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Corrigendum to “Belief Functions Contextual Discounting
and Canonical Decompositions” [International Journal of
Approximate Reasoning 53 (2012) 146–158]

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Abstract

Proposition 4 and Theorem 1 of the article “Belief Functions Contextual Discounting and Canonical Decompositions” [International Journal of Approximate Reasoning 53 (2012) 146–158] provide an erroneous result. We give here the true result with a correct proof.

Keywords: Belief functions, Contextual Discounting.

We hereby correct Proposition 4 and Theorem 1 in [2], which contained erroneous results.

Let us first recall the problem. A source S of information provides to agent Ag a piece of information represented by a mass function m_S^Ω (with $\Omega = \{\omega_1, \dots, \omega_K\}$), simply denoted by m in this corrigendum. Let \mathcal{A} be a non empty set of subsets of Ω called contexts. Agent Ag owns a meta-knowledge regarding the reliability of S conditionally on each set $A \in \mathcal{A}$. Formally, for all $A \in \mathcal{A}$, we suppose that

$$\begin{cases} m_{Ag}^{\mathcal{R}}[A](\{R\}) &= 1 - \alpha_A = \beta_A \\ m_{Ag}^{\mathcal{R}}[A](\mathcal{R}) &= \alpha_A, \end{cases} \quad (1)$$

where $\alpha_A \in [0, 1]$ and $\mathcal{R} = \{R, NR\}$ (R meaning the source is reliable, NR otherwise), and the notation $m[\cdot]$ denotes conditioning.

With the same reasoning as in [1] (where \mathcal{A} was supposed to form a partition of Ω), the knowledge m_{Ag}^Ω held by agent Ag on Ω , based on the in-

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formation m provided by S and his metaknowledge regarding S represented by (1) for all $A \in \mathcal{A}$, can be obtained by the following computation,

$$\left(m^\Omega[\{R\}]^{\uparrow\Omega \times \mathcal{R}} \circledast_{A \in \mathcal{A}} m^\mathcal{R}[A]^{\uparrow\Omega \times \mathcal{R}} \right)^{\downarrow\Omega}, \quad (2)$$

where symbol \uparrow and \downarrow denote, respectively, the deconditioning and projection operations, and $m^\Omega[\{R\}] = m$.

It is stated in [2] that, for $\mathcal{A} = 2^\Omega$ (Proposition 4) and more generally for any set \mathcal{A} of contexts (Theorem 1), Equation (2) is equivalent to

$$m \circledast \left(\bigoplus_{A \in \mathcal{A}} A_{\beta_A} \right). \quad (3)$$

This statement is incorrect. In the general case, for any non empty \mathcal{A} , Equation (2) is equivalent to

$$m \circledast \left(\bigoplus_{A \in \mathcal{A}} \overline{A}^{\alpha_A} \right), \quad (4)$$

as shown by the following proof, which corrects Theorem 1 from [2]. The fact that, in general, (4) is not equivalent to (3) (and particularly when $\mathcal{A} = 2^\Omega$), and therefore (2) is not equivalent in general to (3), is shown below by Example 1.

Proof 1. Let us denote by A_i , $i \in I = \{1, \dots, n\}$, the contexts present in \mathcal{A} , and let us write β_{A_i} simply by β_i , for all $i \in I$. For all $A_i \in \mathcal{A}$, the deconditioning of $m^\mathcal{R}[A_i]$ over $\Omega \times \mathcal{R}$ is given by

$$m^\mathcal{R}[A_i]^{\uparrow\Omega \times \mathcal{R}}(A_i \times \{R\} \cup \overline{A_i} \times \mathcal{R}) = \beta_i, \quad (5a)$$

$$m^\mathcal{R}[A_i]^{\uparrow\Omega \times \mathcal{R}}(\Omega \times \mathcal{R}) = \alpha_i. \quad (5b)$$

Moreover, for all $(A_i, A_j) \in \mathcal{A}^2$, such that $j \neq i$,

$$\begin{aligned} & (A_i \times \{R\} \cup \overline{A_i} \times \mathcal{R}) \cap (A_j \times \{R\} \cup \overline{A_j} \times \mathcal{R}) \\ = & (A_i \cap A_j) \times \{R\} \cup (A_i \cap \overline{A_j}) \times \{R\} \cup (\overline{A_i} \cap A_j) \times \{R\} \cup (\overline{A_i} \cup \overline{A_j}) \times \mathcal{R} \\ = & (A_i \cup A_j) \times \{R\} \cup \overline{(A_i \cup A_j)} \times \mathcal{R}. \end{aligned}$$

With \mathcal{A} composed of two elements denoted by A_i and A_j , we then have

$$\left\{ \begin{array}{ll} (m^\mathcal{R}[A_i]^{\uparrow\Omega \times \mathcal{R}} \circledast m^\mathcal{R}[A_j]^{\uparrow\Omega \times \mathcal{R}})((A_i \cup A_j) \times \{R\} \cup \overline{(A_i \cup A_j)} \times \mathcal{R}) & = \beta_i \beta_j \\ (m^\mathcal{R}[A_i]^{\uparrow\Omega \times \mathcal{R}} \circledast m^\mathcal{R}[A_j]^{\uparrow\Omega \times \mathcal{R}})(A_i \times \{R\} \cup \overline{A_i} \times \mathcal{R}) & = \beta_i \alpha_j \\ (m^\mathcal{R}[A_i]^{\uparrow\Omega \times \mathcal{R}} \circledast m^\mathcal{R}[A_j]^{\uparrow\Omega \times \mathcal{R}})(A_j \times \{R\} \cup \overline{A_j} \times \mathcal{R}) & = \alpha_i \beta_j \\ (m^\mathcal{R}[A_i]^{\uparrow\Omega \times \mathcal{R}} \circledast m^\mathcal{R}[A_j]^{\uparrow\Omega \times \mathcal{R}})(\Omega \times \mathcal{R}) & = \alpha_i \alpha_j \end{array} \right. .$$

In other words, all the focal elements of $\odot_{A \in \mathcal{A}} m^{\mathcal{R}}[A]^{\uparrow \Omega \times \mathcal{R}}$ are the elements $C \times \{R\} \cup \overline{C} \times \mathcal{R}$ with C composed of a union of elements A_i in \mathcal{A} , I' being the set of indices of the A_i 's, which means with $C = \cup_{i \in I' \subseteq I} A_i$. Moreover, each focal element has a mass equal to $\prod_{i \in I'} \beta_i \prod_{j \in I \setminus I'} \alpha_j$. Let us note that this latter result is also true if \mathcal{A} is composed of one element $A \subseteq \Omega$ (directly from Equations (5)).

By induction, we can show that this property remains true with \mathcal{A} composed of n contexts A_i , $i \in I = \{1, \dots, n\}$. Indeed, let us suppose the property true with \mathcal{A} composed of $n-1$ contexts A_i , $i \in I = \{1, \dots, n-1\}$, we then have for all focal elements $C \times \{R\} \cup \overline{C} \times \mathcal{R}$ of $\odot_{i \in I} m^{\mathcal{R}}[A_i]^{\uparrow \Omega \times \mathcal{R}}$, with $C = \cup_{i \in I' \subseteq I} A_i$,

$$\begin{aligned} & (\odot_{i \in I} m^{\mathcal{R}}[A_i]^{\uparrow \Omega \times \mathcal{R}} \odot m^{\mathcal{R}}[A_n]^{\uparrow \Omega \times \mathcal{R}})((C \cup A_n) \times \{R\} \cup \overline{(C \cup A_n)} \times \mathcal{R}) \\ &= \beta_n \prod_{i \in I'} \beta_i \prod_{j \in I \setminus I'} \alpha_j = \prod_{i \in I' \cup \{n\}} \beta_i \prod_{j \in (I \cup \{n\}) \setminus (I' \cup \{n\})} \alpha_j, \end{aligned}$$

and

$$\begin{aligned} & (\odot_{i \in I} m^{\mathcal{R}}[A_i]^{\uparrow \Omega \times \mathcal{R}} \odot m^{\mathcal{R}}[A_n]^{\uparrow \Omega \times \mathcal{R}})(C \times \{R\} \cup \overline{C} \times \mathcal{R}) \\ &= \alpha_n \prod_{i \in I'} \beta_i \prod_{j \in I \setminus I'} \alpha_j = \prod_{i \in I'} \beta_i \prod_{j \in (I \cup \{n\}) \setminus I'} \alpha_j, \end{aligned}$$

which means that focal elements of $\odot_{i \in \{1, \dots, n-1\}} m^{\mathcal{R}}[A_i]^{\uparrow \Omega \times \mathcal{R}} \odot m^{\mathcal{R}}[A_n]^{\uparrow \Omega \times \mathcal{R}}$ are also of the form $C \times \{R\} \cup \overline{C} \times \mathcal{R}$, with $C = \cup_{i \in I' \subseteq I} A_i$, $I = \{1, \dots, n\}$, $A_i \in \mathcal{A}$, and have for mass: $\prod_{i \in I'} \beta_i \prod_{j \in I \setminus I'} \alpha_j$.

Besides, for all $B \subseteq \Omega$,

$$m^{\Omega}[\{R\}]^{\uparrow \Omega \times \mathcal{R}}(B \times \{R\} \cup \Omega \times \{NR\}) = m(B),$$

and, for all $B \subseteq \Omega$, for all $C = \cup_{i \in I' \subseteq I} A_i$,

$$(C \times \{R\} \cup \overline{C} \times \mathcal{R}) \cap (B \times \{R\} \cup \Omega \times \{NR\}) = B \times \{R\} \cup \overline{C} \times \{NR\}.$$

Therefore, after the projection on Ω , $(m^{\Omega}[\{R\}]^{\uparrow \Omega \times \mathcal{R}} \odot_{A \in \mathcal{A}} m^{\mathcal{R}}[A]^{\uparrow \Omega \times \mathcal{R}})^{\downarrow \Omega}$ consists in transferring a part $\prod_{i \in I'} \beta_i \prod_{j \in I \setminus I'} \alpha_j$ of each mass $m(B)$, $B \subseteq \Omega$, from B to $B \cup \overline{C}$, for all $C = \cup_{i \in I' \subseteq I} A_i$.

On the other hand, $m^{\odot}(\odot_{A \in \mathcal{A}} \overline{A}^{\alpha_A})$ can be written as

$$m^{\odot}(\odot_{i \in I} \overline{A_i}^{\alpha_i}) = m^{\odot}\left(\odot_{i \in I} \begin{cases} \Omega & \mapsto \alpha_i \\ \overline{A_i} & \mapsto \beta_i \end{cases}\right).$$

As for all $(i, j) \in I^2$ s.t. $i \neq j$, $\overline{A_i} \cap \overline{A_j} = \overline{A_i \cup A_j}$, it can be shown (with an induction for example) that the focal elements of $\bigoplus_{i \in I} \overline{A_i}^{\alpha_i}$ are the elements \overline{C} with $C = \cup_{i \in I' \subseteq I} A_i$ and have a mass equal to $\prod_{i \in I'} \beta_i \prod_{j \in I \setminus I'} \alpha_j$.

Consequently, operation $m \bigoplus (\bigoplus_{i \in I} \overline{A_i}^{\alpha_i})$ also consists in transferring a part $\prod_{i \in I'} \beta_i \prod_{j \in I \setminus I'} \alpha_j$ of each mass $m(B)$, $B \subseteq \Omega$, from B to $B \cup \overline{C}$, for all $C = \cup_{i \in I' \subseteq I} A_i$. We can then conclude that Equations (2) and (4) are equivalent for any non empty set of contexts \mathcal{A} .

□

Example 1. Let us consider $\Omega = \{\omega_1, \omega_2\}$ and $\mathcal{A} = 2^\Omega$, and let us denote $\alpha_{\{\omega_1\}}$ by α_1 , $\alpha_{\{\omega_2\}}$ by α_2 , and α_Ω by α_{12} . Equation (4) gives

$$\begin{aligned}
& m \bigoplus (\bigoplus_{A \in \mathcal{A}} \overline{A}^{\alpha_A}) \\
= & m \bigoplus (\overline{\emptyset}^{\alpha_\emptyset} \bigoplus \overline{\{\omega_1\}}^{\alpha_1} \bigoplus \overline{\{\omega_2\}}^{\alpha_2} \bigoplus \overline{\Omega}^{\alpha_{12}}) \\
= & m \bigoplus (\{\omega_2\}^{\alpha_1} \bigoplus \{\omega_1\}^{\alpha_2} \bigoplus \emptyset^{\alpha_{12}}) \\
= & m \bigoplus \left(\begin{array}{l} \{\omega_2\} \mapsto \beta_1 \bigoplus \{\omega_1\} \mapsto \beta_2 \bigoplus \{\emptyset \mapsto \beta_{12}\} \\ \Omega \mapsto \alpha_1 \bigoplus \Omega \mapsto \alpha_2 \bigoplus \Omega \mapsto \alpha_{12} \end{array} \right) \\
= & m \bigoplus \left\{ \begin{array}{l} \emptyset \mapsto \beta_1 \beta_2 \alpha_{12} + \beta_{12} \\ \{\omega_1\} \mapsto \alpha_1 \beta_2 \alpha_{12} \\ \{\omega_2\} \mapsto \beta_1 \alpha_2 \alpha_{12} \\ \Omega \mapsto \alpha_1 \alpha_2 \alpha_{12} \end{array} \right. .
\end{aligned}$$

In contrast, Equation (3) leads to

$$\begin{aligned}
& m \bigoplus (\bigoplus_{A \in \mathcal{A}} A \beta_A) \\
= & m \bigoplus \emptyset_{\beta_\emptyset} \bigoplus \{\omega_1\}_{\beta_1} \bigoplus \{\omega_2\}_{\beta_2} \bigoplus \Omega_{\beta_{12}} \\
= & m \bigoplus \{\omega_1\}_{\beta_1} \bigoplus \{\omega_2\}_{\beta_2} \bigoplus \Omega_{\beta_{12}} \\
= & m \bigoplus \left\{ \begin{array}{l} \emptyset \mapsto \beta_1 \bigoplus \emptyset \mapsto \beta_2 \bigoplus \{\emptyset \mapsto \beta_{12}\} \\ \{\omega_1\} \mapsto \alpha_1 \bigoplus \{\omega_2\} \mapsto \alpha_2 \bigoplus \Omega \mapsto \alpha_{12} \end{array} \right. \\
= & m \bigoplus \left\{ \begin{array}{l} \emptyset \mapsto \beta_1 \beta_2 \beta_{12} \\ \{\omega_1\} \mapsto \alpha_1 \beta_2 \beta_{12} \\ \{\omega_2\} \mapsto \beta_1 \alpha_2 \beta_{12} \\ \Omega \mapsto \alpha_1 \alpha_2 \beta_{12} + \alpha_{12} \end{array} \right. .
\end{aligned}$$

To summarize, in [1], the equivalence was shown between (2) and (3) when \mathcal{A} forms a partition of Ω . This corrigendum shows that this equivalence does not hold for any \mathcal{A} , and that (2) is actually equivalent to (4) for any (non empty) \mathcal{A} .

References

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