphase systems. The equations valid in a particular phase can be spatially smoothed in order to obtain a set of equations valid everywhere. For example, if we consider the process of transient heat transfer in porous media, of interest are the temperature of the fluid and the temperature of the solid at a given point in space and time in the porous medium depending on boundary conditions, the initial condition and every physical phenomenon that needs to be taken into account concerning the problem as a whole. The direct analysis of this problem, in terms of equations valid at the pore scale, is not possible in general because of the complex structure of a porous medium and, even with the computational power available nowadays, it is usually impossible to simulate directly all the physical processes that arise in a porous medium. Instead of considering the equations and boundary conditions valid at the pore scale, we can use this information to derive local volume averaged equations that are valid everywhere, cheaper on a numerical point of view and giving all the necessary information for engineering purpose.

### 2.2. Volume averaging

The upscaling problem is illustrated in Fig. 1 which represents a porous medium made of a solid phase,  $\sigma$ , filled with a fluid phase,  $\beta$ . The pore-scale characteristic length is  $l_\beta$  while the scale of the entire domain is  $L$ . To obtain a macro-scale description every point in space is associated with an averaging volume  $V$  of dimension  $r_0$  as illustrated in Fig. 1. We consider a quantity  $c_\beta$  associated with

the  $\beta$ -phase and we define in a classical manner two spatial averages, the superficial average defined by:

$$\langle c_\beta \rangle = \frac{1}{V} \int_{V_\beta} c_\beta dV \quad (1)$$

and the intrinsic average defined by:

$$\langle c_\beta^\beta \rangle = \frac{1}{V_\beta} \int_{V_\beta} c_\beta dV \quad (2)$$

where  $V_\beta$  represents the volume of the  $\beta$ -phase contained within the averaging volume. These two averages are related by:

$$c_\beta = \varepsilon_\beta \langle c_\beta^\beta \rangle \quad (3)$$

where  $\varepsilon_\beta$  is the  $\beta$ -phase volume fraction (equal to the porosity or the volume fraction of the void space if only one phase saturates the pore space), i.e.:

$$\varepsilon_\beta = \frac{V_\beta}{V} \quad (4)$$

The assumption of scale separation is classically expressed by the length-scale constraints:

$$l_\beta \ll r_0 \ll L \quad (5)$$

where  $r_0$  is the characteristic length-scale of  $V$  considered as a REV of the porous medium under consideration. Several simplifications made in the upscaling process use the length-scale constraints expressed by Eq. (5).

To apply the method of volume averaging, the physical problem (system of equations, boundary conditions) must be written at the pore scale and spatial smoothing is applied to this system of equations, i.e. we apply the spatial averaging represented by Eq. (1) to the equations and the variables are decomposed as:

$$C_\beta = C_\beta + \tilde{c}_\beta \quad (6)$$

where  $C_\beta$  is the intrinsic average variable and  $\tilde{c}_\beta$  is the pore-scale spatial deviation variable. Once this is done, we obtain the averaged equations expressed in terms of the intrinsic averaged variables and the spatial deviation variables. These averaged equations are not in a closed form because they have terms involving pore-scale deviations. In order to obtain fully macroscopic expressions of the different balance equations, we need to find a relation between the pore-scale deviations and the averaged variables. This requires

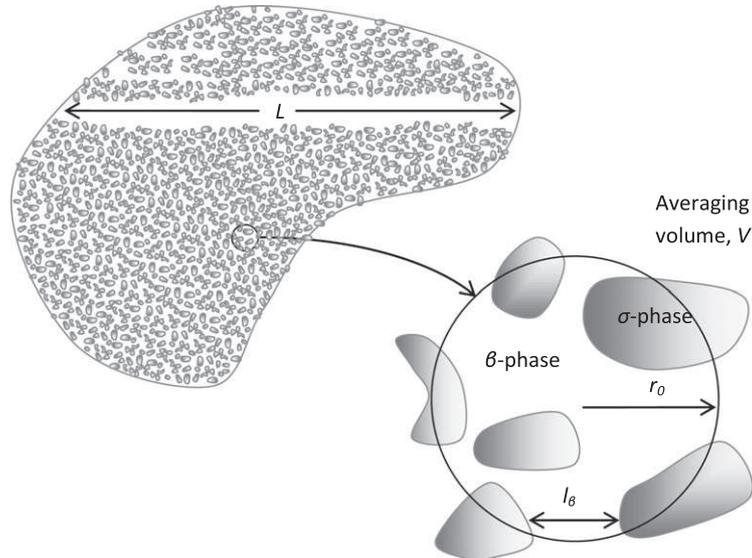


Fig. 1. Macroscopic region of a porous medium and averaging volume collar laminations.

writing governing equations for the pore-scale deviations. These equations are simply the original pore-scale equations in which deviations have been injected. Simplifications are made using the previously obtained average equations. Then we obtain what is called the closure problem [4–9] and, depending on the type of closure problem, we can express the deviations as functions of the averaged variables. Spatial periodicity is most often used for solving the closure problem. In this case, the periodicity determines the size of a Unit Cell (UC). The REV extends over several unit cells. The closure problem permits computing effective parameters, like the permeability in Darcy’s law. The last step is to come back to our problem by inserting the expression of the deviations in the averaged equations to obtain the closed form of the macroscopic equations, for instance, Darcy’s law for the pore-scale Stokes problem [9].

### 3. Method of volume averaging applied to heat transfer problems for superconducting accelerator magnets

#### 3.1. Superconducting magnets

In the particle accelerator context we are going to explore, the coils are maintained mechanically by metallic collars as can be seen in Fig. 2. These collars are laminations with a thickness of a few millimetres and typically hundreds of micrometres spaced. Around the collars is an iron yoke which serves to increase the magnetic field strength and to limit stray fields. The yoke is also made up of laminations with a thickness of a few millimetres (not necessary the same as that of the collars) and typically of the order of hundreds of micrometres spaced. The very low temperatures needed to have the coil in its superconducting state are achieved by using liquid helium as coolant. This helium fills all the voids left by the mechanical structure, in particular the space between collar and yoke laminations, and the space around the inner annulus or beam tube.

#### 3.2. Identification of the $\sigma$ and $\beta$ phases

All the mechanical parts of the magnet constitute the  $\sigma$ -phase. For the purpose of this paper we limit ourselves to the coils, the collars and the yoke and do not consider any finer construction details. Their relevance will have to be checked by direct numerical simulations.

The liquid helium coolant filling all the voids constitutes the  $\beta$ -phase. Depending on the type of cooling, the helium can be in different thermodynamic states: pressurized superfluid helium, supercritical helium or saturated normal helium. The thermodynamic state of the helium has a direct impact on the method of volume averaging since the governing equations are different. In general we will consider one thermodynamic state at a time.

Situations with change of thermodynamic state, induced by high heat discharge when a magnet transits from the superconducting to the normal conducting state, can in principle be dealt with once the equations for the various thermodynamic states are known.

#### 3.3. Unit cell identifications

The length scale constraint Eq. (5) plays a key role in the method of volume averaging and expresses the constraint of separation of scale.

An accelerator magnet is typically several meters long, therefore  $L$  is in the order of 10 m.

The space between laminations (collars and yoke) is of the order of hundreds of micrometres, therefore  $l_\beta$  is of the order of 0.1 mm.

The laminations provide the spatial periodicity needed for solving the closure problems. The sum of the lamination thicknesses and spacing defines the unit cell lengths. The REV must contain roughly at least 10 unit cells to be representative of the porous medium. If we consider laminations over a length of  $r_0 = 0.1$  m, we have more than ten unit cells, whether it concerns the collars or the yoke, and the length scale constraint Eq. (5) is valid in this case. Note, however, that the laminations of the collars and of the yoke do not have the same thickness and, therefore, they do not have the same periodicity. The collars and the yoke thus give rise to two different porous media which need to be coupled via, for instance, specific boundary conditions. The coils could in principle be treated as a third porous medium as well, but this will not be discussed in this paper.

Concerning radial transfers, the diameter of superconducting magnets we are interested in is of the order of 0.6 m. This is to be compared to the lamination thickness, about a few millimetres. As a consequence the macroscale fields (temperature, pressure) are almost linear with respect to the size of the UC. Therefore the yoke or the collar UC will be of a stratified type, i.e. periodic in the axial direction due to periodicity of laminations (collars and yoke), and invariant by translation in the radial direction since a radial translation does not change the problem to be solved.

We see at once here the interest in using a porous-medium-like-description. Let us consider the 10 m long magnet with constant boundary conditions along the axis. The 10 m long pore-scale 3D problem may be transformed into a still 3D problem for a section, the size of about a few laminations. With the same invariance of the boundary conditions along the axis the porous medium scale description is transformed into a 2D cross-section. The need for a 3D porous medium scale description will only arise when the boundary conditions become heterogeneous along the axis, or, for instance, heat source terms in the equations become heterogeneous. In that case, the 3D problem is still easier to solve numerically since the pore-scale characteristic length description is not

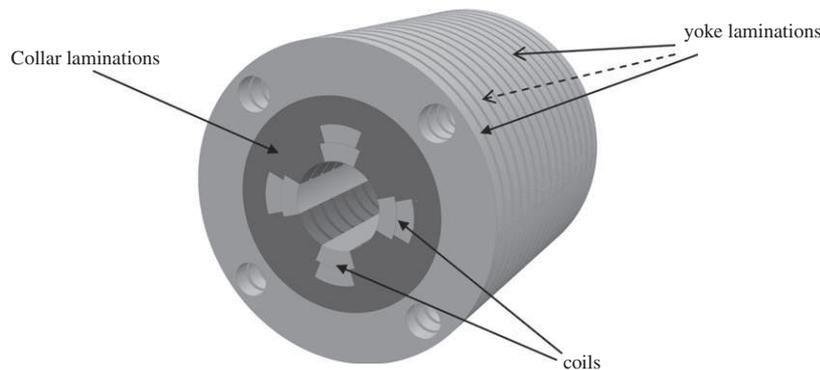


Fig. 2. Simplified model of a typical superconducting accelerator magnet (quadrupole).

necessary and we consider macro-scale equations. The required mesh-size will only depend on the characteristic length-scale of the boundary conditions or source term heterogeneity.

#### 4. Example of magnet cooled by superfluid helium

##### 4.1. Volume averaging method applied to He II

The volume averaging method has been applied for the study of heat transfer in porous media saturated with superfluid helium where the two-fluid model has been considered for the microscopic equations [10]. It concerns the transport of heat via the helium through a porous medium only, excluding heat transfer between the solid and the superfluid. The method was applied considering the equations in the Landau regime. For the steady state the following set of macroscopic equations was obtained: Continuity equation

$$\nabla \cdot (\rho_n(T^\beta) \mathbf{v}_n + \rho_s(T^\beta) \mathbf{v}_s) = 0 \quad (7)$$

Momentum equation for the superfluid component

$$0 = \frac{\rho_s(T^\beta) s(T^\beta)}{\tau} \nabla T^\beta - \frac{\rho_s(T^\beta)}{\rho(T^\beta)} \frac{1}{\tau} \nabla P^\beta + \rho_s(T^\beta) \mathbf{g} \quad (8)$$

Momentum equation for the normal component

$$0 = -\frac{\rho_s(T^\beta) s(T^\beta)}{\tau} \nabla T^\beta - \left(1 - \frac{\rho_s(T^\beta)}{\rho(T^\beta)}\right) \frac{1}{\tau} \nabla P^\beta - \frac{\eta(T^\beta)}{K} \mathbf{v}_n + \rho_n(T^\beta) \mathbf{g} \quad (9)$$

Entropy equation

$$\nabla \cdot (\varepsilon_\beta \rho(T^\beta) s(T^\beta) \mathbf{v}_n) = 0 \quad (10)$$

The structure of the macro-scale equations involves two ‘‘effective parameters’’,  $\tau$  and  $K$ , that depend only on the geometry of the porous medium.  $K$  is the permeability, the same that appears in Darcy’s law for a classical fluid and  $\tau$  represents a tortuosity parameter.

##### 4.2. Extension of the method to magnets

The above is not sufficient to accurately describe the heat flow in a superconducting magnet because of the solid–liquid heat transfer. The heat is mainly produced in the solid (coils, collars and the yoke) and this heat is driven to a heat sink via the superfluid helium. This means that there is heat transfer from the solid to the liquid. Therefore, we have, at the solid surface, the well-known relation in superfluid helium:

$$\mathbf{q} = \rho s T \mathbf{v}_n \quad (11)$$

Then the heat flux flowing through the wall generates momentum of  $\mathbf{v}_n$  at the surface and this changes the spatial smoothing applied to the normal component momentum equation. In [10], the classical no-slip boundary condition for  $\mathbf{v}_n$  is considered. If  $\mathbf{v}_n$  is not zero at the surface of the solid, this is likely to change the closure problem and this case is currently under development. Furthermore, if we have a heat flux at the wall, we would have to consider the Kapitza resistance which implies a discontinuity of temperature at the solid surface. How much? This discontinuity of temperature will be evaluated in the following by direct numerical simulations. Another point that needs to be addressed for solving heat transfer problems in magnet is the extension of the macroscopic model given by Eqs. (7)–(10) to the Gorter–Mellink

regime. For that simulations with a superfluid code, based on the two-fluid model, are necessary, but this is beyond the scope of the present paper.

##### 4.3. Preliminary heat transfer macro-scale model including the solid phase

As mentioned in the previous sections, there are still pending problems to obtain a full transport model for superfluid helium in a porous structure. In this section we adopt a simplified point of view, i.e. the use of a superfluid heat diffusion model to derive a simplified macro-scale model that takes into account transport through/from the solid phase.

Starting with the heat diffusion model [11] we have:

$$\rho C_p \frac{\partial(T)}{\partial t} + \rho C_p \mathbf{v} \cdot \nabla(T) = \nabla \cdot \left( \frac{1}{f(T)} \nabla T \right)^{1/3} + q_{vol} \quad (12)$$

This equation takes into account the convective term so, to solve this equation, we need to know the velocity and, for that, we need to solve the two fluid model of He II. For the moment, let us consider that the resulting Péclet number is small and we only consider the diffusive part in the equation. Furthermore, several authors used a trick to bring this equation to the following classical heat diffusion equation [12–14]:

$$\rho C_p \frac{\partial(T)}{\partial t} = \nabla \cdot \left[ \left( \frac{1}{f(T) \cdot |\nabla T|^2} \right)^{1/3} \nabla T \right] + q_{vol} \quad (13)$$

In which we have an effective thermal conductivity given by

$$k_{eff} = \left( \frac{1}{f(T) \cdot |\nabla T|^2} \right)^{1/3}.$$

At this point, we have returned to a classical (aside from the specific  $k_{eff}$  nonlinearity) porous medium heat transfer problem. It must be noticed that the following is valid for normal liquid helium. The only thing that should be looked at more precisely is that He I must be taken to be compressible in order to take into account of the convection in the magnet. To see what can be done for He I, the interested reader should refer to the results summarized below and look at the work done by Quintard and Whitaker for slightly compressible fluids where they consider a state equation for the density [15,16].

We briefly summarize below the development of the macro-scale model for the simplified heat transfer problem:

$$(\rho C_p)_\beta \frac{\partial T_\beta}{\partial t} = \nabla \cdot (k_\beta \nabla T_\beta), \text{ in the } \beta\text{-phase} \quad (14)$$

$$\text{B.C.1 } T_\beta = T_\sigma, \text{ at } A_{\beta\sigma} \quad (15)$$

$$\text{B.C.2 } \mathbf{n}_{\beta\sigma} \cdot k_\beta \nabla T_\beta = \mathbf{n}_{\beta\sigma} \cdot k_\sigma \nabla T_\sigma + \Omega, \text{ at } A_{\beta\sigma} \quad (16)$$

$$(\rho C_p)_\sigma \frac{\partial T_\sigma}{\partial t} = \nabla \cdot (k_\sigma \nabla T_\sigma) + \Phi_\sigma, \text{ in the } \sigma\text{-phase} \quad (17)$$

where  $\Omega$  and  $\Phi_\sigma$  are respectively homogeneous and heterogeneous source terms that we have to consider for the case of superconducting magnets. In this development, we do not consider an interfacial heat transfer resistance (on the basis of direct numerical simulation results presented later in this paper). We refer the reader to Gobbé et al. (1998) [17] for the introduction of interfacial thermal barriers in the framework of the volume averaging technique.

This problem has been examined by several authors and the two cases of local equilibrium, i.e.  $\langle T_\beta \rangle^\beta = \langle T_\sigma \rangle^\sigma = \langle T \rangle$  and non-equilibrium were considered. The former case leads to the following one-equation model [18–20]:

$$\langle \rho \rangle C_p \frac{\partial \langle T \rangle}{\partial t} = \nabla \cdot (\mathbf{K}^* \cdot \nabla \langle T \rangle) + a_v \langle \Omega \rangle_{\beta\sigma} + \varepsilon_\sigma \langle \Phi_\sigma \rangle^\sigma \quad (18)$$

where  $a_v$  is the specific area and  $\Omega_{\beta\sigma}$  is the area averaged value of the heterogeneous thermal source. The heat capacity per unit volume is given by:

$$\langle \rho \rangle C_p = \varepsilon_\beta (\rho C_p)_\beta + \varepsilon_\sigma (\rho C_p)_\sigma \quad (19)$$

The effective thermal diffusion tensor  $\mathbf{K}^*$  is obtained explicitly from the closure problem [3].

In many situations, the assumption of local thermal equilibrium is not valid and non-equilibrium models have been proposed in the form of two-equation models. The interested reader is referred to [3,18,20–23] for much more detail.

The choice between the two types of models depends on several geometrical and physical characteristics. In this paper, we evaluate the validity of the one-equation model by using direct numerical simulations over simple domains involving a small number of unit cells.

#### 4.4. Direct numerical simulations (DNSs) to explore the validity of the local equilibrium assumption

A numerical code has been implemented using the software Comsol Multiphysics in a steady-state situation. A standard heat diffusion equation like Eq. (17) is considered for the solid and for the superfluid helium, using in this case the effective thermal conductivity  $k_{\text{eff}} = \left( \frac{1}{f(T) + |\nabla T|^2} \right)^{1/3}$ . The function taken for  $f(T)$  comes from the fit done by Kashani et al. [13]. In order to study a heat transfer problem as close as possible to the case of a superconducting accelerator magnet, we consider the geometry illustrated in Fig. 3: the domain is made of two flat plates of thickness  $e$  with a 0.2 mm space filled with superfluid helium. These two flat plates represent either the collars or yoke lamination. At the top, we consider a fixed temperature ( $T_{\text{heatsink}}$ ) in order to represent the heat sink. Lines (a), (b) and (c) are to simplify the interpretation of Figs. 4 and 5. For the considered geometry, two types of simulations have been carried out: the first simulation assumes that there is a volume heat source ( $\Phi_\sigma$ ) in the solid. The second simulation assumes that the heat source is a surface heat flux ( $\Omega$ ) applied at the bottom, a flux originating from heat generated in the coil and in the beam-pipe and flowing through the solid and helium towards the heat sink, i.e. a cooling pipe.

For both simulations, we consider two types of boundary conditions at the solid/He II surface: first we assume continuity of the temperature and that the heat flux is expressed by Eq. (15) and (16). The other boundary condition used is the Kapitza resistance,

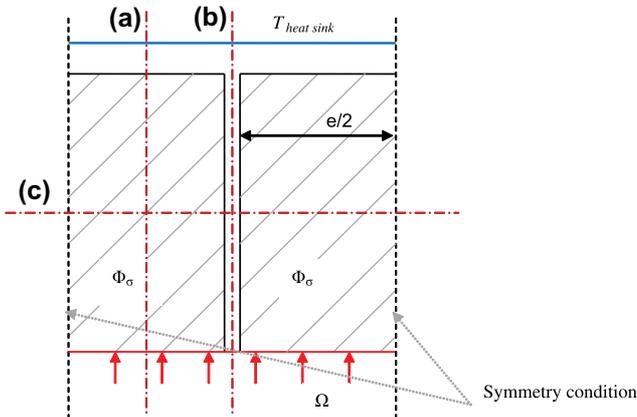


Fig. 3. Simple domain representative of heat transfer between two plates.

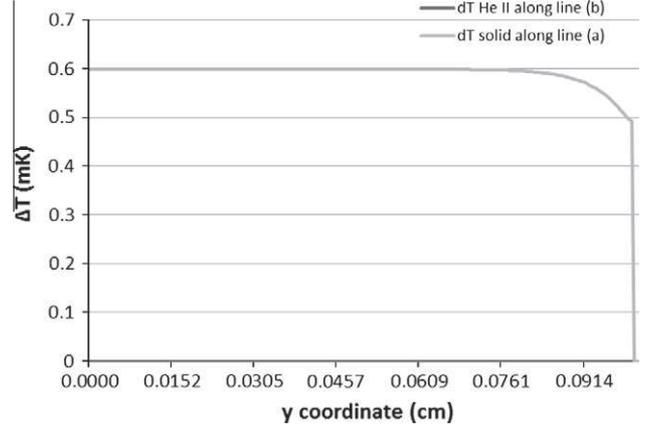


Fig. 4. Difference in temperature ( $T - T_{\text{heatsink}}$ ) along line (a) and (b) with and without Kapitza resistance for the volume heat source ( $\Phi_\sigma$ ).

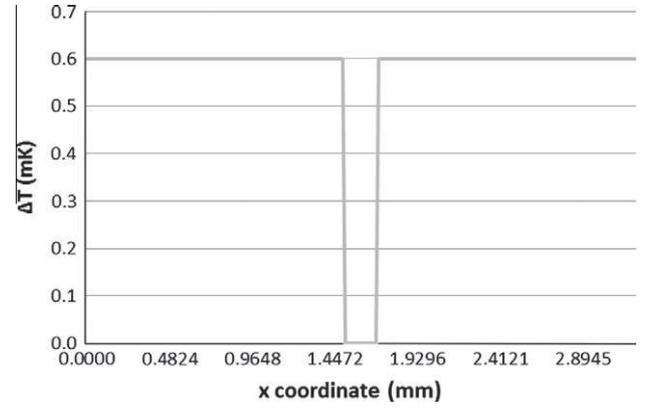


Fig. 5. Difference in temperature ( $T - T_{\text{heatsink}}$ ) along the line (c) with and without Kapitza resistance for the volume heat source ( $\Phi_\sigma$ ).

Table 1

Values of the parameters used in the simulations.

Parameters	Value	Unit
$k_{\text{solid}}$ (thermal conductivity of the solid)	109	W/m K
$h_k$	5000	W/m <sup>2</sup> K
$\Omega$	1000	W/m <sup>2</sup>
$\Phi_\sigma$	2000	W/m <sup>3</sup>
$e$	$1.5 \times 10^{-3}$	m

expressed by  $q = h_k \Delta T_s$  where  $h_k$  is the Kapitza conductance and  $\Delta T_s$  the temperature difference between the solid and the liquid at the boundary [11]. The values of the different parameters used in the simulations are summarized in Table 1. Typical values arising in heat transfer situations in superconducting magnets [24,25] were taken for ( $\Omega$ ) and ( $\Phi_\sigma$ ).

Fig. 4 presents the difference in temperature along lines (a) (in the solid) and (b) (in the liquid) with and without Kapitza resistance. In the liquid (along line (b)), the difference in temperature with regards to the heat sink temperature is null so the Kapitza resistance does not change the temperature. In the solid (along line (a)), there is a difference in temperature of 0.6 mK which is to be compared to the maximum temperature difference ( $T_{\text{max}} - T_{\text{heatsink}}$ ) between the solid and the heat sink and this corresponds to a difference of 9%.

Fig. 5 presents the difference in temperature along line (c) with and without Kapitza resistance. It is observed that the Kapitza

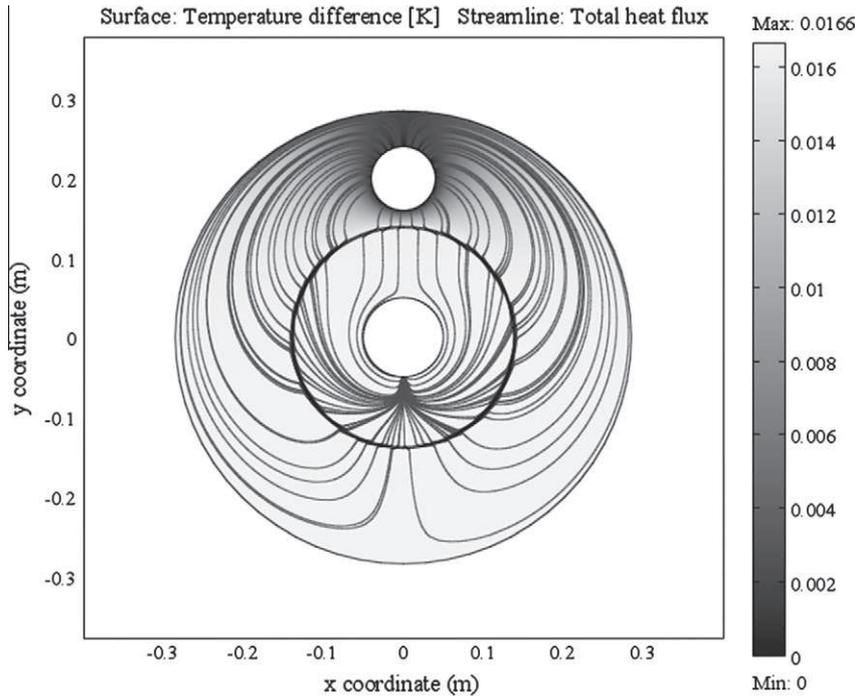


Fig. 6. Two dimensional view of the field of temperature difference ( $T - T_{heatsink}$ ) and heat flux lines in a superconducting magnet with one heat exchanger.

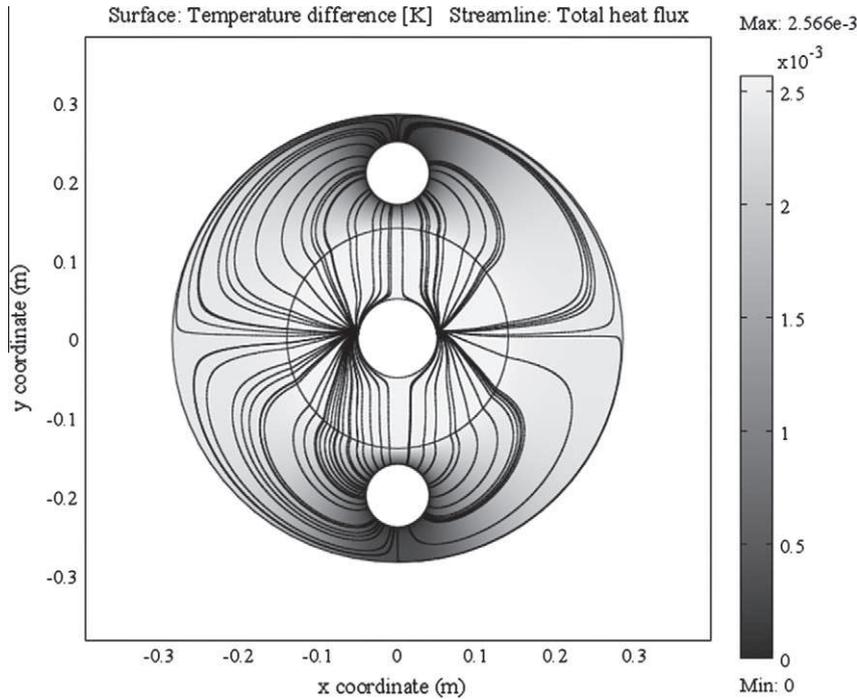


Fig. 7. Two dimensional view of the field of temperature difference ( $T - T_{heatsink}$ ) and heat flux lines in a superconducting magnet with two heat exchangers.

resistance induced a discontinuity of temperature at the solid/fluid interface of 0.6 mK which corresponds also to a difference of 9% with regards to the maximum temperature difference between the solid and the heat sink.

The same type of simulations has been done for a surface heat flux ( $\Omega$ ) and the same conclusion arises: the difference in temperature with and without Kapitza resistance is 0.8% different compared to the maximum temperature difference between the solid and the heat sink.

It must be reminded here that we consider at this point a steady-state situation. At any given position in the porous medium the temperatures of the solid and the liquid have become equal, and, since the conductivity is much larger in He II, the contribution of the solid to the heat flux is small. This conclusion might be wrong in the early stages of a transient problem since heat is generated in the solid phase and must be transferred to the liquid. This latter process being mainly determined by the heat transfer resistance through the solid phase, the characteristic time for such a process

may be estimated as  $l^2(\rho C_p)_\sigma/\lambda_\sigma = 20 \mu\text{s}$  for aluminum collars and 1.5 ms for iron yoke which is to be compared to the characteristic time of the source term variations.

Another point that is clarified with these simulations is the path of the heat flux: in all simulations considered until now, the heat flux has been calculated in the solid and in the liquid. Each time, more than 99.5% of the heat flows through the liquid to the heat sink. This means that, for the steady state situations under consideration, the role of the solid is negligible compared to the role of the liquid.

Following the previous simulations, we can consider that the difference in temperature between the liquid and the solid is the same with less than 9% error, the Kapitza resistance does not really influence heat transfer and the heat is brought to the heat exchanger via the liquid. All of these results confirm that the one equation model proposed by the literature for local thermal equilibrium (Eq. (18)) is adequate for the problem under consideration.

#### 4.5. Application to MQXC magnet design (magnet considered for LHC upgrade)

If we look at a 2D view of a superconducting magnet, the thermal diffusion tensor is reduced in our case to a diagonal matrix with  $(\varepsilon_\beta \times k_{eff})$  for coefficient where  $\varepsilon_\beta$  is the porosity of the liquid. Since the porosity of the aluminum collars is 6% and 2% for the yoke, we have to consider, at least, two porous media. These two porous media are connected, technologically speaking, through a thin layer of He II. How should we couple the temperature and fluxes between the two domains? Do we need to add explicitly a thin fluid layer? We carried out simulations with and without this liquid layer. The results of our simulations show that the model can be simplified to a classical coupling between the two porous domains by assuming continuity of macro-scale temperature and fluxes.

Taking into account all the results presented before, we present in Figs. 6 and 7 a two dimensional view of the temperature field in a superconducting accelerator magnet. We consider a volume heat source that takes place in the aluminum collars. For the case presented in Fig. 6, we have considered one heat exchanger and two in Fig. 7. We can see in Fig. 6 that the flux lines have a big node below the vacuum tube or beam tube (empty disk in the middle), which is not the case with two heat exchangers. The other point that is very important is the fact that we have one order of magnitude on the maximum temperature differences between the two cases, which gives a first idea of the impact of considering one or two heat exchangers.

## 5. Conclusion

In this paper, we introduce the method of volume averaging in order to study heat transfer in superconducting accelerator magnets. For this goal, we considered the interior of this type of magnet as a porous medium and we identified the different length-scales that allow us to apply the method in order to obtain the macro-scale equations suitable for numerical simulations. As a starting point, we apply this method to the heat diffusion model of superfluid helium in steady-state and use direct numerical simulations to show that a one equation model for local thermal equilibrium is valid under the case of consideration. This permits us to calculate the temperature field in a section of a magnet and to see the influence of considering one or two heat exchangers. This model is a

preliminary model and it does not take into account the convection in a magnet. To extend this model, we must apply the method of volume averaging to superfluid helium with heat exchange between the solid and the liquid and use direct numerical simulations to see, for instance, if we can consider local equilibrium or non-local equilibrium between the solid and the liquid, especially for transient phenomenon.

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