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COMCI: A CONstrained Method for lensless Coherent Imaging of thin samples

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Lensless in-line holography can produce high resolution images over a large field of view (FoV). In a previous work [1], we have shown that (i) the actual FoV can be extrapolated outside of the camera FoV and (ii) effective resolution of the setup can be several times higher than the resolution of the camera. In this paper, we present of a reconstruction method to recover high resolution with an extrapolated FoV images of the phase and the amplitude of a sample from aliased intensity measurements taken at lower resolution.

OCIS codes: 100.5070, 100.3020, 100.3190, 100.6640

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1. INTRODUCTION

Lensless inline holography is an imaging technique where diffracted light is recorded without any optical parts between the sample and the camera. Given the simplicity, the compactness, the robustness and the relatively low cost of this setup [2–4], inline digital holography is successfully employed in many applications such as lensfree microscopy[3–6] or metrology[7]. However, unlike direct imaging methods, the recorded hologram cannot be directly visualized. Instead, computational algorithms are required to image the phase and the amplitude of the sample or to extract parameters of interest.

The naive backpropagation of the measured hologram to the sample plane leads to the well-known twin-image artifacts caused by the loss of information about phase. Phase retrieval methods rely either on phase diversity and/or prior knowledge on the sample to estimate the missing phase. Phase diversity [8] consists in the recording of several images with an additional known phase variation (*i.e.*, varying illumination angle [9], wavelength [10–13], or sample-camera distance[10, 14–20]). The priors can be enforced either by constraining the solution to belong to a given subspace (*e.g.*, the subspaces of phase- or absorption-only objects or the subspace of objects with a given spatial support [21]) or by imposing a priori statistics on the object. The most generic way to account for prior knowledge is to introduce a so-called regularization function that favors reconstructions with desirable properties (*e.g.*, smoothness). In diffraction imaging, the most popular priors are the sparsity [17, 22–27], total variation [28–31], and learned priors such as dictionary-based [32] or deep-learning plug-and-play [33] regu-

larizations.

The earliest phase-retrieval algorithm is due to Gerchberg and Saxton [14]. It reconstructs a complex wavefront in the object plane from intensity measurements taken at two different depths. This alternating-projection strategy was further improved by Fienup [21, 34] and gave birth to a large family of (non-convex) successive projection algorithms [35–39]. The proposed CONstrained Method for lensless Coherent Imaging (COMCI) originates from these projection methods as its optimization scheme is the alternating-direction-of-multipliers-method (ADMM) [40], where each step makes use of generalized projections (the so-called proximity operator). This ADMM scheme was already successfully used in a phase-retrieval context[41–44].

For a decade, several authors [23–26] have also proposed to address the phase-retrieval problem through convex relaxation. More recently deep-learning-based methods were also proposed [45]. A state-of-the-art review of phase-retrieval methods can be found in [30, 46].

One of the main advantages of lensless inline holography is that it can produce high-resolution images over a large field of view (FoV) which can be even larger than the camera FoV. Indeed, while most works [41, 47–54] restrict the reconstructed FoV to that of the sensor, other works[22, 31, 55, 56] achieve, for similar measurements, the retrieval of information over a much larger FoV.

In a previous work[1], we have shown that (i) the actual FoV can be extrapolated outside of the camera FoV and (ii) the resolution does not depend on the sampling by the detector (for instance in terms of sampling rate, pixel shape, pixel fill-factor)

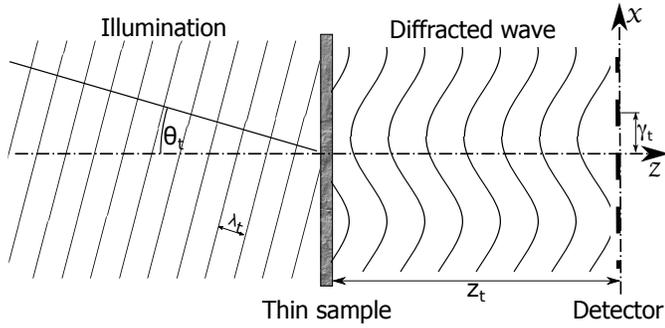


Fig. 1. Setup.

and is determined only by the propagation distance z and the illumination parameters (wavelength, coherence, incidence angle). From these parameters, we have proposed some estimates of the FoV and the resolution of any given setup.

The estimated [1] effective resolution of the setup can be several times higher than that of the camera even though the recorded signals are often undersampled and the Nyquist criterion not fulfilled. This induces aliasing issues ignored in many works [41, 47–51] that reconstruct the object at the same resolution as that of the detector. To overcome this limitation, several methods have proposed to recover aliased spatial frequencies on super-resolved holograms built from subpixels-shifted low-resolution holograms [9, 31, 52–54, 57]. Other works directly reconstruct the object from aliased intensity using prior knowledge on its shape or bandwidth [58–61].

In this paper, we present a reconstruction method that recovers high-resolution, large FoV images of the phase and amplitude of a sample from intensity measurements taken at a lower resolution and over a smaller FoV. It does not need the estimation of superresolution holograms before use. The presented framework is versatile enough to be suitable to a wide range of applications with varying depth, wavelength, and illumination. It derives from a similar framework for image reconstruction in optical long-baseline interferometry [43, 62] and was already successfully applied in lensless microscopy [3, 4].

COMCI has the same objective as the propagation phasor approach proposed in [63], but it has only one stage and no need for super-resolution methods while still being able to extrapolate the FoV. Moreover, based on an inverse-problem approach, COMCI is optimal in a likelihood-related statistical sense.

2. IMAGE FORMATION MODEL

A. Notations

We typeset symbols in lowercase for functions and scalars (e.g., o and λ), in boldface lowercase for vectors (e.g., \mathbf{o}), in uppercase calligraphic for operators acting on functions (e.g., \mathcal{M}), and in boldface uppercase for matrices (e.g., \mathbf{H}). In this context, *vectors* are collections of values (e.g., dataset, sampled object) while *matrices* are linear mappings that produce vectors when applied to vectors. We denote by x^\top the adjoint (i.e., the conjugate transpose) of x , $\langle x, y \rangle = x^\top y$ is the scalar product between vectors x and y and $x \times y$ their element-wise (Hadamard) product. The Euclidean norm of x being $\|x\|_2 = \sqrt{x^\top x}$ while $\|x\|_{\mathbf{W}} = \sqrt{x^\top \mathbf{W} x}$ is a weighted version of the norm (with \mathbf{W} Hermitian positive semidefinite). With these notations, a wave, say $w \in \mathcal{W}$, is a complex-valued function of the 2D lateral coordinates $x = (x_1, x_2)$. Due to the finite amount of power trans-

ported by a physically realistic wave, w is square-integrable, hence \mathcal{W} is the Hilbert space of square-integrable functions from \mathbb{R}^2 to \mathbb{C} . The discretized version of this wave is the vector $\mathbf{w} = (w_1, \dots, w_N) \in \mathbb{C}^N$ ordered in lexicographical order of N samples. Functions and vectors with a hat (i.e., \hat{w}) and without a hat (i.e., w) are in the Fourier and in the space domains, respectively. We note $\omega = (\omega_1, \omega_2) \in \mathbb{R}^2$ the 2D angular frequency and by \mathcal{F} the continuous (non-unitary) 2D Fourier-transform operator defined as

$$\hat{f}(\omega) \triangleq \mathcal{F}\{f\}(\omega) = \iint_{\mathbb{R}^2} f(x) e^{-jx^\top \omega} dx. \quad (1)$$

B. Setup

We consider the lensless setup depicted in Fig. 1. There a thin (2D) sample, described by the function $o : \mathbb{R}^2 \rightarrow \mathbb{C}$, is placed orthogonally to the optical axis at $z = 0$. For each observation $t = \{1, \dots, T\}$, the sample is illuminated by the wave u_t of wavelength λ_t (or a wavenumber $k_t = n_0 \frac{2\pi}{\lambda_t}$) arriving at the incidence angle $\theta_t = (\theta_{1,t}, \theta_{2,t})$ relatively to the optical axis. After propagation in a medium of refractive index n , the diffracted wave w_t is recorded by a detector of size $(\ell_1 \times \ell_2)$ placed at a distance z_t , orthogonally to the optical axis. The detector produces the discrete measurements $\mathbf{d}_t \in \mathbb{R}^P$. The detector is of size $\ell = (\ell_1 \times \ell_2)$ with a square pixel pitch of Δ and a number of pixels $P = \ell_1 \ell_2 / \Delta^2$.

C. Varying Parameters

The phase-retrieval problem can be solved only by varying the parameters between observations to provide sufficient diversity. The parameters that can vary are:

- the **distance** z_t between the sample and the detector;
- the **wavelength** λ_t or the wavenumber $k_t = n_0 \frac{2\pi}{\lambda_t}$ of the illumination light;
- the illumination wave u_t and its **angle** $\theta_t = (\theta_{1,t}, \theta_{2,t})$ with the optical axis. In addition to providing diversity to solve the phase retrieval, an inclined illumination also increases the spatial resolution through aperture synthesis;
- the lateral **shift** $\gamma_t = (\gamma_{x,1}, \gamma_{y,2})$ of the detector. As no phase diversity is introduced, detector shifts do not add information for phase retrieval.

3. CONTINUOUS MODEL

To interpret the measurements \mathbf{d}_t , one has to derive a rigorous model that accounts for the totality of the measured information. This model is summarized in the forward operator $\mathcal{H}_t : L^2(\mathbb{R}^2) \rightarrow \mathbb{R}^P$ that acts on the Hilbert space of square-integrable functions L^2 and links the object o to the measurements $\mathbf{d} \in \mathbb{R}^P$ as in

$$\mathbf{d}_t = \mathcal{H}_t(o) + \mathbf{e}_t, \quad (2)$$

where \mathbf{e}_t is an error term.

A. Sample Illumination

The complex electrical field v_t right after a sample illuminated by a wave u_t with an incident angle θ_t is

$$v_t(x) = o(x) u_t(x) \exp(jk_t x^\top \sin(\theta_t)). \quad (3)$$

The fact that the incidence is tilted induces a modulation that can be expressed as a $k_t \sin(\theta_t)$ shift in the Fourier domain, defining $\sin(\theta_t) = (\sin(\theta_{1,t}), \sin(\theta_{2,t}))$

B. Propagation

As lensless setups can have a high numerical aperture, the propagation from the sample plane to the detector plane is modeled by the mean of the angular-spectrum (AS) propagation model [48]. Under a tilted illumination, this propagation kernel writes [1]

$$\hat{h}_t^{\text{AS}}(\omega) = \begin{cases} e^{jz_t \sqrt{k_t^2 - \|\omega + k_t \sin(\theta_t)\|^2}}, & \|\omega + k_t \sin(\theta_t)\|^2 \leq k_t^2 \\ 0, & \text{otherwise.} \end{cases} \quad (4)$$

C. Detector Shift

From one measurement to another, the detector may experience lateral shifts γ_t with respect to the system of coordinates used in the propagation model. Thus, in the detector plane, the relation between the electrical field in the system of coordinates q_t of the sample and the electrical field in the system of coordinates w_t of the detector is modeled as

$$q_t(x + \gamma_t) = w_t(x). \quad (5)$$

Equivalently, it can also be expressed as a modulation in the Fourier domain:

$$\hat{w}(\omega) = \hat{q}(\omega) e^{-j\gamma_t^\top \omega}. \quad (6)$$

D. Overall Model

Taking into an account illumination, propagation, and detector shifts, the model that links the object and the measurements in the detector plane is

$$w_t(x) = \mathcal{F}^{-1} \left\{ \hat{h}_t \times \mathcal{F} \{ u_t \times o \} \right\} (x) e^{jkx^\top \sin(\theta_t)}, \quad (7)$$

with the kernel \hat{h}_t given by

$$\hat{h}_t(\omega) = \begin{cases} e^{j \left(z_t \sqrt{k_t^2 - \|\omega + k_t \sin(\theta_t)\|^2} - \gamma_t^\top \omega \right)}, & \|\omega + k_t \sin(\theta_t)\|^2 \leq k_t^2 \\ 0, & \text{otherwise.} \end{cases} \quad (8)$$

If the object spatial-frequency bandwidth is narrow or if the paraxial-approximation hypothesis is fulfilled (see [1]), this propagation model can be approximate by

$$\hat{h}_t^{\text{F}}(\omega) = e^{j \left(k_t z_t - \gamma_t^\top \omega - \frac{z_t}{2k_t} \|\omega + k_t \sin(\theta_t)\|^2 \right)}. \quad (9)$$

E. Sensing

The intensity of the diffracted wave w_t is sampled on P pixels by the detector according to

$$d_{p,t} = \iint \varphi_{p,t}(x) |w_t(x)|^2 dx + e_{p,t}, \quad (10)$$

where $\varphi_{p,t}(x)$ is the response of the pixel p of the observation t at the position x . The error term $e_{p,t}$ accounts for the noise of detector, for the photon noise, and for approximation errors such as quantization.

4. DISCRETIZATION

A. Field of View and Superresolution

In practice, to numerically model the propagation, one has to establish a discrete versions of the equations found in section 3. To that end, it is important to determine the field of view and the spatial-frequency bandwidth probed by the setup. Due to the bandlimited nature of the propagation kernel given in Eq. (4), the theoretical field of view of a lensless setup is infinite and its angular spatial-frequency bandwidth is $B = 2k$, centered on $k \sin(\theta)$. In a recent paper [1], we have derived more realistic bounds on the size ℓ'_t of the FoV and the bandwidth B'_t for the set $\{z_t, \lambda_t, \theta_t, \gamma_t\}$ of setup parameters. These bounds can be easily derived from few setup properties, namely, the noise level and the spatio-temporal coherence of the source. In this work, we define the field of view and the bandwidth of the object o as the maxima $\ell' = \max_t(\ell'_t)$ and $B' = \max_t(B'_t)$, over all T observations.

To ensure the separability of the likelihood term in the reconstruction algorithm, we impose commensurability between the sampling rates of the detector and of the object o . Consequently, if we sample the object on a grid, its pixel pitch Δ' is given by an integer superresolution factor S such that

$$\Delta' = \Delta/S, \quad (11)$$

$$S = \left\lceil \frac{\Delta B'}{2\pi} \right\rceil. \quad (12)$$

At optical wavelengths, detectors cannot measure complex amplitudes of the electric field but only the intensity of the light. The Fourier spectrum of the intensity is the auto-correlation of the Fourier spectrum of the complex amplitude [48]. As consequence, the bandwidth of the intensity can be twice as wide as the bandwidth B' of the diffracted wave w . Thus, in the modeling (Eq. (10)) of the sensing, w must be oversampled by a factor two along each dimension before to being converted to intensity. Hence for an object o sampled on N pixels, the electric field w in the detector plane must be oversampled on $4N$ pixels.

Among all possible interpolation functions, we choose to sample the functions of the object o , the waves w_t and the operators and \hat{h}_t using Dirac delta functions to form the vectors

$$o_n = o(x_n), \quad (13)$$

$$u_{t,n} = u(x_{t,n}), \quad (14)$$

$$w_{t,n} = w_t(x_n), \quad (15)$$

$$\hat{h}_{t,n} = \hat{h}_t(\omega_n). \quad (16)$$

There, x_n is the position of the n th pixel and ω is the angular frequency of the n th frequel. It is possible to use better sampling functions when permitted by the spatio-frequency properties of the observed sample [64].

B. Discrete Forward Model

With discretization of Sec. A, for each observation t the complex electrical field on the surface of the detector is given by

$$w_t = \mathbf{H}_t o, \quad (17)$$

where the forward operator $\mathbf{H}_t \in \mathbb{C}^{4N \times N}$ is a propagation operator given by

$$\mathbf{H}_t = \mathbf{F}_{4N}^{-1} \cdot \mathbf{Q} \cdot \text{diag}(\hat{h}_t) \cdot \mathbf{F}_N \cdot \text{diag}(u_t). \quad (18)$$

where \mathbf{F}_N is the 2D discrete Fourier operator of size N . $\mathbf{Q} \in \mathbb{C}^{4N \times N}$ is a zero-padding operator accounting for the spatial over-sampling of w .

From Eq. (10), this intensity is linked to the measured image intensities $d_t \in \mathbb{R}_+^P$ by

$$d_{p,t} = \sum_{n \in \mathcal{S}_p} \varphi_{p,t,n} |w_{n,t}|^2 + e_{p,t}. \quad (19)$$

As the wave is sampled at a rate that is higher than that of the detector, \mathcal{S}_p is the set of samples of the wave $w_{n,t}$ in the sensitive area of the detector pixel p and $\varphi_{p,t,n} = \varphi_{p,t}(x_n)$ is the discretized response of the pixel p in the observation t for a wave sample at position x_n .

5. INVERSE PROBLEM

The goal of this work is to estimate the complex object o from the observation of the diffracted intensity, which is an inverse problem. It is classically solved in a variational framework where one estimates o^+ minimizing a cost function built as the sum of the likelihood term \mathcal{L} and a regularization term \mathcal{R} like in

$$o^+ = \arg \min_{o \in \mathbb{D}^N} (\mathcal{L}(o) + \mu \mathcal{R}(o)), \quad (20)$$

where \mathbb{D} is the subspace of \mathbb{C} where lies the object. In this approach, known as penalized *maximum likelihood* or *maximum a posteriori*, the data term is defined according to the forward model and the statistics of the noise, whereas the regularization function is designed to enforce some prior knowledge about the object such as support, non-negativity or smoothness.

A. Likelihood

The likelihood term \mathcal{L} is computed according to the forward model described in Sec. B and the statistics of the noise. We assume that the noise e_p is independent and there is no crosstalk between pixels ($\mathcal{S}_p \cap \mathcal{S}_{p'} = \emptyset; \forall p \neq p'$). As consequence, the likelihood term is separable with

$$\mathcal{L}(o) = \sum_{t=1}^T \sum_{p \in \mathcal{P}_t} \mathcal{L}_{p,t} \left(\sum_{n \in \mathcal{S}_p} \varphi_{p,t,n} |w_{n,t}|^2 \right). \quad (21)$$

The set \mathcal{P}_t provides an easy way to cope with badly measured pixels, such as saturated ones which do not belong to \mathcal{P}_t .

In most applications, only two types of noise are considered: non-stationary Gaussian noise or Poisson noise in the low-flux case. In the Gaussian case, the likelihood term writes

$$\mathcal{L}_{p,t}(x) = \frac{1}{\sigma_{p,t}^2} (x - d_{p,t})^2, \quad (22)$$

where $\sigma_{p,t}^2$ is the variance of the noise at the pixel p of the t th image.

In the photon-counting case, the noise follows a Poisson distribution and the likelihood function writes

$$\mathcal{L}_{p,t}(x) = x - d_{p,t} \log(x + \beta_{p,t}), \quad (23)$$

where $\beta_{p,t}$ is the expectation of some spurious independent Poisson process that accounts for incoherent background emission and detector dark current at the pixel p of the t th image.

B. Regularization

The regularization function \mathcal{R} enforces some prior knowledge about the observed sample. In this work, we use the total variation [28]. For a complex object o , it is:

$$\mathcal{R}(o) = \|\mathbf{B} o\|_{2,1} \quad (24)$$

$$= \sum_{n=1}^N \|\mathbf{B} o\|_n, \quad (25)$$

while \mathbf{B} is the spatial gradient operator

$$\mathbf{B} = \begin{bmatrix} \nabla_1 \\ \nabla_2 \end{bmatrix}, \quad (26)$$

where ∇_1 and ∇_2 are finite-difference operators along the horizontal and vertical directions, respectively. Under circulant boundaries condition, these finite-difference operators can be defined in the Fourier domain as

$$\mathbf{B} = \begin{bmatrix} \mathbf{F}^{-1} \cdot \text{diag}(\hat{b}_1) \\ \mathbf{F}^{-1} \cdot \text{diag}(\hat{b}_2) \end{bmatrix} \mathbf{F}. \quad (27)$$

where \hat{b}_1 and \hat{b}_2 are spatial differentiation operator expressed in the Fourier domain along dimension 1 and 2 respectively.

Compared to the total variation applied separately on the phase and modulus of o , as in [65], this complex regularization function enforces a correlation between edges in both the real and imaginary parts. This means that different area of the sample are supposed differ in their real and imaginary parts or, equivalently, for both the modulus and phase. That makes sense as different media may differ at the same time in their refractive index and in their absorption.

C. Constraints

Depending on the object, various constraints can be chosen to define the subspace \mathbb{D} . We define the characteristic function of the subspace \mathbb{D} as

$$\mathcal{C}_{\mathbb{D}}(x) = \begin{cases} 0, & x \in \mathbb{D} \\ +\infty, & \text{otherwise.} \end{cases} \quad (28)$$

No amplification As no emission occurs within the sample, the modulus of the object must be lower than 1, so that

$$\mathbb{D}_1 = \{x \in \mathbb{C}; |x| \leq 1\}. \quad (29)$$

Phase-only object Biological samples are often transparent and can be modeled as phase-only object. In this case, the modulus of the object is constrained to have a unit modulus leading to

$$\mathbb{D}_2 = \{x \in \mathbb{C}; |x| = 1\}. \quad (30)$$

Absorption-only object If the sample is composed of opaque structures, then its phase is null and its magnitude must be either 0 or 1. Such prior is highly non-convex; we relax it to the convex constraint

$$\mathbb{D}_3 = \{x \in \mathbb{R}; 0 \leq x \leq 1\}. \quad (31)$$

Support Constraint The constraint can also be a support constraint, as classically done in phase retrieval [34]. It constrains the object to be 1 outside of the support \mathcal{G} of the object

$$\mathbb{D}_4 = \{x_n \in \mathbb{C}; x_n = 1 \text{ if } n \notin \mathcal{G}\}. \quad (32)$$

6. MINIMIZATION SCHEME

From the forward model of Section B and the separable likelihood in Eq. (21), the Equation 20 can be rewritten in the constrained form, introducing the auxiliary variables $\mathbf{y} \in \mathbb{C}^N$ and $\mathbf{z} \in \mathbb{C}^{2N}$

$$\begin{aligned} \mathbf{o}^+ = \arg \min_{\mathbf{o}} & \sum_{t=1}^T \sum_{p=1}^P \mathcal{L}_{p,t} \left(\sum_{m \in \mathcal{S}_p} \phi_{p,n} |w_{m,t}|^2 \right) \\ & + \mu \|\mathbf{z}\|_{2,1} + \sum_n \mathcal{C}_D(y_n) \\ \text{subject to} & \begin{cases} \mathbf{H}_t \mathbf{o} = \mathbf{w}_t, \forall t \\ \mathbf{o} = \mathbf{y} \\ \mathbf{z} = \mathbf{B} \mathbf{o}, \end{cases} \end{aligned} \quad (33)$$

These constraints are enforced by means of an augmented-Lagrangian formulation. The augmented-Lagrangian is

$$\begin{aligned} \mathcal{J}(\mathbf{o}, \mathbf{w}, \mathbf{y}, \mathbf{z}, \mathbf{q}, \mathbf{v}, \mathbf{r}) = & \sum_{t=1}^T \sum_{p=1}^P \mathcal{L}_{p,t} \left(\sum_{m \in \mathcal{S}_p} |w_{m,t}|^2 \right) \\ & + \mu \sum_n \|\mathbf{z}_n\|_{2,1} + \sum_n \mathcal{C}_n(|y_n|^2) + \frac{\rho_1}{2} \sum_{t=1}^T \|\mathbf{H}_t \mathbf{o} - \mathbf{w}_t + \mathbf{q}_t\|_2^2 \\ & + \frac{\rho_2}{2} \|\mathbf{o} - \mathbf{y} + \mathbf{v}\|_2^2 + \frac{\rho_3}{2} \|\mathbf{B} \mathbf{o} - \mathbf{z} + \mathbf{r}\|_2^2, \end{aligned} \quad (34)$$

where $\mathbf{q}_t \in \mathbb{C}^N$, $\mathbf{v} \in \mathbb{C}^N$, $\mathbf{r} \in \mathbb{C}^{2N}$ are the scaled Lagrange multipliers and $\rho_1 > 0$, $\rho_2 > 0$, and $\rho_3 > 0$ the augmented-penalty scalar parameters. As there is no theoretical guidelines to set these augmented penalty, we choose to rely on the empirical rules described in Sec. E

Under this form, the problem can be solve by the mean of the alternating-direction method of multiplier (ADMM)[40], solving at each iteration k the series of problems

$$\mathbf{w}_t^{(k+1)} = \text{prox}_{1/\rho_1 \mathcal{L}} \left(\mathbf{H}_t \mathbf{o}^{(k)} + \mathbf{q}_t^{(k)} \right), \quad (35)$$

$$\mathbf{y}^{(k+1)} = \text{prox}_{1/\rho_2 \mathcal{C}_D} \left(\mathbf{o}^{(k)} + \mathbf{v}^{(k)} \right), \quad (36)$$

$$\mathbf{z}^{(k+1)} = \text{prox}_{\mu/\rho_3 \|\cdot\|_{2,1}} \left(\mathbf{B} \mathbf{o}^{(k)} + \mathbf{r}^{(k)} \right), \quad (37)$$

$$\begin{aligned} \mathbf{o}^{(k+1)} = \arg \min_{\mathbf{o} \in \mathbb{C}^N} & \frac{\rho_1}{2} \sum_{t=1}^T \left\| \mathbf{H}_t \mathbf{o} - \mathbf{w}_t^{(k)} + \mathbf{q}_t^{(k)} \right\|_2^2 \\ & + \frac{\rho_2}{2} \left\| \mathbf{o} - \mathbf{y}^{(k)} + \mathbf{v}^{(k)} \right\|_2^2 + \frac{\rho_3}{2} \left\| \mathbf{B} \mathbf{o} - \mathbf{z}^{(k)} + \mathbf{r}^{(k)} \right\|_2^2. \end{aligned} \quad (38)$$

There, $\text{prox}_{1/\rho_1 \mathcal{L}}$, $\text{prox}_{1/\rho_2 \mathcal{C}_D}$, and $\text{prox}_{\mu/\rho_3 \|\cdot\|_{2,1}}$ are proximity operators (*a.k.a.* Moreau's proximal mapping operators) of the likelihood function, the constraints characteristic function, and the regularization function, respectively.

At each iteration, the scaled Lagrange multipliers are updated using the rule

$$\mathbf{q}_t^{(k+1)} = \mathbf{q}_t^{(k)} + \mathbf{H}_t \mathbf{o}^{(k+1)} - \mathbf{w}_t^{(k+1)}, \quad (39)$$

$$\mathbf{v}^{(k+1)} = \mathbf{v}^{(k)} + \mathbf{y}^{(k+1)} - \mathbf{o}^{(k+1)}, \quad (40)$$

$$\mathbf{r}^{(k+1)} = \mathbf{r}^{(k)} + \mathbf{B} \mathbf{o}^{(k+1)} - \mathbf{z}_t^{(k+1)}. \quad (41)$$

A. Likelihood Sub-Problem

The solution of the first inner problem Eq. (35) is given by the proximity operator of the likelihood function \mathcal{L} . As the likelihood function is separable, its solution is given by the proximity operator of the function $\mathcal{L}_{p,t}$ for each observation t and each low resolution pixel p

$$\text{prox}_{1/\rho \mathcal{L}_{p,t}}(\mathbf{y}) = \arg \min_x \left(\mathcal{L}_{p,t}(x) + \frac{\rho}{2} \|\mathbf{x} - \mathbf{y}\|_2^2 \right). \quad (42)$$

For Gaussian and Poisson likelihoods, and without crosstalk between pixels, this proximity operator admits closed form solutions [66] that are used further in the paper. A generalization of these proximity operators in presence of crosstalk is proposed in [67].

B. Constraints Sub-Problem

The solution of the second inner problem Eq. (36) is given by the proximity operator of the characteristic function \mathcal{C}_D of the constraints. All the constraints described in Section C are separable and the solution is given for each pixel p by

$$\text{prox}_{1/\rho \mathcal{C}}(y_n) = \arg \min_x \left(\mathcal{C}_D(x) + \frac{\rho}{2} \|x - y_n\|_2^2 \right). \quad (43)$$

For each constraint described in Section C, this proximity operator is a simple projection on the subspace \mathbb{D} and has a closed-form solution.

C. Regularization Sub-Problem

The solution of the third inner problem Eq. (37) is given by the proximity operator of the mixed norm $\|\cdot\|_{2,1}$. This proximity operator has the closed-form solution [68]

$$\left[\text{prox}_{1/\rho \|\cdot\|_{2,1}}(\mathbf{y}) \right]_{n,d} = y_{n,d} \left(1 - \left(\rho \sqrt{y_{n,1}^2 + y_{n,2}^2} \right)^{-1} \right)_+, \quad (44)$$

with $(x)_+ = \max(0, x)$, $x \in \mathbb{R}$. The pixel index is given by n and $d \in \{1, 2\}$ is the direction of the derivative.

D. Fourth (Consensus) Sub-Problem

The fourth problem Eq. (38) is a quadratic problem. It has the closed-form solution

$$\begin{aligned} \mathbf{o}^+ = & \left[\frac{\rho_1}{2} \sum_{t=1}^T \mathbf{H}_t^\top \mathbf{H}_t + \frac{\rho_2}{2} + \frac{\rho_3}{2} \mathbf{B}^\top \mathbf{B} \right]^{-1} \\ & \left[\frac{\rho_1}{2} \sum_{t=1}^T \mathbf{H}_t^\top (\mathbf{w}_t - \mathbf{q}_t) + \frac{\rho_2}{2} (\mathbf{y} - \mathbf{v}) + \frac{\rho_3}{2} \mathbf{B}^\top (\mathbf{z} - \mathbf{r}) \right]. \end{aligned} \quad (45)$$

Unfortunately, in the general case, owing to the size of \mathbf{H} and \mathbf{B} , the matrix inversion of the first term cannot be performed in practice. However, this problem can be solved approximately by conjugate-gradient steps as is classically done[40]. In this case, the number M of Fourier transforms of size N computed per iteration is:

$$M = M_{\text{CG}} (4T + 2) + 2T + 3, \quad (46)$$

where M_{CG} is the number of conjugate-gradient steps. In addition, at each iteration, there are $(T \times P)$ estimations of a root of a cubic or quadratic polynomial needed at Equation 35.

The use of the conjugate gradient makes the computational burden very high and prevents the use of the algorithm for large problem. However there are some cases where the exact solution of Equation 45 can be efficiently estimated: when the illumination is a plane wave and when the illumination is uniform.

D.1. Plane Wave Illumination

When the sample is illuminated by plane waves, the term u_t in Eq. (3) is constant: $u_{t,p} = a_t, \forall p$. In this case, for each observation t , the matrix $\mathbf{H}_t^\top \mathbf{H}_t$ becomes diagonal in the Fourier domain

$$\mathbf{H}_t^\top \mathbf{H}_t = |a_t|^2 \mathbf{F}^{-1} \text{diag} \left(|\widehat{\mathbf{h}}_t|^2 \right) \mathbf{F}. \quad (47)$$

The solution of Eq. (38) can be written in the Fourier domain as

$$\widehat{\mathbf{o}}^+ = \left(\rho_1 \sum_{t=1}^T a_t^2 + \rho_2 + \rho_3 \sum_{d \in \{1,2\}} |\widehat{\mathbf{h}}_d|^2 \right)^{-1} \left(\rho_1 \sum_{t=1}^T a_t \widehat{\mathbf{h}}_t^* (\widehat{\mathbf{w}}_t - \widehat{\mathbf{q}}_t) + \rho_2 \sum_{d \in \{1,2\}} \widehat{\mathbf{b}}_d^* (\widehat{\mathbf{z}}_d - \widehat{\mathbf{r}}_d) + \rho_3 (\widehat{\mathbf{y}} - \widehat{\mathbf{v}}) \right) \quad (48)$$

Let us notice that, in that case, it is possible to spare Fourier transform computations by performing all ADMM steps (Equations (35) to (38)) in the Fourier domain. Equations (37) and (36) are solved in accordance with the following property of the proximal operator [69]:

$$g(x) = f(\mathbf{F}x) \ \& \ \mathbf{F}\mathbf{F}^\top = \mathbf{I} \Rightarrow \text{prox}_g(x) = \mathbf{F}^\top \text{prox}_f(\mathbf{F}x). \quad (49)$$

The number of Fourier transforms per iteration is reduced to

$$M = 2T + 6. \quad (50)$$

D.2. Uniform Illumination and Fresnel Propagation

In the Fresnel regime, the Fresnel propagation operator \mathbf{h}^F described in Eq. (9) has a unit modulus. Whatever the phase of the illumination wave, if its magnitude is uniform ($|u_{t,p}| = b_t, \forall p$) the expression $\mathbf{H}_t^\top \mathbf{H}_t$ becomes

$$\mathbf{H}_t^\top \mathbf{H}_t = b_t^2 \mathbf{I}. \quad (51)$$

The matrix inversion Eq. (45) can then be computed efficiently in Fourier domain. The number of Fourier transforms per iteration becomes

$$M = 4T + 5. \quad (52)$$

E. Setting Parameters

In addition to the hyperparameter μ common to all maximum a posteriori (MAP) methods, there are three more parameters to be set in this method: the augmentation parameters ρ_1, ρ_2 , and ρ_3 . As the overall problem is not convex, its solution depends on them. The optimum for these four parameters cannot be found by trials and errors with such iterative methods. Fortunately, after investigating the range of possible parameters, we found empirically the rules:

$$\rho = 100 \mu \quad (53)$$

$$\rho_1 = \rho / (T I_0) \quad (54)$$

$$\rho_2 = \rho \quad (55)$$

$$\rho_3 = \rho, \quad (56)$$

where I_0 is the mean intensity per pixel. These rules are used throughout all our results, leaving only the hyperparameter μ to tune.

The superresolution parameter s is usually set according to the rules derived in [1]. We experimentally found that the hyperparameter μ is approximately independent of this superresolution parameter s . Using this observation, one can rapidly estimate the hyperparameter at a small superresolution before to perform a full reconstruction at larger superresolution.

7. NUMERICAL RESULTS

The proposed method covers a wider panel of applications that we can deal with. Nevertheless, we present in this section some results on synthetic data covering typical setups. All simulations and reconstructions presented in this paper were done in MATLAB™ using the GLOBALBIOIM library [70]. The open-source code used is available in the COMCI repository [71]. To assess its FoV extrapolation and super-resolution capabilities, all COMCI reconstruction were performed on a larger, extended FoV, with smaller pixels than prescribed in [1].

The computation time is dominated by the Fourier transform involved in the propagation modeling. Due to the GPU implementation of the FFT algorithm, it is possible to maintain acceptable processing times despite the method being computationally intensive: a handful to tens of minutes on an NVIDIA TITAN X GPU. However, given the superresolution and the extrapolation capabilities of COMCI, the dimension of the estimated vectors can reach tens of million pixels. It is limited by the amount of memory of GPU cards to less than $(10\,000 \times 10\,000)$ pixels, which corresponds to a few mm^2 sampled at $\lambda/2$.

Whether COMCI was already used in its preliminary form on real experiments ([3, 4], its performance was never assessed. We assessed it in terms of actual FoV, resolution, and reconstruction signal-to-noise ratio (SNR). This SNR is computed only on the camera FoV in order to be comparable with the reconstruction SNR of the Error Reduction (ER) algorithm, when possible. The ER algorithm [14, 34] is an alternating projection algorithm. It consists in propagating the wavefront back and forth between object and detector plane, constraining the modulus of the wavefront to be equal to the square root of the measured intensities in the detector plane and enforcing prior knowledge on the object by the mean of constraints as defined in section C.

The half-pitch resolution of the reconstructed target is given by the finest-resolved element of the target. It can be compared to the theoretical half-pitch resolution at the center of the FoV given by [1]

$$R = \frac{\lambda}{2n_0} \left(\frac{\ell/2}{\sqrt{(\ell/2)^2 + z^2}} + \sin(|\theta|) \right)^{-1}, \quad (57)$$

with $\ell = \max(\ell_1, \ell_2)$ the largest width of the detector and $\theta = \max(\theta_1, \theta_2)$ the higher illumination angle.

A. Simulating Data

To assess the performance of COMCI, we have built a synthetic vectorial image of a USAF-1951 resolution target [72] shown on Fig. 2. This vectorial image contains all resolution groups from group -1 to group 11. The smallest resolution element (11-6) has a width of 65 nm.

To avoid the so-called *inverse crime*, where one reconstructs the object from simulated data with exactly the same numerical model as the one used for the simulation, the propagation is simulated using bandlimited angular spectrum [73] with very high resolution over a very large field of view that is much larger than the camera FoV. This prevents the border artifacts

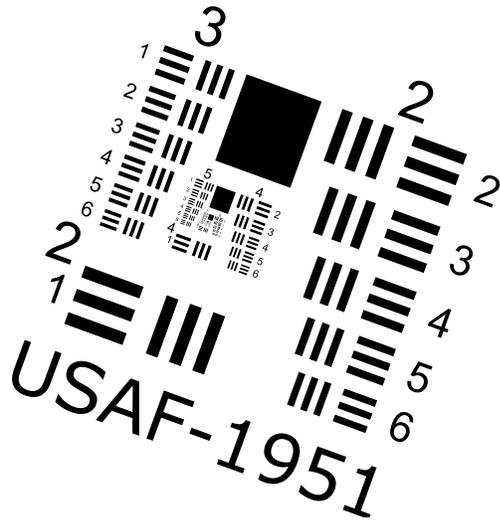


Fig. 2. Vectorial USAF-1951 resolution chart.

caused by propagation over a small FoV, as well as the frequency aliasing caused by the undersampling by the camera.

B. Low-Light Single Exposure of Absorbing Object

To assess the performance of COMCI under various noise conditions, we reconstruct an absorbing object from a single exposure for two levels of illumination: a low-light condition with 1 photon per pixel in average and an intense-light condition with 10^6 photons per pixel. The hologram of an opaque USAF-1951 target is simulated at $z = 1.5$ mm for a normal illumination at $\lambda = 532$ nm and a refractive index of 1. To prevent aliasing, the propagation is done over (23000×23000) square pixels of width $\Delta_s = 140$ nm. The modeled intensity in the detector plane is then downsampled on pixels of width $\Delta = 2.24$ μ m with unit fill-factor ($\phi_{p,n} = 1$ in Eq. (21)). The central area of (512×512) pixels is then corrupted by Poisson noise and used as the simulated measurements.

The target is reconstructed in 3000 iterations under an absorption-only object as described in Sec. C and a likelihood term adapted to photon counting (Eq.23). The data and the reconstructed targets are shown in Fig. 3 and compared to the result of the ER algorithm [74] that appears, in this case, still competitive with the state of the art [75]. Reconstruction parameters and quantitative assessments in terms of size of the FoV, resolution, and reconstruction SNR are given in Tab.1. The modulus of the reconstructed target is shown on Fig. 3 with zooms on high-resolution group on Fig. 4 and Fig. 5. Due to the absorption-only constraint the phase is 0 everywhere.

In the high-flux (low-noise) case, the reconstructed target by the ER algorithm is qualitatively good but suffer from border artifacts near the edges of the camera. Its resolution is limited by the sampling of the camera to a half-pitch of 2.19 μ m (group 7-6). By comparison, the COMCI reconstruction enjoys a much higher SNR (36.25 dB vs 19.92 dB) and, above all, has high resolution and a larger FoV as predicted from [1]. The extrapolation is clearly visible on Fig. 3. As stated in [1], lensless setups cannot capture low-spatial frequencies from regions outside of the FoV; but the edges of the resolution elements that contain high frequencies are clearly visible as far as 210 μ m away from the detector FoV. The smallest element being resolved by the reconstruction is the group 8-6, with a half-pitch resolution of 1.1 μ m

	Mean number of photons per pixel	
	1	10^6
reconstruction parameters		
Δ' (reconstruction)	560 nm	
FoV	(3120 \times 3120) pixels (1.7 \times 1.7) mm ²	
μ	0.2	10^{-6}
COMCI		
Extrapolation along each directions	210 μ m	14 μ m
FoV	(1.57 \times 1.57) mm ²	(1.12 \times 1.12) mm ²
half-pitch resolution	7.81 μ m	1.1 μ m
SNR	14.76 dB	36.25 dB
Error reduction		
Δ	2.24 μ m	
FoV	(1.1 \times 1.1) mm ²	(1.1 \times 1.1) mm ²
SNR	4.07 dB	19.92 dB
half-pitch resolution	13.92 μ m	2.19 μ m

Table 1. Reconstruction parameters and assessment of the reconstruction quality for the two illumination-flux cases.

(NA= 0.24) that corresponds to a superresolution factor $S = 2$ and a space-bandwidth product of the reconstruction almost 8 times larger than that of the camera. It is still larger than the theoretical half-pitch resolution at the center of the FoV of $R = 760$ nm.

In the very-low-flux case, the ER algorithm fails to reconstruct the target. COMCI achieves the recovery of low-resolution elements down to the 6-1 group. This corresponds to a half-pitch of 7.8 μ m (NA= 0.03), similar to the theoretical half-pitch resolution at the center of the FoV of 8 μ m estimated from [1]. The noise in the measurements almost prevents extrapolation and limits it to few pixels (14 μ m).

C. Multi-Angles Illumination

By combining holograms with various illumination angles, it is possible to increase the resolution by aperture synthesis. We test the performance in resolution of COMCI by simulating 9 holograms under various incidence angles, varying $\theta_i \in [-50 : 25 : 50]^\circ$ along both axis separately. The simulated target is a transparent USAF-1951 target whose phase is either 0 or 1. The illumination at $\lambda = 700$ nm and a refractive index of 1.52. The propagation to $z = 0.1$ mm is done over (23000×23000) pixels of width $\Delta_s = 120$ nm. The modeled intensity in the detector plane is then downsampled on pixels of width $\Delta = 1.2$ μ m with unit fill-factor ($\phi_{p,n} = 1$ in Eq. (21)). The central area of (308×308) pixels is then corrupted by 60 dB Gaussian noise and used as the simulated measurements.

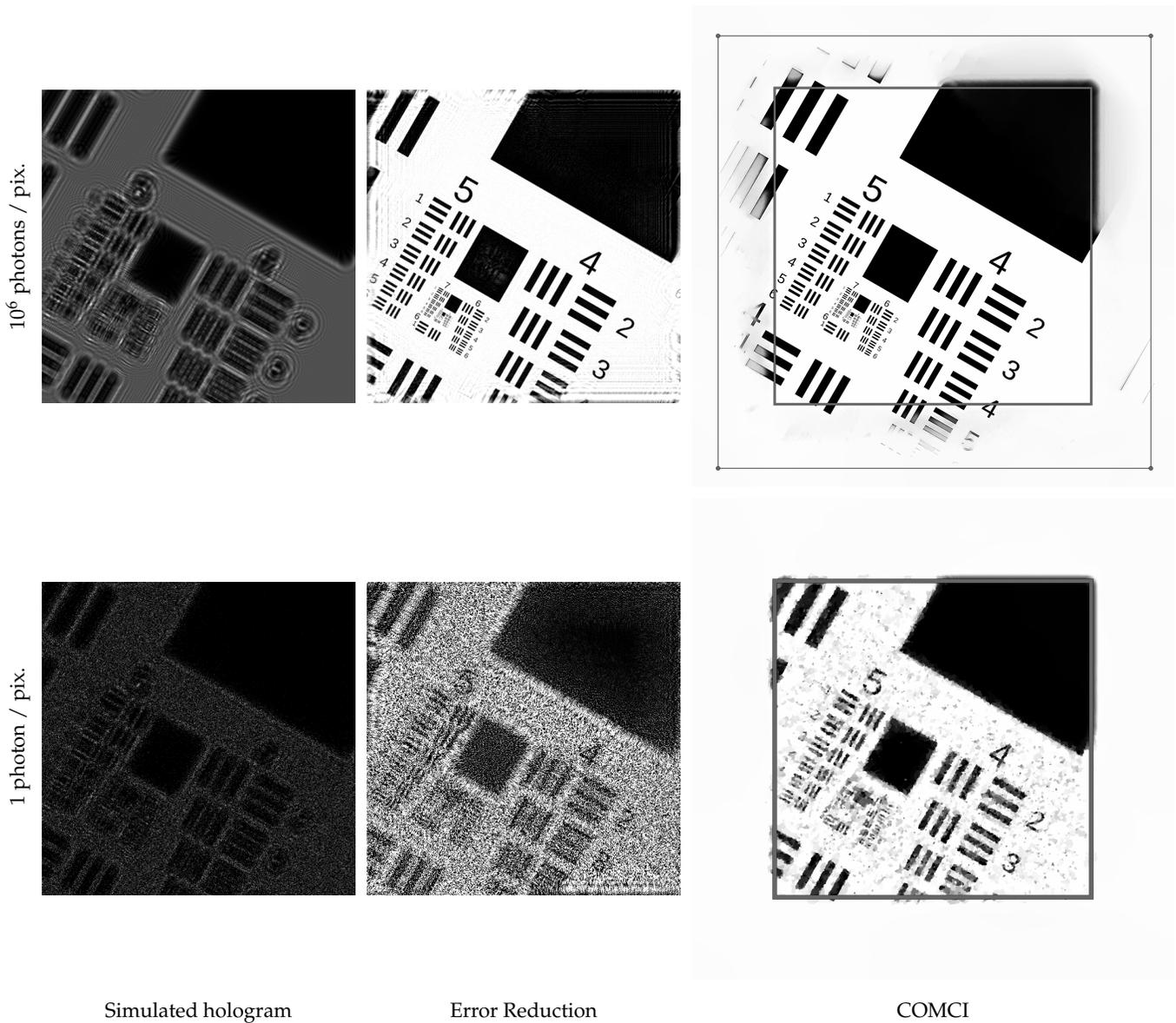


Fig. 3. Simulated hologram and modulus of the reconstructed target by the ER and COMCI algorithms for two levels of flux. On the COMCI reconstruction, the inner square represents the camera FoV, the outer square represents the extrapolated FoV. As a scalebar, an element of the group 5-1 is of size $78 \times 16 \mu\text{m}^2$.

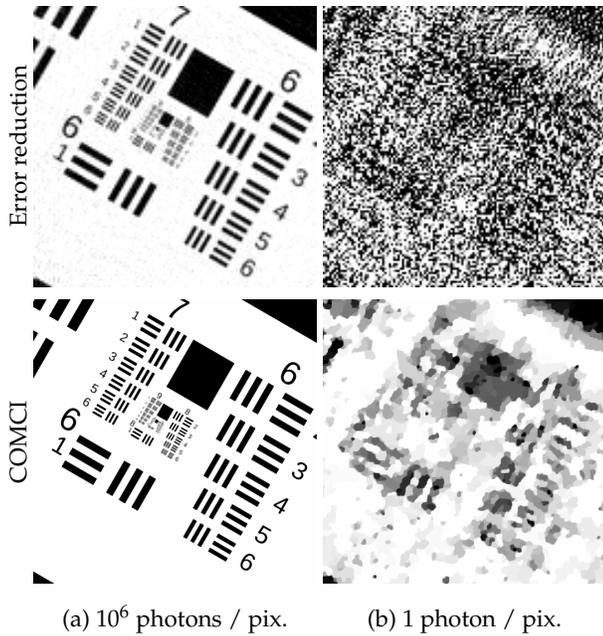


Fig. 4. Zoom on the group 6 and 7 of the modulus of the reconstructed target by the ER and COMCI algorithms. As a scalebar, an element of the group 6-1 is of size $39 \times 7.8 \mu\text{m}^2$.

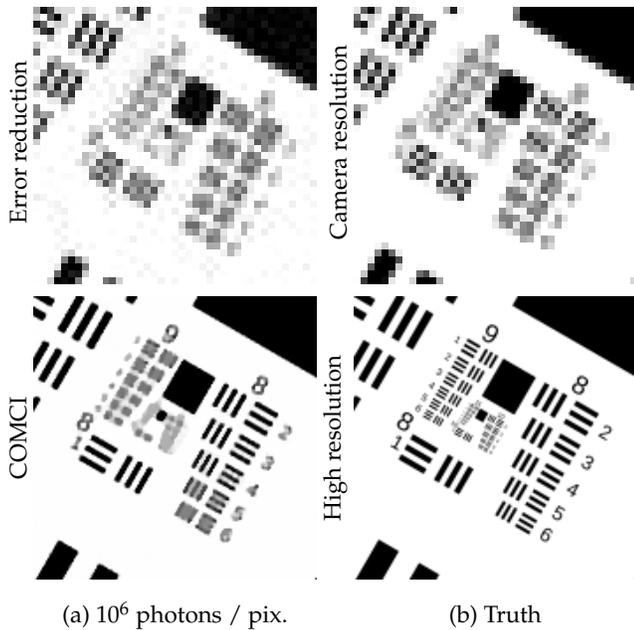


Fig. 5. Zoom on the group 8 and 9 of (a) Fig. 3 in the 10^6 photons / pixel case compared to the ground truth (b) rasterized at the camera resolution (top) and the resolution of the COMCI reconstruction (bottom). As a scalebar, an element of the group 6-1 is of size $9.75 \times 1.95 \mu\text{m}^2$.

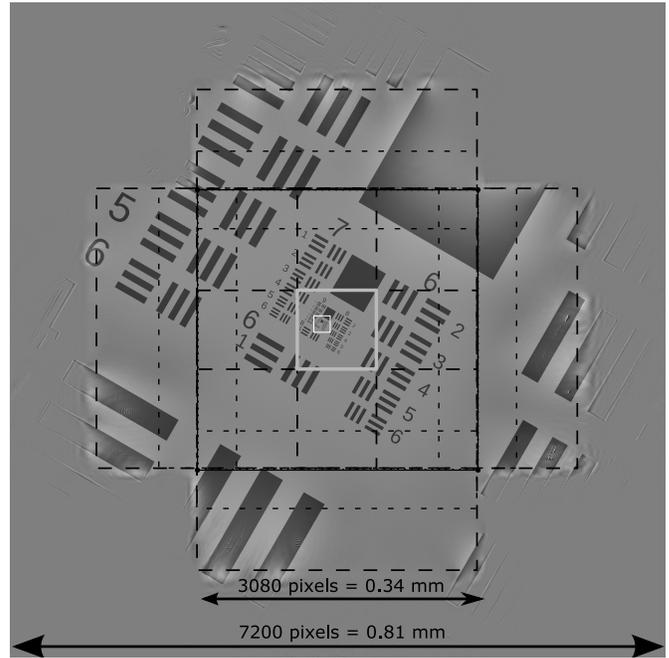


Fig. 6. Reconstructed target in the multiangle case (9 angles). The 9 black squares represent the FoV of the 9 different measurements. The central gray square depicts the area common to the 9 holograms. The very small inner white square covers the zoom on the group 10 shown Fig. 7.

The target is reconstructed with $\Delta' = 120 \text{ nm}$ pixel-pitch over a FoV of (7200×7200) pixels. Its phase, after 3000 iterations, is shown in Fig. 6 with a zoom on the group 10 shown in Fig. 7. As a tilted illumination shifts the hologram, the camera FoV varies from one frame to the next. Only a small area of $(104 \times 104) \mu\text{m}^2$ centered on the group 8 is visible on the nine holograms. However, the structures are well recovered even in area in the FoV of only a single hologram. The edges are visible well outside of the FoV of all hologram, which sets the actual reconstructed FoV to $(810 \times 810) \mu\text{m}^2$, more than 5 times the detector area. The finest resolved element on Fig. 7 is the group 10-6, giving a half-pitch resolution of 275 nm ($\text{NA} = 1.25$), at twice the theoretical resolution of $R = 138 \text{ nm}$. This corresponds an actual superresolution factor $S = 4$ and gives a space-bandwidth product of the reconstruction that is 20 times larger than the camera space-bandwidth product. Even at such high resolution, the reconstructed phase is accurate, as can be seen on the profile plotted on Fig. 8 where the phase varies on a range of 1 radian in accordance with the ground truth.

D. Multi-height reconstruction

To show the versatility of the COMCI framework, we reconstruct in this section an object with amplitude and phase from measurements with two different distance between the sample and the sensor: $z = \{250, 270\} \mu\text{m}$. The data was simulated with parameters similar to [63]: a normal incidence illumination at $\lambda = 530 \text{ nm}$, a refractive index of $n_0 = 1$ and a camera of (500×500) square pixels of width $\Delta = 1.12 \mu\text{m}$ (camera FoV of $(560 \times 560) \mu\text{m}^2$). To prevent aliasing we simulate data over a FoV of $(1.3 \times 1.3) \text{ mm}^2$ with pixels of width 93 nm . As gigapixels images was needed, we build a ground truth image from a NASA MODIS earth observation color image, using the hue and

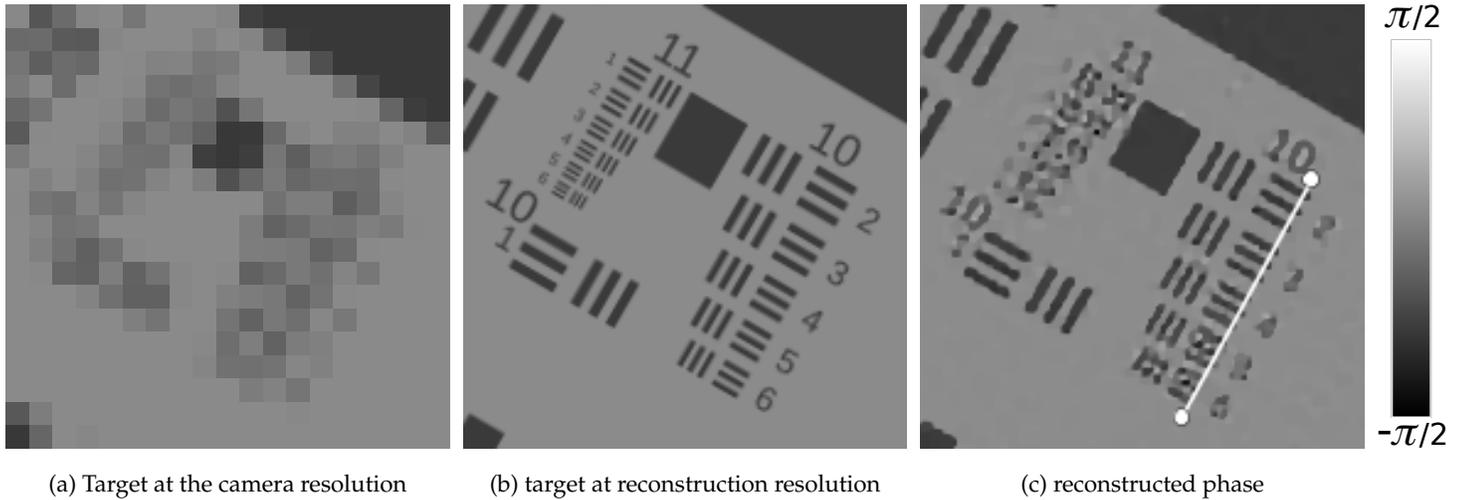


Fig. 7. Zoom on the groups 10 and 11 of the reconstructed target shown as the inner white square in Fig. 6 compared to the ground truth.

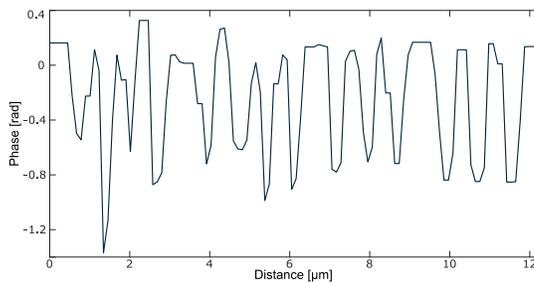


Fig. 8. Profile of the phase of the reconstructed target along the line plotted on Fig. 7.(c) along the group 10 of the resolution target.

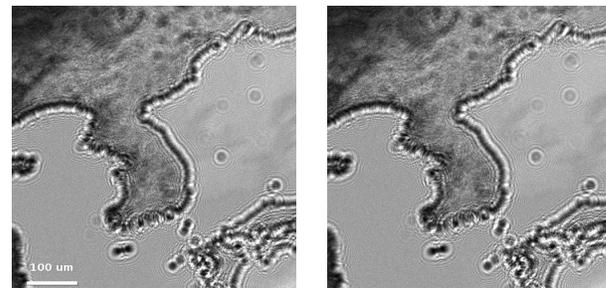


Fig. 9. Simulated data at two different heights.

saturation channels as the phase and amplitude, respectively. The simulated data is corrupted with 30 dB Gaussian noise and is shown in Fig. 9.

According to [1] prescription, the target is reconstructed with $\Delta' = 560$ nm pixel-pitch (superresolution $S = 2$) over a FoV of (0.8×0.8) mm² (1432×1432 pixels). As the object has features both in phase and modulus, it was reconstructed under "no amplification" constraint only enforcing a modulus equal to or below 1. The hyperparameter was set to $\mu = 10^{-2}$. The reconstruction after 3000 iterations is shown on Fig. 10. It shows that on an object much more complex than a USAF-1951 target, the COMCI provides qualitatively good reconstruction both in term of resolution and FoV extrapolation.

8. CONCLUSION

The key contributions of this article are twofold. Firstly, we have shown that, as we suggested in [1], it is possible to perform lensless holographic image reconstruction over a FoV larger than the camera FoV with a half-pitch resolution that is several times finer than that of the camera without relying on subpixel-shifted holograms. This means that images are reconstructed with a space-bandwidth product that is an order of magnitude greater than that of the camera (*i.e.*, there are an order of magnitude fewer data than unknowns). Therefore, some prior knowledge must be introduced into the reconstruction process. Secondly, we

have proposed a quantitative reconstruction method (COMCI) that allows us to recover an image over an area 5 times larger than the camera FoV, with a resolution 4 times better. COMCI is an open-source generic framework that can be applied to a wide variety of lensless imaging setups (*e.g.*, multi-heights, multi-wavelength, multiangle). It follows a complete "inverse problems" approach that involves three terms: a likelihood term that is adapted to the noise statistics (either Gaussian or Poisson noise); a constraint term that prevents unphysical solutions; a regularization term that enforces more generic knowledge about the object statistics. The regularization adopted in this work was the general-purpose Total Variation regularization which appears to be well suited to the USAF-1951 target. It can easily be replaced by any regularization involving a proximity operator such as learned plug-and-play priors [33]. It is likely that such a regularization will be more effective on biological samples.

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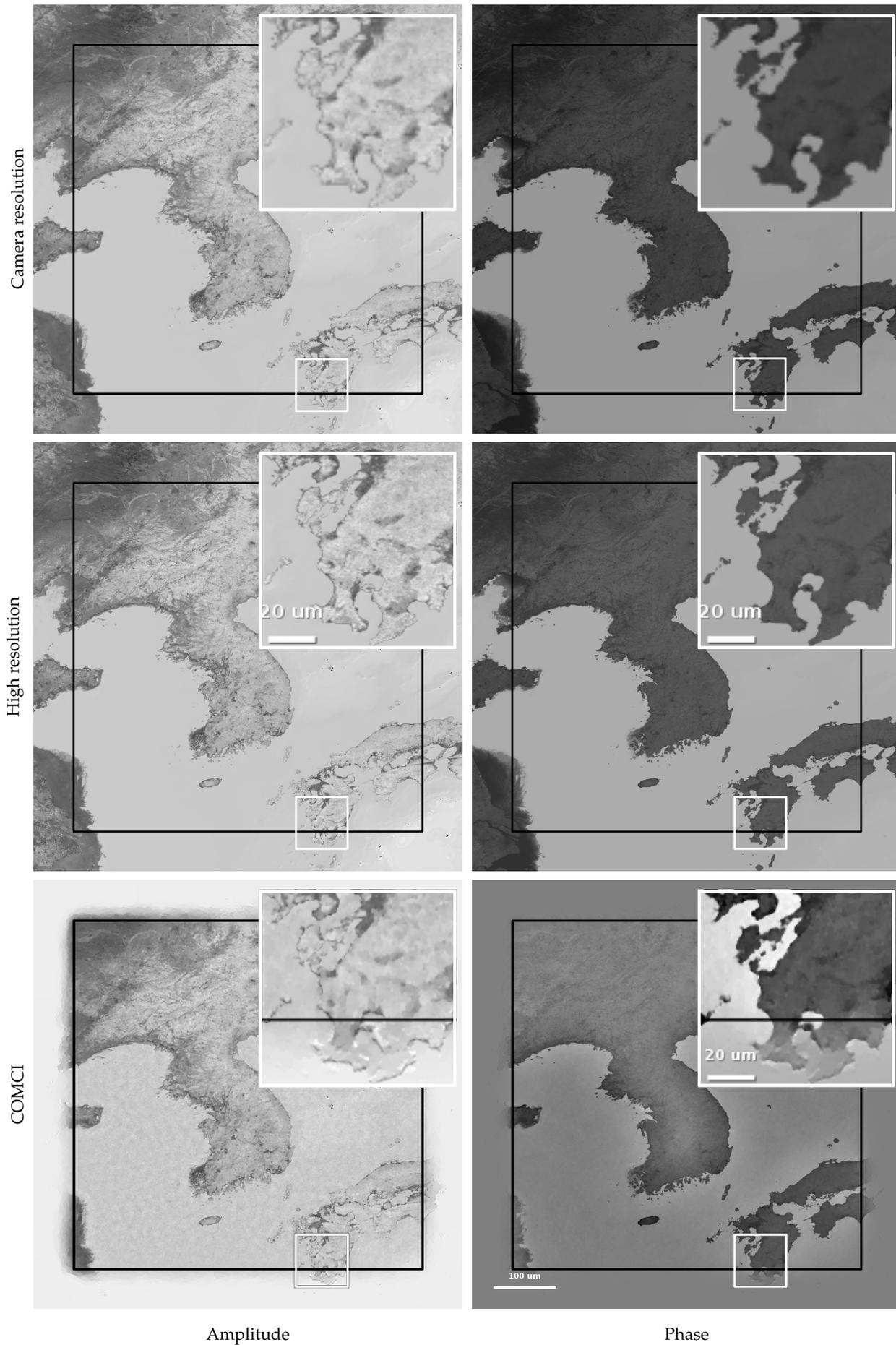


Fig. 10. Amplitude and phase of the reconstructed object from the multi-height data given Fig. 9. The black square and the white square cover the camera field of view and the area zoomed in the inset, respectively.

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