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► To cite this version:

Isidoros Marampoutis, Marina Vinot, Lorraine Trilling. Multi-objective vehicle routing problem with flexible scheduling for the collection of refillable glass bottles: A case study. *EURO journal on decision processes*, 2022, 10, pp.100011. 10.1016/j.ejdp.2021.100011 . hal-03521985

HAL Id: hal-03521985

<https://hal.science/hal-03521985>

Submitted on 8 Jan 2024

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Multi-objective vehicle routing problem with flexible scheduling for the collection of refillable glass bottles: A case study

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ARTICLE INFO

Keywords:

Vehicle routing problem

Reverse logistic

Reusable glass container

Waste collection

Multi-objective optimization

Integer linear programming

ABSTRACT

Deposit return systems have started making their reappearance as more environmentally conscious consumers seek ways to effectively reduce their carbon footprint. An example is the management of refillable glass bottles which requires a well-organized collection network with inventory management. A collection planning with an efficient algorithm and information system has to be applied. This paper investigates, using integer linear programming, a vehicle routing problem with time constraints to provide flexibility as well as priority rules to avoid inventory saturation at collection points. The model presented, based on a real-life application in the city of Lyon and surrounding areas, includes several objectives with specific assumptions. The result of the optimization is a vehicle routing plan with flexible scheduling based on time slots. Numerical experiments are conducted on instances of different scales making it possible to model the current problem as well as its future evolution. These experiments consider several instances, using a single vehicle among three vehicle types (cargo-bicycle, car and van) and a network composed of 20 stores/clients to collect bottles from. The results show the impacts of the priority rules on the solution obtained and additional indicators are proposed in order to analyze more precisely the quality of the solution in terms of financial cost and environmental impact. The proposed model and program will help make appropriate decisions in planning and scheduling the routes of the vehicles for the refillable glass bottle collection, especially in urban areas.

1. Introduction

A shift towards eco-friendlier alternatives, both in industrial and commercial scale, has become more prominent in our societies. For example, 96% of European citizens agree that companies should take more initiatives to limit plastic waste and extend reuse and recycling (European Commission, 2014). Customers are often actively looking to reduce their own carbon footprint and waste production by selecting products that offer these options. In beverage market, the usage of refillable containers, and glass containers more precisely, have started making a comeback in recent years. This practice is often referred to as a deposit-refund system (DRS). The customer pays a small deposit upon purchasing the drink that is included in the price. Upon the return of the empty bottle at a local collection point or at the store where the bottle was purchased from, he receives the deposit back. The bottles are then washed and reused (Zhou et al., 2020).

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7 Today returnable and refillable bottles can be a valid sustainable alternative to single-use packages and considered
8 as a real opportunity, on both economic and environmental aspects, as a life cycle assessment in an Italian network
9 shows (Tua et al., 2020). Zhou et al. (2020) recently provided a comparison of 40 DRS in the world and confirm
10 the positive environmental impact of this practice worldwide. European studies mentioned that refillable bottles
11 show better environmental performances than single-use bottles, in the case of local markets (Bø et al., 2013) for a
12 minimum of around 20 reuses (CM Consulting and Reloop, 2016). Furthermore, Simon et al. (2016) highlights that
13 collection is a crucial life cycle stage in DRS and Coelho et al. (2020) alerts on the negative impacts that could occur
14 when switching to reusable packaging, due to additional resource consuming activities. It is therefore crucial to
15 carefully manage the transport movements for all steps of this complex logistics: production, distribution, collection
16 and cleaning.

24 This article investigates the collection problem in a local area for refillable glass bottles. The implementation of
25 such a network raises several issues related to real-life applications. At strategic and tactical levels, concerning the
26 network design, the issue is to define how many collecting points should be included in the network, their position,
27 their capacity and how one can ensure the robustness of the network when demand changes. At an operational level,
28 the main question is how to organize the collection of used glass bottles (stored in crates) and the simultaneous
29 delivery of the same number of empty crates by visiting each collection point. An additional issue is to figure out
30 which type of vehicle is best suited to perform the task. This paper focuses on the collection phase at the operational
31 level. In this phase a set of clients/stores have to be collected (crates full of empty glass bottles) by scheduling routes
32 with a single vehicle. The duration of the collection/routing is often the objective to minimize but other criteria
33 could be used. We explore particularly cost, environmental impact of transportation, time of operation and client
34 satisfaction, positively measured if a client is visited shortly after he needs it. In our approach, some expressions will
35 be considered as weighted objectives to be optimized (time of operation, client satisfaction), whereas others will be
36 assessed after resolution (cost, environmental impact). We mention that each collected crate is in fact exchanged with
37 another empty crate, where the bottles returned by the consumers will be stored. The delivery phase is simultaneous
38 to the collection phase and does not modify the nature of the problem studied. In the following, we use the term
39 "collection", for covering both the collection and delivery activities, to simplify the writing.

52 This problem of scheduling the collection of empty bottles is modeled as a Vehicle Routing Problem (VRP) and solved
53 using optimization techniques such as Integer Linear Programming (ILP). Each collecting point needs to manage its
54 own level of inventory, *i.e.* avoids shortage of empty crates (saturation of available storage capacity), and needs to
55 ask for a visit in order to collect all crates full of empty bottles. Hence this problem includes inventory management
56 constraints. However it is not a typical Inventory Routing Problem (IRP), in which the producer/collector decides
57 how much to deliver to which client at each time period (Savelsbergh and Song, 2008).

7 This work has been conducted with a local organization in an urban area around the city of Lyon (France), with
8 the aim of setting up a network to collect, clean, control and deliver returnable glass bottles. This paper tends to model
9 and solve a real-life problem based on an existing network (early stage) which includes only a few collecting points,
10 where reused full bottles are sold and empty reusable bottles are collected. The number of collecting points tends
11 to increase as the organization involves more and more partners (stores/clients, beverage producers using washed
12 bottles). Bottles collected are mainly beer bottles reused by local breweries, but other producers are progressively
13 integrating the network (oil, wine and more). This leads us to consider in our numerical experiments different scales
14 instances, designated as small, medium and large, corresponding to the original size of the network, the current state
15 and its near future. Each employee is responsible for multiple tasks in the organization with the collection being one
16 of them. A flexible collection scheduling is required to allow for the remaining tasks to be completed effectively.

24 The VRP applications have evolved over the years as they have become one of the most recognizable problem
25 in operational research (Mor and Speranza, 2020). Several studies on VRP with time constraints seem to address
26 similar problems with flexible home delivery problem (Strauss et al., 2020), autonomous vehicle fleet sizing problem
27 (Allahviranloo and Chow, 2019) or vehicle routing problem for online police patrol management (Saint-Guillain
28 et al., 2020). Nevertheless, in all these papers, the slots are defined by a start time and an end time or by a subset
29 of time windows. The time constraint proposed in this paper differs by allowing some flexibility by not restricting
30 the collection time within a time window. We define a number of time slots available over the time horizon and
31 a duration for each time slot. Routing problems with inventory management are also extensively studied. Recent
32 papers, focusing on solving the transportation problem and inventory problem simultaneously, attempt to propose
33 new models based on complex inventory costs definition (Sherif et al., 2021). In our case the financial cost of the
34 inventory is not a decisive criterion. However, a well-suited inventory management, thanks to an efficient collection
35 scheduling, is decisive to prevent overstocking of empty glass bottles and improve client satisfaction.

45 The main contributions of this paper are the following:

- 46 1. The introduction and definition of a new routing problem which has a relevant practical application.
- 47 2. A mathematical model based on a VRP with some additional considerations:
 - 48 (a) Time flexibility: The planning of the collection routes is flexible and adaptable, depending on the avail-
49 ability of the employee in charge of the collection.
 - 50 (b) Inventory management: The collection point (clients) must not reach saturation. Appropriate rules and
51 priorities are defined and integrated in a multi-objective function.
- 52 3. A decision-making tool based on an Integer Linear Programming (ILP) with additional indicators to analyze
53 precisely the quality of the solution (financial cost and environmental impact).
- 54 4. The production of several real datasets and the analysis of the results obtained from the resolution of compre-
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7 hensive instances based on the number of clients in the network, the time slots configuration and the vehicle
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9 type (van, car, cargo-bicycle).

10 The paper is composed as follows. Related research papers are reviewed in section 2. Section 3 focuses on
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12 problem description with its specific features. In section 4, the mathematical model formulation is given with all
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14 notations, the objective function and the constraints. Numerical experiments are described and discussed in section
15
16 5. In section 6, conclusions are derived.

19 **2. Literature review**

21 Vehicle Routing Problem has the same foundation as the TSP (Traveling Salesman Problem) but considers a
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23 vehicle with a limited capacity, available at a depot, and produces numerous routes to pass through all clients (Clarke
24 and Wright, 1964). The aim of VRP is to satisfy all demand while minimizing the total traveled distance. Current
25 VRP models are very different from the one introduced by Dantzig and Ramser (1959) and Clarke and Wright
26 (1964), as they aim to incorporate complex real-life constraints, such as for instance time-dependent travel times,
27 time windows for pickup and delivery, or other information (e.g. inventory management, penalty costs for delays or
28 non-serviced customers). The VRP is an NP-hard problem (Lenstra and Rinnooy Kan, 1981) and all added features
29 bring along substantial complexity.

31 In order to introduce all the features of our problem, we reviewed Vehicle Routing Problems (VRPs) literature
32 with several variants and extensions. In sections 2.1, 2.2 and 2.3, we focus our attention on three variants dealing
33 with time constraints, inventory management and multi-objective functions.

41 **2.1. Routing problem with time constraints**

43 In the literature several time constraints are considered in a routing problem. The most studied one is the VRP
44 with Time Windows (VRPTW), since it is often needed in real-world applications. In this problem, each client has
45 a time interval during which he is available for pickup or delivery operations. This problem has been intensively
46 studied and a large number of solving methods have been published (Desaulniers et al., 2014). Time constraints can
47 also be expressed with time dependencies in routing problems. The Time Dependent VRP (TDVRP) tends to model
48 real-world problems, by considering non constant travel times, allowing to model daily phenomena in traffic such as
49 congestion (Mancini, 2017). A detailed survey on this problem can be found in the paper of Gendreau et al. (2015).

51 Many other constraints are addressed in routing problems, among them precedence or synchronization constraints
52 (Garaix et al., 2018), occurring among others in the optimization of service planning (technician, home health care
53 personnel, etc.). Moreover, several time constraints are often considered in a single routing problem, such as time
54 windows and time dependencies (Ferani et al., 2020), justified by the transportation of perishable products, or time

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7 windows, precedence and synchronization constraints addressed by Cappanera et al. (2020).

10 **2.2. Routing problem with inventory management**

11 The inventory-routing problem (IRP) is a problem integrating vehicle routing, inventory management and de-
12 livery scheduling decisions. In most classical IRP variants, the inventory management lies in the definition of an
13 optimal delivery scheduling to avoid critical inventory levels. This kind of problem comes from the vendor-managed
14 inventory (VMI) problem based on the optimization of replenishment decisions for products (Angulo et al., 2004).
15 Some variants focus on inventory management with pickups and deliveries such as Archetti et al. (2018). Their
16 problem named IRP-PD (inventory routing problems with pickups and deliveries) is a many-to-many problem since
17 commodities have multiple origins and multiple destinations. The objective is to minimize the sum of transportation
18 cost and inventory cost at all locations, dealing with a single commodity and a single vehicle.

19 Inventory management is also considered in waste collection problems where the stake is to never reach the
20 breaking point (container full). In some studies, predictions to estimate the inventory/waste level are modeled or
21 simulated based on history of observations (Markov et al., 2020) but other methods can be proposed. In transportation
22 problems, the definition of priority levels can be introduced to manage some risks (Taslimi et al., 2020). In Hannan
23 et al. (2020), a mixed-integer linear programming (MILP) model is proposed in order to respond to the management
24 of the bin status (quantity of waste), using a small time horizon since routes must be provided daily with recent
25 bin status information to avoid the stochastic aspect. In this case, the advantage of MILP is that a solution can be
26 provided and recalculated within a very short period of time.

41 **2.3. Multi-objective functions in routing problems**

42 In order to model real-life problems and properly evaluate the solution quality, a well-defined objective function
43 has to be proposed. In some cases a simple cost can be chosen. Often, transportation problems must handle several
44 objectives usually in conflict, justifying the need of a multi-objective function. In multi-objective optimization, three
45 variants of model and resolution methods can be found according to Hwang and Masud (1979) and Zajac and Huber
46 (2020):

- 51 1. A priori approach is based on the combination of several objectives with the aim of achieving a single preferred
52 solution.
- 53 2. A posteriori approach often seeks several compromising solutions. They are based on the definition of a Pareto
54 front which offers a set of optimal solutions.
- 55 3. Interactive approaches take into account the different objectives in order to guide a search algorithm towards
56 a preferential solution.

57 The objectives can be taken into account in many ways but they are also manifold. The paper of Zajac and
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7 Huber (2020) proposes a review of multi-objective routing problems with over 100 contributions. In this study, a
8 classification summarizes more than 60 objectives used in routing problems from classical objectives (depending on
9 cost, distance, time, etc.) to newly identified objectives (CO₂/GHGE-GreenHouse Gas Emissions, Traffic volume,
10 Inventory cost, etc.). These new objectives, not yet widely studied, are often conflicting objectives with classical
11 objectives leading to new research opportunities. In Mostafayi Darmian et al. (2020) the problem addressed deals
12 with the collection of municipal solid waste. The study provides a multi-objective sustainable location-districting
13 model with three objectives based on cost, environmental impact and social dissatisfaction. This article is based
14 on several case studies and shows the importance of multi-objective modeling for complex problems which tend to
15 model real-life applications.
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24 The purpose of the present paper is to explain how the collection problem of refillable glass bottles can be turned
25 into an operational model with specific constraints and then solved using an integer linear programming (ILP) solver.
26 In Rabbani et al. (2020) the usage of an optimization tool, also based on ILP, has shown a real efficiency in solving
27 waste management issues, including routing, in the automotive industry. One advantage of ILP is that the model can
28 be tuned with a lot of parameters, and the solutions provided can be readily applied in practice. Additional analyses
29 based on environmental impacts are discussed since the initial motivation of setting up a bottle collection network,
30 with a view to reusing them instead of recycling or throwing them away, is to reduce the impact on the environment.
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37 This paper contributes to the literature by introducing a new transportation problem with several real-life con-
38 straints and a study of the environmental impact of the resulting network in order to take into account these aspects
39 in the decision-making process.
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43 **3. Problem description**

44 The Vehicle Routing Problem variant studied in this paper is inspired by a real-life application: a deposit and
45 reuse network for glass bottle collection in order to reduce waste. The logistic network of a classical reusable bottle
46 can be found in figure 1. This cycle can be modeled by a network with several places and actors. Starting at the
47 manufacturer (not shown in figure 1), the bottle is transported to the brewing company where it is filled and shipped
48 to the retail store. The consumer then buys the full bottle and is prompted to return it when empty thanks to a
49 deposit-refund system. Upon consumption of the beverage, the consumer returns the bottle and receives his deposit
50 back. The empty bottle placed in a crate is then collected from the store and transferred to a warehouse, waiting to be
51 transported to a washing station where it is sanitized. Finally, the bottle is sold back to the brewery where the cycle
52 starts over.
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61 The flows in this cycle can be managed by the organization proposing the reuse system. The major challenge
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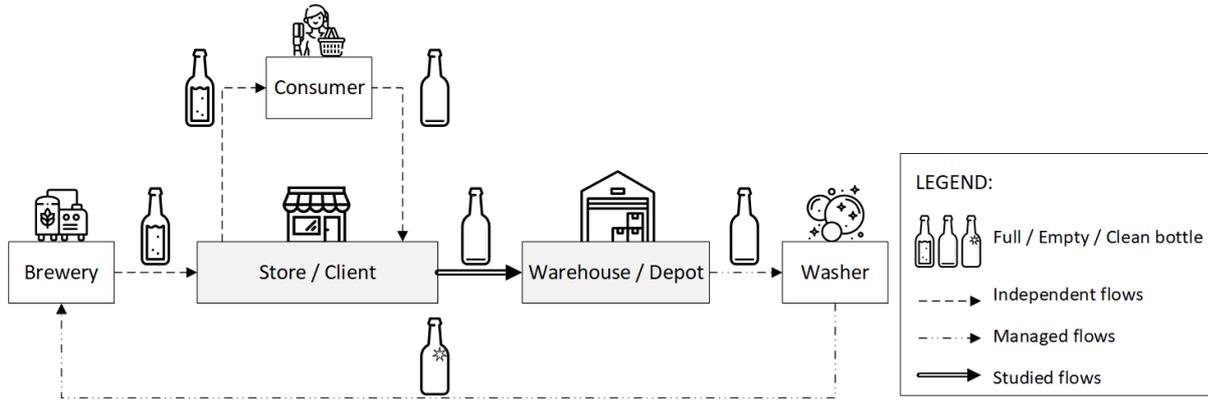


Figure 1: Simplified logistic network for reusable glass bottles

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faced in this network is the efficient collection of the glass bottles from the retail stores, and their transportation to a warehouse. The network studied in this paper is focused on the flow between the stores (considered as clients) and the warehouse (also named depot) in figure 1. In order to build a viable business model for an organization proposing the reuse system, the selling price of each clean bottle at the brewery should cover all the expenses including the purchase of empty bottles from the stores, the transportation, the storage, the washing, etc. On this hypothesis, an income can be calculated for each bottle collected (assuming it is necessarily resold). The optimization of the transportation part aims to ensure the competitiveness of the process in terms of routing cost while maximizing the store/client satisfaction (avoiding stock saturation).

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Following the taxonomy proposed by Braekers et al. (2016), the major scenario characteristics of the VRP modeled here can be defined as:

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- a known number of stops for the routing (number of clients),
- a load with splitting not allowed (only one visit for each client),
- a deterministic client service demand quantity,
- an onsite deterministic service time (dependent of the demand),
- a time horizon based on multi-period,
- a set of nodes with pickups requests.

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The routing problem considered in this paper is based on a one-to-many problem since all commodities are collected in multiple locations but bring back to a single depot with a single vehicle. Although the delivery requests are not explicitly modeled in our study, each collected crate full of empty bottles is exchanged for an empty one. In this sense, the nodes characteristics could be easily extended to simultaneous pickups and deliveries requests. The problem studied can also be generalized to other kinds of resources within a collection and transportation network, more characteristics are detailed in the next section.

3.1. Routing considerations

To facilitate the description of the routing problem and its modeling, basic assumptions are summarized in this section. The assumptions are classified to account for all the aspects considered.

Transported resources assumptions: All collected resources are identical and undamaged during the collection and the transportation. In this problem, the collected resources are crates, each crate contains 12 empty bottles to wash. Crates are collected only when full.

Network, node and depot assumptions: The network is comprised of several nodes modeling the stores where the resources are collected with a single depot, modelling the warehouse. It is assumed that the inventory level and remaining storage capacity of each store is known at the beginning of the time horizon and remains constant throughout the collection process. The depot is considered with an infinite capacity. Finally, all distances and time traveled are calculated with GPS data and the calculation of the geodesic distance between two nodes is stored in a symmetric matrix.

Vehicle and driving assumptions: A single vehicle (van, car or bicycle) is available/used per time horizon and is defined by a specific capacity and velocity. External interference such as accidents or traffic jams are not considered for this problem. The vehicle velocity remains constant, since all trips are made in an urban area.

Pickup operations assumptions: At the start of the time horizon, the stores communicate their current inventory level and can request a pickup. These stores will be prioritized and visited. Each collection route comprises of trips linking one node to another, starts and ends at the depot. Each client can be visited only once within the time horizon. Operation times are considered for all pickup operations: a fixed collection time is defined for all stores/clients and a variable collection time proportional to the number of crates collected is considered.

3.2. Specific considerations

Specific assumptions characterize the problem and underline our contributions.

Time slots: Time slots are predefined intervals of time in which the collection can take place. These time slots can be scheduled at any moment during the time horizon potentially leading to idle time (for the routing problem) as modeled by the hatched areas in figure 2b. This flexibility is not considered in a classical problem with a multi-period time horizon (figure 2a). However, in the case where the modeling of the problem is not indexed on time but on periods, time slots and periods can be modeled in the same way (figure 2c), our problem modeling is then similar to a classical multi-period time horizon routing problem. Moreover, in classical problem dealing with a time horizon with multi-period (Archetti et al., 2015), clients may require service on a specific period but the service may be postponed to another period by charging a penalty. This kind of assumption is also taken into account in our study with the criterion of client satisfaction.

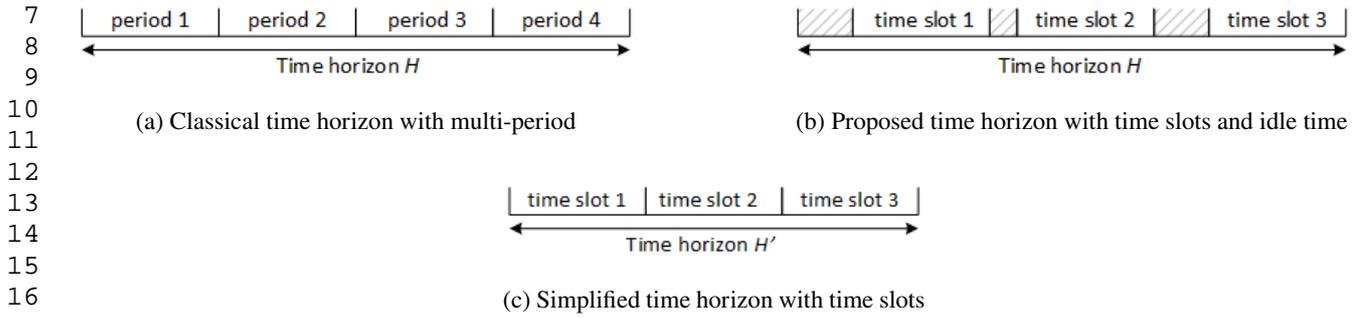


Figure 2: Representation of time horizons with multi-period vs. time slot

Moreover, contrarily to time windows, time slots are not bound by a starting and ending time but only defined by a duration. For example, for a traditional time window, a collection time is defined between 14:00 and 16:00, leading to a duration of 2 hours. For time slots, the start and end times are not fixed and only the duration of the action is determined, in this case 2 hours. This implies that as long as the operation is completed within a 2-hour frame, it is considered a success. Such time slots are especially useful for non-specialized employees that need to undertake multiple types of tasks including driving among others. Time slots offer the ability to propose a flexible scheduling.

In this paper we define in the time horizon a fixed number of time slots available, which are characterized by a time limit and a maximum number of routes. The maximum number of routes is defined in order to reduce the combination possibilities in the model and could be relaxed by defining the maximum number of routes equal to the total number of clients. Not all time slots nor routes will be necessarily used in the final solution. For example, in a weekly time horizon, the number of time slots can be equal to 5 with a duration of 8 hours if the organization responsible of the crates collection is willing to do the collection at most 5 times in the week (once per day for example) and during at most 8 hours each time. Other scheduling frameworks could be defined, for example 10 time slots of 2 hours each, if this policy is more suitable for the employee carrying out the bottle collection. The length of the time slots is also a limitation of the total duration of one route.

Clients satisfaction: Some considerations about the clients satisfaction are taken into account in order to avoid inventory saturation problems (too many crates in one store). Priority rules, expressed by penalty costs in the objective function, are proposed in order to serve some clients earlier in the cycle, depending on two criteria: (i) the request of a pickup by the store, (ii) the level of stock communicated by the store at the start of each time horizon.

Objective function: The implementation of multiple optimization objectives in the objective function is proposed and an analysis of the weights of all these objectives is detailed in section 5.3. The main aim of the problem is to minimize the time necessary for the collection of the glass bottles by taking into account for every route, travel time, as well as the fixed and variable collection time. Nevertheless, in order to model more precisely the real-life constraints, the prioritization of stores with the two criteria are considered as objectives, the idea is to schedule pri-

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7 ority stores at earlier time slots. The last objective tends to minimize the number of time slots used over the time
8 horizon.
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10 11 12 **4. Mathematical Model** 13

14 The approach proposed to solve the problem in this paper is based on an integer linear program (ILP). The math-
15 ematical model is provided in this section. In section 4.1, all notations are provided from set and data to variables.
16 In sections 4.2 and 4.3, the mathematical model is detailed with the objective function and all the constraints.
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19 20 **4.1. Notations** 21

22 All notations for sets, indices, parameters and variables can be found in table 1. The mathematical model proposed
23 is based on four sets: the set of nodes including all the clients and the depot \mathcal{N} (or without the depot \mathcal{N}^*), the set of
24 routes \mathcal{R} , the set of time slots \mathcal{T} , the set of routes in each time slot t denoted \mathcal{R}^t defined between $1 + (t - 1) \times NRT$
25 and $t \times NRT$ with NRT the number of routes in a time slot. Indices i, j are used to represent the nodes, index k is
26 used to model routes and index t is used to define time slots (table 1a). The depot would always take the subscript 0.
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29 Each client i is then defined by a capacity C_i , a pickup demand D_i and its coordinates X_i and Y_i . These data are
30 used to calculate the distance between i and j , $Dist_{ij}$. Using D_i and C_i , the filling rate τ_i is calculated ($\tau_i = D_i/C_i$).
31 A binary parameter is also introduced to represent the request of a pickup by a client (A_i). NC is the maximum
32 number of clients that could be visited during the time horizon (table 1b).
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35 The vehicle is defined by its capacity VC and its velocity VV . Using VV and $Dist_{ij}$, the travel time TT_{ij} between
36 nodes i and j is calculated. In addition to the travel time, other parameters are defined to simulate a real life collection
37 process. A fixed collection time FCT is applied at every location upon arrival and a collection time per crate CTC is
38 used to show that larger pickups require more time. NT defined the number of time slots allowed. Each time slot is
39 defined by a time limit TD and a maximum number of routes NRT . It is now possible to define the total number of
40 possible routes over the whole time horizon as $NR = NT \times NRT$.
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43 Weights are also introduced in the model with the aim of adjusting the objectives easily. Four values are defined
44 to represent different aspects of the problem. α is used for the routing cost, β is used to prioritize the clients depending
45 on τ_i , γ is used to prioritize clients that requested a pickup A_i and δ is used to minimize the number of time slots
46 used. Finally, M is a large enough number explained in the following sections.
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49 All variables (binary and integer) used in the model are detailed in table 1c, they allow to build the routes of the
50 vehicle, avoiding sub-tours, and the assignment of customers into time slots.
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\mathcal{N}	set of nodes including the depot, = $\{0, \dots, NC\}$	
\mathcal{N}^*	set of nodes without the depot, = $\{1, \dots, NC\}$	
\mathcal{R}	set of routes, = $\{1, \dots, NR\}$	$i, j \in \mathcal{N}$
\mathcal{T}	set of time slots, = $\{1, \dots, NT\}$	$k \in \mathcal{R}$
\mathcal{R}^t	set of routes in time slot t , = $\{1 + (t-1) \times NRT, \dots, t \times NRT\}$	$t \in \mathcal{T}$

(a) Set and Indices

NC	number of clients	VC	vehicle capacity in crates
(X_i, Y_i)	client coordinates	VV	vehicle velocity
$Dist_{ij}$	distance between clients i and j	FCT	fixed collection time for every client
TT_{ij}	travel time between clients i and j	CTC	collection time per crate.
D_i	current demand of client i	NT	number of time slots allowed
C_i	capacity of client i	NRT	number of routes per time slot allowed
τ_i	filling rate for client i , $\in [0, 1]$	NR	= $NT \times NRT$ total number of routes allowed
A_i	= 1 if client i request a pickup, else 0	TD	time slot duration

(b) Parameters / Data

x_{ijk}	= 1 if the trip between clients i and j belongs to route k , else 0	binary	$\llbracket 0 ; 1 \rrbracket$
m_k	= 1 if the route k exists, else 0	binary	$\llbracket 0 ; 1 \rrbracket$
p_t	= 1 if the time slot t is used, else 0	binary	$\llbracket 0 ; 1 \rrbracket$
y_i	load of the vehicle upon arrival at node i	integer	$\llbracket 0 ; VC \rrbracket$
z_i	time slot assigned to client i	integer	$\llbracket 0 ; NT \rrbracket$

(c) Decision Variables

Table 1

Notation used in the mathematical model

4.2. Objective function

The aim of this model is to minimize Z defined by four components (1), Z_1 the routing cost equal to the necessary time for the transportation and the collection of the crates, Z_2 the filling priority cost, Z_3 the requesting priority cost and Z_4 the total number of time slot used.

$$\text{Minimize } Z = \alpha Z_1 + \beta Z_2 + \gamma Z_3 + \delta Z_4 \quad (1)$$

$$\text{with } Z_1 = \sum_{i \in \mathcal{N}} \sum_{j \in \mathcal{N}} \sum_{k \in \mathcal{R}} (TT_{ij} + D_i \times CTC + FCT)x_{ijk} \quad (2)$$

$$Z_2 = \sum_{i \in \mathcal{N}^*} \tau_i(z_i - 1) \quad (3)$$

$$Z_3 = \sum_{i \in \mathcal{N}^*} A_i(z_i - 1) \quad (4)$$

$$Z_4 = \sum_{t \in \mathcal{T}} p_t \quad (5)$$

The first component Z_1 calculates the total time required to collect the clients. For every route, the travel time TT_{ij} , fixed collection time FCT and crate collection time $D_i \times CTC$ are summed together to give the total operation time. As the depot has a capacity and a demand of 0, only FCT was considered and represents the unloading time.

The second component Z_2 is the prioritization of clients that are nearing or have reached maximum capacity. This

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7 is done to compensate the assumption that D_i is constant during the time horizon. By summing the filling rate τ_i
8 multiplied by the time slot z_i allocated to node i , the model would try to assign clients with a high filling rate to the
9 first time slots. This would be achieved by pairing nodes of lower τ_i with higher z_i values and vice versa.

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12 The parameter Z_3 is the prioritization of clients that requested a pickup A_i . The model would place clients that did
13 not request a pickup at latter time slots.

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15 Finally, Z_4 simply aims at reducing the number of time slots used by directly asking the model to minimize the sum
16 of p_i values.

20 4.3. Constraints

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22 The constraints can be split into several categories. The first category deals with classical vehicle routing con-
23 straints. Constraints (6) and (7) ensure that each route starts and ends at the depot. Constraint (8) prevents routes
24 from having trips between clients with the same origin and destination. Constraints (9) ensure that a trip between two
25 clients i and j can be used at most once, whatever the route and the direction. Flow conservation is represented by
26 constraints (10), checking that the total number of trips leaving a node i are the same as the number of trips arriving
27 at a node i . For example, this constraint implies that $x_{123} = 1$ and $x_{253} = 1$ is possible (*i.e.* trip from 1 to 2 and trip
28 from 2 to 5 belong to route 3) but $x_{123} = 1$ and $x_{252} = 1$ would not be valid since client 2 cannot belong to route 3
29 and route 2.

$$39 \quad \sum_{j \in \mathcal{N}^*} x_{0jk} = m_k \quad \forall k \in \mathcal{R} \quad (6)$$

$$42 \quad \sum_{i \in \mathcal{N}^*} x_{i0k} = m_k \quad \forall k \in \mathcal{R} \quad (7)$$

$$45 \quad \sum_{k \in \mathcal{R}} \sum_{i \in \mathcal{N}} x_{iik} = 0 \quad (8)$$

$$48 \quad \sum_{k \in \mathcal{R}} x_{ijk} + \sum_{k \in \mathcal{R}} x_{jik} \leq 1 \quad \forall i \in \mathcal{N}^* \quad \forall j \in \mathcal{N}^* \quad (9)$$

$$51 \quad \sum_{j \in \mathcal{N}} x_{ijk} - \sum_{j \in \mathcal{N}} x_{jik} = 0 \quad \forall k \in \mathcal{R} \quad \forall i \in \mathcal{N}^* \quad (10)$$

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7 The second category deals with additional routing constraints taking into account the vehicle and permitting the
8 avoidance of partial routes. Concerning the vehicle capacity, three constraints are used. The first one simply imposes
9 that upon visiting the depot, the used capacity y_0 is equal to 0, constraints (11). This applies to both the start and
10 end of routes. Constraints (12) are used to define the vehicle capacity by imposing that all cumulative capacity y_i
11 should remain under the vehicle capacity VC . Therefore, the vehicle will not visit a new location if the value of
12 y_i exceeds VC . This forces the vehicle to return to the depot. Constraints (13) ensure that all collected crates D_i
13 from nodes visited over a single route should never exceed VC . Constraints (14) and (15) use the big M method to
14 ensure sub-route elimination by forcing the model to make sure that all nodes visited have an increasing y_i value.
15 In constraints (14), if a route x_{ijk} exists between nodes i and j for any value of k , then $y_i + D_j \geq y_j$. Similarly
16 the statement stands for (15) and the lower limit. If $x_{ijk} = 0$ then, with the big M, the constraints are deactivated.
17 To avoid cutting off valid solutions, the value of the big M needs to be chosen carefully. In constraints (14 and 15),
18 the lower bound of M is equal to $VC + \max_{i \in \mathcal{N}} D_i$ since the variable y_i models the quantity of resources load on a
19 vehicle with a limited capacity VC .

$$y_0 = 0 \quad (11)$$

$$y_i \leq VC \quad \forall i \in \mathcal{N} \quad (12)$$

$$\sum_{i \in \mathcal{N}} \sum_{j \in \mathcal{N}} x_{ijk} \times D_i \leq VC \quad \forall k \in \mathcal{R} \quad (13)$$

$$y_i + D_j + M(1 - \sum_{k \in \mathcal{R}} x_{ijk}) \geq y_j \quad \forall i \in \mathcal{N} \quad \forall j \in \mathcal{N}^* \quad (14)$$

$$y_i + D_j - M(1 - \sum_{k \in \mathcal{R}} x_{ijk}) \leq y_j \quad \forall i \in \mathcal{N} \quad \forall j \in \mathcal{N}^* \quad (15)$$

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The third category models the time slots, which are the novelty proposed. Time slots are defined with a duration and a maximum number of routes. If there were 2 time slots with 2 routes each, the total number of routes in the time horizon would be $RN = 4$. A time slot could be therefore defined as the summation of its routes. For example, using the previous example, the routes belonging to time slot $t = 1$ are routes $k = 1$ and $k = 2$, and the routes belonging to time slot $t = 2$ are routes $k = 3$ and $k = 4$. Therefore, $\sum_{k \in \mathcal{R}^t} x_{ijk}$ is used to define the routes that belonged to a single time slot t . Constraints (16) is used to define the maximum duration of the time horizon by adding all the time parameters associated with the routes contained in a single time slot t and imposing them to be lower than the allowable value TD . The process is repeated for all time slots. Constraints (17) are used to define the binary variable z_i that determines if client i belongs to route t . If a trip containing client i belongs to a route k , then the value of t is stored as a lower limit of z_i . In constraints (18), the binary variable p_t is defined using the big M method. If any trips present in the range of routes k that constituted the time slot t are explored, then the value of p_t have to be positive. Nevertheless, p_t is kept as low as possible since it appears in the objective function with a minimization. In the same trade of constraints (14 and 15), to avoid cutting of valid solutions, the lower bound of M that could be considered in constraint (18) is equal to $\mathcal{N} + NRT - 1$, modeling the maximum number of trips needed to collect all clients during a time slot.

$$\sum_{i \in \mathcal{N}} \sum_{j \in \mathcal{N}} \sum_{k \in \mathcal{R}^t} (TT_{ij} + FCT + CTC \times D_i) x_{ijk} \leq TD \quad \forall t \in \mathcal{T} \quad (16)$$

$$\sum_{j \in \mathcal{N}} \sum_{k \in \mathcal{R}^t} x_{ijk} \times t \leq z_i \quad \forall t \in \mathcal{T} \quad \forall i \in \mathcal{N}^* \quad (17)$$

$$\sum_{i \in \mathcal{N}} \sum_{j \in \mathcal{N}} \sum_{k \in \mathcal{R}^t} x_{ijk} \leq p_t \times M \quad \forall t \in \mathcal{T} \quad (18)$$

Finally, constraints (19), (20) and (21) establish the variable domains.

$$x_{ijk}, m_k, p_t \in \{0, 1\} \quad \forall i, j \in \mathcal{N}, k \in \mathcal{R} \quad (19)$$

$$0 \leq y_i \leq VC \quad \forall i \in \mathcal{N} \quad (20)$$

$$0 \leq z_i \leq NT \quad \forall i \in \mathcal{N} \quad (21)$$

The big M value of the whole model has to be chosen accordingly to all constraints, leading to the inequality $M \geq \max(VC + \max_{i \in \mathcal{N}} D_i, \mathcal{N} + NRT - 1)$. In order to illustrate the problem and the model proposed above, a small numerical example is detailed in the appendix with several solutions.

5. Results and Discussion

5.1. Data assumptions

Since this paper is based on a real-life application, some assumptions in the numerical model have been provided by the organization in charge of the collection of the crates full of refillable glass bottles. It is assumed that the collection time of one crate (*CTC*) takes 0.25 minute and that for each stop at a client, independently of the number of crates collected, a fixed time (*FCT*) of 4 minutes is required.

Moreover, additional economic and environmental indicators are provided in the next sections to measure the quality of the solution. In order to support this analysis, some financial information is necessary. Three financial aspects are taken into account:

1. the cost linked to the vehicle used,
2. the cost linked to the employee who drives and collects crates at each stop,
3. the income linked to the number of bottles collected.

The income can be calculated considering that each crate collected contains 12 bottles and that each collected bottle brings in 0.17 €. The cost linked to the employee is based on the gross hourly rate of the minimum wage in France, which is 10.15 € since January 1, 2020. This cost is calculated using the total time needed to collect all the stores visited within the time horizon. The cost linked to the vehicle is based on the rent cost provided by a French multinational automobile manufacturer (*Renault*, a). The chosen vehicles correspond to a light commercial vehicle (*Kangoo* model) or a classical van (*Trafic* model) for greater collection capacity. The respective rent prices of these two models are equal to 7€/hour and 9€/hour.

An environmental indicator based on CO_2 emissions is also provided in the next sections. The CO_2 emissions are assumed to be perfectly proportional to the distance traveled and dependent on the vehicle used. This indicator is calculated from the data obtained from the manufacturer's technical sheet with the CO_2 emissions in grams per kilometers (g/km) reported on the entire routing operation of the solution. The average CO_2 emissions for the *Kangoo* and the *Trafic* model are equal to 145 g/km and 188 g/km respectively (*Renault*, b).

5.2. Instances

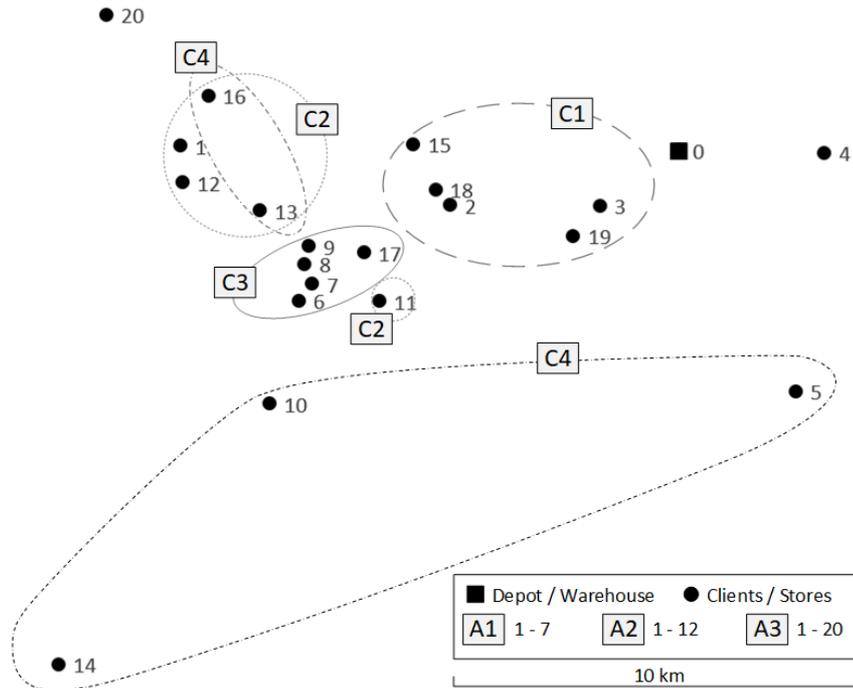
The case study includes 20 instances of crate collections for refillable glass bottles in the city of Lyon, France. Each instance has a weekly planning horizon starting on Monday and finishing on Friday. Only part of the time slots will be selected and associated with one or several routes. It is then possible to schedule each time slot selected at any time within the scheduling horizon, the relative order of time slots remains unchanged. This assumption provides flexibility to the solution.

The clients/nodes visited are separated into 4 clusters (C1, C2, C3 and C4) with a depot numbered 0 represented

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7 in figure 3. The first one (C1) is a cluster close to the depot represented by a square in figure 3, while C2 and C3
8 are located further away and more or less spread out. The fourth and final cluster C4 is a set of nodes located far
9 away from the depot and from each other. Other subsets of clients, named A1, A2 and A3, are used in instances of
10 different scales, and composed of clients number 1 to 7, 1 to 12 and 1 to 20 respectively. These subsets group several
11 clients from several clusters in order to propose a random distribution, approaching real demands and locations. The
12 distances in this network have been calculated as the geodesic distance using the GPS coordinates of the clients.

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41 **Figure 3:** Simplified network from real data with the depot and all clients divided in 4 clusters (C1, C2, C3, C4) and 3
42 subsets (A1, A2, A3) providing several datasets

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Three scales of instances are studied in the next sections. The small-scale instances (C1, C2, C3, C4 and A1) make
it possible to model the problem of scale corresponding to the launch of the company, the medium-scale instances
(A2) allow us to provide a solution for the current scale of the real-life problem and the large-scale instances (A3)
aim to study the scalability of the model and to anticipate the development of the organization by incorporating more
stores into the network.

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In the experiment, the datasets include the variation of the weights associated to each component of the objective
function (α , β , γ and δ), as well as the number of time slots considered in the time horizon (NT), the number of
routes authorized per time slot (NRT) and the time slots duration (TD).

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5.2.1. Small-scale instances

Eight small-scale instances are provided in table 2. The impact of stores location, with four clusters, is explored for the first four instances (S1–S4). In these instances, the number of clients is equal to 5, their demands D_i (capacities C_i) have been randomly generated between 3 and 10 (6 and 10) checking that $D_i \leq C_i$. No client has requested a pickup ($A_i = 0, \forall i \in \mathcal{N}$) and the time slots duration is large enough to allow the collection of all clients in the cluster. The analysis of the impact of the weights is also possible, thanks to instance S5 tested with various weights (see section 5.3). Two clients (1 and 5) have requested a pickup operation in this instance, symbolized by Y (Yes) in column A_i in table 2.

Instance S5 is a datum instance for the next small-scale instances since the client information is fixed. Several vehicle types (bicycle, car) are then explored in instances S6, S7 and S8 with various routes and time slot configurations.

id	Clients / Stores					Vehicle			Routes/Time Slots		
	NC	D_i	C_i	A_i	Location	Type	VV	VC	NT	NRT	TD
S1	5	{6,6,6,8,6}	{8,6,6,8,10}	N	C1	Car	30	40	2	2	420
S2	5	{6,6,4,3,6}	{7,7,7,10,7}	N	C2	Car	30	40	2	2	420
S3	5	{8,5,8,10,10}	{10,8,9,10,10}	N	C3	Car	30	40	2	2	420
S4	5	{5,4,3,3,6}	{9,7,10,7,7}	N	C4	Car	30	40	2	2	420
S5	7	{8,7,10,6,5,4,8}	10	Y	A1	Car	30	40	2	2	120
S6	7	{8,7,10,6,5,4,8}	10	Y	A1	Car	30	40	2	2	240
S7	7	{8,7,10,6,5,4,8}	10	Y	A1	Bicycle	10	10	5	2	120
S8	7	{8,7,10,6,5,4,8}	10	Y	A1	Bicycle	10	10	3	2	240

Table 2
Small-scale instances

5.2.2. Medium-scale instances

Eight medium-scale instances are provided in this section (table 3). This table contains the main medium-scale instances characteristics, including the average demand (\widetilde{D}_i) and the average capacity (\widetilde{C}_i) of all clients. Medium-scale instances include 12 clients located in the set of clients A2 modeled in figure 3. The first two instances M1 and M2 differ only in time slot duration TD , in order to measure the impact of this parameter. Then instances M3 to M8 have the same clients to collect with the same demands, capacity, request and location. The differences between instances rely on the vehicle assigned (car or van) and the route/time slot configurations. These datasets allow us to discuss the impact of the vehicle/route/time slot configurations on the solution quality and identify the best one.

5.2.3. Large-scale instances

Four large-scale instances are provided in this section (table 4). Large-scale instances include the 20 clients represented in figure 3. The instances are based on the same set of clients but allowing the use either of the car or the van (with a larger capacity), either with few or plenty of time slots and routes.

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id	Clients / Stores					Vehicle			Routes/Time Slots		
	NC	\widetilde{D}_i	\widetilde{C}_i	A_i	Location	Type	VV	VC	NT	NRT	TD
M1	12	6.33	10	Y {3,7,8,12}	A2	Car	30	40	2	2	120
M2	12	6.33	10	Y {3,7,8,12}	A2	Car	30	40	2	2	240
M3	12	9.15	11.23	Y {2,3,6,7,9,11,12}	A2	Car	30	40	4	2	240
M4	12	9.15	11.23	Y {2,3,6,7,9,11,12}	A2	Car	30	40	5	1	240
M5	12	9.15	11.23	Y {2,3,6,7,9,11,12}	A2	Car	30	40	2	2	420
M6	12	9.15	11.23	Y {2,3,6,7,9,11,12}	A2	Van	30	110	4	2	120
M7	12	9.15	11.23	Y {2,3,6,7,9,11,12}	A2	Van	30	110	4	2	420
M8	12	9.15	11.23	Y {2,3,6,7,9,11,12}	A2	Van	30	110	4	2	720

Table 3
Medium-scale instances

id	Clients / Stores					Vehicle			Routes/Time Slots		
	NC	\widetilde{D}_i	\widetilde{C}_i	A_i	Location	Type	VV	VC	NT	NRT	TD
L1	20	9.40	11.50	Y {2,3,6,7,9,11,12,19}	A3	Car	30	40	5	2	420
L2	20	9.40	11.50	Y {2,3,6,7,9,11,12,19}	A3	Van	30	110	2	2	420
L3	20	9.40	11.50	Y {2,3,6,7,9,11,12,19}	A3	Van	30	110	6	1	120
L4	20	9.40	11.50	Y {2,3,6,7,9,11,12,19}	A3	Van	30	110	6	1	720

Table 4
Large-scale instances

5.3. Numerical experiments

Numerical experiments of all instances are detailed below in tables 5, 6 and 7. All results tables provide the weights (α , β , γ and δ) associated with each instance, the best found solution BFS with the objective function Z composed of Z_1 , Z_2 , Z_3 , Z_4 , the total time TT to solve the instance, the gap to optimality provided by the solver and finally the order and assignment of each client to the routes and time slots. In the last column, the clients collected in the same route are in braces and routes in the same time slot are in brackets.

All experiments were carried out on Python program, using Spyder and a Windows 10 Pro operating system on a computer with an AMD Ryzen 5 3600 CPU at 3.6 GHz and 16 Gb of RAM. Gurobi 9.0 was the solver used to solve the integer linear program.

5.3.1. Small-scale results

In table 5, the experimental results of small-scale instances are detailed. All provided solutions are optimal and obtained in less than 0.5 second. The first four instances (S1, S2, S3 and S4) allow us to measure the impact of the clients' location on the BFS. Comparing the routing cost of S1 and S4 solutions, we logically note that cluster C1 (closest to the depot) offers the best results, whereas cluster C4 (with nodes located far away from the depot and from each other) has a routing cost twice as high. Also, comparing the routing cost of S2 and S3 solutions, it appears that the collection of clients in cluster C2 is more efficient than in cluster C3. However, looking at figure 3, the more compact cluster C3 might seem more suitable for collection. The difference in routing cost can be explained

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7 by the clients demand, with an average demand (\widetilde{D}_i) equal to 5 for S2 and to 8.2 for S3. With these observations and
8
9 knowing that the vehicle capacity $VC = 40$, we can make an assumption on the average number of clients per route,
10 which is equal to 8 for S2 while it is equal to 4.9 for S3. This gap, leading to a solution for S2 with only one route
11 whereas the S3 is composed of two routes with necessarily two returns to the depot and leading to higher costs.
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id	Weights				BFS					TT (s)	Gap (%)	Sets of routes in time slots
	α	β	γ	δ	Z	Z_1	Z_2	Z_3	Z_4			
S1	1	10	10	1	60.02	59.02	0	0	1	0.23	0	[[{3, 19, 2, 18, 15}]]
S2	1	10	10	1	82.33	81.33	0	0	1	0.34	0	[[{16, 1, 12, 13, 11}]]
S3	1	10	10	1	107.39	106.39	0	0	1	0.30	0	[[{9, 8, 6, 7}]] [[{17}]]
S4	1	10	10	1	127.79	126.79	0	0	1	0.27	0	[[{16, 13, 10, 14, 5}]]
S5	1	10	10	1	140.83	128.83	1.0	0	2	0.34	0	[[{4, 5, 7, 6, 1, 2}]] - [[{3}]]
S5 β	1	10 ³	10	1	651.28	139.28	0.5	1.0	2	0.20	0	[[{3, 7, 6, 1, 2}]] [[{4}]] - [[{5}]]
S5 γ	1	10	10 ³	1	140.83	128.83	1.0	0	2	0.33	0	[[{2, 1, 6, 7, 5, 4}]] - [[{3}]]
S5 $\beta\gamma$	1	10 ³	10 ³	1	741.28	139.28	0.6	0	2	0.45	0	[[{5}]] [[{2, 1, 6, 7, 3}]] - [[{4}]]
S6	1	10	10	1	128.29	127.29	0	0	1	0.32	0	[[{4, 5}]] [[{3, 7, 6, 1, 2}]]
S7	1	10	10	1	infeasible					-	-	-
S8	1	10	10	1	611.22	555.22	4.3	1.0	3	0.16	0	[[{6, 5}]] [[{3}]] - [[{1}]] [[{2}]] - [[{7}]] [[{4}]]
Avg										0.29	0	

28 **Table 5**
29 Small-scale instances results with routes solution
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32 Some comments on instance S5 with its variations can be noted. Instance S5 is tested with different weights of
33 the objective function. Instance S5 β has the same weights that instance S5 excepted weight β , we follow the same
34 reasoning for S5 γ and S5 $\beta\gamma$. The solutions of S5, S5 β , S5 γ and S5 $\beta\gamma$ can be compared to each other. Routing cost
35 Z_1 is minimized for S5, whereas in S5 β , more time is spent to collect all clients in order not to be too penalized by
36 the filling cost. In solution of S5 β client 3 is then integrated into the first time slot (table 5). The optimal solution of
37 S5 γ has the same cost as S5 since no penalties are applied to the requesting cost (clients 5 and 1 are integrated into
38 the first time slot). Then in the solution of S5 $\beta\gamma$, the filling cost and requesting cost are strongly taken into account,
39 nevertheless, the client with the smallest filling rate and not having made a request is still placed in the first time slot
40 (client 4). This phenomenon can be explained by the duration of the time slot that does not allow a route, including
41 client 4 without removing two other clients from the route. The best solution, minimizing Z_2 and Z_3 , schedules all
42 clients except client 4 in the first time slot.
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46 The weights modifications therefore generate significant alteration in the cost of the solutions and in particular change
47 the routes and the composition of the time slots. Finally, solution of instance S6 shows that all clients can be collected
48 within one time slot, by simply increasing the time slot duration. Instance S7 is infeasible since the time slot duration
49 is not adapted to the vehicle type: with the use of a bicycle (low velocity), the collection of client 1 for example is
50 not possible in less than 120 minutes (table 5). With twice the time and by reducing the number of routes available
51 per time slot (instance S7), the instance became feasible and a solution is provided.
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54 All these instances, with their results and solutions, perfectly illustrate the complexity of the problem. The results
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7 showed that, by including the assumption of the filling rate and the requests of the clients in the objective function,
8 the optimal solutions with the routes scheduled are modified. In all real-life applications, many criteria must be
9 taken into account, it is a question of finding the best compromise. Additional indicators will be developed in section
10 5.4, in order to compare all these solutions with numerical information interesting for the organization in charge of
11 collection.
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id	Weights			BFS					TT (s)	Gap (%)	Sets of routes in time slots
	α	$\beta = \gamma$	δ	Z	Z_1	Z_2	Z_3	Z_4			
M1	1	10	1	230.09	202.09	2.6	0	2	876	0	[[{1, 12, 9, 8, 7, 2}{3}] - [{11, 6, 10, 5, 4}]]
M2	1	10	1	190.55	189.55	0	0	1	744	0	[[{2, 7, 8, 9, 12, 1}{4, 5, 10, 6, 11, 3}]]
M2 α	1	0	0	189.55	189.55	4.0	3	2	1712	0	[[{3, 11, 6, 10, 5, 4}] - [{2, 7, 8, 9, 12, 1}]]
M3	1	10	1	264.62	237.62	2.5	0	2	3266	0	[[{1, 12, 6, 7}{3, 11, 8, 9, 2}] - [{10, 5, 4}]]
M4	1	10	1	343.69	238.69	8.2	2	3	102	0	[[{2, 9, 7, 11, 3}{1, 12, 8, 6}] - [{4, 5, 10}]]
M5	1	10	1	264.62	237.62	2.5	0	2	578	0	[[{2, 9, 8, 11, 3}{1, 12, 6, 7}] - [{10, 5, 4}]]
M6	1	10	1	226.09	199.09	2.5	0	2	695	0	[[{2, 1, 12, 9, 8, 7, 6, 11, 3}] - [{10, 5, 4}]]
M7	1	10	1	173.98	172.98	0	0	1	183	0	[[{5, 4}{2, 1, 12, 9, 8, 7, 6, 10, 11, 3}]]
M8	1	10	1	173.98	172.98	0	0	1	104	0	[[{5, 4}{2, 1, 12, 9, 8, 7, 6, 10, 11, 3}]]
Avg									1235	0	

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30 **Table 6**
31 Medium-scale instances results with routes solution

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35 **5.3.2. Medium-scale results**

36 In table 6, the experimental results of medium-scale instances are detailed. All provided solutions are optimal
37 and obtained in 20 minutes (1235 seconds) on average. The solutions obtained for instances M1 ($TD = 120$) and
38 M2 ($TD = 120$), proved that the optimal routes are modified and not just moved from one time slot to another by
39 increasing the time slot duration. In the solution of instance M2, with a time slot duration twice as big as M1, all
40 clients can be collected in one time slot. Consequently, Z_2 and Z_3 are zero and the routing cost can be minimized
41 ignoring all other costs. The solution obtained for M2 α confirms this reasoning with an objective function minimizing
42 only Z_1 due to the weights adopted. Therefore, the optimal route scheduling has a cost equal to 189.55. The routes
43 in solution of M2 and M2 α are symmetrical. Solutions detailed for instance M3 and M5 are interesting, since they
44 are identical with the same costs. Nevertheless, a notable difference in computation time can be observed. It appears
45 that increasing the time slot duration improves the speed of problem solving. The same behavior is observed for
46 solutions of instances M7 and M8.
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56 All these medium-scale instances solutions highlight the cost reduction obtained by using a vehicle with greater
57 capacity. By using a van, the number of routes necessary to collect all clients can be reduced, the total distance
58 traveled is therefore reduced as well as the number of returns to the depot with induced unloading activities.
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7 **5.3.3. Large-scale results**

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9 In table 7, the experimental results of large-scale instances are detailed. All provided solutions are not proven
10 to be optimal and sometimes reach the time limit of 3600 seconds with an average gap provided by the solver equal
11 to 9.7%. By testing instance L1 with two weights, we can notice that the computational time can be divided by at
12 least one hundred. The complexity of the problem studied in this paper is linked to the compromise between all the
13 costs with a complex routing problem. Looking at a part of the problem (for example, only Z_1 , or Z_2 and Z_3), this
14 becomes much easier to solve. Finally, the hypothesis proposed for medium-scale instances with the time slot dura-
15 tion is also investigated for large-scale instances with L3 and L4. When the duration of time slot is less restrictive,
16 the speed of convergence towards the optimal solution is better. The better performance observed is also due to the
17 vehicle capacity constraint, that limits the number of clients collected in one route, thus the length/duration of the
18 route, and finally the duration of time slot to which the route must be assigned.

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id	Weights				BFS					TT (s)	Gap (%)	Sets of routes in time slots
	α	β	γ	δ	Z	Z_1	Z_2	Z_3	Z_4			
L1	1	10	1		522.39	396.39	11.3	1	3	3600	24	[[{11, 6, 7, 9, 17}{18, 2, 19, 3}] – [[{14, 10, 8, 13}{12, 1, 20, 16}] – [[{15, 5, 4}] [[{3, 11, 7, 9, 2}{19, 6, 12, 20}] – [[{15, 17, 14, 4}{18, 13, 1, 10}] – [[{5, 8, 16}]
L1 $\beta\gamma$	1	10 ⁴	1		109457.79	454.79	10.9	0	3	36	0	[[{2, 18, 17, 9, 8, 13, 12, 1, 20, 16, 15}] {3, 19, 11, 7, 6, 10, 14, 5, 4}] [[{3, 19, 11, 6, 7, 8, 9, 13, 15, 18, 2}] [[{17, 10, 12, 1, 20, 16}] – [[{4, 5, 14}]]
L2	1	10	1		288.33	287.33	0	0	1	3600	11	[[{3, 19, 11, 6, 7, 9, 13, 12, 1, 20, 16, 2}] – ([15, 18, 17, 8, 10, 14, 5, 4])
L3	1	10	1		449.87	341.87	9.5	1	3	3600	19	[[{3, 19, 11, 6, 7, 8, 9, 13, 15, 18, 2}] [[{17, 10, 12, 1, 20, 16}] – [[{4, 5, 14}]]
L4	1	10	1		360.61	294.61	6.4	0	2	3600	4	[[{3, 19, 11, 6, 7, 9, 13, 12, 1, 20, 16, 2}] – ([15, 18, 17, 8, 10, 14, 5, 4])
L4 $\beta\gamma$	1	10 ⁴	1		59307.46	305.46	5.9	0	2	170	0	[[{3, 19, 2, 17, 11, 6, 7, 8, 9, 13, 12, 1, 20}] – [[{4, 5, 14, 10, 16, 15, 18}]]
Avg										2434	9.7	

43 **Table 7**
44 Large-scale instances results with routes solution

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47 Figure 4 illustrates the routes created by the ILP for four large-scale instances. The impact of the vehicle type (car
48 or van) on the routes construction can be clearly observed. The routes scheduled for L1 $\beta\gamma$ seems a lot more anarchic
49 than L1, with a lot of overlap and especially a lack of optimization of the distances traveled due to the weights chosen.
50 The two routes scheduled for L2 and L4 are very similar but looking at table 7, the routes provided for L2 have a
51 smaller traveling time for the collection of all clients.
52

53 All results detailed in this subsection underline three important phenomena that must be noticed to solve real-life
54 problems. We observe that the solving gets easier when considering:
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- 56 1. a significant weight on the penalties induced by client's request and filling rate;
- 57 2. a large number of time slots made up of a single route;

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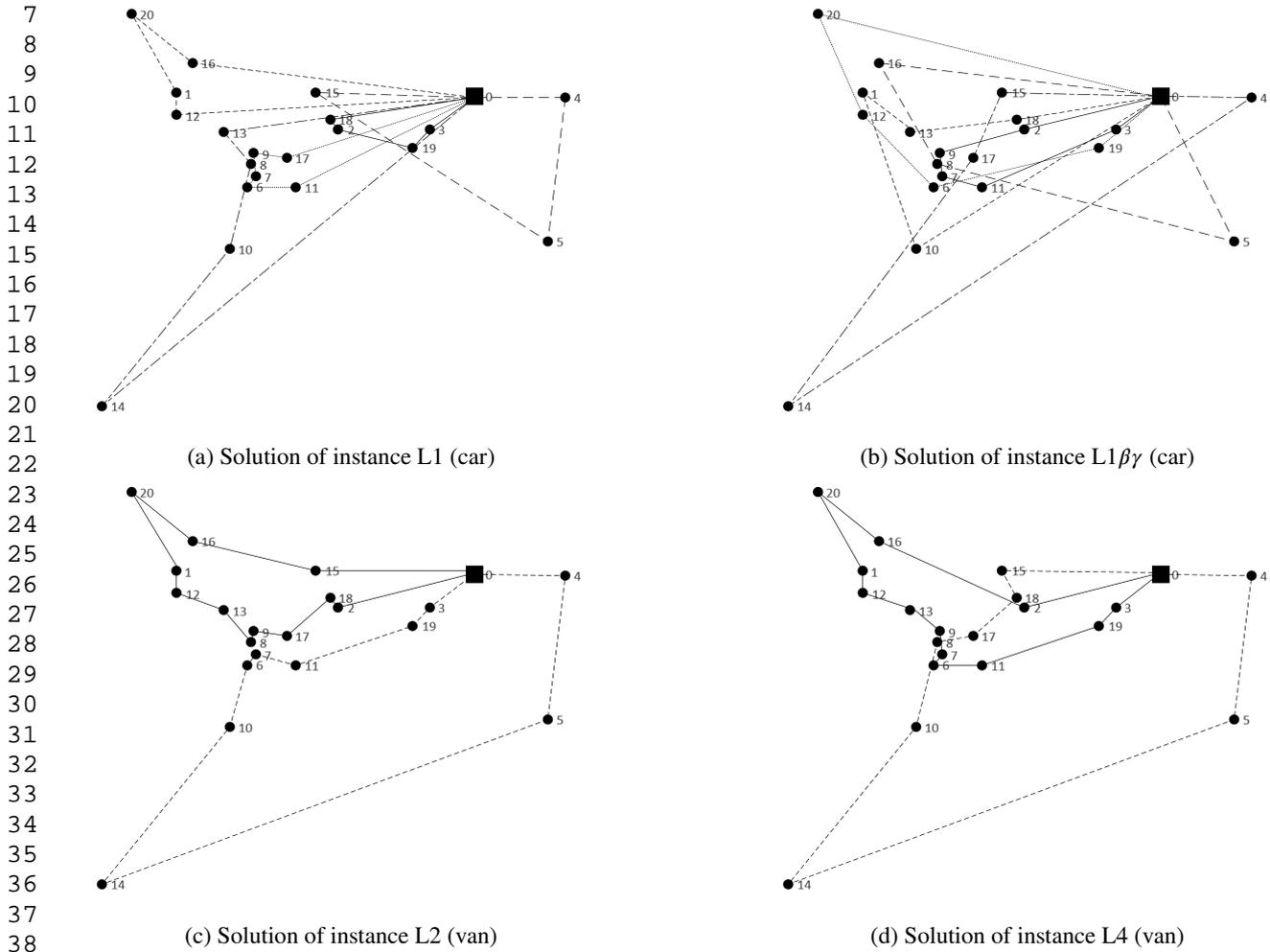


Figure 4: Graphical representation of several routing solution for large-scale instances

3. large time slot duration when the vehicle capacity sufficiently restricts the size of the routes.

Nevertheless, the study of real-life problems cannot be limited to the objective function cost in order to measure the quality of the solution and the routes configuration. The assessment of the solution quality through additional indicators appears to be necessary.

5.4. Discussion and analysis of the indicators

Tables 8, 9 and 10 provide additional information, named indicators, to characterize the solutions obtained. The numerical information related to the data assumption in order to calculate these indicators are detailed in section 5.1. The aim of these indicators is to compare the solutions with more concrete information, allowing the assessment of the quality of the solution according to different dimensions: operational, economic, environmental.

Each one of the tables contains the following information for each solution:

1. the value of the solution obtained (BFS);

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2. the cost linked to the routing, depending on the total duration of the routing multiplied by the cost of using the vehicle and the salary of the employee carrying out the routes with the collections;
3. the number of bottles collected calculated from the number of crates collected;
4. the income only linked to the number of bottles collected;
5. the balance sheet equal to the difference between the income and the routing cost;
6. the CO_2 emission related to the use of the vehicle;
7. the scheduling of routes in time slots or their number.

All these figures can also be used by the collecting organization, in order to define routing rules. For example, it will be possible to consider the collection of some clients into the next time horizon or to consider a reconfiguration of the network.

In table 8, the balance between financial cost and environmental impact can be observed. All the solutions obtained have a positive balance sheet even the one using a bicycle (S8) with a very high transportation time resulting in expensive routing costs. Among all these solutions, the worst is certainly associated to instance S4, with a strong carbon impact 7,07kg and a low financial profitability +0.39€. These results can be explained by two factors: a low number of bottles to collect and clients too far from each other. Two solutions are then possible for the collecting organization: waiting for the next time horizon to collect more bottles or rethinking the network with another location for the depot and/or other depots. Defining a minimum number of crates to be collected at each client during a time slot is currently being considered by the organization.

id	<i>BFS</i>	Routing cost (€)	Bottles collected	Income (€)	Balance sheet (€)	CO_2e (kg)	Sets of routes in time slots
S1	60.02	16.98	384	65.28	+48.30	1.96	[[3, 19, 2, 18, 15]]
S2	82.33	27.76	300	51.00	+23.24	3.70	[[16, 1, 12, 13, 11]]
S3	107.39	32.00	492	83.64	+51.64	4.94	[[16, 1, 12, 13, 11]]
S4	127.79	42.45	252	42.84	+0.39	7.07	[[16, 13, 10, 14, 5]]
S5	140.83	42.79	576	97.92	+55.12	5.86	[[4, 5, 7, 6, 1, 2]] - [[3]]
S5 β	651.28	44.56	576	97.92	+53.36	6.33	[[3, 7, 6, 1, 2]]{4} - [[5]]
S5 γ	140.83	42.79	576	97.92	+55.12	5.86	[[2, 1, 6, 7, 5, 4]] - [[3]]
S5 $\beta\gamma$	741.28	44.56	576	97.92	+53.36	6.33	[[5]]{2, 1, 6, 7, 3} - [[4]]
S6	128.29	42.53	576	97.92	+55.39	5.75	[[4, 5]]{3, 7, 6, 1, 2}
S8	611.22	93.92	576	97.92	+3.99	0.00	[[6, 5]]{3} - [[1]]{2} - [[7]]{4}

Table 8
Small-scale instances indicators and solutions

In table 9, three categories of instances/solutions can be observed according to the different costs calculated. The first category with M1, M2, M2 α shows us how close these three solutions are in terms of financial cost and environmental cost, although the routes scheduled are not identical. These results push us to favor instance M1 or M2 over M2 α since the request and the filling rate of clients are taken into account without deteriorating the quality

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7 of the solution. The second category with M3, M4, M5 and M6 also provides very close solutions in terms of cost
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9 and emissions. This makes it possible to highlight favorable properties of some instances. For example, M5 solution
10 takes into account the client requests with all of them in the first time slot thanks to a time slot duration $TD = 420$.
11
12 By increasing the duration of time slots the computational time decreases, it also allows the collection of more clients
13
14 with a request without negatively impacting the cost. Then, the third category with M7 and M8 allows us to notice a
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16 beneficial impact related to the use of a van for the collection of bottles, with a positive balance sheet and low CO_2
17
18 emissions. However, this impact is all the more important as the duration of the time slots is large, in order to allow
19
20 the collection of a large number of bottles and thus to use the capacity of the vehicle at best.
21
22

id	<i>BFS</i>	Routing cost (€)	Bottles collected	Income (€)	Balance sheet (€)	CO_2e (kg)	Sets of routes in time slots
M1	230.09	62.19	912	155.04	+92.85	8.92	[[{1, 12, 9, 8, 7, 2}{3}] - [{11, 6, 10, 5, 4}]]
M2	190.55	60.07	912	155.04	+94.97	8.30	[[{3, 5, 4}{7, 8, 9, 12, 1}] - [{11, 10, 6, 2}]]
M2 α	189.55	60.07	912	155.04	+94.97	8.30	[[{3, 11, 6, 10, 5, 4}] - [{2, 7, 8, 9, 12, 1}]]
M3	264.62	68.20	1344	228.48	+160.28	10.85	[[{1, 12, 6, 7}{3, 11, 8, 9, 2}] - [{10, 5, 4}]]
M4	343.70	68.38	1344	228.48	+160.10	10.93	[[{2, 9, 7, 11, 3}{1, 12, 8, 6}] - [{4, 5, 10}]]
M5	264.62	68.20	1344	228.48	+160.28	10.85	[[{2, 9, 8, 11, 3}{1, 12, 6, 7}] - [{10, 5, 4}]]
M6	226.09	69.68	1344	228.48	+158.80	10.82	[[{2, 1, 12, 9, 8, 7, 6, 11, 3}] - [{10, 5, 4}]]
M7	173.98	56.26	1344	228.48	+172.21	8.36	[[{5, 4}{2, 1, 12, 9, 8, 7, 6, 10, 11, 3}]]
M8	173.98	56.26	1344	228.48	+172.21	8.36	[[{5, 4}{2, 1, 12, 9, 8, 7, 6, 10, 11, 3}]]

34 **Table 9**
35 Medium-scale instances indicators and solutions
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38 Finally, in table 10 the study of all costs illustrates the links between the balance sheet and the CO_2 emission.
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40 Solution obtained for instance L1 $\beta\gamma$ provides the lowest balance sheet with the highest CO_2 emissions, and at the
41
42 opposite, L2 solution combines efficiency and durability with the best profitability for the organization and the lowest
43
44 CO_2 emissions. By analyzing the instances L2, some hypotheses and advice can be proposed to the organization, and
45
46 especially their choice could head towards the use of a van when a sufficient number of clients have to be collected
47
48 during the time horizon but with large size time slots.
49
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id	<i>BFS</i>	Routing cost (€)	Bottles collected	Income (€)	Balance sheet (€)	CO_2e (kg)	Number of time slots / total routes used
L1	522.39	116.06	2256	383.52	+267.46	18.08	3 / 5
L1 $\beta\gamma$	109447.38	132.94	2256	383.52	+250.58	22.32	3 / 5
L2	288.33	93.61	2256	383.52	+289.91	14.32	1 / 2
L3	449.87	111.83	2256	383.52	+271.69	19.07	3 / 3
L4	360.06	94.84	2256	383.52	+288.69	15.00	2 / 2
L4 $\beta\gamma$	59301.52	105.67	2256	383.52	+277.85	16.02	2 / 2

59 **Table 10**
60 Large-scale instances indicators and solutions
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7 The numerical experiments conducted in that section highlight the weights' impact on the solutions and costs,
8 proving the difficulty to manage such problems. By taking into account the stores request and filling rate, the routing
9 cost, the balance sheet and the CO_2 emissions are negatively impacted. The selected weights $\alpha = 1$, $\beta = 10$, $\gamma = 10$
10 and $\delta = 1$, seem to combine the minimization criteria related to time and transport costs while favoring clients with
11 a high filling rate and/or a collection request. The ILP provides the optimal solution for small and medium instances.
12 This size of problem corresponds to the demands and the functioning of the organization studied and for which a
13 decision support tool has been considered based on the model presented in this paper.
14
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16 It is also interesting to notice that the sub-optimal solutions, obtained for large-scale instances, offer better cost
17 indicators, than the optimal solutions obtained by simplifying the optimization problem using the weights (β, γ) only.
18 In reality obtaining an optimal solution is not a priority, a good solution with a low computing time is often preferable,
19 allowing more flexibility when unforeseen events arise. With an average time of less than 20 minutes, the proposed
20 method satisfied this criterion of speed, the goal being to solve the problem once or twice a week maximum for the
21 real-life application.
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24 Further discussion on the environmental impact of the collection system for the reuse of glass bottles could be
25 developed in order to justify such organization. However, despite the repeated transport of these bottles, the process
26 remains ecological because it avoids the manufacture of new bottles, which are much more environmentally costly
27 (Tua et al., 2020).
28
29

30 6. Conclusions

31 This paper studies a vehicle routing problem variant inspired by a real-life application of a deposit-refund collect-
32 ing system in France. The contribution of this paper is the model of a transportation problem with several real-life
33 constraints and a study of the environmental impact of such network, making it possible to take into account these
34 aspects in the decision-making process. This problem is modeled with several complex assumptions to fit with the
35 problem faced by our partner, including time constraints and inventory management. We propose an integer lin-
36 ear program based on a multi-objective approach. Comprehensive instances with various numbers and positions of
37 clients in the network, values of times slots parameters and vehicle types have been solved and the results analyzed.
38 Numerical experiments of the optimization model, with several weights associated with each component of the ob-
39 jective function, provide optimal solutions for small and medium-scale instances (corresponding to the current size
40 of the problem currently encountered). It is now possible for the decision maker to generate the optimal routing for
41 the collection over a time horizon. The analysis of performance indicators provided for these instances (transporta-
42 tion cost, environmental impact with CO_2 emission) also helps the decision maker to choose the best organization
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7 planning and the most adapted vehicle. For large-scale instances, the study of the solutions obtained highlights the
8 complexity of the problem. In order to solve larger-scale instances, the scheduling of several smaller time horizon
9 could be preferred in order to take into account a smaller proportion of clients, and thus break down the problem.
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11

12 Due to the complexity of this routing problem, with time-constraints and possible stochastic information, several
13 avenues of research remain to be explored. First, the time considerations could be extended to the opening time of
14 each store (clients) and thus, the model could be enriched with time-windows constraints. Secondly, in this study the
15 travel time from one point to another is deterministic. In order to create more accurate routes, the real-time travel
16 time taking into account the congestion and velocity of the vehicle could be considered. Another additional feature
17 could be to design the routes not with only one type of vehicle but with an heterogeneous fleet of vehicles available.
18 The model could then propose to the decision maker the most adapted vehicle according to the network of clients
19 to visit and the number of bottles to collect at each point. This would still be close to a real-life problem, where the
20 vehicle is not necessarily owned by the user, but might be rented and then changed from one day to another. With
21 the increase of the combinatorial complexity ILP could show some limits, especially with large-scale instances, and
22 the track of meta-heuristics or other non-exact methods should be explored.
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33 References

- 34
35 Allahviranloo, M., Chow, J.Y., 2019. A fractionally owned autonomous vehicle fleet sizing problem with time slot demand substitution effects.
36 Transportation Research Part C: Emerging Technologies 98, 37–53.
37
38 Angulo, A., Nachtmann, H., Waller, M., 2004. Supply chain information sharing in a vendor managed inventory partnership. Journal of Business
39 Logistics 25, 101–120.
40
41 Archetti, C., Christiansen, M., Grazia Speranza, M., 2018. Inventory routing with pickups and deliveries. European Journal of Operational
42 Research 268, 314–24.
43
44 Archetti, C., Jabali, O., Speranza, M.G., 2015. Multi-period vehicle routing problem with due dates. Computers & Operations Research 61,
45 122–134.
46
47 Braekers, K., Ramaekers, K., Van Nieuwenhuysse, I., 2016. The vehicle routing problem: State of the art classification and review. Computers &
48 Industrial Engineering 99, 300–313.
49
50 Bø, E., Hammervoll, T., Tvedt, K., 2013. Environmental impact of refillable vs. non-refillable plastic beverage bottles in norway. International
51 Journal of Environment and Sustainable Development 12, 379–395.
52
53 Cappanera, P., Requejo, C., Scutellà, M., 2020. Temporal constraints and device management for the skill vrp: Mathematical model and lower
54 bounding techniques. Computers & Operations Research 124.
55
56 Clarke, G., Wright, J.W., 1964. Scheduling of vehicles from a central depot to a number of delivery points. Operations Research 12, 568–582.
57 CM Consulting and Reloop, 2016. Deposit systems for one-way beverage containers: Global overview URL: <https://www.reloopplatform.org/wp-content/uploads/2017/05/BOOK-Deposit-Global-24May2017-for-Website.pdf>.
58
59 Coelho, P.M., Corona, B., ten Klooster, R., Worrell, E., 2020. Sustainability of reusable packaging—current situation and trends. Resources,
60 Conservation & Recycling: X 6, 100037.
61
62
63
64
65

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3
4

- 7 Dantzig, G.B., Ramser, J.H., 1959. The truck dispatching problem. *Management Science* 6, 80–91.
- 8
9 Desaulniers, G., Madsen, O., Ropke, S., 2014. The vehicle routing problem with time windows, in: Toth, P., Vigo, D. (Eds.), *Vehicle Routing: Problems, Methods, and Applications*. MOS-SIAM series on optimization, Philadelphia. volume 18, pp. 119–159.
- 10
11 European Commission, 2014. Attitudes of europeans towards waste management and resource efficiency URL: http://ec.europa.eu.docelec.insa-lyon.fr/public_opinion/flash/fl_388_en.pdf.
- 12
13
14 Ferani, E., Kuo, R., Dwiyantri, Y., 2020. A many-objective gradient evolution algorithm for solving a green vehicle routing problem with time windows and time dependency for perishable products. *Journal of Cleaner Production* 242.
- 15
16
17 Garaix, T., Gondran, M., Lacomme, P., Mura, E., Tchernev, N., 2018. Workforce scheduling linear programming formulation, in: 16th IFAC Symposium on Information Control Problems in Manufacturing INCOM 2018, IFAC-PapersOnLine, Bergamo, Italy. pp. 264–269.
- 18
19
20 Gendreau, M., Ghiani, G., Guerriero, E., 2015. Time-dependent routing problems: A review. *Computers & Operations Research* 67, 189–197.
- 21
22 Hannan, M., Begum, R., Al-Shetwi, A.Q., Ker, P., Al Mamun, M., Hussain, A., Basri, H., Mahlia, T., 2020. Waste collection route optimisation model for linking cost saving and emission reduction to achieve sustainable development goals. *Sustainable Cities and Society* 62.
- 23
24
25 Hwang, C.L., Masud, A., 1979. *Multiple objective decision making - methods and applications: A state-of-the-art survey*, Springer, Berlin.
- 26
27 Lenstra, J.K., Rinnooy Kan, A.H.G., 1981. Complexity of vehicle routing and scheduling problems. *Networks* 11, 221–227.
- 28
29 Mancini, S., 2017. A combined multistart random constructive heuristic and set partitioning based formulation for the vehicle routing problem with time dependent travel times. *Computers & Operations Research* 88, 290–296.
- 30
31 Markov, I., Bierlaire, M., Cordeau, J.F., Maknoon, Y., Varone, S., 2020. Waste collection inventory routing with non-stationary stochastic demands. *Computers & Operations Research* 113.
- 32
33
34 Mor, A., Speranza, M., 2020. Vehicle routing problems over time: a survey. *4OR* 18, 129–149.
- 35
36 Mostafayi Darmian, S., Moazzeni, S., Hvattum, L.M., 2020. Multi-objective sustainable location-districting for the collection of municipal solid waste: Two case studies. *Computers & Industrial Engineering* 150, 106965.
- 37
38 Rabbani, M., Amirhossein Sadati, S., Farrokhi-Asl, H., 2020. Incorporating location routing model and decision making techniques in industrial waste management: Application in the automotive industry. *Computers & Industrial Engineering* 148, 106692.
- 39
40
41 Renault, a. Hourly rental rates by vehicle category. <https://www.renault-mobility.com/tarifs>. Accessed: 2020-10-22.
- 42
43 Renault, b. Technical sheet of each renauld vehicle. <https://www.renault.fr>. Accessed: 2020-10-22.
- 44
45 Saint-Guillain, M., Paquay, C., Limbourg, S., 2020. Time-dependent stochastic vehicle routing problem with random requests: Application to online police patrol management in brussels. *European Journal of Operational Research* .
- 46
47 Savelsbergh, M., Song, J.H., 2008. An optimization algorithm for the inventory routing problem with continuous moves. *Computers & Operations Research* 35, 2266–2282.
- 48
49
50 Sherif, S.U., Asokan, P., Sasikumar, P., Mathiyazhagan, K., Jerald, J., 2021. Integrated optimization of transportation, inventory and vehicle routing with simultaneous pickup and delivery in two-echelon green supply chain network. *Journal of Cleaner Production* 287, 125434.
- 51
52
53 Simon, B., Amor, M.B., Földényi, R., 2016. Life cycle impact assessment of beverage packaging systems: Focus on the collection of post-consumer bottles. *Journal of Cleaner Production* 112, 238–248.
- 54
55
56 Strauss, A., Gülpınar, N., Zheng, Y., 2020. Dynamic pricing of flexible time slots for attended home delivery. *European Journal of Operational Research* .
- 57
58
59 Taslimi, M., Batta, R., Kwon, C., 2020. Medical waste collection considering transportation and storage risk. *Computers & Operations Research* 120.
- 60
61
62 Tua, C., Grosso, M., Rigamonti, L., 2020. Reusing glass bottles in italy: A life cycle assessment evaluation. *Procedia CIRP* 90, 192–197.
- 63
64
65

2
3
4

7 Zajac, S., Huber, S., 2020. Objectives and methods in multi-objective routing problems: a survey and classification scheme. *European Journal*
8 *of Operational Research* .
9

10 Zhou, G., Gu, Y., Wu, Y., Gong, Y., Mu, X., Han, H., Chang, T., 2020. A systematic review of the deposit-refund system for beverage packaging:
11 Operating mode, key parameter and development trend. *Journal of Cleaner Production* 251.
12
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15
16
17
18
19
20
21
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7 A. Appendix

8 A.1. Numerical example: instance example and solutions

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11 This appendix describes the main characteristics of the problem, with an example instance, and introduces several
12 solutions to illustrate the key points of the optimization problem. Several decisions are considered in the problem
13 studied in this paper: (i) define the shortest routes in order to visit the stores/clients with a demand, (ii) prioritize
14 stores/clients demands accordingly to their filling rate, in order to avoid inventory saturation, and (iii) prioritize
15 demands of clients requesting a pickup. A client with a high priority demand should be assigned at a time slot as
16 early as possible in the time horizon. To solve these multi-objective stakes, compromises must be done since, the
17 best solution for (i) is not the best solution for (ii) and (iii). In this section, an example using the mathematical model
18 notations is proposed, in order to highlight these difficulties.

19
20 The example proposed is characterized by an instance example summarized in tables 11 and 12. In this example
21 five clients need to be served (crate collection), they have various capacities and demand with a corresponding filling
22 rate (*Demand/Capacity*). Some client request a pickup in order to be collected as a priority in the time horizon
23 (column 4 in 11a). In this example instance, a cargo-bicycle is chosen among the 3 types of vehicles available (car,
24 van, cargo-bicycle) with a specific capacity and velocity. The driver in charge of the crate collection is available
25 only twice 120 minutes during the time horizon (one week). This information is detailed in table 11b with the
26 number and the duration of time slots. During each time slot they do not want to do more than 2 routes (a route is
27 a routing sequence starting and finishing at the depot, going through one or several clients). A route is also defined
28 by a duration depending on the riding time (calculated thanks to distances and vehicle velocity) and on collection
29 time (with a fixed and a variable cost linked to the number of crates to collect 11b). Figure 5 models a graph of the
30 network, with nodes (clients and the depot) and non-oriented arcs weighted with the distances (km) and the durations
31 (min) of each possible trip.

Node	Capacity (crate)	Demand (crate)	Pickup request	Filling rate
Depot 0	infinite	–	–	–
Client 1	8	3	no	0.375
Client 2	12	10	yes	0.833
Client 3	8	7	yes	0.875
Client 4	20	6	no	0.300
Client 5	10	8	no	0.800

(a) Capacity and demand of each node/client

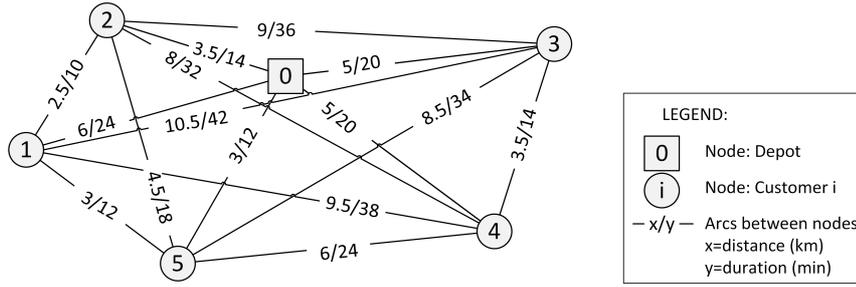
Vehicle capacity	14	crates
Vehicle velocity	15	km/h
Collection time per node	4	min
Collection time per crate	0.25	min
Number of time slots	2	
Time slots duration	120	min
Number of routes per time slot	2	
Total number of routes	4	

(b) General data (vehicle, route, time slot)

56 **Table 11**
57 Instance example for 5 clients, 1 depot and 1 vehicle (cargo-bicycle)

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17 **Figure 5:** Network with distances(km)/duration(min) between all clients and depot

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With all these data, several solutions can be computed. A solution is then defined by a set of routes assigned to the available time slots. The objective (A.1) of the problem is to minimize a function Z which takes into account four components detailed in 4.2. In this instance example, the weights $\alpha, \beta, \gamma, \delta$ are respectively equal to 1, 10, 10, 1.

$$\text{Minimize } Z = \alpha Z_1 + \beta Z_2 + \gamma Z_3 + \delta Z_4 \tag{A.1}$$

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A.1.1. Minimizing only the first component

$$\text{Minimize } Z = Z_1 \tag{A.2}$$

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We can first define several solutions of the routing problem, by measuring component Z_1 ($\alpha = 1$) only. The objective function is then given by (A.2). Three solutions are detailed in (table 12) in order to measure their quality for the objective (A.2).

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To build the first solution (table 12a), we choose to serve as soon as possible the two clients with a pickup request (clients 2 and 3), since their cumulative demand exceeds the vehicle capacity, they have to be split between two routes. Then the remaining clients have to be served within two routes, we choose to gather clients 1 and 5 given their distance. In the second solution (table 12b), we try to improve solution 1 by gathering clients 3 and 4, this modification decreases the routing time and improves the quality of service of client 4 (not measure in the objective function). Finally, in the third solution (table 12c), we decide to minimize the routing time to create the routes and then to divide them between the two time slots.

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In these solutions, the routes duration is obtained by summing the riding time and the collection time. For

Time Slot	Route	Clients Collected	Quantity Collected	Distance (km)	Route Duration (min)	Total Routes Duration (min)
Time Slot 1	Route 1.1	$0 \Rightarrow 3 \Rightarrow 0$	7	10	49.75	88.25
	Route 1.2	$0 \Rightarrow 2 \Rightarrow 0$	10	7	38.50	
Time Slot 2	Route 2.1	$0 \Rightarrow 5 \Rightarrow 1 \Rightarrow 0$	11	12	62.75	112.25
	Route 2.2	$0 \Rightarrow 4 \Rightarrow 0$	6	10	49.50	
Total			34	39	200.50	200.50

(a) Solution 1: good solution ($Z = 200.5$)

Time Slot	Route	Clients Collected	Quantity Collected	Distance (km)	Route Duration (min)	Total Routes Duration (min)
Time Slot 1	Route 1.1	$0 \Rightarrow 3 \Rightarrow 4 \Rightarrow 0$	13	13.5	69.25	107.75
	Route 1.2	$0 \Rightarrow 2 \Rightarrow 0$	10	7	38.50	
Time Slot 2	Route 2.1	$0 \Rightarrow 5 \Rightarrow 1 \Rightarrow 0$	11	12	62.75	62.75
Total			34	33.5	170.50	170.50

(b) Solution 2: better solution ($Z = 170.5$)

Time Slot	Route	Clients Collected	Quantity Collected	Distance (km)	Route Duration (min)	Total Routes Duration (min)
Time Slot 1	Route 1.1	$0 \Rightarrow 2 \Rightarrow 1 \Rightarrow 0$	13	13.5	63.25	97.25
	Route 1.2	$0 \Rightarrow 5 \Rightarrow 0$	8	6	34	
Time Slot 2	Route 2.1	$0 \Rightarrow 3 \Rightarrow 4 \Rightarrow 0$	13	13.5	69.25	69.25
Total			34	33	166.50	166.50

(c) Solution 3: optimal solution ($Z = 166.5$)**Table 12**

Solutions for the example data set by taking into account the total routing time (riding and collecting time)

example, in solution 2 (table 12b) route 1.1 going through clients 3 and 4 has a duration of 69.25 minutes. This route duration includes:

- the riding time equal to $20 + 14 + 20 = 54$ min,
- the fixed collection time equal to $3 \times 4 = 12$ min since two clients have to be collected and the vehicle has to be unloaded at the end of the route,
- the variable collection time depending on the number of crate equal to $13 \times 0.25 = 3.25$ min.

Solution 2 from 12b is also represented graphically in figure 6. This figure highlights the distribution of routes on the two time slots with the total riding time indicated on the depot, since the collections start on the depot at time 0 on figures 6a and 6b. These figures also illustrate the moves of the vehicle in time and space with arriving time and departure time at each node. The duration between arrival and departure time matches the collection time necessary for the client crates collection.

The three solutions described in table 12 are of good quality according to the objective function (A.2). The solution provided in table 12c is even the optimal solution, minimizing the duration of the routing. Nevertheless, in the problem studied in this paper, some additional components must be taken into account in order to control the risks associated with the inventory management.

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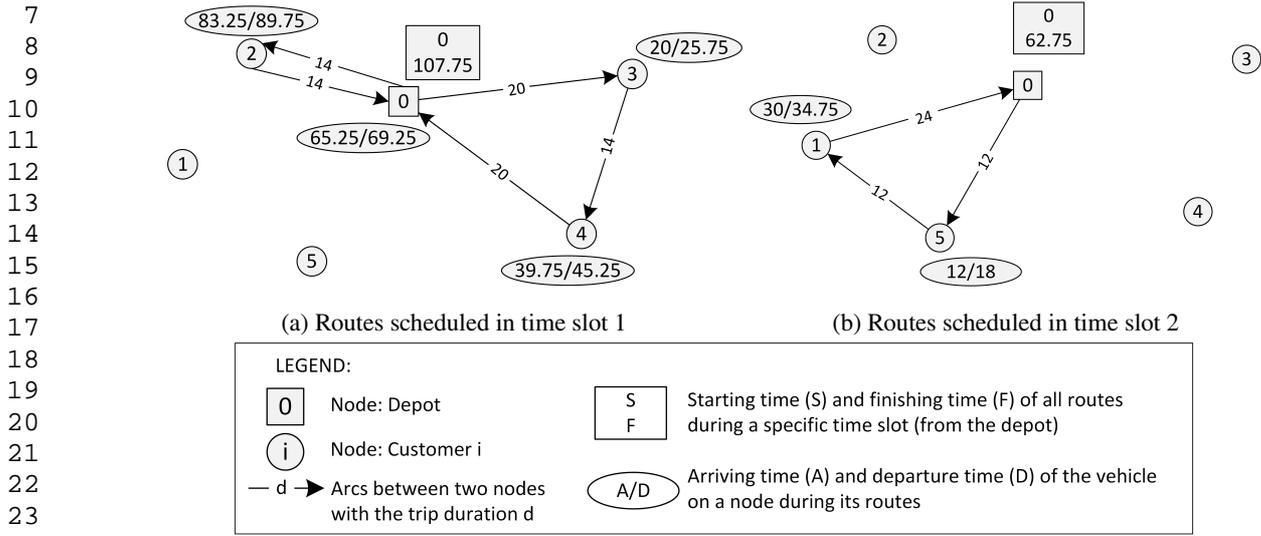


Figure 6: Route network of solution 2 with the two time slots and the duration including riding time and collecting time

A.1.2. Minimizing the multi-objective problem

$$\text{Minimize } Z = Z_1 + 10 Z_2 + 10 Z_3 + Z_4 \quad (\text{A.3})$$

For each of the solutions obtained in table 12, three additional costs are now considered to evaluate the objective function (A.3).

Filling priority cost ($10 Z_2$):

This cost favors the collection as soon as possible of the clients with a high filling rate. The problem being based on a minimization criterion, this cost increases with the filling rate and the time slot assigned to collect a customer. The cost is then calculated for each client with the formula: $10 \times \text{Filling rate} \times (\text{Time Slot Number} - 1)$ and summed for all clients. In solution 1, this cost is equal to 14.75 (table 13) since the clients 2 and 3 are collected during the first time slot ($\text{Time Slot Number} = 1$) and the three other clients (5, 1 and 4) are served in the second time slot ($\text{Time Slot Number} = 2$). Considering all the clients in alphanumeric order for solution 1, $10 Z_2 = 3.75 \times 1 + 8.33 \times 0 + 8.75 \times 0 + 3.00 \times 1 + 8.00 \times 1 = 14.75$.

Requesting priority cost ($10 Z_3$):

This cost favor the collection as soon as possible of the clients with pickup request. With the same reasoning as before, this cost can be quite simply equal to $10 \times (\text{Time Slot Number} - 1)$ for each client asking for a request. In

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7 solution 3, $10 Z_3 = 10$ (table 13) since the client 3 request a pickup but is served in the second time slot and not in
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12 *Time slot number (Z_4):*
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14 The number of time slots used is added in order to favor solutions with the minimum number of time slots. In all the
15 solutions, detailed above, the two time slots are used, $Z_4 = 2$.
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19 All the previous components are summed in order to obtain the total cost (Z) expressed in the last column of
20 table 13. Solution 2 (table 13) is the optimal solution of our problem based on the objective function (A.3). We
21 can first notice that an optimal solution for the objective (A.2) does not lead to an optimal solution for the global
22 objective function. Some discussions are proposed in section 5.3 about the additional costs with different weights
23 and then measure the impact of each one on the solutions properties.
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Solution	Routing Cost (riding and collecting time)	Filling Priority Cost	Requesting Priority Cost	Time Slot Number	Total Cost
Solution 1	200.50	14.75	0	2	217.25
Solution 2	170.50	11.75	0	2	*184.25
Solution 3	166.50	11.75	10	2	190.25

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35 **Table 13**
36 Detailed solutions for the example instance with the objective function (A.3) (*optimal solution)
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Declaration of Competing Interests

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.