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Design of Reverse Supply Chains under Uncertainty: the Lexicographic R_* criterion for exploring opportunities

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Abstract

Reverse Supply Chains (RSC) have been increasingly implemented in the last years to answer the new regulations aiming to manage the growing flow of solid waste generated by End-Of-Life products (EOL), as well as minimize their environmental impact. Furthermore, recent research has shown that the implementation of RSC also benefits to job creation, allows savings in raw materials, and creates income from the sales of re-manufactured products. On the other hand, designing RSC requires to deal with many sources of uncertainty, as the reverse flow of EOL products is often hard to predict. To model these uncertainties, we assume that a set of equally possible scenarios can be built by the decision maker. In previous models for scenario sets, the decision has been often influenced by the negative scenarios while neglecting existing opportunities. We propose a new risk/opportunity approach based on the R_* criterion to give more weight to positive scenarios in the decision making process. This criterion is used in order to distinguish zones of risk and opportunity and guide the decision making process accordingly to the existing zones. In this study, we develop a lexicographic approach for the consideration of existing scenarios and we propose two methods to compute the optimal solution for lexicographic R_* criterion: the first method is in the form of an algorithm, the second one is in the form of a Mixed-Integer Program (MIP). The performance of the developed approaches is demonstrated

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on a case study for a reverse facility location problem.

Keywords: Supply chain, Uncertainty, Reverse logistics

1. Introduction

Reverse Logistics (RL) manages the flows from consumers towards manufacturing facilities (in some cases to the point of origin) with the aim to recapture value of the used products [Rogers and Tibben-Lembke, 1998]. A Reverse Supply Chain (RSC) includes all facilities concerned by these reverse flows of End-Of-Life (EOL) products (i.e. collection centers, dismantling centers, recycling centers...).

In the actual context of awareness about the climate change and growing pollution, the implementation of RSC is actively encouraged by institutional actors in order to decrease the accumulation of solid waste all over the world. The abundant examples of recent initiatives include: G7 Declaration made in June 2015: “The G7 Alliance on Resource Efficiency promotes circular economy, refurbishment and recycling strategic actions to limit consumption of natural resources and reduce waste”, documents “Roadmap on the circular economy” (April 2015) and “Towards a circular economy: zero waste program for Europe” (July 2014) written by the European Commission, as well as the public consultation on the circular economy carried out by the European Union (EU) in August 2015.

Furthermore, the benefits of RSC have been demonstrated not only for environment but also for businesses. The McKinsey Center for Business and Environment particularly showed that developing circular economy may reduce the cost of primal resources in Europe by 25% in the next 10 years. In addition, [Toffel, 2004] highlighted the possibilities of new economic benefits arising from reusing materials or components, [de Brito and Dekker, 2003] and [Duman et al., 2016] pointed out due to the rising awareness of the consumers regarding environmental concerns, reprocessing EOL products has a positive effect on the public image of industries, which is indirectly another source of economic benefits. The study of [Perey et al., 2018] illustrates the positive economic effects of RL by reviewing several cases of organisations which changed their business models to integrate waste as a resource and took advantage of this change.

However, RSC design is a strategic and not straightforward decision. Generally, at the moment when the RSC is designed, precise information about

future amount of EOL products cannot be available [Govindan et al., 2017] and this lack of information should be integrated in the decision making process about RSC [Bing et al., 2016]. The assumptions made have a significant impact on the efficiency of RSC put in place. New models are needed in order to take into account the uncertain context of RSC related to the quantity but also quality of returned products, as well as a greater variety in flow sources, more complex cost functions, and new market opportunities [Almaktoom et al., 2016, Aras et al., 2015, Hanafi et al., 2007].

For this purpose, many approaches have been proposed in the literature. The most common ones are stochastic and robust optimization. Their advantages and drawbacks are discussed in details in Section 2.

The motivation for this research comes from the observation that none method in the literature was able to guide the decision maker in accordance to the psychological evidence that the choice of a solution is different regarding if the decision maker believes to be in a opportunistic zone or a risky zone of solutions [Grabisch, 2006].

In order to provide such a solution approach, we develop a new criterion which selects solutions differently regarding the zone of risks or opportunities. Such zones of solutions are defined in regard to the optimism of the decision maker and her/his risk aversion. The objective of this new approach is to make it possible for the decision maker to explore the opportunities while controlling the level of taken risks ¹.

This paper is organized as follows. An overview of the literature regarding existing methods to take into account uncertainty in RSC design and their drawbacks is given in Section 2. The proposed solution approach in order to overcome these drawbacks is developed in Section 3. It includes the formulation of the bipolar R_* criterion as well as the Lexicographic R_* algorithm and the lexicographic R_* MIP. In Section 4, both methods are applied to a generic facility location problem for a RSC in order to illustrate their behaviour and provide managerial insights for the decision makers. The conclusions of the study are presented in Section 5.

¹This paper is a extended version of a conference paper [Krug et al., 2019], including a new formulation of the lexicographic R_* criterion in the form of a Mixed Integer Program (MIP) as well as a deeper analysis of the results

2. Related literature

As aforementioned, it is well recognized that uncertainty is one of the most challenging factors to take into account when designing RSC [Govindan et al., 2017]. To deal with uncertainty, several approaches have been proposed in the literature on RSC design. The most commonly used approaches are stochastic and robust optimization.

With stochastic optimization, the decision maker needs to consider a probability distribution for the uncertain parameters and then apply a stochastic solution method [Amin et al., 2017, Ayvaz and Bolat, 2014, Ayvaz et al., 2015, Habibi et al., 2017, Zhang and Unnikrishnan, 2016, Soleimani and Govindan, 2014]. While providing the best expected value for the objective in average, this method offers no guarantee for the worst cases as the objective value can be very high in the best cases and compensates for very bad values in the worst cases. Furthermore, stochastic models have two main disadvantages: first, stochastic programs are often hard to solve computationally [Shapiro and Nemirovski, 2005]. Second, it can often be difficult to know the exact distribution of probability of the uncertain parameters, and, a too broad estimation can lead to inaccuracies in the solutions. Moreover, the most of RSC have not ever existed before, the data about the uncertain parameters is not always either available or reliable. As a result, stochastic optimization methods often suffer from the impossibility to get statistical data of product returns if the RCS has not existed at all before.

In order to overcome the absence of reliable historical data, it is common to define a discrete set of equally possible scenarios corresponding to expected realizations of the uncertain parameters. In order to take a decision over this set of scenarios, the commonly used approach is the robust optimization.

Robust approach needs no distribution of probability: the DM expects that one scenario $s \in S$ will occur, but does not know which one. The solution has to be taken without knowing which one will occur in reality later on. To avoid the consequences of the worst scenario, usually, in the literature, the *Maxmin* criterion is used [Ahuja, 1985, Bertsimas et al., 2011, Gorissen et al., 2015]. This criterion will try to find the most satisfying solution when the worst case happens [Pishvaei et al., 2011, Ramezani et al., 2013, De Rosa et al., 2013]. As this approach gives a lot of importance to the worst case, it is very pessimistic and often neglects the opportunities potentially occurring in better cases [Dubey et al., 2015].

In order to take into account the optimism of the decision maker, in

decision science, more sophisticated criteria has been discussed such as the Ordered Weighted Average (OWA) [Yager, 2004] and the Hurwicz criterion [Hurwicz, 1951]. These two criteria allow taking into account better case scenarios, but they are compensatory. The OWA criterion is applied by ranking all the costs of the scenarios from the best one to the worst one, and then calculating a weighted average on the ranked costs where the weights are based on the position of ranking. The Hurwicz criterion is a particular case of the OWA when the weight for the best case scenario is α , the weight for the worst case scenario is $1 - \alpha$ and all the weights for other scenarios are 0. As a consequence, a good case scenario mitigates the impact of a bad case one and reciprocally, i.e. it is possible that such a criterion selects as the best one a solution which will be very good for one scenario and very bad for another one. According to their definition, such a compensation remains always possible and the decision maker cannot make a compromise between the level of risk taken and the corresponding opportunities found [Krug et al., 2020]. As a result, it is impossible to control the level of risk and search for opportunities at the same time.

We propose in Table 1 a recap of the criteria presented above, their advantages and their drawbacks:

Method	Advantages	Drawbacks
<i>Stochastic optimization</i>	Best expected value in average	Need for historical data Often hard to solve computationally
<i>Maxmin</i>	No need for historical data Risk-resistant	No search for opportunities
<i>OWA</i>	No need for historical data Takes into account the DM's optimism	Compensatory No risk control
<i>Hurwicz</i>	No need for historical data Takes into account the DM's optimism	Compensatory No risk control
<i>Our proposal</i>	No need for historical data Takes into account the DM's optimism Possibility to control taken risks	

Table 1: Comparison of existing criteria

We can conclude that the models found in the literature to take uncertainty into account in a context of complete ignorance of the DM are risk-oriented and never distinguish hazard from opportunity. In order to offer

to the decision makers an approach that suits better their natural behavior, in this paper, we introduce R_* criterion and $LexiR_*$ criterion for the RSC design problem. This criterion reflects the following behaviour: the DM is pessimistic in an hazardous zone and optimistic in an opportunity zone. The definition of such zone depends on the level of optimism of the decision maker and her/his risk aversion. The mathematical modelling of such a criterion is proposed in the next section.

3. Solution approach

The criterion R_* we propose to use belongs to the class of bipolar operators [Grabisch, 2006]. Such operators include a neutral value (here denoted as e). All the values above the e are considered as "good" scores and all the values below as "bad" scores. We can thus consider that the interval $] - +\infty, e]$ is an interval of hazard while the interval $]e, +\infty[$ is an interval of opportunity. The neutral value e corresponds to the minimum profit expected by the DM in the considered uncertain context, in our case, the design of RSC. The criterion R_* uses the minimum operator in the risky area and the maximum operator in the opportunity area. The formal definition of this criterion is given below:

Let $F(x, s)$ be the evaluation of the objective function for solution x over a scenario $s \in S$,

$$R_*((F(x, s))_{s \in S}, e) = \begin{cases} \min_{s \in S} F(x, s) & \text{if } \exists F(x, s) \leq e \\ \max_{s \in S} F(x, s) & \text{otherwise} \end{cases} \quad (1)$$

It should be noted that by its definition this criterion can be seen as a generalization of the robust one, since if e is equal to the value of the worst case or above, the robust solution will be selected. To provide a better understanding of how this operator aggregates possible solutions, an illustrative example is given here below.

Example 1. *Let $s_1, s_2 \in S$ be two possible return amount scenarios. Let F be the function of profit that we aim to maximize. We denote $F(x, s_1)$ (resp $F(x, s_2)$) the profit gained with solution x when s_1 (resp s_2) occurs. The solution space of the problem is shown in Fig.1.*

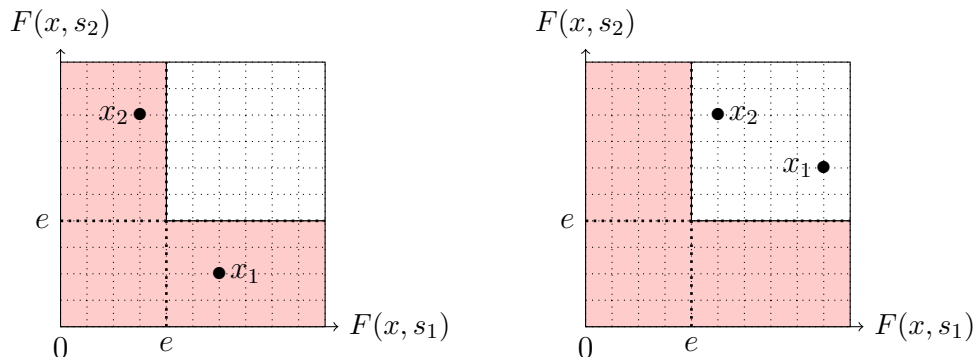


Figure 1: R_* criterion solution space

Two solutions x_1 and x_2 are presented in this example, their coordinates give the profit obtained for scenario s_1 and s_2 respectively, e defines the profit expected by the decision maker from the created RSC. Based on this value, the risky area is colored in red oppositely to the opportunistic area kept in white. In the first case (left figure), both solutions are in the risky area. The R_* operator therefore selects the solution with the highest minimum profit regarding the two scenarios (or robust solution). Here, we have $F(x_1, s_1) < F(x_2, s_2)$, thus, solution x_2 is selected. In the second case (right figure), both solutions are in the opportunistic area. Hence, the R_* operator selects the highest maximum profit over the two scenarios (or optimistic solution). Here, we have $F(x_1, s_1) > F(x_2, s_2)$: solution x_1 is selected.

Through its characteristics, R_* brings to the DM the possibility to choose a threshold of robustness while still searching for opportunities in opportunistic areas. In order to make this search even more exhaustive, we develop a lexicographic R_* approach denoted in this paper as $LexiR_*$.

Originally, R_* only pays attention to the best and worst cases profits and oversees all cases in between: two solutions with the same best case profit and the same worst case profit are considered equivalent even if they give different profits for other scenarios. This comparison scenario by scenario can be realized with $LexiR_*$ criterion which uses a lexicographic minimum (*leximin*) in the risky area and a lexicographic maximum (*leximax*) in the opportunistic area. *Leximin* operator selects a solution by maximizing the one with the highest minimum profit overall scenarios. If multiple solutions exist with the same minimum profit, the criterion is applied to maximize the

second minimum profit, then the third, etc... until one solution dominates the others. *Leximax* operator works in the same manner but maximizing the highest profit, then the second highest profit, then the third, etc...([Ogryczak, 1997, Ogryczak and Śliwiński, 2003]).

The formal definitions of *leximax* and *leximin* are given below ([Yager, 1997]):

Definition 1: Let $N = 1, \dots, n$ be the number of possible scenarios. Let $x = (F(x, 1), \dots, F(x, n))$ and $y = (F(y, 1), \dots, F(y, n))$ be two possible solutions. The *Leximax* order (noted $<_{leximax}$) is defined as:

$$x <_{leximax} y \iff \exists k \geq 1 \text{ such as } \forall i < k, x_i = y_i \text{ and } x_k < y_k$$

$$x =_{leximax} y \text{ if } x_i = y_i \forall i = 1, \dots, n$$

In the same way, the *Leximin* order (noted $<_{leximin}$) is defined as:

$$x <_{leximin} y \iff \exists k \geq 1 \text{ such as } \forall i < k, x_i = y_i \text{ and } x_k > y_k$$

$$x =_{leximin} y \text{ if } x_i = y_i \forall i = 1, \dots, n$$

The *LexiR** criterion can be written formally as:
 Be $S^- = \{s \in S | F(x, s) \leq e\}$ and $S^+ = S \setminus S^-$:

$$LexiR_*((F(x, s))_{s \in S}, e) = \begin{cases} Leximin_{s \in S^-} F(x, s) \\ \text{Then} \\ Leximax_{s \in S^+} F(x, s) \end{cases} \quad (2)$$

An example of resolution with *LexiR** criterion is presented in Example 2.

Example 2. Let $S = \{s_1, s_2, s_3, s_4\}$ be a discrete set of scenarios and F represent the profit obtained for each scenario. Let us consider two solutions x and y which provide the following profits for 4 considered scenarios: $F_s^x = (2, 3, 8, 10)$ and $F_s^y = (2, 5, 7, 10)$. We can observe that solutions x and y are considered equal by the R_* criterion since they offer the same best case

profit and worst case profit. However, they are distinguished with $LexiR_*$ criterion. We consider 3 different cases of the DM optimism reflected by the risk threshold e .

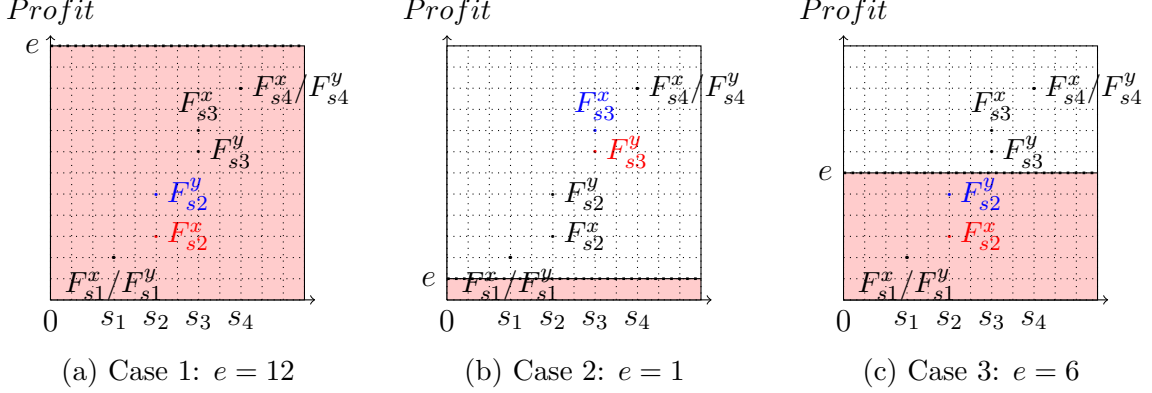


Figure 2: Resolution with $LexiR_*$ criterion

1. *Case 1: The DM expects the profit $e = 12$ (see Fig.2 (a)). All possible profit values for considered scenarios are in the risky area for both solutions. Based on this observation and in accordance with the definition of $LexiR_*$ approach, leximin criterion is applied to select the best solution. First, the worst case scenario (here s_1) is considered. We have $F_{s_1}^x = F_{s_1}^y$, therefore x and y are not differentiated. Then the second worst case scenario is considered (here s_2). We have $F_{s_2}^y > F_{s_2}^x$, therefore solution y is preferred to x by leximin.*
2. *Case 2: The DM expects the profit equal to $e = 1$ (see Fig.2 (b)). All possible profit values for considered scenarios are in the opportunistic area for both solutions. Based on this observation and in accordance with the definition of $LexiR_*$ approach, leximax criterion is thus applied to select the final solution. For the best case scenario (here s_4), we have $F_{s_4}^x = F_{s_4}^y$, therefore x and y are not differentiated. For the second best case scenario (here s_3), we have $F_{s_3}^x > F_{s_3}^y$, therefore solution x is preferred to y .*
3. *Case 3: The DM expects the profit $e = 6$ (see Fig.2 (c)). Two profit values are in the opportunistic area and two in the risky area for both*

solutions. Based on this observation and in accordance with the definition of $LexiR_*$ approach, firstly, the risky values are compared with $leximin$ criterion, as in Case 1. Therefore, solution y is selected.

3.1. Lexicographic R_* Algorithm

In this section, we present the formal mathematical formulation for $LexiR_*$ approach for RSC design according to its definition in the previous section.

First, $LexiR_*$ criterion is applied taking into account all possible scenarios. The obtained values of profit found at this step for all scenarios are compared to the threshold e . If all of them are above the threshold, then the algorithm continues with the application of $leximax$ criterion to select the final solution. Otherwise, if for some scenarios, the objective value found is below e , the worst case scenario is expressed as a constraint for further resolution. The R_* criterion is then re-applied on the reduced set of scenarios taking into account a new constraint for the worst case. It means that among all solutions having this value of the profit in the worst case, we are looking for solutions providing the best opportunities in all other cases. As before, if the objective value found is above e , a $leximax$ criterion is applied to select the final solution and on the contrary if it is below e , it is recorded and then the second worst case scenario is expressed as a constraint. This process continues until all scenarios have been treated.

Notation	Description	Notation	Description
R_k	Linearization of min	Y_s	Binary variable
r_k	Linearization of max	δ_s^k	Binary variable
k	Number of iteration	γ_s^k	Binary variable
e	Risk threshold	M	Big value

Table 2: Notations used in Algorithm 1.

Algorithm 1.

- *Step 0:*
 $k \leftarrow 0, k' \leftarrow 0, N \leftarrow \{s_1, \dots, s_n\}$.
- *Step 1:*

Solve Model (3)

$$\begin{aligned}
& \max R_k + r_k && (3) \\
& S.t \\
& (a) \quad R_k \leq F((x, y), s) && \forall s \in N, \\
& (b) \quad R_k \leq e \\
& (c) \quad F((x, y), s) \geq -M * Y_s + e(1 - Y_s) && \forall s \in N \\
& (d) \quad F((x, y), s) \leq e * Y_s + (1 - Y_s)M && \forall s \in N \\
& (e) \quad r_k \leq (1 - Y_s)M && \forall s \in N \\
& (f) \quad \sum_{s=1}^S \delta_s^k = 1 \\
& (g) \quad r_k \leq F((x, y), s) + (1 - \delta_s^k)M && \forall s \in N
\end{aligned}$$

- *Step 2:*
 If $R_k + r_k \leq e$
 then $R_k = R_k^*$, $k = k + 1$ and go to *Step 3*.
 Otherwise,
 define $R_k = e$, $r_1 = r_k^*$, $k' = 2$ and go to *Step 5*.

- *Step 3:*
 Solve Model (4)

$$\begin{aligned}
& \max R_k + r_k && (4) \\
& S.t \\
& (a) \quad R_i \leq F((x, y), s) + M(1 - \gamma_s^i) && \forall s \in N, i \in 1..k \\
& (b) \quad \sum_{s=1}^S \gamma_s^i = S - i && \forall i \in 1..k \\
& (c) \quad R_k \leq e \\
& (d) \quad F((x, y), s) \geq -M * Y_s + e(1 - Y_s) && \forall s \in N \\
& (e) \quad F((x, y), s) \leq e * Y_s + (1 - Y_s)M && \forall s \in N \\
& (f) \quad r_k \leq (1 - Y_s)M && \forall s \in N \\
& (g) \quad \sum_{s=1}^S \delta_s^k = 1 \\
& (h) \quad r_k \leq F((x, y), s) + (1 - \delta_s^k) * M && \forall s \in N
\end{aligned}$$

- *Step 4:*
 If $S - k = 0$ then go to *Step 7*.
 Otherwise:

If $R_k \leq e$, then $R_k = R_k^*$, $k = k + 1$ and return to Step 3.
 If $R_k + r_k > e$, then $R_k = e$, $r_1 = r_k^*$ define $k' = 2$ and go to Step 5.

- *Step 5:*
Solve Model (5)

$$\begin{aligned}
 & \max r_{k'} && (5) \\
 & \text{S.t} \\
 & (a) \quad R_i \leq F((x, y), s) + M(1 - \gamma_s^i) \quad \forall s \in N, \forall i \in 1..k \\
 & (b) \quad \sum_{s=1}^S \gamma_s^i = S - i, \quad \forall i \in 1..k \\
 & (c) \quad r_i \leq F((x, y), s) + M(1 - \delta_s^i) \quad \forall s \in N, \forall i \in 1..k' \\
 & (d) \quad \sum_{s=1}^S \delta_s^{k'} = k'
 \end{aligned}$$

- *Step 6:*
If $S - k - k' - 1 = 0$ then go to Step 7.
Otherwise $r_{k'} = r_{k'}^*$, $k' = k' + 1$, and go to Step 5.

- *Step 7:*
Stop.

The steps of Algorithm 1 can be detailed in the following manner:

Step 0: We define k as the number of iterations performed in the risky area, k' as the number of iterations performed in the opportunistic area and $N = s_1, \dots, s_n$ is the discrete set of scenarios considered. At the start of the algorithm, k and k' are initialized by 0.

Step 1: The decision problem is solved using R_* criterion through the resolution of Model 3 (only the best and worst case scenario are taken into account). This model is a MIP. Constraints (b) to (e) differentiate the case where the robust solution is selected (i.e there exist no possible solution for which all objective values are above e overall scenarios) and the case where the opportunistic solution is selected (otherwise). In the former case, constraints (a) help to maximize the minimum objective value. In the latter case, constraints (f) and (g) help to maximize the maximum objective value.

Step 2: The objective value found at Step 1 is analysed if it lower than e , then k is implemented and the process goes to Step 3. If it is higher than e , then k' is implemented and the process goes to Step 5.

Step 3: Model 4 is used in order to maximize the second worst profit, then the third and so on depending of the number k of iterations previously solved.

Step 4: This step checks if the value of the objective found through Model 4 is lower or higher than e . If this objective value is lower, and if the number k of iteration is lower than the number of scenario (namely S), then k is implemented and Model 4 is solved one more time. If $S = k$, then all scenarios have been treated and Algorithm proceeds to Step 7. Otherwise, if the objective value is higher than e , k' is implemented and Algorithm goes to Step 5.

Step 5: Model 5 is used to maximize the maximum profit, then the second one and so on depending on the number of iterations k' previously performed.

Step 6: This Step checks if all scenarios have been treated, if not, Step 5 is repeated. Otherwise, the algorithm goes to Step 7.

Step 7: The algorithm stops.

3.2. Lexicographic R_* MIP: approximated method

Algorithm 1 offers an exact resolution for the application of $LexiR_*$ approach. However, it includes the resolution of 3 different MIP that can require relatively long solution time. If the solution time is relatively limited, the following approximate compact method in the form of a single MIP can be used instead to implement $LexiR_*$ approach. We denote it as MIP- $LexiR_*$. Its resolution provides an approximated solution and cannot guarantee the optimum, but the solution time is reduced significantly.

Notation	Description	Notation	Description
R_k	Linearization of min	Y_s	Binary variable
r_k	Linearization of max	$\delta_{s,s'}$	Binary variable
k	Number of iteration	γ_s^k	Binary variable
A_k	Binary variable	μ_k	Binary variable
$C_{s,k}$	Binary variable	$D_{s,k}$	Binary variable
e	Risk threshold	M	Big value

Table 3: Notations used in MIP- $LexiR_*$

All notations needed for the description of MIP- $LexiR_*$ are given in Table 3.

Let $\omega_k = (\omega_1, \dots, \omega_n)$ be a vector of weights with $\omega_1 > \omega_2 > \dots > \omega_n$. We denote s, s' two indexes representing scenarios $\in N$ with $s \neq s'$. K is a vector of the same size of N and $k \in K$ is an index helping to browse all scenarios, S is the number of scenarios.

The proposed MIP is detailed in Model 6.

$$\begin{aligned}
& \max \sum_k (\omega_k * (R_k + r_k)) && (6) \\
& S.t \\
(a) \quad & R_k \leq F((x, y), s) + M * (1 - \gamma_s^k) && \forall s \in N, k \in K \\
(b) \quad & R_k \leq e && \forall k \in K \\
(c) \quad & \sum_{s=1}^S \gamma_s^k = S - k + 1 && \forall k \in K \\
(d) \quad & F((x, y), s) \geq -M * (1 - Y_s) + e * Y_s && \forall s \in N \\
(e) \quad & F((x, y), s) \leq e * (1 - Y_s) + Y_s * M && \forall s \in N \\
(f) \quad & F((x, y), s) \leq F((x, y), s') + M * \delta_{s,s'} && \forall s, s' \in N \\
(g) \quad & F((x, y), s') - M * (1 - \delta_{s,s'}) \leq F((x, y), s) && \forall s, s' \in N \\
(h) \quad & r_k \leq A_k * M && \forall k \in K \\
(i) \quad & \sum_{s'=1}^S \delta_{s,s'} \leq k + (1 - C_{s,k}) * M && \forall s \in N, k \in K \\
(j) \quad & \sum_{s'=1}^S \delta_{s,s'} \geq k - (1 - C_{s,k}) * M && \forall s \in N, k \in K \\
(k) \quad & D_{s,k} \leq Y_s && \forall s \in N, k \in K \\
(l) \quad & D_{s,k} \leq C_{s,k} && \forall s \in N, k \in K \\
(m) \quad & A_k \leq \sum_{s=1}^S D_{s,k} && \forall k \in K \\
(n) \quad & \sum_s \mu_{s,k} = \sum_{i=1}^k A_i && \forall k \in K \\
(o) \quad & r_k \leq F((x, y), s) + (1 - \mu_{s,k}) * M && \forall s \in N, k \in K
\end{aligned}$$

Constraints (a), complementary with constraint (c) are the linearization of the minimum profit, then second minimum profit, then third and so on as long as the minimum profit is lower than e . Otherwise, the value for R_k is set to e in constraint (b).

Constraints (d) and (e) define the following:

$$Y_s = \begin{cases} 0 & \text{if } F(x, s) \leq e \\ 1 & \text{if } F(x, s) > e \end{cases}$$

Constraints (f) and (g) define the following:

$$\delta_{s,s'} = \begin{cases} 0 & \text{if } F(x, s) \leq F(x, s') \\ 1 & \text{Otherwise} \end{cases}$$

The sum on s of variable $\delta_{s,s'}$ represents the number of scenarios above and below $F(x, s)$ for each k . This helps us to define constraints (i) and (j) in order to give the following information:

$$C_{s,k} = \begin{cases} 1 & \text{if } \sum_s \delta_{s,s'} = k \\ 0 & \text{Otherwise} \end{cases}$$

Constraint (k) and (l) are then used to define:

$$D_{s,k} = \begin{cases} 1 & \text{if } Y_s = 1 \text{ and } C_{s,k} = 1 \\ 0 & \text{Otherwise} \end{cases}$$

It should be noted that $D_{s,k}$ is only activated if $k - st$ profit $F(x, s)$ is higher than e .

Constraint (m) defines

$$A_k = \begin{cases} 1 & \text{if } \sum_s D_{s,k} = 1 \\ 0 & \text{Otherwise} \end{cases}$$

It should be noted that A_k is only activated is $k - st$ profit $F(x, s)$ is higher than e . This is used to set the value of r_k to 0 if profit $F(x, s)$ is lower than e in constraint (h).

Then constraint (n) is used to count how many times variable A_k has been activated: the number of times where profit $F(x, s)$ has been higher than e using variable $\mu_{s,k}$.

Constraint (o) is the linearization of the maximum profit, then second maximum profit and so on as long as it is higher than e due to variable $\mu_{s,k}$

4. Case Study

In this section, we present a case study in order to illustrate the application of the developed *LexiR** approach to a design problem of RSC. The framework of case study is very generic in order to represent a situation that can occur in different industrial contexts. In order to facilitate the understanding of the solution approaches, the structure of the RSC has been simplified. Therefore, the attention is given to the decision making process

and not to the organizational details of the RSC. This simplification is only made for seek of illustration, the proposed approach can be applied to the design problem of any RSC independently of the industrial context and its complexity. The only mandatory condition is to be able to collect enough information about the RSC in order to define the corresponding optimization problem. Another limitation is the centralized decision making process: all decisions about all facilities are taken by a centralized decision maker who is a single person or a group of persons capable of reaching a consensus and acting for the common objective of maximizing the total profit.

The context of the study is the following. A third party logistic company is willing to design a new RSC. The goal is to decide which new facilities (collection centers, re-manufacturing centers and disposal centers) have to be installed and where as well as the organization of the transportation flows between them in order to maximize the total profit gained from the RSC. Figure 3 shows the structure of the RSC under design. Three parameters are considered uncertain, namely the quantity of returned EOL products by the customers, the quantity of demand for re-manufactured products and the reprocessing time of EOL products.

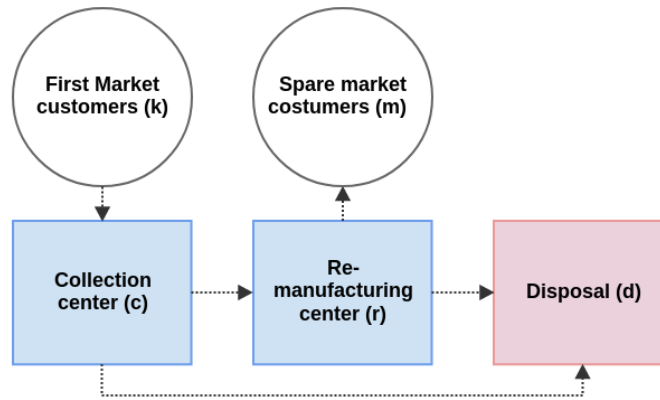


Figure 3: Reverse Supply Chain under design

4.1. Mathematical model

This optimization problem is formulated through the following mathematical model. The indexes, parameters and variables of the problem are defined as follows:

	Indexes
$k = 1..K$	Index of customers
$m = 1..M$	Index of spare markets
$c = 1..C$	Index of collection centers
$r = 1..R$	Index of re-manufacturing centers
$d = 1..D$	Index of disposal sites
$s = 1..S$	Index of scenarios
	Demand
$D_{m,s}$	of spare market m for scenario s
	Capacity
$CapC_c$	of collection center c
$CapR_r$	of re-manufacturing center r
	Distance between...
$DKC_{k,c}$	customer k and collection center c
$DCR_{c,r}$	collection center c and re-manufacturing center r
$DCD_{c,d}$	collection center c and disposal d
$DRD_{r,d}$	re-manufacturing center r and disposal d
$DRM_{r,m}$	re-manufacturing center r and spare market m
	Time parameters
$Tremanu_s$	Unit dismantling time
	Unit operational cost
$Coph_c$	at collection center c
$Cdis_p$	at re-manufacturing center p
$Ceco$	tax for non-reprocessed products
TC	transportation cost for 1 kilometer
	Rate parameters
R_s	Quantity of return for scenario s
Rr	Re-manufacturing rate after collection
Rd	Disposal rate after re-manufacturing
	Unit selling price parameters
RSP_m	of product at spare market m
	Fixed opening cost parameters
CFC_c	for collection center c
CFR_r	for re-manufacturing center r
CFD_d	for disposal d
	Positives variables (<i>Flow from . to . for s</i>)
$XKC_{k,c,s}$	customer k to collection center c
$XCR_{c,r,s}$	collection center c to re-manufacturing center r
$XCD_{c,d,s}$	collection center c to disposal d
$XRD_{r,d,s}$	re-manufacturing center r to disposal d
$XRM_{r,m,s}$	re-manufacturing center r to spare market m
	Binary variables
YC_c	collection center c is opened or not
YR_r	re-manufacturing center r is opened or not
YD_d	Disposal d is opened or not

To make the model more readable, some expressions are defined below.

The total income: it includes all sales revenues. It is scenario dependent and can be formulated as:

$$Income_s = \sum_{m=1}^M (\sum_{r=1}^R (RSP_m * XRM_{r,m,s})) \quad (7)$$

The total operational cost: it includes all production costs, assembling costs, buying costs, dismantling costs or distribution costs from/to all centers of the chain. It is scenario dependent and can be defined as follows:

$$\begin{aligned} OpCost_s = & \sum_{k=1}^K (\sum_{c=1}^C (Coph_c * XKC_{k,c,s})) \\ & + \sum_{c=1}^C (\sum_{r=1}^R (Cdis_r * XCR_{c,r,s})) \\ & + \sum_{f=1}^F (\sum_{c=1}^C (Ceco * (XRD_{r,d,s} + XCD_{c,d,s}))) \end{aligned} \quad (8)$$

The total fixed cost: it is the sum of the set-up costs of facilities:

$$\begin{aligned} FixedCost = & \sum_{c=1}^C (CFC_c * YC_c) \\ & + \sum_{p=1}^P (CFR_r * YR_r) \\ & + \sum_{d=1}^D (CFD_d * YD_d) \end{aligned} \quad (9)$$

The total transportation costs: it is the sum of travel costs between connected points of the Supply Chain. It is scenario dependent and can be written as:

$$\begin{aligned} TrtCost_s = & \sum_{k=1}^K (\sum_{c=1}^C (TC * DKC_{k,c} * XKC_{k,c,s})) \\ & + \sum_{r=1}^R (\sum_{c=1}^C (TC * DCR_{c,r} * XCR_{c,r,s})) \\ & + \sum_{d=1}^D (\sum_{c=1}^C (TC * DCD_{c,d} * XCD_{c,d,s})) \\ & + \sum_{r=1}^R (\sum_{d=1}^D (TC * DRD_{r,d} * XRD_{r,d,s})) \\ & + \sum_{r=1}^R (\sum_{d=1}^D (TC * DRSM_{r,m} * XRM_{r,m,s})) \end{aligned} \quad (10)$$

4.1.1. Objective

The objective is to maximize the total profit calculated as:

$$TotalProfit_s = Income_s - OpCost_s - FixedCost - TrtCost_s \quad (11)$$

4.1.2. Constraints

$$\begin{aligned}
(1) \quad & \sum_{k=1}^K XKC_{k,c,s} \leq CapC_c * YC_c & \forall c \in C, s \in S \\
(2) \quad & \sum_{c=1}^C XCR_{c,r,s} * Tremanu_s \leq CapR_r * YR_r & \forall r \in R, s \in S \\
(3) \quad & \sum_{c=1}^C XCD_{c,d,s} + \sum_{r=1}^R XRD_{r,d,s} \leq B * YF_f & \forall d \in D, s \in S \\
(4) \quad & \sum_{r=1}^R XRM_{r,m,s} \leq D_{m,s} & \forall m \in M, s \in S \\
(5) \quad & \sum_{c=1}^C XKC_{k,c,s} \leq R_s & \forall k \in K, s \in S \\
(6) \quad & \sum_{k=1}^K XKC_{k,c,s} * Rr = \sum_{r=1}^R XCR_{c,r,s} & \forall c \in C, s \in S \\
(7) \quad & \sum_{c=1}^C XCR_{c,r,s} * Rd = \sum_{d=1}^D XRD_{r,d,t,s} & \forall r \in R, s \in S \\
(8) \quad & \sum_{k=1}^K XKC_{k,c,s} = \sum_{r=1}^R XCR_{c,r,s} + \sum_{d=1}^D XCD_{c,d,s} & \forall c \in C, s \in S \\
(9) \quad & \sum_{c=1}^C XCR_{c,r,s} = \sum_{d=1}^D XRD_{r,d,s} + \sum_{m=1}^M XRM_{r,m,s} & \forall r \in R, s \in S
\end{aligned}$$

Constraints (1) to (3) are capacity constraints on the number of products each center is able to process at each period. Constraint (4) certifies that the production does not exceed the demand. Unsatisfied demand is considered lost. Constraint (5) assures that the quantity of collected EOL products cannot be superior to the quantity of returned products from the consumer. Constraints (6) and (7) calculate the quantity of dismantled, repaired and recycled products according to the predefined rates Rr and Rd . Constraints (8) and (9) check the balance of the flows between centers.

For our numerical experiments, this model has been initiated with the following data. We consider 10 possible locations for each type of center (collection, re-manufacturing and disposal), 10 first market consumers and 10 spare market consumers. The distances between centers are comprised between 1 and 500 kilometers. The related parameters are defined in Table 4. Other parameters used: $Ceco=1$, $TC=1$, $Rd=80\%$, $Rr=80\%$.

Parameter	Value
$CapC$	[55000, 107000, 190000, 58000, 100000, 200000, 90000, 150000, 170000, 121200]
$CapR$	[56100, 50500, 47060, 28000, 29100, 53500, 31000, 56100, 38000, 42100]
$Coph$	[3, 2, 5, 2, 1, 4, 5, 6, 1, 3]
$Cdis$	[2, 2, 6, 8, 7, 4, 5, 3, 1, 2]
RSP	[100, 150, 200, 320, 140, 210, 220, 110, 100, 150]
CFC	[10000, 20000, 60000, 90000, 70000, 80000, 15000, 12000, 17000, 18000]
CFR	[40000, 60000, 50000, 45000, 55000, 47000, 57000, 42000, 52000, 40000]
CFD	[42000, 52000, 40000, 50500, 40000, 60000, 50000, 40000, 55000, 40000]

Table 4: Deterministic Parameters

Three parameters are considered uncertain, namely the quantity of returned EOL products by the customers, the quantity of demand for re-manufactured products and the reprocessing time of EOL products. Their values are not known at the moment of the decision making and are estimated by experts. Each parameter has its "high" and "low" estimation. For example, the quantity of returned EOL products by each of 10 customers is within the interval of $([12000,17500])$ if the level of return is low and this returned quantity is considered within the interval $([22000, 27500])$ if the return is high. The same is for the demand for re-manufactured products R for second market actors: it is within the interval $([12000,17500])$ if the demand is low or within the interval $([22000, 27500])$ if the demand is high, finally the remanufacturing time depends on the quality of the returned products, it is long if the quality is low, i.e. $Tremanu$ is within the interval $([7,8])$ and short if the quality is high, i.e. within the interval $([1,2])$. According to those levels, four scenarios are defined and presented in Table 5.

Parameter	s_1	s_2	s_3	s_4
D	high	low	high	high
R	low	low	high	high
$Tremanu$	short	long	long	short

Table 5: Scenarios

Then, one value for each parameter is randomly selected from the intervals presented above. The final values considered for the resolution are presented in Table 6.

4.2. Analysis of results and discussion

The analysis of the results is presented in three parts. First, we solve the problem with a simple R_* criterion and we compare its performances with the models the most frequently used in the literature on RSC: stochastic average approach and robust optimization.

The resolution was conducted with IBM-ILOG CPLEX 12.6.3 on an Intel Core 2.60 gigahertz machine with 15 gigabyte RAM.

4.2.1. Comparison with R_* criteria

Table 7 presents the results for the stochastic average, robust and R_* models. In this Section, we compare the results obtained with robust and

Scenario	Parameter	Value
s_1	D	[24967, 25401, 25073, 23365, 26290, 23216, 26261, 26850, 23033, 26346]
	R	12038
	$Tremanu$	2
s_2	D	[13726, 12222, 12072, 13870, 16114, 15414, 15311, 17264, 14782, 12971]
	R	12993
	$Tremanu$	7
s_3	D	[26557, 23532, 25311, 23601, 24251, 23588, 22349, 26758, 25860, 23204]
	R	23879
	$Tremanu$	7
s_4	D	[23450, 24732, 22067, 24776, 26893, 26328, 25329, 24128, 22458, 26663]
	R	23588
	$Tremanu$	1

Table 6: Final values of Uncertain Parameters

Stochastic average approaches versus R_* criterion, never used before in RSC design. We show the advantages of the latter, particularly regarding the control of the level of risk taken by the DM and found opportunities.

The first column of Table 7 indicates the model used, the second column provides the value of e for R_* criterion. From a practical point of view, the value of e is set up as a percentage of the solution provided by the robust method. This method allows us to analyze which percentage of the profit the DM must accept to loose in order to access some opportunities. The results of the profit for each scenario are presented in Columns 3 to 6. They are colored in green when R_* criterion brings an improvement compared to the robust model and in red when it brings a deterioration.

The results presented in Table 7 show that the stochastic average model gives very good results in the two best cases, but at the same time, the profit made in the worst case is the lowest compared to the other solutions. We can conclude that using this method, the DM is unable to control the level of taken risks.

Table 7 shows that R_* criterion returns the robust solution if the value e is equal to the "MinMax" solution or above. It illustrates that R_* is a generalization of the robust criterion and is able to protect the DM from high risks. The more the value of e decreases, i.e. the more the DM is willing to take risks, the more the profit in the worst case scenario declines, but the profit in the best case scenario shows a considerable improvement and the payoff is much more important than the losses. The two middle case

Model	e	$s_1(\text{€})$	$s_2(\text{€})$	$s_3(\text{€})$	$s_4(\text{€})$
Average	-	3786470	1829385	7276791	7180619
Robust	-	2133440	2133395	2133381	2230306
R_*	-0%	2133440	2133395	2133381	2230306
R_*	-1%	2845669	2132842	6553907	3690808
R_*	-2%	2678834	2112049	6553922	5115179
R_*	-3%	2888210	2069437	6553922	2090713
R_*	-4%	2083069	2048049	6553948	4515563
R_*	-5%	2069412	2026713	6553994	2026719
R_*	-6%	2005422	2005398	6553994	2026712
R_*	-7%	2005372	1984142	6554003	4313810
R_*	-10%	2984174	1984057	6554003	2663382

Table 7: Comparison of the profits obtained with robust and stochastic average approaches versus R_* criterion for the case of 4 scenarios

scenarios can either be improved or degraded depending on the case.

These results show that with the choice of value for e , the DM can control the robustness of the solution while exploring the zone of opportunities. In the case of the design of RSC, the readiness for opportunities follows to opening more collection and recycling centers, i.e. more returned products can be treated if the return level is high and remanufacturing is profitable. If the decision is made with the use of a robust approach, the RSC can be under-sized inducing a loss of opportunities. Thus, the robust approach may slow down the development of RSCs.

4.2.2. Comparison with $LexiR_*$

In Table 8, we compare the results found with *Average*, *Leximax*, *Leximin* and $LexiR_*$ models. The table is built in the same way as Table 7. Here again the profits made in each scenario are colored in green when $LexiR_*$ criterion brings an improvement compared to the robust model and in red when it brings a deterioration.

The following observations are made:

- The results obtained with the *Average* criterion are close to the ones found with $LexiR_*$ criterion when e is set to -50% of the maxmin solution. Nevertheless in this case, the $LexiR_*$ criterion allows a better control on the level of risk taken in the worst case scenario.
- The higher the value of e (i.e. the more pessimistic the DM), the closer the results obtained with $LexiR_*$ to the results obtained with *Leximin*.

Model	e	$s_1(\text{€})$	$s_2(\text{€})$	$s_3(\text{€})$	$s_4(\text{€})$
Average	-	3786470	1829385	7276791	7180619
<i>Leximin</i>	-	3880421	2133395	6542653	6519706
<i>LexiR*</i>	+20%	3409469	2133395	6553907	6519830
<i>LexiR*</i>	0%	3409469	2133395	6553907	6519830
<i>LexiR*</i>	-5%	2672418	2026856	6553994	6519900
<i>LexiR*</i>	-10%	2133544	1984173	6554003	6519900
<i>LexiR*</i>	-50%	2048230	1829419	7276812	7180453
<i>Leximax</i>	-	0	6	7276860	7180744

Table 8: Compared Profit between Robust and *LexiR** approach for the case of 4 scenarios

- If e decreases, but is still equal to or above the robust "MinMax" solution, the *LexiR** model improves the three best case scenarios compared to the robust solution and the two best case scenarios compared to *Leximin* criterion. In this case, applying the *LexiR** allows the DM to find new opportunities without taking any risks compared to the robust method. It also allows to find new opportunities in comparison to *Leximin* criterion, but in this case, some risks must be taken on the third best case scenario.
- The more e decreases in comparison to the "MinMax" solution, the worse are the results for the worst case scenarios, however, more opportunities are revealed in the two best case scenarios. The results reveal more opportunities found with *LexiR** criterion than with R_* criterion in every case.
- When the value of e is very low (close to 0), i.e. the DM is very optimistic, the results obtained with *LexiR** criterion are very close to the results obtained with *Leximax* criterion.
- The *LexiR** model gives better results for the two worst case scenarios compared to the R_* model. Therefore, the risks taken by the DM are better controlled.

To give a better view of the amplitude of the solutions, we present in Fig 4 the values of the profit for each scenario depending on the value of e . Each color represents a scenario: the worst case scenario (s_2) is colored in red while the best case scenario (s_3) is colored in yellow.

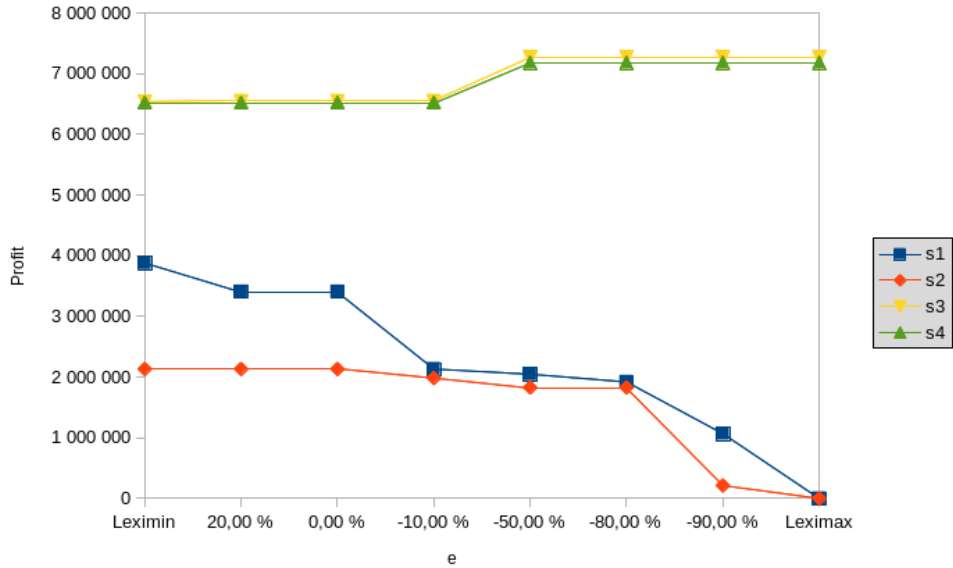


Figure 4: Amplitude of profits for 4 scenarios depending on e

We can see from this graph that *Leximin* criterion has the smallest amplitude between the scenarios contrary to *Leximax* criterion which has the highest one. *LexiR** criterion's amplitude depends on the value of e : the higher the value of e the less important the amplitude between the scenarios. Some jumps can be visible in the amplitudes of the solutions: for instance when $e = -10\%$, the profit for scenario s_1 is significantly reduced, or when $e = -50\%$ the profit for scenarios s_3 and s_4 are significantly increased. These jumps correspond to a change in the strategy of the DM regarding the design of the RSC. Indeed, Table 9 shows that the number of opened centers has increased every time a jump has been visible. Thus, we can assume that when the DM takes more risks he increases her/his investments and opens more centers. Therefore, he is able to reprocess more EOL products. Such a strategy will benefit the RSC when a good case scenario happens (higher return and demand rates).

According to these results, we propose here that:

Proposition 1. *If a company implements a RSC with the use of a $LexiR_*$ approach (with e the value guaranteeing the sustainability of the RSC), it allows to take advantage of all context-related opportunities, while avoiding*

Model	e	Collection centers	Recycling centers	Disposal
$LexiR_*$	-10%	5	4	5
$LexiR_*$	-50%	6	5	5
$LexiR_*$	-80%	6	5	6
$LexiR_*$	-90%	6	6	6

Table 9: Number of opened centers

as much as possible difficult financial situations.

4.3. Comparison and discussion on the proposed method: $LexiR_*$ algorithm and $LexiR_*$ MIP

In this section, the technical choice between the two proposed method are discussed. First, we compare the computation time and the quality of the approximation. Then, we advise on the choice between the two approaches depending on the DM's priorities.

Number of scenarios	$LexiR_*$ Algorithm	$LexiR_*$ MIP
4	5.28	4.54
8	547.29	35.35
16	2222.19	300.40
32	17000.28	1454.73
64	85271.36	12839.46

Table 10: Comparison of $LexiR_*$ methods' solving times (in seconds)

Table 10 shows the solution times for both $LexiR_*$ algorithm and $LexiR_*$ MIP for different numbers of scenarios. The scenarios have been randomly generated for the purpose of the experiment. Each instance has been solved with both methods for 10 different values of e (also randomly generated). When four scenarios are considered, the two methods are able to solve the problem quite fast and in comparable amount of time (see Table 10). Then, the more the number of scenarios increases, the more the solution time extends. The solution time of the $LexiR_*$ MIP increases significantly slower than the one of the $LexiR_*$ algorithm. On the other hand, the $LexiR_*$ MIP being an approximate solution method, it does not always allow to find the exact optimum. Furthermore, the $LexiR_*$ algorithm is solved by iterations, and the number of iterations is the same than the number of scenarios

considered. Each iteration takes approximately the same time to solve. For instance, when 32 scenarios are solved, the total solution time of the algorithm is 17000.28 seconds. Therefore, it is approximately $17000.28 \div 32 \approx 531$ seconds for each iteration. The DM can choose to stop the resolution at any iteration. In this case, the found solution will be optimized for the firsts considered scenarios and approximate the other ones.

To illustrate this point, we compared the results found for the optimal solution when 32 scenarios are considered with the results obtained when the Algorithm is stopped at the 4th, 8th, 16th and 24th iterations. We analyzed the deviation from the optimal solution depending on the number of iteration solved. We also compared with the results found with the *LexiR** MIP. The table presenting the results is available in Appendix A.

The results show that the MIP gives a very good approximation of the optimal solution: only 2 scenarios deviates slightly from the optimum. When the model is solved with the algorithm stopped at the 24th, 16th, 8th, or 4th iterations, the following observations can be made:

- The more the number of iterations solved is high, the more the found solution is close to the optimal
- As previously explained in Section 3.1, the *LexiR** algorithm first optimizes the worst cases scenarios with a *Leximin* before moving on to the best cases scenarios with a *Leximax*. Thus, as long as the value of e is higher than the profit of the scenario considered, the DM is risk adverse and opportunities are not revealed. As so, if the number of iterations solved is lower than the number of scenarios in the risky area, opportunities cannot be found. This effect is visible in our case when the algorithm is stopped at 4 or 8 iterations: only the worst cases scenarios are optimized and the better cases scenarios can deviate up to 27% from the optimum.
- If the number of iterations solved is high enough for scenarios in the opportunity area to be considered, then opportunities are revealed on the best cases.

Therefore, we propose that:

Proposition 2. *If the DM is willing to solve the problem with an exact method and if the duration of the computational time is not an issue, he/she should use the *LexiR** algorithm to solve it.*

Proposition 3. *If the DM would prefer a more rapid solution, even if this one is not optimal, he/she can either use the $LexiR_*$ MIP or the $LexiR_*$ Algorithm and interrupt it at a satisfying number of iterations.*

Remark 1. *When using the $LexiR_*$ Algorithm and interrupting it at a chosen number of iterations, the DM must be careful that either the number of iterations solved is high enough for opportunities to be revealed, or that the value of e is low enough for the number of scenarios in the risky area to be lower than the number of iteration solved, in order to disclose new opportunities.*

5. Conclusion

In order to help the decision makers to design a successful RSC under uncertainty, we suggest a new bipolar criterion, namely $LexiR_*$, able to differentiate risks from opportunities depending on the optimism of the DM. We compare this criterion with classic criteria from the literature to show its advantages and propose a comparison of each criteria in terms of practical results. We particularly show that the $LexiR_*$ criterion better explores opportunities compared to a classic robust solution approach, while still allowing the DM to control the level of risk he is willing to take.

We develop two solution methods to apply this criterion. The first one is in the form of an algorithm and another is in the form of a MIP. We show that the use of $LexiR_*$ criterion helps to better explore opportunities compared to a robust model without losing control over robustness. We compare the results obtained with $LexiR_*$, $Leximin$ and $Leximax$ criteria. We analyse the solution time for both $LexiR_*$ algorithm and $LexiR_*$ MIP and show that the $LexiR_*$ algorithm takes a significantly longer time, but allows to find an exact solution, contrary to $LexiR_*$ MIP that offers only an approximate solution but obtained in shorter time.

From the methodological point of view, the further research will be oriented to the generalization of $LexiR_*$ criterion to a continuous set of scenarios. Furthermore, a generalized study including both forward and reverse supply chains will be performed for the case of creating a Closed-Loop Supply Chain. In addition to the economic performance of such supply chains, their environmental and social performances should be evaluated as well, in order to be more in line with the reality of industrial concerns.

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Appendix A. *LexiR** Algorithm versus MIP

Table A.11 is constituted as follows: Column 1 is the scenario considered (the scenarios are ranked from the worst case to the best case). Column 2 is the optimal solution, i.e. the solution found when the *LexiR** algorithm is used to solve the problem without being stopped at any iteration. Column 3 to 6 are respectively the deviation from the optimum solution when the model is solved using the *LexiR** algorithm interrupted at 24, 16, 8, and 4 iterations. The last Column is the deviation from the optimal solution when the *LexiR** MIP is used to solve the problem.

Model	<i>LexiR*</i> Algorithm					<i>LexiR*</i> MIP
Number of iterations	32	24	16	8	4	-
Scenario	Profit(€)	Deviation	Deviation	Deviation	Deviation	Deviation
s ₁	2427710	0,00%	0,00%	0,00%	0,00%	0,00%
s ₂	2610029	0,00%	0,00%	0,00%	0,00%	0,00%
s ₃	2651913	0,00%	0,00%	0,00%	0,00%	0,00%
s ₄	2683295	0,00%	0,00%	0,00%	0,00%	0,00%
s ₅	2700265	0,00%	0,00%	0,00%	0,00%	-0,01%
s ₆	2782584	0,00%	0,00%	0,00%	0,00%	-2,96%
s ₇	2787666	0,00%	0,00%	0,00%	0,00%	-3,14%
s ₈	2845463	0,00%	0,00%	0,00%	0,00%	-5,11%
s ₉	2948545	0,00%	0,00%	0,00%	0,00%	-8,42%
s ₁₀	2950168	0,00%	0,00%	0,00%	-0,05%	-8,47%
s ₁₁	2950906	0,00%	0,00%	0,00%	-0,08%	-8,50%
s ₁₂	3013577	0,00%	0,00%	0,00%	-2,16%	-10,40%
s ₁₃	3077298	0,00%	0,00%	0,00%	-4,18%	-9,58%
s ₁₄	3120285	0,00%	0,00%	0,00%	-5,50%	-10,66%
s ₁₅	3175696	0,00%	0,00%	0,00%	-7,15%	-10,40%
s ₁₆	3265788	0,00%	0,00%	0,00%	-9,71%	-10,12%
s ₁₇	3297803	0,00%	0,00%	0,00%	-10,59%	-10,69%
s ₁₈	3395708	0,00%	0,00%	0,00%	-13,16%	-13,10%
s ₁₉	3398593	0,00%	0,00%	0,00%	-13,17%	-11,33%
s ₂₀	3519252	0,00%	0,00%	0,00%	-14,51%	-14,06%
s ₂₁	3533295	0,00%	0,00%	0,00%	-13,12%	-12,66%
s ₂₂	3582553	0,00%	0,00%	0,00%	-12,90%	-13,50%
s ₂₃	3764976	0,00%	0,00%	0,00%	-16,46%	-17,12%
s ₂₄	4080441	0,00%	0,00%	-4,70%	-22,17%	-23,44%
s ₂₅	4335246	0,00%	0,00%	-5,88%	-23,93%	-27,77%
s ₂₆	4427709	-0,12%	-1,87%	-25,49%	-26,32%	0,00%
s ₂₇	4427710	-0,10%	-0,21%	-24,65%	-25,52%	0,00%
s ₂₈	4427710	-0,09%	-0,21%	-24,50%	-25,12%	0,00%
s ₂₉	4427714	-0,08%	-0,01%	-23,49%	-20,66%	0,00%
s ₃₀	4472278	0,00%	0,00%	-22,59%	-21,16%	0,00%
s ₃₁	4487614	0,00%	0,00%	-20,93%	-14,11%	0,00%
s ₃₂	4518240	0,00%	0,00%	-14,32%	-12,66%	0,00%

Table A.11: Comparison of results *LexiR** Algorithm versus MIP for 32 scenarios