The study of teachers' mathematical and didactic praxeologies as a tool for teacher education

Gisèle Cirade, Anne Crumière

The study of teachers' mathematical and didactic praxeologies as a tool for teacher education

Gisèle Cirade and Anne Crumière
UMR EFTS, University Toulouse Jean Jaurès, France

To introduce the use of the ATD in research on teacher education, we set out to explore the didactic infrastructures that are best suited to mathematics teacher education throughout a scientifically-based project which aims to engage students in the construction of mathematical and didactic praxeologies. We focus on teachers' praxeologies and try to identify what strategies to develop to enrich their praxeological equipment.

This short chapter has two sections: the first examines the notion of "teaching trade" and the difficulties that appear in the exercise of the trade; the second looks at the role of teacher training or teacher education in the evolution of this trade towards a more professional status.

1 The teaching trade and its difficulties

Before discussing the *training* of mathematics teachers, let us begin by talking about the *trade* of mathematics teaching based on Chevallard (2013). Having specified that the word *trade* would be used "in a sense specific to the ATD", the author states:

> The *trade* of mathematics teaching is what anyone *in* this trade must do. To practise the trade of general practitioner means, in particular, receiving patients in your office, questioning them, examining them, asking them for their health card, writing prescriptions; and it also means visiting patients at home, etc. Similarly, practising the trade of mathematics teaching leads to what I would call the *gestures of the trade*—for example, "teaching", "preparing your course", "giving homework", "correcting homework", "writing an answer sheet", "filling out school reports", etc. (p. 86, our translation)

This sketched list of *gestures of the trade* raises the question of the *praxeological equipment* of teachers. Obviously, the analysis of such equipment will depend strongly on the authority, personal or institutional, that proposes it. Let's take a quick example, in France, with the "referential of professional competences of teaching trades and education" (Ministère de l'Éducation Nationale, 2013), to situate what is happening from the employer's point of view. In the first part, we find the "competences common to all teachers and educational personnel"; there are 14 of them, we list five of them:

> 1. Share the values of the Republic; 3. Know the students and the learning processes; […] 8. Use a foreign language in situations required by his trade; […] 10. Cooperate as part of a team; […] 14. Engage in an individual and collective approach to professional development.

This referential then proposes five "competencies common to all teachers", the last three being indicated as being addressed to them as "expert practitioners of learning":

> P1. Master disciplinary knowledge and its didactics
> P2. Master the French language as part of your teaching

P3. Build, implement and facilitate teaching and learning situations that take into account the diversity of students

P4. Organise and ensure a group operating mode that promotes student learning and socialisation

P5. Assess student progress and achievements (*Ibid.*, our translation)

The introduction to the referential specifies that "each competence [...] is accompanied by items that detail its components and specify its scope"; for example, consider one of the five generic items accompanying competence P3:

> Know how to prepare the class sequences and, for this, define plans and progressions; identify objectives, content, devices, didactic obstacles, support strategies, training and assessing modalities. (*Ibid.*, our translation)

Here we can identify an emblematic type of tasks: preparing a sequence, as well as another one, which is directly related to it: defining a plan and a progression. We also see the appearance of subtypes of tasks of the first one: identifying objectives, identifying contents, etc. Without pursuing the analysis of the above-mentioned professional competences, we note that we see the emergence of what will constitute, for the employer, the didactic stake in teacher training.

After introducing the concept of a *trade* in ATD, Yves Chevallard (2013, p. 86) specifies: "What matters now is this: the exercise of a trade encounters *difficulties*". It is these *difficulties*, and how to identify them, that we will now focus on. The question of identifying the difficulties encountered in the trade is crucial, and it is a delicate one. The dialectic of the individual and the collective plays an essential role here because it is through the difficulties experienced by *some* professionals that we will be able to identify those faced by *the* professionals—the former revealing the latter, which are in fact the main focus of our research.

Let us introduce a system aimed at bringing to light the difficulties encountered by teachers. Implemented in some initial teacher training courses under the name of the *questions of the week*, this system can be presented succinctly as follows:

> Every working week, during a working session with the entire cohort, student teachers, whether they are preparing for CAPES[1] in the first year or are trainee teachers in the second year, are invited to write down, individually, a difficulty they have encountered and the questions it raises for them. (Cirade, 2006, p. 63, our translation)

For example, we propose below some of the questions of the week formulated by a 2nd-year student teacher, Margot, throughout her teacher-training year 2014/15 (Cirade, 2006, pp. 119–133, the numbers correspond to the week of training). As can be seen, the difficulties encountered are diverse:

> 3. The majority of students in my class do their exercises but some do not. What to do about it? Punish them? // 7. I start a chapter on the configurations of the plan mainly based on reviewing parts of lower secondary mathematics to learn how to perform proofs. How to "schematise" a geometric proof? How can one explain a type of reasoning? Can we talk about logic? // 8. In terms of generalities about functions, what techniques should students know about function variation?

1. In France, the *Certificat d'Aptitude au Professorat de l'Enseignement du Second degré* (CAPES) is the competition for the recruitment of certified teachers for general subjects. The winners of this competition then follow a second-year training course at the *École Supérieure du Professorat et de l'Éducation* (ESPE) as trainee teachers.

> My book only uses reading graphs. Is the "algebraic" method (showing that, if $a \leq b$ then $f(a) \leq f(b)$, etc.) still on the syllabus of grade 10? // 11. A student goes to the blackboard to solve a geometry exercise. He writes his solution, it is correct, but there is a faster way. One student points this out orally. Should this other reasoning be apparent on the blackboard? Do students have to take note of this? // 14. In the chapter *Locating in the plan*, it is recommended to make markings in a spreadsheet. How to create an activity about marking in a spreadsheet? What can be its mathematical interest? // 16. About the coordinates of a vector, the row data $\vec{u} = (a; b)$ and column data $\vec{u} = \begin{pmatrix} a \\ b \end{pmatrix}$ appear in various documents. What is the "right" writing? // 22. Let ABC be any triangle. Note P the intersection point of the inner bisector of *A* with [BC]. How to prove that $\frac{PB}{PC} = \frac{AB}{AC}$? // 24. How to develop a "common" assignment at the end of the year?

We will not comment on these questions because we wish to emphasise the following: the difficulties mentioned by *a* particular *individual* (in this case Margot) are, in fact, an indication of the difficulties faced by *the collective* of student teachers and, beyond that, of the teaching *profession*. In the context of the *forum of questions and answers* where the *questions of the week* were approached, it was clearly mentioned to the students that the aim of the work was not answering *Margot*, but the *questions* she was brought from the field, which participants must learn to see as questions of the *profession*.

If the system of the *questions of the week* makes it possible to highlight the *difficulties* encountered in the trade, this is of course not the only way. The work carried out by André Pressiat (chapter 13) describes difficulties that have been identified in different contexts (visits of a teacher in training, meetings at the ministry, publications, etc.), even if they are usually ignored or at least not approached as difficulties as such. We now examine what can happen after identifying difficulties encountered by teachers, beginners or not, in the practice of their trade. To do this, let us look at how Chevallard (2013) models the inquiry that can occur from the moment a difficulty is recognised:

> A difficulty having been recognised by a person or institution ξ, it can be transformed, for a person or institution ξ^*, into a *question* to which an *answer* must be given. [...] The recognition that a difficulty affects the practice of the trade, its transmutation into a question *Q*, the construction of an answer *A* and the control of the validity and value of this answer are by definition within the *noosphere of the trade*. (p. 88, our translation)

Pressiat (2017) places itself *de facto* in the *noosphere* (see Glossary) of the trade when he identifies a difficulty and transforms it into a *question* to be studied. When the question is studied and disseminated, it acquires the status of a *problem for the noosphere of the trade*—that is, what is called, in short, a *problem of the profession*. The same applies, of course, to the system of the questions of the week in the context of initial teacher training. It should also be noted that, if we look at all the questions of the week asked over several years and focus on a particular difficulty, in some cases the questions of the week abound—for example those relating to homework (Cirade, 2011)—while in other cases they emerge sporadically—this is the case for those relating to alternating internal angles (Cirade, 2008).

Chevallard (2013) specifies what he calls the level of professionalisation of the noosphere of a trade or, in short, the *level of professionalisation of a trade*. We can start by assuming that the noosphere of a trade is a profession if it satisfies a certain number of criteria from which various lists have been drawn up in a convergent manner: fulltime occupation, the establishment of training or university schools, a national association of professional ethics, state licensing laws, etc. Recalling the work carried out by the American sociologist Amitai Etzioni (1969), which makes it possible to define what is called *semi-professions* but also other levels of professionalisation of the trade, Chevallard mentions trades for which the noosphere does not satisfy these criteria:

> Let us add that, of course, there are trades, "small trades", whose degree of professionalism is lower, even much lower than that of semi-professions. Etzioni has thus created the term *McJob* (from McDonald's restaurants) to designate, according to the eponymous article in *Wikipedia*, jobs "where little training is required, staff turnover is high, and workers' activities are tightly regulated by managers". (Chevallard, 2013, p. 90, our translation)

To simplify, we use the term *profession problem* to refer to a problem dealt with in the noosphere of the trade, even if the *level of professionalization* of this noosphere is not such that it can be described as a profession. It should also be noted that "it is by assuming the difficulties of the teaching trade as *problems*, the resolution of which requires appropriate fundamental and applied research, that the noosphere of the teaching trade can give rise to a true profession" (*ibid*., p. 90, our translation).

2 The professionalisation of the trade

The question of building a "university vocational training" for teaching (Chevallard & Cirade, 2009) continues to be an insistent one, and it provides an opportunity to strongly reformulate the links between training and research:

> The university would fail in its mission if it only offered teacher training to *readers* in science that is safe, but without scope for contextual intelligibility and practical effectiveness *in professional matters*. We have to go backwards. Building appropriate answers $A^{\blacktriangledown}$ requires the development at *new cost*, without false economies, of teaching *techniques* that will be tested in a thousand ways, *technologies* that project on them an adequate intelligibility, rooted in *theories* that cannot be expected to exist ready-made in the realm of academic knowledge and that must, therefore, continue to be produced. The university must not only provide safe *lectores* for teacher training. It is vital that it accepts the exciting adventure of becoming a collective *auctor*, without arrogance, without boastfulness, with the generosity due to professions that, every day, contribute as much as possible to giving society its own intelligibility and each of its members the intelligence of the situations experienced. (p. 55, our translation)

We will now repeat the problem of the praxeological equipment of teachers, starting not from the competences of the referential mentioned in section 1, but from the questions to be studied in the didactic systems of teacher training that are mainly produced by the question Q_f: how to occupy the position of a teacher in the school didactic systems? (Artaud, Bourgade, Cirade & Sémidor, 2016). The authors specify that this implies identifying a certain number of types of tasks $T_{\pi i}$ to be performed in this position and answering the associated questions $Q_{\pi i}$: how to perform $T_{\pi i}$? The problem of the didactic transposition of the praxeologies to teach then arises,

as in any didactic system, and is deployed in multiple sub-problems, among which we can mention: what are the questions to be studied? What are the answers to be provided? Where did they come from? What conditions and constraints influence the dissemination of questions and answers developed in institutions for the production of relevant knowledge, including research in didactics?

These are questions relating to *teaching* praxeologies, and similar questions can be stated for the *training* praxeologies: How to help the study community to study the questions $Q_{\pi i}$? This is the second problem trainers have to face, which is not unrelated to the previous one.

2.1 The difficulties of the trade of teacher educator

We will now consider a reflection on the *difficulties* encountered by *teacher trainers or educators* in the exercise of their trade. Until now we have essentially considered the *teaching* trade, but the developments proposed by Chevallard (2013) allow us to place ourselves within the general framework of the exercise of *a* trade. We choose here to place ourselves within the framework of the trade of *trainer* by now considering the reflection mentioned above:

> I have great difficulty in justifying the need to carry out a mathematical praxeological analysis—to explain the types of tasks, techniques and technologies of a mathematical organisation—to students preparing for the teaching trade, as well as the didactic praxeological analysis and the relationship between the two. Students find this work excessive. How to conduct didactic analysis in the context of the ATD should be developed. (Cirade, 2019, p. 6, our translation)

In this statement, two aspects can be distinguished: first, the *raisons d'être* of a didactic analysis; second, the *technique* to be used to carry out a didactic analysis—it should obviously be noted that, beyond the technique to be used, it is the praxeology itself that must be considered. In the immediate term, let us try to identify why it is useful, or even essential, to include didactic analysis in the praxeological equipment of future teachers. To study this question, we will place ourselves within the framework of the *primordial* problematic, which can be stated as follows:

> The *primordial* problematic is to determine, for a given instance (personal or institutional) u, *the praxeological equipment* which would be considered adequate, by an instance v (with possibly $v = u$), under constraints K and in conditions C, for the design and implementation of a given project Π. This type of questioning, therefore, aims to determine the set $\wp$ of praxeologies $\mathcal{p}$ [...] considered, by the authority v, *useful or essential* to u to engage in the project Π under the constraints K and in conditions C. (Chevallard, 2017, pp. 31–32, our translation)

Consider the case where $u = (I, p)$ is the institutional position of trainer in a teacher-training institution, the project Π consisting in training teachers and v a body composed of didacticians coming to occupy a certain position in the noosphere of the trainer's trade. The question is to determine the praxeologies $\mathcal{p}$ useful or essential to carry out such a project—under constraints K and in conditions C which we will not detail here. In this didactic institution I, it is a question of setting up teaching praxeologies. A didactic system is created around the generating question Q_f: "How to occupy the position of teacher in school didactic systems?", with students X and study aids Y, to bring together a didactic milieu M and confront it in order to produce an answer $A^{\heartsuit}$. Other didactic systems will follow, around questions that will appear during the study and

which, for some, may come from the transmutation of difficulties that will have been recognised in the training (see section 1.2). In any case, this study process can be modelled by the Herbartian schema (see Glossary) presented here in its semi-developed form:

$$[S(X; Y; Q) \rightharpoonup M] \hookrightarrow A^{\heartsuit}.$$

The didactic milieu, *M*, which is thus brought together is of course evolving, and it is throughout the study process that it is enriched. Let us take up again what Chevallard says (2019, pp. 22–23) about the creation of this milieu *M* which will make it possible to produce the answer $A^{\heartsuit}$:

> The first is the search—"in the literature" and, in particular, on the internet—for existing answers offered by other persons or institutions. Such answers are usually denoted by $A^{\diamondsuit}$ […] At his stage, the milieu *M* is therefore to be written thus: $M = \{A_1^{\diamondsuit}, A_2^{\diamondsuit}, \ldots, A_m^{\diamondsuit}, \ldots\}$.
>
> To draw upon the answers $A_i^{\diamondsuit}$ $(1 \leq i \leq m)$, the didactic system has recourse to works of various kinds, like theories, experiments, historiographical narratives, etc. Therefore the milieu is now to be written: $M = \{A_1^{\diamondsuit}, A_2^{\diamondsuit}, \ldots, A_m^{\diamondsuit}, W_{m+1}, W_{m+2}, \ldots, W_n, \ldots\}$. To use these works, the student […] needs to study them. What does it mean to study a work *W* which is not itself a question? Such a study consists in studying a number of *questions* Q_w about the work under study […] So that the milieu *M* takes on the following appearance: $M = \{A_1^{\diamondsuit}, A_2^{\diamondsuit}, \ldots, A_m^{\diamondsuit}, W_{m+1}, W_{m+2}, \ldots, W_n, Q_{n+1}, Q_{n+2}, \ldots, Q_p\}$.

For many of the didactic systems studied in training, an answer $A_i^{\diamondsuit}$ can be a class report, a textbook extract, a teacher's website, a document distributed to students, etc. These answers will be analysed and assessed to develop instructional products. In training, we will, therefore, have to analyse the teaching praxeologies, by approaching both the *praxeological analysis of the content to teach* and the *praxeological analysis of the teaching activities*, as mentioned in the reflection proposed at the beginning of section 2.1. More broadly, it is a question of *constituting and studying corpuses of works*, which can be of a diverse nature, and of integrating some *praxeologies* into the milieu *M* to study these *corpuses*.

2.2 Infrastructures for didactic analysis

We will now briefly present two examples based on our work carried out as part of teacher training processes at the ESPE Toulouse Midi-Pyrénées. The first example (Cirade, 2019, section 2.1) concerns the *mathematics to teach*, in the case of the theme entitled *Weighted averages* in the curriculum for the 4th grade (students aged 13–14) in force at the beginning of the 2009 school year in France. In training, a didactic system is built around a question *Q* such as "What is the mathematics to teach about 'weighted average'?" As the study progresses, the milieu *M* will be enriched with answers $A_i^{\diamondsuit}$—a curriculum extract, textbooks extracts, etc.—questions Q_k—"What does the syllabus and textbooks say about this?", "What is a weighted average?", etc.—as well as other works W_j. Among the works W_j there is a mathematics book on averages, but others are introduced that are less "visible" but essential because they are the ones that will allow the study of the above-mentioned documents (curriculum extract, textbooks extracts, etc.): the elements needed to perform a *praxeological analysis* of the mathematics to teach. This analysis will clarify what is problematic for student teachers—namely what is a *weighted average* and why the programme introduces this notion. In this case, the questions addressed focused, in a non-independent way, on the four components of the praxeology under

consideration. We could then see that the topic "Weighted averages" consists only of one type of tasks; that the differences between average and weighted average are of a technical nature and relative to the numerical series considered (their size and, above all, the way they are presented: as a list, a distribution table, a graph). We could also see that there were not two definitions, but a definition and a *property*, and that the property can be justified; that, in all cases, a series of values can be summarised by a number that equidistributes the values in the sample; etc. What matters here is that the initial question—"What is the mathematics to teach?"—can be studied *scientifically*: the notion of praxeology providing a *model* for analysing, and therefore better *understanding*, what is mentioned in the curriculum, what is found in textbooks, etc., and thus for changing the students' relations to the mathematical objects considered.

The second example (Crumière & Cirade, in press) is related to the block of content—a *sector* in the scale of codeterminacy (see Glossary)—called "Data organisation and management" in lower secondary school in France (students aged 12–15) at the beginning of the 2016 school year. Trainers from the ESPE Toulouse Midi-Pyrénées offered a workshop to the student teachers with the following instruction: "Choose a statement to start an 'activity' to teach a mathematical organisation around the type of tasks 'Determine a median'." Before this workshop, an analysis of the mathematical praxeologies of this sector was carried out with the teacher students. During the workshop, the students produced about 15 proposals—which were all answers $A_i^{\lozenge}$ to the question "How to determine a median?". The analysis of these responses revealed a number of difficulties. In fact, student teachers had the impression that, by offering highly guided activities, they achieved greater control over the time and progress of the study and, through this, better classroom management—which is often the main concern of beginning teachers. However, this management is very closely linked to the robustness of the proposed activities and, in particular, to the proper management of the progress of the study through crucial questions. The analysis of these responses $A_i^{\lozenge}$ also highlighted the absence of *raisons d'être* for the median in the work proposed by the student teachers. In addition, the analysis of the corresponding mathematical praxeologies revealed the following points. Even if the sector "Data organisation and management" occupies a significant place in lower secondary school curricula in France, it is often ignored by the profession because it is considered to be of low mathematical level and therefore not a priority. Some technological elements are "non-stabilised", and there is persistent uncertainty regarding the basic elements of statistical models (data, values, variables, series, etc.), which confirms a certain *pejoration* of statistical knowledge. In conclusion, the didactic analysis revealed *recurrent difficulties* encountered by teachers in the exercise of their trade, even if the inquiry started from those encountered by a group of student teachers. Furthermore, it also provided some tools for the development of new teaching praxeologies.

2.3 Conditions and constraints

Within the framework of a training project aimed at encouraging the involvement of student teachers in the design of mathematical and didactic praxeologies in a *scientifically sound manner*, the question of *didactic infrastructures* in training arises, in the dual sense of the content at stake (teaching praxeologies) and of the direction of the study (training praxeologies). Many conditions and constraints must be taken into account, at the different levels of the scale

of didactic codeterminacy (see Glossary). At the lowest levels of the scale, the conditions related to the themes, sectors, domains and the (mathematical) discipline itself should be further studied. However, upper-level conditions are also significant. For example, *the denial of the need for scientifically based praxeological equipment*, which seems to us to be essentially due to the *pejoration* of didactics in our societies. This constitutes in our opinion one of the most important constraints in the context of such a project because in the trade of teaching and, more generally, in the corresponding semi-profession, the situation described by Chevallard (1997) still seems to be relevant:

> we must note the absence of a language rich enough and widely enough shared to allow an objective (and not simply personal) analysis of even the most common professional situations, with consequently a weak collective and individual capacity to communicate, debate, think even, about the objects of an activity that can easily be enclosed, for that, in the repetition of gestures and in technical solipsism. (p. 23, our translation)

This has a very significant impact on teacher training and creates many difficulties for trainers, such as the one mentioned in section 2.1, which points to the difficulty of justifying to students the need to rely on scientifically based tools—it should be noted that the question does not arise regarding the discipline they are supposed to teach.

A source of difficulties is obviously due to the complexity of the didactic stake in training—namely, *teaching praxeologies.* However, the low development of *teachertraining praxeologies* is of greater importance because it does not facilitate the creation and dissemination of solid teaching infrastructures for teacher training.

References

Artaud, M., Bourgade, J.-P., Cirade, G. & Sémidor, P. (2016). *Analyser des praxéologies de formation : apports de la TAD*. Symposium conducted at the 4e colloque international de l'ARCD, "Analyses didactiques des pratiques d'enseignement et de formation : quelles perspectives ?", Toulouse, France. https://arcd2016.sciencesconf.org/browse/session?sessionid=17180

Chevallard, Y. (1997). Familière et problématique, la figure du professeur. *Recherches en didactique des mathématiques, 17*(3), 17–54.

Chevallard, Y. (2013). L'évolution du paradigme scolaire et le devenir des mathématiques. Questions vives et problèmes cruciaux. In A. Bronner et al. (eds), *Questions vives en didactique des mathématiques : problèmes de la profession d'enseignant, rôle du langage* (pp. 85–120). Grenoble, France: La pensée sauvage.

Chevallard, Y. (2017). La TAD et son devenir : rappels, reprises, avancées. In G. Cirade et al. (Eds), *Évolutions contemporaines du rapport aux mathématiques et aux autres savoirs à l'école et dans la société* (pp. 27–65). https://citad4.sciencesconf.org

Chevallard, Y. (2019). Praxeological Issues in the Development, Reception and Use of ATD: Some Remarks. *Educação Matématica Pesquisa, 21*(4), 1–7.

Chevallard, Y. & Cirade, G. (2009). Pour une formation professionnelle d'université. Éléments d'une problématique de rupture. *Recherche et formation, 60*, 51–62.

Cirade, G. (2006). *Devenir professeur de mathématiques : entre problèmes de la profession et formation en IUFM. Les mathématiques comme problème professionnel* (Doctoral dissertation). http://tel.archives-ouvertes.fr/tel-00120709/fr/

Cirade, G. (2008). Les angles alternes-internes : un problème de la profession. *Petit x, 76*, 5–26.

Cirade, G. (2011). Un passé qui ne passe pas. Le cas des « devoirs à la maison ». In M. Bosch et al. (eds), *Un panorama de la TAD* (pp. 299–320). Barcelone, Spain: CRM.

Cirade, G. (2019). Infrastructures didactiques pour la formation des professeurs : le cas de l'étude de praxéologies d'enseignement. *Educação Matématica Pesquisa, 21*(4), 338–356.

Crumière, A. & Cirade, G. (in press). L'organisation et la gestion de données au cycle 4 : quelles difficultés ? In H. Chaachoua et al. (Eds), *Méthodes de recherche en TAD*.

Etzioni, A. (ed.) (1969). *The Semi-Professions and Their Organization: Teachers, Nurses, Social Workers*. New York, NY: The Free Press.

Ministère de l'Éducation Nationale. (2013). Référentiel des compétences professionnelles des métiers du professorat et de l'éducation. *Bulletin officiel n° 30 du 25 juillet 2013*. http://www.education.gouv.fr/pid285/bulletin_officiel.html?cid_bo=73066

Pressiat, A. (2017). Éléments sur les apports de la théorie anthropologique du didactique à la profession et leur réception. In G. Cirade et al. (Eds), *Évolutions contemporaines du rapport aux mathématiques et aux autres savoirs à l'école et dans la société* (pp. 679–725). https://citad4.sciencesconf.org