Revision and update in multiagent belief structures

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1 Motivation

When we want to model the evolution of the beliefs of a set of agents several difficulties arise that come with the following characteristics of multiagent systems:

(1) Both an agent's *perception* and *beliefs* can be incomplete and even erroneous.

(2) There are both *ontic actions* (with effects on the 'physical' world) and *epistemic actions* (no effects on the 'physical' world, but on the agents' beliefs only).

(3) An agent must have procedures to *update* his beliefs in order to predict action effects, including his higher-order beliefs (beliefs about other agents' beliefs).

(4) An agent must have procedures to not only expand, but also *revise* his beliefs upon observations, including his higher-order beliefs.

None of the logics existing in the literature can currently deal with all these characteristics: Baltag et col.'s approach [4, 3] offers a satisfactory handling of (1) and (3), but lacks (2) (there are only epistemic actions), and (4) (agents cannot revise beliefs). Gerbrandy et col.'s approach [6, 8, 7] and van der Hoek et col.'s approach [14] contain a satisfactory account of (2) and (3), while they lack (1) (an agent cannot have erroneous beliefs about inexecutability of actions, else his beliefs may get inconsistent), and (4) (agents cannot revise beliefs). Gärdenfors et col.'s belief revision [1, 5] is the standard approach for (4), but there are neither ontic actions nor updates, and it does not deal with higher-order beliefs. Van der Meyden [20] and Aucher [2] have a more general account of multiagent belief revision, but they do not allow for ontic actions. Katsuno and Mendelzon's belief update [13] is the standard approach for (3), but it does not account for revision, and does not deal with higher-order beliefs. Halpern and Lakemeyer [9] do not account for revision, just as van der Meyden [21, 15] and van Ditmarsch [22] who only considers reliable perception. Scherl, Levesque et col. [17] have an account of (2) and (3), and the integration of belief revision has been proposed in [18], but the approach still lacks an account of incomplete and erroneous perception. Tallon et al. [19] propose an account of multi-agent revision with higher-order beliefs, but under the strong hypothesis that agents communicate all their beliefs, i.e. their belief state as a whole.

In this paper we propose a logic which addresses all the issues in our list. The only shortcoming is that for the time being we have no account of update and revision by beliefs about other agents' beliefs: we only consider perception in terms of objective (boolean) formulas.

The logic combines Baltag's account allowing for different perceptions of the same action, Scherl and Levesque's solution to what they have called the epistemic frame problem, and ordering-based belief revision à la Gärdenfors. It can be viewed as an extension of Baltag et col.'s approach by integrating both ontic actions and belief revision into it. It can also be viewed as an extension of Shapiro et al.'s approach by allowing for partial and erroneous perception.

We use standard possible worlds models for each agents' beliefs. For the sake of the exposition we suppose there are only two agents 1 and 2. Then a *belief model* $\mathcal{M}$ for $\mathsf{KD45}_2$ is a tuple $\mathcal{M} = \langle W, val, R_1, R_2 \rangle$ where

- $W = \{w, v, \ldots\}$ is a set of possible worlds;
- $val : W \to 2^{AtProp}$ maps possible worlds to sets of atoms;
- R_1, R_2 are relations on W that are serial, transitive, and Euclidean.

The modal operators of individual belief $\mathbf{B}_1$ and $\mathbf{B}_2$, and the modal operator of common belief $\mathbf{CB}_{1,2}$ are interpreted in such structures in the standard way (the accessibility relation for $\mathbf{CB}_{1,2}$ being the transitive closure of $R_1 \cup R_2$). A *pointed belief model* is a belief model $\mathcal{M}$ together with an actual world $w* \in W$.

Let φ is a propositional formula built on $AtProp$, then $Mod(\varphi)$ denotes the set of models of φ, that is, the set of all valuations of 2^{AtProp} that make φ true. Valuations will be denoted by sets of literals in the following form: $[a, \neg b, c]$ denotes the valuation assigning a and c to true and b to false.

2 Action structures

We use models for action that differ slightly from that of dynamic logic, and are closer to representations used in the reasoning about actions field. An atomic action $\beta \in AtAct$ is viewed as a 'state transformer', i.e. a transition relation on states, alias valuations: to every α there is associated a function $(.)^\alpha$ mapping valuations $s \subseteq AtProp$ to sets of valuations. As often done in reasoning about actions, we suppose that $s^\alpha \neq \emptyset$ in order to simplify our account.[1] If α is deterministic then s^α is a singleton for every s, else α is nondeterministic.

[1] In fact, this hypothesis makes that we need not introduce revision when reasoning about action effects: when an agent believes that α is inexecutable and subsequently learns about

In order to represent actions in a natural and economic way we have to integrate a solution of the frame problem. This can be done in a modular way, as we have proposed in [11], where we have investigated progression and regression reasoning methods that can be 'plugged' into the mapping $(.)^\alpha$. We thus do not rely on the specificities of particular solutions. The key feature is that $(.)^\alpha$ does not relate possible worlds as in dynamic logic, but directly the associated valuations.

In the examples, we make use of three actions α, β and γ:

- α is the action whose transition model is as follows:
  - for all $s \neq [p, q]$, $s^\alpha = \{s\}$;
  - $[p, q]^\alpha = \{[p, \neg q], [\neg p, q]\}$.

  α can be viewed as an *update by* $(\neg p \vee \neg q)$ [13], namely, as the action of making $(\neg p \vee \neg q)$ true while minimizing change (where the measure of change between two states is here considered to be the number of propositional variables that are flipped).

- β switches the truth value of p; it defined by $[p, q]^\beta = \{[\neg p, q]\}$, $[\neg p, q]^\beta = \{[p, q]\}$, $[p, \neg q]^\beta = \{[\neg p, \neg q]\}$ and $[\neg p, \neg q]^\beta = \{[p, \neg q]\}$.

- λ is the void action, defined by $s^\lambda = \{s\}$ for all s.

Following ideas of among others Moore, Scherl and Levesque, and Gerbrandy, the update of a belief model $\mathcal{M}$ by an action α is a belief model $\mathcal{M}^\alpha = \langle W^\alpha, val^\alpha, R_1^\alpha, R_2^\alpha \rangle$ where $W^\alpha = \{\langle w, s \rangle \mid w \in W, s \in val(w)^\alpha\}$, $val^\alpha(\langle w, s \rangle) = s$, and $\langle w, s \rangle R_i^\alpha \langle v, t \rangle$ iff wR_iv. Then the update of a pointed belief model $\langle \mathcal{M}, w* \rangle$ by an action α is the *set* of pointed belief models

$$\langle \mathcal{M}, w* \rangle^\alpha = \{\langle \mathcal{M}^\alpha, \langle w*, s \rangle \rangle \mid s \in val(w*)^\alpha\}$$

Such an account presupposes that action occurrences are perceived completely and correctly by every agent. In order to relax this constraint we extend Baltag et col.'s ideas and use *epistemic action structures* $\mathcal{A} = \langle A, act, S_1, S_2 \rangle$ where

- $A = \{a, b, \ldots\}$ is a set of possible worlds;
- $act : A \rightarrow AtAct$ maps possible worlds to actions;
- S_1, S_2 are relations on A that are serial, transitive, and Euclidean.

S_i relates an action a to agent i's 'subjective versions' of a: if aS_ib and a occurs then in i's view b is one of the actions that might have happened. In this way one can model incomplete and erroneous perception.

A *pointed action structure* is an action structure $\mathcal{M}$ together with an actual action $a* \in A$.

the execution of α then he must revise his beliefs (cf. [10, 12] for an integration of revision in that case).

3 Updating belief models

The *update* of a belief model $\mathcal{M} = \langle W, val, R_1, R_2\rangle$ by an epistemic action structure $\mathcal{A} = \langle A, act, S_1, S_2\rangle$ is a belief model $\mathcal{M}^{\mathcal{A}} = \langle W^{\mathcal{A}}, val^{\mathcal{A}}, {R_1}^{\mathcal{A}}, {R_2}^{\mathcal{A}}\rangle$ defined by

- $W^{\mathcal{A}} = \{\langle w, a, s\rangle \mid w \in W, a \in A, s \in val(w)^{act(a)}\}$;
- $val^{\mathcal{A}}(\langle w, a, s\rangle) = s$;
- $\langle w, a, s\rangle {R_i}^{\mathcal{A}} \langle v, b, t\rangle$ iff wR_iv and aS_ib.

It can be checked that each ${R_i}^{\mathcal{A}}$ is serial, transitive, and Euclidean.

The *update of a pointed belief model* $\langle \mathcal{M}, w*\rangle$ by a pointed action structure $\langle \mathcal{A}, a*\rangle$ is a *set* of pointed belief models

$$\langle \mathcal{M}, w*\rangle^{\langle \mathcal{A}, a*\rangle} \{\langle \mathcal{M}^{\mathcal{A}}, \langle w*, a*, s\rangle\rangle \mid s \in val(w*)^{act(a*)}\}$$

Example 1 *Let* $AtProp = \{p, q\}$ *and* $\mathcal{M} = \langle W, R_1, R_2, w^*, val\rangle$ *where* $W = \{w_0, w_1, w_2\}$ *and* val*,* R_1 *and* R_2 *are shown on the following table:*

	val	R_1	R_2
w_0	$[p, q]$	$\{w_1, w_2\}$	$\{w_0\}$
w_1	$[p, q]$	$\{w_1, w_2\}$	$\{w_1\}$
w_2	$[p, \neg q]$	$\{w_1, w_2\}$	$\{w_2\}$

In this Kripke model, 1 does not know the truth value of q *but knows that 2 believes whether* q *holds or not (and this is common belief); moreover, it is common belief that* p *holds.*

We consider the epistemic action structure $\mathcal{A}$ *where it is common knowledge that* α *is performed:* $\mathcal{A} = \langle A, act, S_1, S_2\rangle$ *where* $A = \{a\}$*,* $act(a) = \alpha$ *and* $S_1(a) = S_2(a) = \{a\}$*.*

The update of $\mathcal{M}$ *by* $\mathcal{A}$ *is the following epistemic action model* $\mathcal{M}^{\mathcal{A}} = \langle W^{\mathcal{A}}, val^{\mathcal{A}}, {R_1}^{\mathcal{A}}, {R_2}^{\mathcal{A}}\rangle$ *defined by*

- $W^{\mathcal{A}} = \{\langle w_0, a, [p, \neg q]\rangle, \langle w_0, a, [\neg p, q]\rangle, \langle w_1, a, [p, \neg q]\rangle, \langle w_1, a, [\neg p, q]\rangle, \langle w_2, a, [p, \neg q]\rangle\}$;
- *for every world* $\langle w, s\rangle \in W^{\mathcal{A}}$*,*
  ${R_1}^{\mathcal{A}}(\langle w, s\rangle) = \{\langle w_1, a, [p, \neg q]\rangle, \langle w_1, a, [\neg p, q]\rangle, \langle w_2, a, [p, \neg q]\rangle\}$;
- ${R_2}^{\mathcal{A}}(\langle w_0, a, [p, \neg q]\rangle) = {R_2}^{\mathcal{A}}(\langle w_0, a, [\neg p, q]\rangle) = \{\langle w_0, a, [p, \neg q]\rangle, \langle w_0, a, [\neg p, q]\rangle\}$;
  ${R_2}^{\mathcal{A}}(\langle w_1, a, [p, \neg q]\rangle) = {R_2}^{\mathcal{A}}(\langle w_1, a, [\neg p, q]\rangle) = \{\langle w_1, a, [p, \neg q]\rangle, \langle w_1, a, [\neg p, q]\rangle\}$;
  ${R_2}^{\mathcal{A}}(\langle w_2, a, [p, \neg q]\rangle) = \{\langle w_2, a, [p, \neg q]\rangle\}$;
- $val^{\mathcal{A}}(\langle w_0, a, [p, \neg q]\rangle) = val^{\mathcal{A}}(\langle w_1, a, [p, \neg q]\rangle) = [p, \neg q]$;
  $val^{\mathcal{A}}(\langle w_0, a, [\neg p, q]\rangle) = val^{\mathcal{A}}(\langle w_1, a, [\neg p, q]\langle) = [\neg p, q]$;
  $val^{\mathcal{A}}(\langle w_2, a, [p, \neg q]\langle) = [p, \neg q]$.

The update of the pointed belief model $\langle \mathcal{M}, w_0\rangle$ *by the pointed action structure* $\langle \mathcal{A}, a\rangle$ *contains the two pointed belief models* $\langle \mathcal{M}, \langle w_0, a, [p, \neg q]\rangle\rangle$ *and* $\langle \mathcal{M}, \langle w_0, a, [\neg p, b]\rangle\rangle$*.*

Example 2 *Let $\mathcal{M}$ as in Example 1 and $\mathcal{A}$ the following action model:*

- $AtAct = \{\beta, \lambda\}$ *(defined earlier);*
- $A = \{a_0, a_1\}$*;*
- $act_A(a_0) = \beta$*;* $act_A(a_1) = \lambda$*;*
- $S_1(a_0) = \{a_0\}$*;* $S_1(a_1) = \{a_1\}$*;* $S_2(a_0) = S_2(a_1) = \{a_1\}$*;*
- $a^* = a_0$.

In this action model, 1 *correctly believes that action β is performed;* 2 *is not aware of this action occurrence and thus believes that the void action is performed, and that this is common belief;* 1 *is aware of this.*

The update of $\mathcal{M}$ by $\mathcal{A}$ is the following epistemic action model $\mathcal{M}^{\mathcal{A}} = \langle W^{\mathcal{A}}, val^{\mathcal{A}}, {R_1}^{\mathcal{A}}, {R_2}^{\mathcal{A}} \rangle$ defined by

- $W^{\mathcal{A}} = \{w'_0 = \langle w_0, a_0, [\neg p, q] \rangle, w''_0 = \langle w_0, a_1, [p, q] \rangle, w'_1 = \langle w_1, a_0, [\neg p, q] \rangle, w''_1 = \langle w_1, a_1, [p, q] \rangle, w'_2 = \langle w_2, a_0, [\neg p, \neg q] \rangle, w''_2 = \langle w_2, a_1, [p, \neg q] \rangle\}$.
- $val^{\mathcal{A}}(w'_0) = [\neg p, q]$*;* $val^{\mathcal{A}}(w'_0) = [p, q]$*; etc.*
- ${R_1}^{\mathcal{A}}(w'_0) = \{w'_1, w'_2\}$*;*
  ${R_1}^{\mathcal{A}}(w''_0) = \{w''_1, w''_2\}$*;*
  ${R_1}^{\mathcal{A}}(w'_1) = \{w'_1, w'_2\}$*;* ${R_1}^{\mathcal{A}}(w''_1) = \{w''_1, w''_2\}$*;*
  ${R_1}^{\mathcal{A}}(w'_2) = \{w'_1, w'_2\}$*;* ${R_1}^{\mathcal{A}}(w''_2) = \{w''_1, w'_2\}$*;*
  ${R_2}^{\mathcal{A}}(w'_0) = \{w''_0\}$*;* ${R_2}^{\mathcal{A}}(w''_0) = \{w''_0\}$*;*
  ${R_2}^{\mathcal{A}}(w'_1) = \{w''_1\}$*;* ${R_2}^{\mathcal{A}}(w''_1) = \{w''_1\}$*;*
  ${R_2}^{\mathcal{A}}(w'_2) = \{w''_2\}$*;* ${R_2}^{\mathcal{A}}(w''_2) = \{w''_2\}$.

The update of the pointed belief model $\langle \mathcal{M}, w_0 \rangle$ by the pointed action structure $\langle \mathcal{A}, a_0 \rangle$ contains the unique pointed belief model $\langle \mathcal{M}, \langle w_0, a_0, [\neg p, q] \rangle \rangle$

4 Revising belief models

4.1 Proximity structures

Just as in the case of structures for updating, structures for revision are not based on possible worlds, but on valuations. A *comparative proximity structure* is a complete preorder $\preceq \subseteq 2^{AtProp} \times 2^{AtProp}$ on valuations. Informally, $(s_1, s_2) \preceq (s_3, s_4)$ means that s_1 is at least as close to s_2 as s_3 is to s_4. We denote by $\prec$ and $\sim$ the strict order and the equivalence induced by $\preceq$ in the usual way. The proximity structure $\preceq$ must satisfy

- if $(s, s) \sim (s, s')$ then $s = s'$;
- $(s, s) \sim (s', s')$;
- $(s, s') \sim (s', s)$.

for all valuations s and s'.

To get a grasp of such structures consider the structure $\preceq_H$ induced by the Hamming distance: $(s_1, s_2) \preceq_H (s_3, s_4)$ iff $d_H(s_1, s_2) \leq d_H(s_3, s_4)$, where $d_H(s, s') = \text{card}((s \setminus s') \cup (s' \setminus s))$ is the cardinality of the symmetric difference between s and s' (i.e., the number of propositional symbols assigned a different value by s and s').

Let $\mathcal{M} = \langle W, val, R_1, R_2 \rangle$ be a belief model and $\preceq$ a proximity structure. For every set of worlds $U \subseteq W$ and set of valuations $S \subseteq 2^{AtProp}$ we define

$$Min_{\preceq}(U \times S) = \{\langle u, s \rangle \in U \times S \mid \langle val(u), s \rangle \preceq \langle val(u'), s' \rangle \text{ for all } \langle u', s' \rangle \in U \times S\}$$

4.2 Common knowledge of observations

To warm up let us assume that agents have common knowledge about what they observe. This corresponds for instance to the case of public announcements of the truth of objective propositions. Each of these common observations corresponds to a consistent propositional formula φ.

Let $\mathcal{M} = \langle W, val, R_1, R_2 \rangle$ be a belief model, $\preceq$ a proximity structure, and φ a consistent propositional formula. The *revision* of $\mathcal{M}$ by φ is a belief model $\mathcal{M}^{\varphi} = \langle W^{\varphi}, val^{\varphi}, {R_1}^{\varphi}, {R_2}^{\varphi} \rangle$ defined by

- $W^{\varphi} \ = \ \bigcup_{w \in W} Min_{\preceq}(\{w\} \times Mod(\varphi))$
- $val^{\varphi}(\langle w, s \rangle) \ = \ s$
- ${R_i}^{\varphi}(\langle w, s \rangle) \ = \ Min_{\preceq}(R_i(w) \times Mod(\varphi))$

Example 3 *We take the proximity relation induced by the Hamming distance. Let M be the following pointed belief model: $W = \{w_0, w_1, w_2, w_3\}$; $w^* = w_0$; val, R_1 and R_2 are shown on the following table:*

	val	R_1	R_2
w_0	$[p, q]$	$\{w_1\}$	$\{w_0\}$
w_1	$[\neg p, q]$	$\{w_1\}$	$\{w_2, w_3\}$
w_2	$[\neg p, q]$	$\{w_2\}$	$\{w_2\}$
w_3	$[\neg p, \neg q]$	$\{w_3\}$	$\{w_3\}$

Intuitively, in this pointed belief model, 1 wrongly believes that p is false and correctly believes that q is true; moreover 1 wrongly believes that 2 does the same mistake as him concerning p and, wrongly as well, that 2 does not believe whether q is true or not. 2 has correct beliefs about everything.

The revision of M by $\varphi = (a \leftrightarrow b)$ is the following pointed model:
$W = \{\langle w_0, [p, q] \rangle, \langle w_1, [p, q] \rangle, \langle w_1, [\neg p, \neg q] \rangle, \langle w_2, [p, q] \rangle, \langle w_2, [\neg p, \neg q] \rangle, \langle w_3, [\neg p, \neg q] \rangle\}$;
$\langle w, s \rangle^* = \langle w_0, [p, q] \rangle$;
val^S, ${R_1}^S$ *and* ${R_2}^S$ *are shown on the following table:*

	val^{φ}	${R_1}^{\varphi}$	${R_2}^{\varphi}$
$\langle w_0, [p, q]\rangle$	$[p, q]$	$\{\langle w_1, [p, q]\rangle, \langle w_1, [\neg p, \neg q]\rangle\}$	$\{\langle w_0, [p, q]\rangle\}$
$\langle w_1, [p, q]\rangle$	$[p, q]$	$\{\langle w_1, [p, q]\rangle, \langle w_1, [\neg p, \neg q]\rangle\}$	$\{\langle w_3, [\neg p, \neg q]\rangle\}$
$\langle w_1, [\neg p, \neg q]\rangle$	$[\neg p, \neg q]$	$\{\langle w_1, [p, q]\langle, \langle w_1, [\neg p, \neg q]\rangle\}$	$\{\langle w_2, [p, q]\rangle, \langle w_2, [\neg p, \neg q]\rangle\}$
$\langle w_2, [\neg p, \neg q]\rangle$	$[\neg p, \neg q]$	$\{\langle w_1, [p, q]\rangle, \langle w_1, [\neg p, \neg q]\rangle\}$	$\{\langle w_2, [p, q]\rangle, \langle w_2, [\neg p, \neg q]\rangle\}$
$\langle w_2, [p, q]\rangle$	$[p, q]$	$\{\langle w_2, [p, q]\rangle, \langle w_2, [\neg p, \neg q]\rangle\}$	$\{\langle w_2, [p, q]\rangle, \langle w_2, [\neg p, \neg q]\rangle\}$
$\langle w_2, [\neg p, \neg q]\rangle$	$[\neg p, \neg q]$	$\{\langle w_2, [p, q]\rangle, \langle w_2, [\neg p, \neg q]\rangle\}$	$\{\langle w_2, [p, q]\rangle, \langle w_2, [\neg p, \neg q]\rangle\}$
$\langle w_3, [\neg p, \neg q]\rangle$	$[\neg p, \neg q]$	$\{\langle w_3, [\neg p, \neg q]\rangle\}$	$\{\langle w_3, [\neg p, \neg q]\rangle\}$

Being initially wrong in the belief that 2 wrongly believes that p is false, after the observation 1 believes (wrongly) that either 2 does not have any belief about p, or that he believes that p is false.

4.3 General case

Such an account presupposes that it is common knowledge that the observation made by a given agent is also made by the other one; moreover, agent 1 may observe φ while believing that 2 observes ψ and that 2 has no idea about what he (i.e., 1) observed, and so on. Similar to action structures, an *objective observation structure* is a tuple $\mathcal{O} = \langle O, obs, T_1, T_2\rangle$ where

- $O = \{o, \ldots\}$ is a set of possible observations;
- $obs : O \to 2^{AtProp}$;
- T_1, T_2 are relations on O that are serial, transitive, and Euclidean.

The *revision* of $\mathcal{M}$ by $\mathcal{O}$ is a belief model $\mathcal{M}^{\mathcal{O}} = \langle W^{\mathcal{O}}, val^{\mathcal{O}}, {R_1}^{\mathcal{O}}, {R_2}^{\mathcal{O}}\rangle$ defined by

- $W^{\mathcal{O}} \;=\; \bigcup_{w \in W}\{\langle u, o, s\rangle \mid o \in O, \langle u, s\rangle \in Min_{\preceq}(\{w\}\times obs(o))\}$
- $val^{\mathcal{O}}(\langle u, o, s\rangle) \;=\; s$
- ${R_i}^{\mathcal{O}}(\langle w, o, s\rangle) \;=\; Min_{\preceq}(R_i(u)\times(\bigcup_{o' \in T_i(o)} obs(o')))$

Some remarks on the construction of $\mathcal{M}^S$.

1. What are pointed observation structures like? First of all, as observations are propositions the actual observation should not be an element of the set of possible observations O, but a subset of it. Second, its seems to us that its nature is a matter of debate: one option is to define the actual observation $o*$ as the set of all valuations. Another option is to constrain it to be a singleton that is equal to the valuation of the actual world in the current belief model. This requires an appropriate definition of the revision of pointed belief models, just as we have discussed for the case of common knowledge of observations.

2. It seems to be against the spirit of revision that the (obvious) definition of revision of pointed models allows for the actual world to change its valuation. This can be avoided by restricting the revision $\langle \mathcal{M}, w*\rangle^S$ to the case where $val(w*) \in S$. This guarantees that $Min_{\preceq}(\{w*\}\times S) = \{w*\}$.

3. In $\mathcal{M}^S$, a lot of worlds are inaccessible from the actual world, and could obviously be deleted. This is a difference with belief update, where the new accessibility is built separately for each world (just by copying the old accessibility relation on their images by β), while for revision the new accessibility relations ${R_i}^S$ are built separately *for each set of worlds simultaneously accessible by* R_i. More generally, $\mathcal{M}^S$ may be simplified by a minimisation process similar to automata minimisation, by clustering worlds (cf. [16]).

4. It is not hard to prove that R'_i is serial, transitive and Euclidean, therefore the revision of $\mathcal{M}$ by φ is a KD45 model.

5 Discussion

A natural question to ask is the relation between our constructions and AGM belief revision. Let us consider only agent 1. We identify revising a set of boolean formulas Γ by a boolean formula φ with the revision $\langle \mathcal{M}, w*\rangle^{Mods(\varphi)}$ of some pointed belief model $\langle \mathcal{M}, w*\rangle$ such that $w* \models \mathbf{B}_1\varphi_i$ for every $\varphi_i \in \Gamma$, where $Mods(\varphi)$ is the set of all valuations s satisfying φ. Under this hypothesis we can show that the AGM postulates are satisfied. This is mainly due to the fact that the underlying comparative proximity structure complies with the AGM framework.

Another thing we can prove easily is that if $S = Mods(\varphi)$ then for common knowledge of observations we have: $\langle \mathcal{M}^S, \langle w*, s\rangle\rangle \models \mathbf{CB}_{1,2}\varphi$ for every pointed belief model $\langle \mathcal{M}, w*\rangle$ and $s \in S$.

The only restriction of our modelling is that new information must correspond to boolean formulas. To overcome it is the subject of ongoing work.

In the revision part, our observations structures are objective because observations consist of objective facts, not of agents' beliefs. Next step will consist in revising belief structures by subjective observations such as "agent 1 believes φ", "agent 1 believes that agent 2 does not believe φ, etc. While this extension from observing objective facts to observing others' beliefs does not pose any particular problem in case of expansion, this is not so for revision.

Another further line of research is the investigation of methods of progression and regression that ultimately will enable planning in multiagent domains.

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