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# Equation-of-motion and Lorentz-invariance relations for tensor-polarized parton distribution functions of spin-1 hadrons

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## Abstract

Structure functions of polarized spin-1 hadrons will be measured at various accelerator facilities in the near future. Recently, transverse-momentum-dependent and collinear parton distribution functions were theoretically proposed at twist 3 and twist 4 in addition to the twist-2 ones, so that full investigations became possible for structure functions of spin-1 hadrons in the same level with those of the spin-1/2 nucleons. Furthermore, twist-3 tensor-polarized multiparton distribution functions were also recently found for spin-1 hadrons. In this work, we show relations among the collinear parton- and multiparton-distribution functions for spin-1 hadrons by using equation of motion for quarks. These relations are valuable in constraining the distribution functions and learning about multiparton correlations in spin-1 hadrons.

**Keywords:** QCD, Quark, Gluon, Polarized structure function, Spin-1 hadron, Equation of motion, Lorentz-invariance relation

## 1. Introduction

High-energy spin physics has been an exciting field in physics since late 1980's for finding the origin of nucleon spin. Now, the longitudinally polarized parton distribution functions (PDFs) are relatively well determined except for the gluon distribution. Partonic angular-momentum contributions are not still determined experimentally, and their studies are in progress with measurements of generalized parton distributions (GPDs) [1]. In addition, the field of finding the origin of hadron masses is also rapidly progressing because similar theoretical formalisms are used and the same GPDs or generalized distribution amplitudes (timelike GPDs) can be used for probing quark and gluon composition of hadron masses [2].

On the other hand, structure functions of spin-1 hadrons are quite interesting in providing different aspects of hadron polarizations because of the existence of new tensor-polarization observables, which do not exist in the spin-1/2 nucleons. As for the tensor-polarized structure functions, there exist four functions  $b_{1-4}$  in charged-lepton deep inelastic scattering from a spin-1 target such as the deuteron [3], and there was a measurement on  $b_1$  [4]. There were theoretical studies on the spin-1 physics on the  $b_1$  sum rule [5], its second moment [6], fragmentation functions [7], tensor-polarized PDFs in the proton-deuteron Drell-Yan process [8, 9], GPDs [10], projection operators on  $b_{1-4}$  [11], optimum tensor-polarized PDFs [12], standard-deuteron model prediction for  $b_1$  [13, 14], effects of

pions and hidden-color state in  $b_1$  [15], and gluon transversity [16, 17, 18]. In addition to twist 2 [19], transverse-momentum-dependent parton distribution functions (TMDs), PDFs, and their sum rules were investigated at twist 3 and twist 4 [20], recently. Furthermore, a twist-2 relation and a sum rule were obtained for the twist-3 function  $f_{LT}$  with investigations on possible twist-3 multiparton distributions [21].

In future, there are experimental projects [22] to investigate structure functions of the spin-1 deuteron at the Thomas Jefferson National Accelerator Facility (JLab), Fermilab (Fermi National Accelerator Laboratory), Nuclotron-based Ion Collider fAcility (NICA), LHC (Large Hadron Collider)-spin, and electron-ion colliders (EIC, EicC). Therefore, the field of spin-1 hadrons could become an exciting topic in hadron physics for investigating exotic aspects of hadrons and nuclei, possibly beyond the simple bound systems of nucleons for the deuteron.

In this work, we show useful relations among the tensor-polarized PDFs and the multiparton distribution functions by using equation of motion for quarks. Then, a so-called Lorentz-invariance relation is derived for the tensor-polarized PDFs and the multiparton distribution functions. This kind of studies have been done for structure functions of spin-1/2 nucleons [23]. Here, we investigate such relations for tensor-polarized spin-1 hadrons. In Sec. 2, we introduce correlation functions necessary for explaining possible tensor-polarized PDFs and multiparton distribution functions. Then, their relations are obtained by using the equation of motion, and a Lorentz-invariance relation is derived in Sec. 3. Our results are summarized in Sec. 4.

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## 2. Correlation functions of spin-1 hadrons

The PDFs and the multiparton distribution functions are defined through correlation functions for spin-1 hadrons, so that we introduce them in this section. The correlation function is related to the amplitude to extract a parton from a hadron and then to insert it into the hadron at a different spacetime point  $\xi$  as given by

$$\Phi_{ij}^{[c]}(k, P, T) = \int \frac{d^4\xi}{(2\pi)^4} e^{ik\cdot\xi} \langle P, T | \bar{\psi}_j(0) W^{[c]}(0, \xi) \psi_i(\xi) | P, T \rangle, \quad (1)$$

where  $\psi$  is the quark field,  $k$  is the quark momentum,  $P$  and  $T$  indicate hadron momentum and tensor polarization,  $W^{[c]}$  is the gauge link necessary for the color gauge invariance, and  $c$  indicates the integral path. In this work, we discuss only the tensor polarization, so that the spin vector  $S$  is not explicitly denoted in Eq. (1). The TMD correlation function is given by integrating Eq. (1) over the lightcone momentum  $k^-$  as

$$\Phi_{ij}^{[c]}(x, k_T, P, T) = \int dk^+ dk^- \Phi_{ij}^{[c]}(k, P, T) \delta(k^+ - xP^+), \quad (2)$$

where  $x$  is the momentum fraction carried by a parton as defined by  $k^+ = xP^+$ , and  $k_T$  is the transverse momentum. The lightcone vectors  $n$  and  $\bar{n}$  defined as

$$n^\mu = \frac{1}{\sqrt{2}}(1, 0, 0, -1), \quad \bar{n}^\mu = \frac{1}{\sqrt{2}}(1, 0, 0, 1), \quad (3)$$

are used in this paper.

Furthermore, if the function is integrated over the transverse momentum, we obtain the collinear correlation function as

$$\begin{aligned} \Phi_{ij}(x, P, T) &= \int d^2k_T \Phi_{ij}^{[c]}(x, k_T, P, T) \\ &= \int \frac{d\xi^-}{2\pi} e^{ixP^+\xi^-} \langle P, T | \bar{\psi}_j(0) W(0, \xi | n) \psi_i(\xi) | P, T \rangle_{\xi^+=0, \vec{\xi}_T=0}. \end{aligned} \quad (4)$$

In investigating various polarized collinear distribution functions, it is useful to define the  $k_T$ -weighted collinear correlation function by [24, 25]

$$(\Phi_\theta^{[\pm]\mu})_{ij}(x, P, T) = \int d^2k_T k_T^\mu \Phi_{ij}^{[\pm]}(x, k_T, P, T). \quad (5)$$

Although some collinear distribution functions vanish due to the time-reversal invariance, the  $k_T$ -weighted distributions could exist. The superscript index  $c = \pm$  indicates the direction of the integral path, namely the plus or minus direction of the coordinate  $\xi^-$  ( $n^-$ ) as shown in Fig. 4 of Ref. [20]. For example, the sign  $+$  and  $-$  are associated with the correlation functions in the semi-inclusive deep inelastic scattering (SIDIS) and the Drell-Yan process, respectively [20]. For discussing twist-3 PDFs, multiparton (three-parton in this work) correlation functions are defined with the gluon field tensor  $G^{+\mu}$ , the gluon field  $A^\mu$ , or the covariant derivative  $D^\mu$  as

$$\begin{aligned} (\Phi_X^\mu)_{ij}(x_1, x_2, P, T) &= \int \frac{d\xi_1^-}{2\pi} \frac{d\xi_2^-}{2\pi} e^{ix_1P^+\xi_1^-} e^{i(x_2-x_1)P^+\xi_2^-} \\ &\quad \times \langle P, T | \bar{\psi}_j(0) Y^\mu(\xi_2^-) \psi_i(\xi_1^-) | P, T \rangle, \\ X(Y^\mu) &= G(gG^{+\mu}), \quad A(gA^\mu), \quad D(iD^\mu), \end{aligned} \quad (6)$$

where the gauge link is not explicitly written and  $g$  is the QCD coupling constant. The field tensor  $G^{\mu\nu}$  is given by the gluon field  $A^\mu$  as  $G^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu - ig[A^\mu, A^\nu]$ , where the color factor is included in the field as  $A^\mu = A_a^\mu \lambda^a / 2$  with the Gell-Mann matrix  $\lambda^a$ . In this paper, the lightcone gauge ( $n \cdot A = A^+ = 0$ ) is used, so that the field tensor  $G^{+\alpha}$  is expressed by the gluon field  $A^\alpha$  as  $G^{+\alpha} = \partial^+ A^\alpha$ . The covariant derivative  $D^\mu$  is given by  $D^\mu = \partial^\mu - igA^\mu$ . These are the correlation functions used in this work for discussing the tensor-polarized PDFs and the multiparton distributions functions.

## 3. Relations among PDFs and multiparton distribution functions for spin-1 hadrons by equation of motion

Using the correlation functions defined in the previous section, we derive relations among the PDFs and the multiparton distribution functions. Since the correlation functions appear repeatedly in this section, we abbreviate the momentum  $P$  and tensor polarization  $T$  hereafter in denoting the correlation functions  $\Phi$ . The transverse gluon field is the only relevant degree of freedom in the lightcone quantization [21, 26], so that the transverse index  $\alpha (= 1, 2)$  is used in the following equations ( $A^\alpha = A_T^\alpha$ ,  $\partial^\alpha = \partial_T^\alpha$ ).

In the  $k_T$ -weighted correlation function of Eq. (5),  $k_T^\alpha$  is expressed by the derivative  $\partial_T^\alpha = \partial / \partial \xi_{T\alpha}$ . Applying this derivative to the gauge link and integrating over  $\vec{k}_T$ , we express the function  $\Phi_\theta^{[\pm]\alpha}$  by the covariant derivative and the gluon field tensor as [24]

$$\begin{aligned} (\Phi_\theta^{[\pm]\alpha})_{ij}(x) &= \int \frac{d\xi^-}{2\pi} e^{ixP^+\xi^-} \left[ \langle P, T | \bar{\psi}_j(0) iD^\alpha \psi_i(\xi^-) | P, T \rangle \right. \\ &\quad \left. - \langle P, T | \bar{\psi}_j(0) \int_{\pm\infty}^{\xi^-} d\eta^- gG^{+\alpha}(\eta^-) \psi_i(\xi^-) | P, T \rangle \right] \\ &= \int \frac{d\xi^-}{2\pi} e^{ixP^+\xi^-} \left[ \langle P, T | \bar{\psi}_j(0) i\partial^\alpha \psi_i(\xi^-) | P, T \rangle \right. \\ &\quad \left. + \langle P, T | \bar{\psi}_j(0) gA^\alpha(\xi^- = \pm\infty) \psi_i(\xi^-) | P, T \rangle \right]. \end{aligned} \quad (7)$$

The second equation is obtained by using  $G^{+\alpha}(\eta^-) = \partial_- A^\alpha(\eta^-)$ . The field-tensor relation  $G^{+\alpha} = \partial^+ A^\alpha$  is inverted as [27]

$$A^\alpha(\xi^-) = \frac{A^\alpha(\infty) + A^\alpha(-\infty)}{2} - \frac{1}{2} \int_{-\infty}^{\infty} d\eta^- \epsilon(\eta^- - \xi^-) G^{+\alpha}(\eta^-), \quad (8)$$

where  $\epsilon(x)$  is the sign function [21], by noting the boundary conditions at  $\pm\infty$ . Using this relation for Eq. (7), we obtain

$$\begin{aligned} (\Phi_\theta^{[\pm]\alpha})_{ij}(x) &= \int \frac{d\xi^-}{2\pi} e^{ixP^+\xi^-} \left[ \langle P, T | \bar{\psi}_j(0) i\partial^\alpha \psi_i(\xi^-) | P, T \rangle \right. \\ &\quad \left. + \langle P, T | \bar{\psi}_j(0) g \left\{ \frac{A^\alpha(\infty) + A^\alpha(-\infty)}{2} \right. \right. \\ &\quad \quad \left. \left. \pm \frac{A^\alpha(\infty) - A^\alpha(-\infty)}{2} \right\} \psi_i(\xi^-) | P, T \rangle \right]. \end{aligned} \quad (9)$$

Here, the first and second lines are T-even terms which are expressed by T-even  $k_T$ -weighted PDFs, whereas the third line is

a T-odd term expressed by T-odd distributions [27]. The  $k_T$ -weighted PDFs are defined in Eq. (12). The T-odd  $k_T$ -weighted PDFs exist although ordinary T-odd PDFs should vanish due to the time-reversal invariance. The T-odd distributions are actually gluonic-pole effects which are reflected as the difference between  $A_T^\alpha(\infty)$  and  $A_T^\alpha(-\infty)$  [24]. If one considers  $\Phi_\theta^{[+]\alpha}(x) + \Phi_\theta^{[-]\alpha}(x)$ , the T-odd distributions vanish, which is related to the TMD relation  $f_{\text{SIDIS}}(x, k_T^2) = -f_{\text{Drell-Yan}}(x, k_T^2)$  between the TMDs of the SIDIS and the Drell-Yan process. If the boundary condition were imposed as  $A^\alpha(\infty) = A^\alpha(-\infty)$ , the T-odd term vanishes in Eq. (9) [24].

Next, we try to obtain the  $k_T$ -weighted correlation function of Eq. (5) in terms of transverse-momentum moments of TMDs. First, the twist-2 TMD correlation function is given as [19, 20]

$$\begin{aligned} \Phi^{[\pm]}(x, k_T)_{\text{twist-2}} = & \frac{1}{2} \left[ -f_{1LT}^{[\pm]}(x, k_T^2) \frac{k_T \cdot S_{LT}}{M} \not{n} \right. \\ & + g_{1LT}^{[\pm]}(x, k_T^2) \frac{\epsilon_T^{\alpha\mu} S_{LT\alpha} k_{T\mu}}{M} \gamma_5 \not{n} + h_{1LL}^{[\pm]}(x, k_T^2) S_{LL} \frac{k_{T\alpha} \sigma^{\alpha\mu}}{M} \bar{n}_\mu \\ & - h_{1TT}^{[\pm]}(x, k_T^2) \frac{k_{T\alpha} S_{TT}^{\alpha\beta} \sigma_{\beta\mu} \bar{n}^\mu}{M} + h_{1TT}^{[\pm]}(x, k_T^2) \frac{k_{T\alpha} S_{TT}^{\alpha\beta} k_{T\beta} k_{T\rho} \sigma^{\rho\mu}}{M^2} \frac{k_{T\rho} \sigma^{\rho\mu}}{M} \bar{n}_\mu \\ & \left. + k_T\text{-even terms} \right], \end{aligned} \quad (10)$$

where  $M$  is the mass of a spin-1 hadron. The path dependence  $[\pm]$  of the TMDs is often abbreviated, for example, as  $f_{1LT}(x, k_T^2)$  instead of  $f_{1LT}^{[\pm]}(x, k_T^2)$ . However, we keep it here to show path-dependent relations in Eq. (13). By the  $k_T$ -weighted integral, the  $h_{1TT}^{[\pm]}$  vanishes because of  $S_{TT}^{11} = -S_{TT}^{22} = -1$ . From Eq. (10), the  $k_T$ -weighted correlation function is expressed by transverse-momentum moments of the remaining four TMDs as [19, 20]

$$\begin{aligned} \Phi_\theta^{[\pm]\alpha}(x) = & \frac{M}{2} \left[ f_{1LT}^{[\pm](1)}(x) S_{LT}^\alpha \not{n} + g_{1LT}^{[\pm](1)}(x) \epsilon_T^{\alpha\mu} S_{LT\mu} \gamma_5 \not{n} \right. \\ & \left. - h_{1LL}^{[\pm](1)}(x) S_{LL} \sigma^{\alpha\mu} \bar{n}_\mu + h_{1TT}^{[\pm](1)}(x) S_{TT}^{\alpha\beta} \sigma_{\beta\mu} \bar{n}^\mu \right]. \end{aligned} \quad (11)$$

Here, only the leading-twist  $k_T$ -weighted PDFs are included by neglecting higher-twist PDFs, and the transverse-momentum moments of the TMDs are given by

$$f^{(1)}(x) = \int d^2 k_T \frac{k_T^2}{2M^2} f(x, k_T^2). \quad (12)$$

The only T-even distribution is  $f_{1LT}^{(1)}$  and the others are T-odd ones [20] to satisfy

$$\begin{aligned} f_{1LT}^{[+](1)}(x) &= f_{1LT}^{[-](1)}(x), & g_{1LT}^{[+](1)}(x) &= -g_{1LT}^{[-](1)}(x), \\ h_{1LL}^{[+](1)}(x) &= -h_{1LL}^{[-](1)}(x), & h_{1TT}^{[+](1)}(x) &= -h_{1TT}^{[-](1)}(x). \end{aligned} \quad (13)$$

By using the gluon-field expression in Eq. (8) and the sign function  $\epsilon(x)$  in Eq. (3.36) of Ref. [21], the multiparton correlation function  $\Phi_A^\alpha(x, y)$  in Eq. (6) is related to another one  $\Phi_G^\alpha(x, y)$  as [27]

$$\begin{aligned} \Phi_A^\alpha(x, y) = & \delta(x-y) \frac{\Phi_{A(-\infty)}^\alpha(x) + \Phi_{A(\infty)}^\alpha(x)}{2} \\ & - \mathcal{P} \left[ \frac{i}{P^+(x-y)} \right] \Phi_G^\alpha(x, y), \end{aligned} \quad (14)$$

where  $\mathcal{P}$  indicates the principal integral. Then, from Eqs. (6), (9), and (14), the multiparton correlation function defined with the covariant derivative  $\Phi_D^\alpha(x, y)$  is given by

$$\Phi_D^\alpha(x, y) = \delta(x-y) \frac{1}{P^+} \tilde{\Phi}^\alpha(x) - \mathcal{P} \left[ \frac{i}{P^+(x-y)} \right] \Phi_G^\alpha(x, y), \quad (15)$$

where  $\tilde{\Phi}^\alpha$  is defined by the average of  $\Phi_\theta^{[+]\alpha}$  and  $\Phi_\theta^{[-]\alpha}$  as

$$\begin{aligned} \tilde{\Phi}_{ij}^\alpha(x) &\equiv \frac{(\Phi_\theta^{[+]\alpha})_{ij}(x) + (\Phi_\theta^{[-]\alpha})_{ij}(x)}{2} \\ &= \int \frac{d\xi^-}{2\pi} e^{ixP^+\xi^-} \left[ \langle P, T | \bar{\psi}_j(0) i\partial^\alpha \psi_i(\xi^-) | P, T \rangle \right. \\ &\quad \left. + \langle P, T | \bar{\psi}_j(0) g \frac{A^\alpha(\infty) + A^\alpha(-\infty)}{2} \psi_i(\xi^-) | P, T \rangle \right]. \end{aligned} \quad (16)$$

This correlation function  $\tilde{\Phi}^\alpha$  is the T-even part of Eq. (9). Using the  $k_T$ -weighted correlation function in Eq. (11), we find that it is given by the T-even function  $f_{1LT}^{(1)}(x)$  as

$$\tilde{\Phi}^\alpha(x) = \frac{M}{2} f_{1LT}^{(1)}(x) S_{LT}^\alpha \not{n}, \quad (17)$$

where  $f_{1LT}^{[+](1)}(x) = f_{1LT}^{[-](1)}(x) \equiv f_{1LT}^{(1)}(x)$ .

The multiparton correlation function  $\Phi_D^\alpha(x, y)$  is expressed by the multiparton distribution functions in the same way with  $\Phi_G^\alpha(x, y)$  of Ref. [21] as

$$\begin{aligned} \Phi_D^\alpha(x, y) = & \frac{M}{2P^+} \left[ S_{LT}^\alpha F_{D,LT}(x, y) + i\epsilon_\perp^{\alpha\mu} S_{LT\mu} \gamma_5 G_{D,LT}(x, y) \right. \\ & \left. + S_{LL} \gamma^\alpha H_{D,LL}^\perp(x, y) + S_{TT}^{\alpha\mu} \gamma_\mu H_{D,TT}(x, y) \right] \not{n}. \end{aligned} \quad (18)$$

Substituting Eqs. (17), (18), and  $\Phi_G^\alpha(x, y)$  in Eq. (3.32) of Ref. [21] into Eq. (15), we obtain the relations between the multiparton distribution functions as

$$\begin{aligned} F_{D,LT}(x, y) &= \delta(x-y) f_{1LT}^{(1)}(x) + \mathcal{P} \left( \frac{1}{x-y} \right) F_{G,LT}(x, y), \\ G_{D,LT}(x, y) &= \mathcal{P} \left( \frac{1}{x-y} \right) G_{G,LT}(x, y), \\ H_{D,LL}^\perp(x, y) &= \mathcal{P} \left( \frac{1}{x-y} \right) H_{G,LL}^\perp(x, y), \\ H_{D,TT}(x, y) &= \mathcal{P} \left( \frac{1}{x-y} \right) H_{G,TT}(x, y). \end{aligned} \quad (19)$$

Similar relations are given for the multiparton distribution functions of the nucleons in Refs. [28] and [29]. From these relations, Eq. (13), and Eq. (3.33) of Ref. [21], we find that the function  $F_{D,LT}(x, y)$  is even under the exchange of  $x$  and  $y$  and the other functions [ $G_{D,LT}(x, y)$ ,  $H_{D,LL}^\perp(x, y)$ ,  $H_{D,TT}(x, y)$ ] are odd as

$$\begin{aligned} F_{D,LT}(x, y) &= F_{D,LT}(y, x), & G_{D,LT}(x, y) &= -G_{D,LT}(y, x), \\ H_{D,LL}^\perp(x, y) &= -H_{D,LL}^\perp(y, x), & H_{D,TT}(x, y) &= -H_{D,TT}(y, x). \end{aligned} \quad (20)$$

The equation of motion for quarks is  $(i\not{D} - m)\psi = 0$ , which is then multiplied by  $i\sigma^{+\alpha}$  and written by the lightcone coordinates as

$$\begin{aligned} i(\gamma^+ D^\alpha - \gamma^\alpha D^+) \psi + i\epsilon_T^{\alpha\mu} \gamma_\mu \gamma_5 iD^+ \psi \\ - i\epsilon_T^{\alpha\mu} \gamma^+ \gamma_5 iD_\mu \psi + im\sigma^{+\alpha} \psi = 0, \end{aligned} \quad (21)$$

where  $m$  is a quark mass. The constraint from this equation of motion is given by traces of collinear correlation functions  $\Phi_D^{\mu(=\alpha,+)}(x)$  as

$$\begin{aligned} & \int \frac{d\xi^-}{2\pi} e^{ixP^+\xi^-} \langle P, T | \bar{\psi}(0) [i(\gamma^+ D^\alpha - \gamma^\alpha D^+) + i\epsilon_T^{\alpha\mu} \gamma_\mu \gamma_5 iD^+ \\ & \quad - i\epsilon_T^{\alpha\mu} \gamma^+ \gamma_5 iD_\mu + im\sigma^{\alpha+}] \psi(\xi^-) | P, T \rangle \\ & = P^+ \text{Tr} [\Phi_D^\alpha(x) \gamma^+] - P^+ \text{Tr} [\Phi_D^+(x) \gamma^\alpha] + P^+ i\epsilon_T^{\alpha\mu} \text{Tr} [\Phi_D^+(x) \gamma_\mu \gamma_5] \\ & \quad - P^+ i\epsilon_T^{\alpha\mu} \text{Tr} [\Phi_{D\mu}(x) \gamma^+ \gamma_5] + im \text{Tr} [\Phi(x) \sigma^{\alpha+}] \\ & = 0. \end{aligned} \quad (22)$$

Here, the collinear correlation function  $\Phi_D^\mu(x)$  is defined by integrating the correlation function  $\Phi_D^\mu(y, x)$  over the variable  $y$  as

$$\Phi_D^\mu(x) \equiv \int_{-1}^1 dy \Phi_D^\mu(y, x). \quad (23)$$

It should be noted that only the transverse ( $\mu = \alpha$ ) correlation functions are associated with the multiparton distribution functions by Eq. (18). In expressing integrals of multiparton distribution functions over  $y$  in the following [Eqs. (26), (27), (28), (32)], the variables  $x$  and  $y$  are interchanged by using Eq. (20). From the definition of Eq. (6), the correlation function  $\Phi_D^+(x, y)$  is expressed by the collinear correlation function  $\Phi(x)$  as

$$\Phi_D^+(x, y) = \delta(x - y) x \Phi(x), \quad (24)$$

where  $\Phi(x)$  is given by [21]

$$\begin{aligned} \Phi(x) = & \frac{1}{2} \left[ S_{LL} \not{n} f_{1LL}(x) + \frac{M}{P^+} S_{LL} e_{LL}(x) \right. \\ & \left. + \frac{M}{P^+} \not{S}_{LT} f_{LT}(x) + \frac{M^2}{(P^+)^2} S_{LL} \not{n} f_{3LL}(x) \right]. \end{aligned} \quad (25)$$

Calculating Eq. (22) with Eqs. (18), (23), (24), and (25), we obtain the relations among the tensor-polarized PDFs and the multiparton distribution functions as

$$x f_{LT}(x) - \int_{-1}^1 dy [F_{D,LT}(x, y) + G_{D,LT}(x, y)] = 0. \quad (26)$$

A similar relation to this equation was obtained in Ref. [17] by using a different parametrization for the tensor polarization. Namely, the collinear twist-3 function  $f_{LT}(x)$  is given by integrating the twist-3 multiparton distribution functions  $F_{D,LT}$  and  $G_{D,LT}$  over one of the momentum-fraction variables. Then, this relation is written in terms of the multiparton distribution functions defined with the field tensor  $G^{\mu\nu}$  by using Eq. (19) as

$$x f_{LT}(x) - f_{1LT}^{(1)}(x) - \mathcal{P} \int_{-1}^1 dy \frac{F_{G,LT}(x, y) + G_{G,LT}(x, y)}{x - y} = 0. \quad (27)$$

Therefore, the function  $f_{LT}(x)$  is also expressed by the  $k_T$ -weighted function  $f_{1LT}^{(1)}(x)$  and other multiparton distribution functions  $F_{G,LT}$  and  $G_{G,LT}$ . Using the relation between the twist-3 structure function  $f_{LT}$  and the twist-2 one  $f_{1LL}$  in

Eqs. (3.41) and (3.42) of Ref. [21], and taking the derivative of Eq. (27) with respect to  $x$ , we obtain

$$\frac{df_{1LT}^{(1)}(x)}{dx} - f_{LT}(x) + \frac{3}{2} f_{1LL}(x) - 2\mathcal{P} \int_{-1}^1 dy \frac{F_{G,LT}(x, y)}{(x - y)^2} = 0. \quad (28)$$

This is a Lorentz-invariance relation for the tensor-polarized structure functions of spin-1 hadrons in the similar way to the ones for the spin-1/2 nucleons [23]. An equation like Eq. (28) is conventionally called a Lorentz-invariance relation. Here, the Lorentz invariance means the frame independence of twist-3 observables [23]. Therefore, Lorentz-invariant relations play an important role in twist-3 studies. Although it is abbreviated in Eq. (1), there is dependence on the lightcone vector  $n$  due to the gauge link in the correlation function [20]. The Lorentz-invariance relation is affected by  $n$ -dependent terms in the correlation function as noticed in the papers of Goeke *et al.* (2003) and Metz *et al.* (2009) in Ref. [23] for the spin-1/2 nucleons. Such effects are included in our formalism.

Multiplying  $\gamma^\mu$  on the left-hand side of the Dirac equation and using the identity  $\gamma^\mu \gamma^\nu = g^{\mu\nu} - i\sigma^{\mu\nu}$ , we obtain  $(iD^\mu - i\sigma^{\mu\nu} iD_\nu - m\gamma^\mu)\psi = 0$ . Then, taking the lightcone component  $+$ , we have another equation of motion as

$$(iD^+ - i\sigma^{+\nu} iD_\nu - m\gamma^+)\psi = 0. \quad (29)$$

The constraint from this equation of motion is written, in the same way with Eq. (22), as

$$\begin{aligned} & \int \frac{d\xi^-}{2\pi} e^{ixP^+\xi^-} \langle P, T | \bar{\psi}(0) [iD^+ - i\sigma^{+\nu} iD_\nu - m\gamma^+] \psi(\xi^-) | P, T \rangle \\ & = P^+ \text{Tr} [\Phi_D^+(x)] - P^+ i \text{Tr} [\Phi_D^+(x) \sigma^{+-}] \\ & \quad - P^+ i \text{Tr} [\Phi_{D\alpha}(x) \sigma^{+\alpha}] - m \text{Tr} [\Phi(x) \gamma^+] \\ & = 0. \end{aligned} \quad (30)$$

Then, calculating the traces with Eqs. (18), (23), (24), and (25), we obtain the integral relation among the twist-3 PDF  $e_{LL}$ , the twist-2 PDF  $f_{1LL}$ , and the twist-3 multiparton distribution function  $H_{D,LL}^\perp$  as

$$x e_{LL}(x) - 2 \int_{-1}^1 dy H_{D,LL}^\perp(x, y) - \frac{m}{M} f_{1LL}(x) = 0. \quad (31)$$

This equation is expressed by another multiparton distribution function  $H_{G,LL}^\perp$  by using Eq. (19) as

$$x e_{LL}(x) - 2\mathcal{P} \int_{-1}^1 dy \frac{H_{G,LL}^\perp(x, y)}{x - y} - \frac{m}{M} f_{1LL}(x) = 0. \quad (32)$$

Because of  $m/M \ll 1$ , the third terms of Eqs. (31) and (32) could be practically neglected. Then, the twist-3 functions is described only by the multiparton distribution function  $H_{D,LL}^\perp(x, y)$  or  $H_{G,LL}^\perp(x, y)$ . We notice that the distribution  $H_{D,LL}^\perp(x, y)$  has the corresponding twist-3 PDF  $e_{LL}(x)$  by Eq. (31). The distributions  $F_{D,LL}(x, y)$  and  $G_{D,LT}(x, y)$  are related to twist-3 PDF  $f_{LT}(x)$  as shown in Eq. (26). In the nucleon case, all the twist-3 PDFs are expressed by twist-3

multiparton distribution functions. One finds this correspondence in Eqs. (11), (16), (19), and (20) of Ref. [30] in the spin-1/2 case. However, we find that the corresponding twist-3 PDF does not exist for the distribution  $H_{D,TT}(x, y)$  of a spin-1 hadron in Eq. (18). This is because the distribution  $H_{D,TT}(x, y)$  is associated with the tensor-polarization parameter  $S_{TT}^{\mu}$ , and there is no twist-3 PDF which is associated with the parameter  $S_{TT}^{\mu}$  [18, 20, 21]. In this case, no twist-3 PDF is related to  $H_{D,TT}(x, y)$  by the equation of motion.

#### 4. Summary

From the equation of motion for quarks, we derived relations among the tensor-polarized distribution functions and twist-3 multiparton distribution functions defined by the field tensor. We found the relations from the equation of motion for quarks as

$$x f_{LT}(x) - f_{1LT}^{(1)}(x) - \mathcal{P} \int_{-1}^1 dy \frac{F_{G,LT}(x, y) + G_{G,LT}(x, y)}{x - y} = 0,$$

$$x e_{LL}(x) - 2 \mathcal{P} \int_{-1}^1 dy \frac{H_{G,LL}^{\perp}(x, y)}{x - y} - \frac{m}{M} f_{1LL}(x) = 0,$$

for the twist-3 PDF  $f_{LT}$ , the trasverse-momentum moment PDF  $f_{1LT}^{(1)}$ , and the multiparton distribution functions  $F_{G,LT}$  and  $G_{G,LT}$ ; for the twist-3 PDF  $e_{LL}$ , the twist-2 PDF  $f_{1LL}$ , and the multiparton distribution function  $H_{G,LL}^{\perp}$ . Then, the Lorentz-invariance relation was obtained as

$$\frac{df_{1LT}^{(1)}(x)}{dx} - f_{LT}(x) + \frac{3}{2} f_{1LL}(x) - 2 \mathcal{P} \int_{-1}^1 dy \frac{F_{G,LT}(x, y)}{(x - y)^2} = 0.$$

These relation are valuable in constraining the tensor-polarized PDFs and the multiparton distribution functions.

In deriving these relations, we also obtained new relations among the multiparton distribution functions defined by the field tensor and the covariant derivatives. First, the function  $F_{D,LT}(x, y)$  is expressed by  $f_{1LT}^{(1)}(x)$  and  $F_{G,LT}(x, y)$  as

$$F_{D,LT}(x, y) = \delta(x - y) f_{1LT}^{(1)}(x) + \mathcal{P} \left( \frac{1}{x - y} \right) F_{G,LT}(x, y).$$

Next, the functions  $G_{D,LT}(x, y)$ ,  $H_{D,LL}^{\perp}(x, y)$ , and  $H_{D,TT}(x, y)$  are expressed only by the corresponding functions defined with the field tensor as

$$G_{D,LT}(x, y) = \mathcal{P} \left( \frac{1}{x - y} \right) G_{G,LT}(x, y),$$

and same equations for  $H_{D/G,LL}^{\perp}(x, y)$  and  $H_{D/G,TT}(x, y)$ .

These studies will be useful for investigating the tensor-polarized structure functions of spin-1 hadrons.

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