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► **To cite this version:**

Frédéric Pichon, Anne-Laure Jusselme. A norm-based view on conflict. 25e Rencontres Francophones sur la Logique Floue et ses Applications, LFA 2016, Nov 2016, La Rochelle, France. pp.153-159. hal-03522109

HAL Id: hal-03522109

<https://hal.science/hal-03522109>

Submitted on 11 Jan 2022

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Voir le conflit comme une norme

A norm-based view on conflict

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Résumé :

Récemment, une fondation axiomatique a été donnée à la mesure du conflit entre fonctions de croyance. Dans ce contexte, il a été montré que le conflit peut être évalué par l'inconsistance résultant de leur combinaison conjonctive. Deux mesures de consistance entre fonctions de croyance ont été définies, donnant ainsi deux mesures de conflit. Dans ce papier, nous mettons en lumière que ces mesures de consistance correspondent aux normes infinies des fonctions de plausibilité et de contour, offrant ainsi une vue géométrique sur les mesures de conflit considérées. De plus, ce résultat nous permet de mettre en évidence que la consistance d'une fonction de croyance n'est rien d'autre que sa distance à l'état d'inconsistance totale. Comme une conséquence directe, le conflit entre fonctions de croyance est égal à un moins la distance entre leur combinaison conjonctive et l'inconsistance totale. Cela donne un nouveau regard sur l'utilisation de distances pour mesurer le conflit.

Mots-clés :

Théorie de Dempster-Shafer, Fonctions de croyance, Conflit, Consistance, Distance, Norme.

Abstract:

Recently, the measurement of conflict between belief functions has been given an axiomatic foundation, resulting in conflict being measured as the inconsistency yielded by their conjunctive combination. Two measures of consistency have been defined, giving two measures of conflict. We bring to light that these measures of consistency correspond to the infinity norms of the plausibility and contour functions, thus providing a norm-based view on the considered conflict measures. Furthermore, building on this geometric view, we lay bare that the consistency of a belief function amounts to its distance to the state of total inconsistency. The conflict between belief functions is then shown to be equal to one minus the distance between their conjunctive combination and total inconsistency. This sheds some new light on whether distances are appropriate for conflict measurement.

Keywords:

Dempster-Shafer theory, Belief functions, Conflict, Consistency, Distance, Norm.

1 Introduction

The detection of conflicting information is one of the main indicators of anomalies. For instance in maritime security, such detection may reveal maritime anomalies such as vessels deviating from normalcy (such as “off-route vessels”, “too fast vessels”) and those possibly spoofing the Automatic Identification System (AIS) signal to hide suspect behaviour [1] (some vessels may hide their actual type, conceal their current position, etc). In this context, it is crucial to have conflict measures with well defined properties and semantics, in order to suit to specific anomalies.

As belief function theory [2] provides a rich uncertainty representation extending both set and probability theories, measuring and managing conflict between belief functions has been an important topic of research. Recently, conflict quantification was given an axiomatic foundation by Destercke and Burger [3], resulting in a proposal to measure conflict between belief functions as the inconsistency yielded by their conjunctive combination. This latter work was then extended by Pichon *et al.* [4] into a general framework able to deal with conflicting sources in a principled and meaningful manner.

Besides the axiomatic approach of [3], a notable alternative path to conflict measurement has followed a geometric perspective, and especially has studied whether distances and angles between belief functions, may be suited

to that purpose [5, 6]. However, as argued recently in [3, 6], it seems that distances between belief functions may actually not be adequate measures of their conflict, because they do not satisfy desirable conflict measure properties such as insensitivity to refinement or imprecision monotonicity (see Section 2). Similarly, it seems that angles are not totally satisfactory as conflict measures [6].

In this paper, we bring together these two important lines of work on conflict – the axiomatic and the geometric approaches – by proposing a norm-based view on Destercke and Burger conflict measures, unveiled in Section 3. We elaborate on this view in Section 4, where we shed some new light on the relation between conflict and distance measures. Overall, these results indicate that geometric concepts such as distances may still be relevant when reasoning about conflict, and they open new perspectives, particularly on conflict measurement, as discussed in Section 5. Basic concepts of belief function theory and results of [3], which are necessary for the exposition of our contributions, are first recalled in Section 2.

2 Belief functions and conflict: necessary background

In this section, basic concepts of belief function theory as well as the axiomatic approach to conflict measurement of Destercke and Burger [3] are recalled.

2.1 Belief function theory

The theory of belief functions is a framework for uncertainty modeling and reasoning. It was originally introduced by Dempster [7, 8] in the context of statistical inference, as a theory of imprecise probabilities. It was extended by Shafer [2] and Smets and Kennes [9] into a model that could also handle subjective uncertainty related to fixed quantities.

In this framework, uncertainty about an ill-known variable x taking its values in a finite

domain $\mathcal{X} = \{x_1, \dots, x_K\}$, is represented by a so-called *mass function* defined as a mapping $m : 2^{\mathcal{X}} \rightarrow [0, 1]$ verifying $\sum_{A \subseteq \mathcal{X}} m(A) = 1$. Due to the two main views on this framework recalled above, a mass function may be given one of two interpretations, called respectively *imprecise probabilistic* and *singular* in [3], depending on what it is intended to model. Accordingly, the mass $m(A)$ represents either the amount of probability to be shared between elements of A without being assigned yet, by lack of knowledge, or the probability of knowing only that $x \in A$ [10, 11]. Subsets A of \mathcal{X} such that $m(A) > 0$ are called *focal sets* of m , and the set of focal sets is denoted by \mathcal{F} . A mass function having \mathcal{X} as its only focal set is called the *vacuous* mass function and represents total ignorance. The set of all mass functions on \mathcal{X} is denoted by \mathcal{M} .

An equivalent representation of a mass function m is the *plausibility function*. It is defined by, for all $A \subseteq \mathcal{X}$,

$$pl(A) = \sum_{B \cap A \neq \emptyset} m(B). \quad (1)$$

$pl(A)$ may be interpreted as the expected possibility of A with respect to m [11, 12], or equivalently the expected consistency of A with m since given information $x \in B$, A is possible if and only if A is consistent with B [12]. This latter interpretation is best seen when one rewrites Equation (1) as:

$$pl(A) = \sum_{B \subseteq \mathcal{X}} \phi(A \cap B) m(B), \quad \forall A \subseteq \mathcal{X},$$

where $\phi(C)$ is the consistency index of $C \subseteq \mathcal{X}$ defined by $\phi(C) = 1$ if $C \neq \emptyset$, and $\phi(C) = 0$ otherwise, \emptyset denoting the empty set. The plausibility function restricted to the singletons of \mathcal{X} is the *contour function* $\pi : \mathcal{X} \rightarrow [0, 1]$ such that $\pi(x) = pl(\{x\})$, for all $x \in \mathcal{X}$.

One of the appeals of this theory is that it makes it possible to combine multiple pieces of information about a variable. The most classical combination rule of the theory is Dempster's

unnormalised rule [7], also known as *conjunctive rule*. Let m_1 and m_2 be two mass functions representing pieces of information about x . Their combination by the conjunctive rule, denoted by \odot , results in the mass function $m_{1\odot 2}$ defined by, for all $A \subseteq \mathcal{X}$,

$$m_{1\odot 2}(A) = \sum_{B \cap C = A} m_1(B)m_2(C),$$

This combination is appropriate when m_1 and m_2 have been provided by two independent and reliable sources.

2.2 Axiomatic approach to conflict measurement

In [3], Destercke and Burger study how to evaluate the consistency of a mass function. They argue that the mass function m such that $m(\emptyset) = 1$, which will be denoted m_\emptyset hereafter, represents a *totally inconsistent* information state. They also consider two definitions of a *totally consistent* mass function: m is *probabilistically consistent* iff $m(\emptyset) = 0$; m is *logically consistent* iff $\bigcap_{A \in \mathcal{F}} A \neq \emptyset$. Then, they define two consistency measures, which we name in this paper A -consistency and x -consistency, associated respectively to the two definitions of a totally consistent mass function:

Definition 1. *The A -consistency $\phi_A(m)$ of a mass function m is the degree*

$$\phi_A(m) = 1 - m(\emptyset).$$

Definition 2. *The x -consistency $\phi_x(m)$ of a mass function m is the degree*

$$\phi_x(m) = \max_{x \in \mathcal{X}} \pi(x).$$

In [3], these measures are not given any specific name, and ϕ_A is denoted by ϕ_m and ϕ_x by ϕ_{pl} . It will become clear in Section 3 why we change these notations.

We have $\phi_A(m) \in [0, 1]$ and $\phi_x(m) \in [0, 1]$ for any $m \in \mathcal{M}$, and ϕ_A and ϕ_x reach their maximum and minimum when m is, respectively,

totally consistent and totally inconsistent. We also have $\phi_A(m) \geq \phi_x(m)$ for any $m \in \mathcal{M}$ [3, Lemma 2]. Moreover, as detailed in [3], ϕ_A agrees with the imprecise probabilistic interpretation of m , whereas ϕ_x agrees with its singular interpretation (yet, ϕ_A may also be useful in a singular interpretation).

Besides, the authors introduce five properties that a measure of conflict $\kappa(m_1, m_2)$ between two mass functions m_1 and m_2 provided by two independent¹ sources, should satisfy (further details on the motivations of these requirements can be found in [3]):

- Property 1 - Extreme conflict values: $\kappa(m_1, m_2) = 0$ if and only if m_1 and m_2 are non-conflicting and $\kappa(m_1, m_2) = 1$ if and only if m_1 and m_2 are totally conflicting;
- Property 2 - Symmetry: $\kappa(m_1, m_2) = \kappa(m_2, m_1)$;
- Property 3 - Imprecision monotonicity: if m_1 is a specialisation [13] of m'_1 , then $\kappa(m'_1, m_2) \leq \kappa(m_1, m_2)$;
- Property 4 - Ignorance is bliss: if m_2 is vacuous, then $\kappa(m_2, m_1) = 1 - \phi(m_1)$;
- Property 5 - Insensitivity to refinement: If ρ is a refinement function from Ω into Θ , define the refined mass assignment m_1 as $m_{\rho(1)}$, such that, for any focal element $E \in \mathcal{F}_1$, we have $m_1(E) = m_{\rho(1)}(\rho(E))$. Then, $\kappa(m_1, m_2) = \kappa(m_{\rho(1)}, m_2)$.

In particular, Property 3 states that the conflict should not increase as the imprecision (defined in terms of specialisation [13]) of a mass function increases. Property 4 states that the vacuous mass function should not conflict with

¹The study in [3] is actually more general and considers the whole spectrum from unknown dependence to known dependence. For clarity of presentation we do not tackle these more general situations in this paper, although we note that our results can easily be extended to this more general case.

any other mass function. Property 5 states that the refinement of a mass function should not change its conflict value with any other mass function.

They further introduce two measures of conflict, which satisfy Properties 1-5 and that we name in this paper A -conflict and x -conflict:

Definition 3. *The A -conflict $\kappa_A(m_1, m_2)$ between two mass functions m_1 and m_2 is the degree $\kappa_A(m_1, m_2) = 1 - \phi_A(m_1 \odot_2) = m_1 \odot_2(\emptyset)$.*

Definition 4. *The x -conflict $\kappa_x(m_1, m_2)$ between two mass functions m_1 and m_2 is the degree $\kappa_x(m_1, m_2) = 1 - \phi_x(m_1 \odot_2) = 1 - \max_{x \in \mathcal{X}} \pi_1 \odot_2(x)$.*

More precisely, two formal versions of Property 1, corresponding to two² different definitions of the notion of non-conflict, are considered in [3]: one is satisfied by κ_A and the other one by κ_x .

Measures κ_A and κ_x evaluate basically the inconsistency (in terms of ϕ_A and ϕ_x respectively) resulting from the conjunctive combination of two mass functions. Let us finally stress that $\kappa_A(m_1, m_2)$ is nothing but the classical measure of conflict $m_1 \odot_2(\emptyset)$.

3 Norm-based view on conflict

In this section, the notion of norm of belief functions is first recalled. Then, it is used to propose a geometric perspective on the conflict measures presented in the previous section.

3.1 Norms of belief functions

Let $\mathcal{E}_{\mathcal{X}}$ denote the Cartesian space \mathbb{R}^{2^K} spanned by the set of vectors $\{\mathbf{e}_A, A \subseteq \mathcal{X}\}$. Any vector \mathbf{v} of $\mathcal{E}_{\mathcal{X}}$ can then be written as $\mathbf{v} = \sum_{A \subseteq \mathcal{X}} v_A \mathbf{e}_A$, with $v_A \in \mathbb{R}$ the coordinate of \mathbf{v} along dimension \mathbf{e}_A . A mass function m

²In the case of independent sources, two out of the three different definitions of non-conflict in [3] are equivalent.

may then be represented as the vector \mathbf{m} of $\mathcal{E}_{\mathcal{X}}$ such that $v_A = m(A)$. Similarly, a plausibility function pl may be represented by the vector $\mathbf{pl} = \sum_{A \subseteq \mathcal{X}} pl(A) \mathbf{e}_A$, with its plausibility values $pl(A)$ as coordinates of \mathbf{pl} .

Let us also denote by \mathcal{E}_x the K -dimensional subspace of $\mathcal{E}_{\mathcal{X}}$ spanned by the set $\{\mathbf{e}_x, x \in \mathcal{X}\}$ of singleton vectors. Then, the contour function π associated to a mass function m may be represented by the vector $\boldsymbol{\pi} = \sum_{x \in \mathcal{X}} \pi(x) \mathbf{e}_x$ of \mathcal{E}_x .

The L^p -norm $\|\mathbf{v}\|_p$ of a vector $\mathbf{v} = \sum_{1 \leq i \leq N} v_i \mathbf{e}_i$ belonging to the Cartesian space \mathbb{R}^N spanned by the set of vectors $\{\mathbf{e}_i, 1 \leq i \leq N\}$ is defined as

$$\|\mathbf{v}\|_p = \left(\sum_{1 \leq i \leq N} |v_i|^p \right)^{\frac{1}{p}}. \quad (2)$$

Of particular interest in this paper is the case where $p = \infty$ (called infinity norm hereafter), for which Eq. (2) reduces to $\|\mathbf{v}\|_{\infty} = \max_{1 \leq i \leq N} |v_i|$.

In the following, we consider the L^{∞} -norms of \mathbf{pl} and $\boldsymbol{\pi}$:

$$\|\mathbf{pl}\|_{\infty} = \max_{A \subseteq \mathcal{X}} pl(A), \quad (3)$$

$$\|\boldsymbol{\pi}\|_{\infty} = \max_{x \in \mathcal{X}} \pi(x). \quad (4)$$

In the next section, we will detail how these two norms can be related to consistency and conflict measures.

3.2 Consistency and conflict in terms of infinity norms

The relation between the ∞ -norm of the contour function (4) and the x -consistency measure (Def. 2) is obvious from their respective definition. We have indeed for any $m \in \mathcal{M}$,

$$\phi_x(m) = \|\boldsymbol{\pi}\|_{\infty}. \quad (5)$$

Similarly, we can easily see that there exists a simple relation between the x -conflict between

belief functions (Def. 4) and the ∞ -norm of the contour function of their conjunctive combination: we have, for any $m_1 \in \mathcal{M}$ and $m_2 \in \mathcal{M}$,

$$\kappa_x(m_1, m_2) = 1 - \|\boldsymbol{\pi}_{1 \odot 2}\|_\infty. \quad (6)$$

Interestingly, similar relations as Eqs. (5) and (6) can be obtained between the ∞ -norm of the plausibility function (3) and the A -consistency and A -conflict measures (Def. 1 and 3 respectively). To obtain these relations, it suffices to remark that for any $m \in \mathcal{M}$,

$$1 - m(\emptyset) = pl(\mathcal{X}) = \max_{A \subseteq \mathcal{X}} pl(A), \quad (7)$$

and thus

$$\phi_A(m) = \max_{A \subseteq \mathcal{X}} pl(A). \quad (8)$$

The A -consistency of m amounts thus to its maximum expected consistency with respect to *all subsets* A of \mathcal{X} . In contrast, its x -consistency amounts to its maximum expected consistency with respect to *all elements* x of \mathcal{X} : This explains the use of the indices A and x in the notations ϕ_A and ϕ_x and in the associated names for the measures. From (8), it is then straightforward to obtain:

$$\begin{aligned} \phi_A(m) &= \|\mathbf{pl}\|_\infty, & (9) \\ \kappa_A(m_1, m_2) &= 1 - \|\mathbf{pl}_{1 \odot 2}\|_\infty. & (10) \end{aligned}$$

In sum, the consistency of a mass function can be written as a norm, and the conflict between belief functions amounts to 1 minus the norm of their conjunctive combination.

Let us remark that Eq. (7) leads to a nice writing of total consistency in terms of the infinity norms of the plausibility and contour functions:

Remark 1. *A mass function m is logically consistent iff $\max_{x \in \mathcal{X}} \pi(x) = 1$ (i.e., if $\|\boldsymbol{\pi}\|_\infty = 1$) and probabilistically consistent iff $\max_{A \subseteq \mathcal{X}} pl(A) = 1$ (i.e., if $\|\mathbf{pl}\|_\infty = 1$). Indeed, from [3, Lemma 1], m is logically consistent iff $\exists x \in \mathcal{X}$ such that $\pi(x) = 1$, which is equivalent to $\max_{x \in \mathcal{X}} \pi(x) = 1$. Furthermore, m is probabilistically consistent iff $m(\emptyset) = 0$, which is equivalent to $pl(\mathcal{X}) = \max_{A \subseteq \mathcal{X}} pl(A) = 1$.*

3.3 Related work

To the best of our knowledge, this constitutes the first³ geometric view on the conflict measures studied in [3]. However, there is at least one piece of work where the notions of inconsistency and norm have already been related. Indeed, George and Pal [14] introduced a measure denoted $TC(m)$ and defined, for any $m \in \mathcal{M}$ such that $m(\emptyset) = 0$, by

$$TC(m) = \sum_{A \subseteq \mathcal{X}} m(A) \sum_{B \subseteq \mathcal{X}} m(B) (1 - J(A, B))$$

where $J(A, B)$ is the Jaccard index defined by $J(A, B) = \frac{|A \cap B|}{|A \cup B|}$. They presented $TC(m)$ as a measure of total ‘‘conflict’’ in m , where their notion of conflict actually pertains to a form of inconsistency between focal sets in m , and specifically a refined form of inconsistency based on Jacquard indices. Indeed, $J(A, B)$ is classically interpreted as a measure of similarity between sets A and B , yet it can also be seen as a refined consistency degree since we have, for any $A \neq \emptyset$ and $B \neq \emptyset$, $J(A, B) = 0$ iff $\phi(A \cap B) = 0$, $J(A, B) > 0$ iff $\phi(A \cap B) = 1$ and in particular $J(A, B) = 1$ iff $A = B$.

Now, let \mathbf{J} be the matrix of Jaccard indices $J(A, B)$. Because \mathbf{J} is positive definite [15], the quantity $\sqrt{\mathbf{m}' \mathbf{J} \mathbf{m}}$ is then the Euclidean norm ($p = 2$) of m in the space $\mathcal{E}_{\mathcal{X}}$ endowed with the inner product $\mathbf{v}_1' \mathbf{J} \mathbf{v}_2$ for any vectors \mathbf{v}_1 and \mathbf{v}_2 of $\mathcal{E}_{\mathcal{X}}$, and we have [16]:

$$TC(m) = 1 - \mathbf{m}' \mathbf{J} \mathbf{m}.$$

In other words, the measure of George and Pal, which evaluates a form of inconsistency in m , can be expressed in terms of a particular norm of m .

4 Relating distances and conflict

As recalled in the introduction, distances between belief functions have been investigated

³In [5], κ_A has been previously written under the form of an inner product although not satisfying the properties of an inner product.

as a means to evaluate their conflict, yet recent works have argued against it. Nonetheless, based on our preceding results relating norms and conflict, we will show in this section that some distances can be related to the κ_A and κ_x conflict measures.

4.1 Norm-induced distances

The L^p -norm induces a distance $d_p(\mathbf{v}_1, \mathbf{v}_2)$ between any two vectors \mathbf{v}_1 and \mathbf{v}_2 of \mathbb{R}^N , defined as the L^p -norm of their difference, i.e., $d_p(\mathbf{v}_1, \mathbf{v}_2) = \|\mathbf{v}_1 - \mathbf{v}_2\|_p$.

This family of distances gives rise to the Minkowski family of evidential distances, when applied to belief functions [5]. We are particularly interested in this paper by those distances relying on the plausibility and contour functions (in which case they are, respectively, a metric and a pseudo-metric [17]) and the case $p = \infty$. They are defined as follows, for any mass functions m_1 and m_2 :

$$\begin{aligned} d_{pl,\infty}(m_1, m_2) &= \|\mathbf{pl}_1 - \mathbf{pl}_2\|_\infty, \\ d_{\pi,\infty}(m_1, m_2) &= \|\boldsymbol{\pi}_1 - \boldsymbol{\pi}_2\|_\infty. \end{aligned}$$

4.2 Consistency and conflict in terms of distances to inconsistency

Let us first relate the consistency measures ϕ_A and ϕ_x to the above distances.

Proposition 1. *For any $m \in \mathcal{M}$, we have*

$$\phi_A(m) = d_{pl,\infty}(m, m_\emptyset), \quad (11)$$

$$\phi_x(m) = d_{\pi,\infty}(m, m_\emptyset). \quad (12)$$

Proof. Eq. (11) follows from (9) and $d_{pl,\infty}(m, m_\emptyset) = \|\mathbf{pl} - \mathbf{pl}_\emptyset\|_\infty = \|\mathbf{pl}\|_\infty$. Eq. (12) follows from (5) and $d_{\pi,\infty}(m, m_\emptyset) = \|\boldsymbol{\pi} - \boldsymbol{\pi}_\emptyset\|_\infty = \|\boldsymbol{\pi}\|_\infty$. \square

In short, this proposition shows that the consistency of a mass function is nothing but its distance to the totally inconsistent knowledge state. Accordingly, the more a mass function

m is “far” from total inconsistency, the more it is considered consistent, which makes sense. It may be worth noting that the proposition holds because \mathbf{pl}_\emptyset and $\boldsymbol{\pi}_\emptyset$ are null vectors in spaces \mathcal{E}_X and \mathcal{E}_x respectively, and coincide thus with the origins of these spaces.

This result may be exploited to easily show the following relation between conflict and distance.

Proposition 2. *For any $m_1 \in \mathcal{M}$ and $m_2 \in \mathcal{M}$, we have*

$$\begin{aligned} \kappa_A(m_1, m_2) &= 1 - d_{pl,\infty}(m_1 \odot m_2, m_\emptyset), \\ \kappa_x(m_1, m_2) &= 1 - d_{\pi,\infty}(m_1 \odot m_2, m_\emptyset). \end{aligned}$$

Informally, the conflict between m_1 and m_2 amounts to 1 minus the distance between their conjunctive combination and the totally inconsistent knowledge state. This brings a new element to the discussion on whether distances may be appropriate for conflict measurement.

5 Conclusions

A geometric perspective on a recent axiomatic approach to conflict measurement was proposed. It was brought to light that Destercke and Burger consistency measures are the infinity norms of the plausibility and contour functions – which echoed a previously established relation between a particular norm of a belief function and the measure of George and Pal. This allowed us to rewrite the conflict measures associated to these consistency measures in terms of infinity norms. The same result was also used to show that the consistency of a mass function is formally equal to its distance to the state of total inconsistency, which corresponds to the origin of the plausibility space. This led us in turn to lay bare a relation between conflict and distance.

Beyond their simplicity, the results presented in this paper open new perspectives on the measurement of conflict and distances between belief functions, as well as on the use of norms

within this theory. This norm-based view of conflict allows us to explore in future works the natural generalisation to other values for the parameter p of the L^p -norm, as well as other transformations of the mass function than the plausibility or contour function ones. Special attention will be given to the semantics of the resulting norms. The extension of our results to other combination operators than the conjunctive rule, that is those induced by other reliability or dependence assumptions (such as the disjunctive rule [13] or the conjunctive combination under unknown dependence [3]), may be considered as well, although this can formally easily be done. Finally, it seems also worth studying more deeply the links between consistency measures, distances and informational orderings – the former two being related in this paper, the latter two in [17], and the former and latter in [4].

Acknowledgements :

The authors wish to thank the NATO CMRE and the Université d'Artois for their support via their visiting scientist programs.

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