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Wave dispersion analysis of three-dimensional vibroacoustic waveguides with semi-analytical isogeometric method

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Abstract

Material and structural non-destructive evaluations using guided-wave (GW) testing techniques rely on the knowledge of wave dispersion characteristics. When studying coupled fluid-solid waveguides having complex geometries using the semi-analytical finite element (SAFE) method, an excessive computational effort may be required, especially at high-frequency ranges. In this paper, we show the robustness of an efficient computational approach so-called the semi-analytical isogeometric analysis (SAIGA) for computing the wave dispersion in 3D anisotropic elastic waveguides coupled with acoustic fluids. This approach is based on the use of Non-Uniform Rational B-splines (NURBS) as the basis functions for the geometry representation as well as for the approximation of pressure/displacement fields. The obtained results are compared with the ones derived from using the conventional SAFE method which uses Lagrange polynomials. It is shown that for computing the dispersion of GWs, using SAIGA leads to a much faster convergence rate than using the conventional SAFE with the same shape function's order. For hollow prismatic structures immersed in fluids, using high-order NURBS (*e.g.*, $p = 8$) is particularly efficient as it only requires a few elements to achieve solutions having the same precision as the ones obtained by SAFE which requires up to five times of DoF number. Moreover, the continuity of normal displacement at fluid-solid interfaces could be significantly improved thanks to the smoothness feature of NURBS, showing the advantage of SAIGA over SAFE in the evaluation of the shape modes of GWs in coupled fluid-solid systems.

Keywords: Immersed waveguides, Guided waves, Dispersion curve, Isogeometric analysis, NURBS basis, Semi analytical finite element (SAFE)

1. Introduction

Guided-wave (GW) technology, which is based on the analysis of the behavior of waves propagating along surfaces or interfaces, is widely known as a robust technique and an economical way for fast non-destructive evaluation of structures [1]. Among the wide variety of applications, one

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33 may cite the near-surface geophysics and geotechnical site characterization [2], the damage detec-
34 tion in composite materials [3], the characterization of fluid loaded structures [4], or more recently
35 the ultrasonic imaging of biological tissues [5, 6, 7].

36 Due to the presence of interfaces and/or free surfaces, the waves guided along the structure have
37 dispersive behavior, in which the phase velocity and attenuation vary with the frequency content
38 of the wave packages. The dispersion of guided waves has been shown to strongly depend on the
39 geometry of the structures, on the heterogeneity of material properties as well as the existence
40 of surrounding media (most commonly fluid). Knowing the dispersion characteristics of guided
41 waves, one may perform inversion problems to identify the geometrical and mechanical properties
42 of the structure. Basically, the inversion procedure is performed by minimizing the error between
43 the dispersion curves obtained from experiments and modeling [1]. Therefore, the development of
44 low-cost and accurate computational methods for evaluating the wave dispersion is very important
45 to enhance GW-based non-destructive techniques.

46 One of the most common models for studying GWs is the cases where the medium could be
47 assumed to be homogeneous along one (or two) directions. In these cases, the analytical methods
48 were usually used for computing the dispersion curves, due to their efficiency in solving wave
49 equations, especially in waveguides with simple cross-section geometry such as plates or cylinders
50 [1]. When the section is not homogeneous but consists of multilayer materials (*i.e* the variation of
51 properties are piecewise constant functions), the analytical solutions may also be derived by using
52 *e.g* the transfer matrix method or the global matrix method [1]. The analytical methods have also
53 been used to consider functionally-graded waveguides [8].

54 The limitations of analytical models can be circumvented by a more versatile numerical ap-
55 proach so-called semi-analytical finite element (SAFE) which has become popular in recent decades
56 [9, 10, 11, 12]. The idea of SAFE method is to assume a harmonic form of the solution in one
57 (or two) direction and employ finite element discretization in the cross-section of the considered
58 waveguide. In many circumstances, the considered waveguide is coupled with one or several fluid
59 media, and we must deal with a vibroacoustic problem. While the displacement-based equation is
60 applied in the elastic solid, the fluid may also be modeled as an elastic material with very weak
61 shear modulus [13], or by an acoustic fluid for which the pressure-based equation is employed
62 [14, 15, 16]. Using the elastic model employs the displacement-based equations for the fluid, which
63 are easier to be implemented in the SAFE formulation, but it may cause spurious modes due to
64 the zero shear modulus. Using pressure-based equations is better for describing the acoustic fluid,
65 but continuity conditions need to be introduced between the pressure field (of the fluid) and the
66 displacement field (of the solid) at the fluid-solid interfaces. Astaneh *et al.* [17, 18] presented a
67 CFEM for fluid coupled waveguides with cross-section which uses linear midpoint-integrated finite
68 elements with specially designed set of complex-valued lengths. The proposed method has fast
69 convergent, but only simple geometry could be considered. Zuo *et al.* in [16] developed SAFE
70 formulation derived from pressure-displacement equations for considering 3D waveguides coupled
71 with a fluid. Basically, the proposed approach is quite general and can be applied for solving
72 any cases with arbitrary cross-section geometries. However, in practice for studying the guided
73 waves in structures with complex cross-section geometry, a significant mesh refinement may need
74 to be required to obtain the converged solutions, especially at high frequencies because of several
75 reasons. First, for considering complex interfaces defined by high-gradient curves, the mesh using

76 conventional Lagrangian finite elements needs to be extensively refined; second, as the equations in
77 the fluid are written in terms of the pressure, the continuity conditions of normal displacement at
78 the fluid-solid interface may not be numerically achieved due to the numerical errors of the differ-
79 entiation approximation. The mesh refinement leads to larger complex-valued eigenvalue problems,
80 causing a significant increase in computational cost.

81 Based on recent innovations, we propose to employ the isogeometric concept instead of con-
82 ventional finite element-based discretization in the context of SAFE analysis. The isogeometric
83 analysis employs the Computer Aided Design (CAD) concept of Non-uniform Rational B-splines
84 (NURBS) tool to represent not only the complex geometries but also to construct approximations
85 for finite element analysis [19, 20, 21]. In the context of the wave propagation problem, the use of
86 NURBS basis functions, yields more accurate solutions compared to the conventional finite element
87 analysis (FEA) using the same number of degrees of freedom [22, 23, 24, 25]. For the simulation
88 of GWs in elastic plates, Willberg et al. [26] compared several higher-order finite element schemes
89 and their convergence when studying the first Lamb modes at low frequencies. In the context of
90 GW's dispersion study, the NURBS basis functions were employed by Gravenkamp et al. [27] in
91 the scaled boundary finite element method for the dispersion analysis of homogeneous 3D solid
92 waveguides with arbitrary cross-section. Liu et al. [28] also applied IGA to numerical investigation
93 of dispersive behavior of waves in helical thread waveguides. In these studies, the advantage of
94 using IGA for the simulation of GWs in complex geometry structures has been studied. However,
95 the presence of fluids has not been considered. Recently, by comparing with the Lagrange based ap-
96 proach for the computation of dispersion curves in 2D plates (with 1D discretization) coupled with
97 fluids, we have shown that NURBS based approach allows to improve significantly the precision
98 and reduce the computational cost, especially at high-frequency ranges [29].

99 In summary, although the conventional SAFE approach has been applied widely for guided-
100 wave analysis, the coupling between 3D prismatic solid and fluid has been much less investigated.
101 Furthermore, to our knowledge, when using isogeometric analysis in SAFE context, most of existed
102 works only considered simple geometries (*e.g* rectangular section) and without fluid coupling. In
103 this work, a semi-analytical isogeometric formulation (SAIGA) was proposed for analyzing the wave
104 dispersion in arbitrary cross-section structures immersed in fluids. To do so, the NURBS basis
105 functions were used within the SAFE formulation established for an anisotropic elastic domain
106 coupled with acoustic fluids in the frequency-wavenumber domain. It is expected that using high-
107 degree NURBS basis functions could significantly improve the accuracy of the numerical solutions
108 of the wave dispersion with a significant reduction of computational cost. The convergence analysis
109 was carefully performed for several cases including an empty/fluid-filled cylinder and an arbitrary
110 cross-section waveguide, which represent a typical cortical long bone geometry, in order to find
111 optimal NURBS order for these cases. We were particularly interested in studying the continuities
112 of stress and displacement at solid-fluid interfaces, which are important for the evaluation of the
113 mode excitability, computed by the proposed NURBS-based analysis.

114 To the best of the author's knowledge, the NURBS-based modeling guided waves in 3D coupling
115 solid-fluid structures has not been investigated in the literature. It is worth to notice that the term
116 "semi-analytical isogeometric analysis" has also been used in the literature but in different
117 contexts. For example, in the so-called IGA-SBFEM method [27], the IGA was used for computing
118 the coefficient matrices of SBFEM (Scaled Boundary Finite Element Method). In [30], the W-IGA

119 (Wave Isogeometric Analysis) method was used for the simulation wave propagation in periodic
 120 media, in which the IGA can be used for dealing with cell problems derived from Floquet-Bloch
 121 theory . In [31], a “semi-analytical isogeometric analysis” approach named as SIGA, was used for
 122 studying two-dimensional Rayleigh waves in layered composite piezoelectric structures. However,
 123 the term SIGA also widely stands for the so-called Stochastic Isogeometric Analysis and hence, we
 124 would like to not use this term to avoid confusion.

125 The paper is organized as follows. Section 2 describes the governing equations for three-
 126 dimensional waveguides coupled with fluid. After introducing the concept of isogeometric analysis
 127 and NURBS basis functions, Section 3 formulates the SAIGA method. Section 4 subsequently
 128 carries out the numerical dispersion analysis through several numerical examples including the free
 129 waveguide, the waveguides coupled with interior and/or exterior fluids. The convergence analysis of
 130 the phase velocities will be studied for these cases. The mode shapes and their continuities will be
 131 carefully investigated for each case. The last section ends with some conclusions and perspectives.

132 2. Problem formulation

133 2.1. Governing equation

134 *Geometry description.* The geometry of an immersed waveguides with arbitrary cross-section is
 135 shown in Fig. 1. The structure is described in the Cartesian coordinate system with an orthogonal
 136 basis $(\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3)$ and the position vector $\mathbf{x} = (x_1, x_2, x_3)$. The cross section of the solid is constant
 137 along \mathbf{e}_3 . The surfaces of the solid body may be free or loaded by two inner and outer fluids (Ω_1^f and
 138 Ω_2^f as shown in Fig. 1). The domains occupied by the solid body and the fluids are denoted by $\Omega^s =$
 139 $\{(x_1, x_2, x_3) | x_3 \in [-\infty, +\infty], (x_1, x_2) \in \bar{\Omega}^s\}$, $\Omega_1^f = \{(x_1, x_2, x_3) | x_3 \in [-\infty, +\infty], (x_1, x_2) \in \bar{\Omega}_1^f\}$ and
 140 $\Omega_2^f = \{(x_1, x_2, x_3) | x_3 \in [-\infty, +\infty], (x_1, x_2) \in \bar{\Omega}_2^f\}$, where $\bar{\Omega}^s$, $\bar{\Omega}_1^f$ and $\bar{\Omega}_2^f$ are the cross-section
 141 of the solid and two fluid domains. The interfaces between Ω^s and Ω_α^f ($\alpha = \{1, 2\}$) are denoted
 142 by Γ_α^{sf} ($\alpha = \{1, 2\}$). As the cross sections are unchanged along \mathbf{e}_3 , the outward directed, normal
 143 vectors of Ω^s at Γ_α^{sf} are always perpendicular to \mathbf{e}_3 and may be represented by $\mathbf{n}^s = \{n_1, n_2, 0\}^T$.
 144 In what follows, the symbol $\partial_i(\star)$ ($i = 1, 2, 3$) stands for the partial derivative of (\star) with respect
 to x_i .

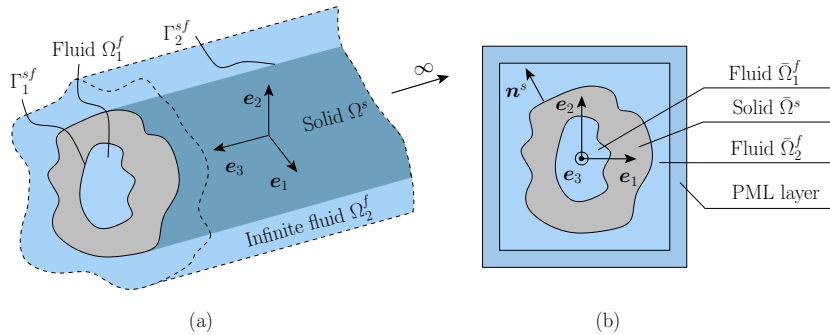


Figure 1: Schematic of (a) fluid-filled solid waveguide with arbitrary cross-section immersed in an infinite fluid (b) cross-section with the PML layer

146 *Dynamic equations in the solid layer.* In the solid domain Ω^s , the infinitesimal displacement vec-
 147 tor at a point \mathbf{x} and at time t is denoted by $\mathbf{u}^s(\mathbf{x}, t) = \{u_1, u_2, u_3\}^T$. For the purpose of conve-
 148 nience, here we use the Voigt notation which represents the stress and strain under the vectorial
 149 form as follows $\mathbf{s}(\mathbf{x}, t) = \{\sigma_{11}, \sigma_{22}, \sigma_{33}, \sigma_{23}, \sigma_{13}, \sigma_{12}\}^T$ and $\mathbf{e}(\mathbf{x}, t) = \{\varepsilon_{11}, \varepsilon_{22}, \varepsilon_{33}, 2\varepsilon_{23}, 2\varepsilon_{13}, 2\varepsilon_{12}\}^T$,
 150 respectively. As $\varepsilon_{ij} = \frac{1}{2}(\partial_i u_j + \partial_j u_i)$, the strain vector \mathbf{e} can be expressed by: $\mathbf{e} = \mathbb{L}\mathbf{u}^s$,
 151 where $\mathbb{L} = \mathbf{L}_1\partial_1 + \mathbf{L}_2\partial_2 + \mathbf{L}_3\partial_3$. The matrices \mathbf{L}_1 , \mathbf{L}_2 and \mathbf{L}_3 are 3×2 matrices of which the
 152 nonzero entries are: $\mathbf{L}_1(1, 1) = \mathbf{L}_1(5, 3) = \mathbf{L}_1(6, 2) = 1$, $\mathbf{L}_2(2, 2) = \mathbf{L}_2(4, 3) = \mathbf{L}_2(6, 1) = 1$ and
 153 $\mathbf{L}_3(3, 3) = \mathbf{L}_3(4, 2) = \mathbf{L}_3(5, 1) = 1$, respectively.

The balance equations of linear momentum at a point $\mathbf{x} \in \Omega^s$ and the linear elastic constitutive law read

$$\rho\ddot{\mathbf{u}}^s - \mathbb{L}^T \mathbf{s} = 0, \quad (1)$$

$$\mathbf{s} = \mathbf{C}\mathbf{e}, \quad (2)$$

154 where ρ is the mass density, $\mathbf{C}_{6 \times 6}$ is the matrix containing the components of the anisotropic elas-
 155 ticity tensor. In this problem, the structure is assumed to be homogeneous along the longitudinal
 156 direction \mathbf{e}_3 but it could be heterogeneous in the section $\bar{\Omega}^s$, i.e. $\rho = \rho(x_1, x_2)$ and $\mathbf{C} = \mathbf{C}(x_1, x_2)$.

Wave equations in the fluids. In the fluid domains Ω_α^f ($\alpha = 1, 2$), the linearized wave equations can be expressed as

$$\rho_\alpha^f \ddot{p}_\alpha - K_\alpha^f \nabla^2 p_\alpha = 0, \quad (3)$$

157 where p_α are the acoustic pressure fields in Ω_α^f , K_α^f and ρ_α^f are the bulk modulus at rest and the
 158 mass density at rest of the Ω_α^f , respectively; $\nabla^2(\star)$ is the Laplace operator. The wave celerity in
 159 Ω_α^f can be defined as $c_\alpha^f = \sqrt{K_\alpha^f / \rho_\alpha^f}$.

Boundary and interface conditions. The boundary conditions of this system (Eqs. 1 & 3) consist of the continuity conditions of the traction and of the normal displacement at the solid-fluid interfaces $\bar{\Gamma}_\alpha^{sf}$ ($\alpha = \{1, 2\}$), and the radiation condition at infinity, i.e.

$$\left. \begin{aligned} \mathbf{t} &= -p_\alpha \mathbf{n}^s \\ \mathbf{u}^s \cdot \mathbf{n}^s &= \mathbf{u}_\alpha^f \cdot \mathbf{n}^s \end{aligned} \right\} \quad \forall \mathbf{x} \in \bar{\Gamma}_\alpha^{sf}, \quad (4)$$

$$p_\alpha \rightarrow 0 \quad \text{when } |\mathbf{x}| \rightarrow \infty, \quad (5)$$

where \mathbf{n}^s is the outward unit vector at the interfaces (Fig. 1), which is opposed to the one of the fluid domain: $\mathbf{n}^s = -\mathbf{n}^f$. It is worth noting that $\mathbf{t} = \boldsymbol{\sigma} \cdot \mathbf{n}^s = (n_1 \mathbf{L}_1^T + n_2 \mathbf{L}_2^T) \mathbf{s}$ and the fluid displacement may be calculated from the pressure field by using the Euler's equation:

$$\rho_\alpha^f \ddot{\mathbf{u}}_\alpha^f + \nabla p_\alpha = 0. \quad (6)$$

160 2.2. Perfectly Matched Layer (PML)

In order to introduce the behavior of the infinite exterior fluid domain, we used the perfectly matched layers (PML) in the cross-section plane, following the procedures proposed in [16, 9].

From a mathematical point of view, the PML can be considered as a result of a mapping into complex coordinates, where the solutions of wave equations decay exponentially [32]. Therefore, the infinite medium can be truncated into a finite domain as shown in Fig. 1. The new stretched coordinates $\tilde{x}_1(x_1)$, $\tilde{x}_2(x_2)$ in the waveguide are defined as

$$\tilde{x}_1(x_1) = \int_0^{x_1} \gamma_1(x_1) dx_1, \quad \tilde{x}_2(x_2) = \int_0^{x_2} \gamma_2(x_2) dx_2, \quad (7)$$

where $\gamma_1(x_1)$ and $\gamma_2(x_2)$ are called PML functions, which satisfy:

$$\gamma_j(x_j) = 1 \text{ for } |x_j| \leq d_j \text{ and } \text{Im}\{\gamma_j(x_j)\} > 0 \text{ for } |x_j| > d_j, \quad \text{for } j = \{1, 2\}, \quad (8)$$

On the exterior boundary of the PML, the boundary condition can be arbitrarily chosen (Dirichlet or Neumann type). The absorption efficiency of leaky waves in the PML strongly depends on the choice of the PML function (γ_1, γ_2), the position of the interfaces (d_1, d_2) and the thickness ($h_1^{\text{pml}}, h_2^{\text{pml}}$) in the \mathbf{e}_1 and \mathbf{e}_2 directions, respectively. There are a number of variants to choose for the γ_1 and γ_2 functions. For this study, a continuous parabolic function for both the real and imaginary parts of the PML function, which has been demonstrated to be efficient in the frequency domain [9, 16], was used:

$$\gamma_j(x_j) = \begin{cases} 1 & \text{if } |x_j| \leq d_j, \\ 1 + \hat{\gamma}_j \left(\frac{|x_j| - d_j}{h_j^{\text{pml}}} \right)^2 & \text{if } |x_j| > d_j, \end{cases} \quad \text{for } j = \{1, 2\}, \quad (9)$$

where $\hat{\gamma}_j = a_j + ib_j$ quantify the PML absorption and will be given explicitly in each case study. As leaky waves grow exponentially in the transverse directions, placing the PML close to the waveguide can reduce the effect of the exponential growth of the leaky modes. In order to estimate the length of the PML, a simplified 2D plane wave propagation model can be used to approximately predict the length of the PML:

$$h_j^{\text{pml}} \geq \frac{6.9}{k_{\text{leak}} b_j}, \quad \text{for } j = \{1, 2\}, \quad (10)$$

161 where k_{leak} represents the wavenumber of the longitudinal wave in the fluid. When the PML
 162 function is given, b_j can be determined as $\text{Im}(\hat{\gamma}_j)$. The length of the PML can be obtained by
 163 calculating the smallest wavenumber in the frequency range of interest [16].

164 It is worth noting that in the context of the SAFE analysis, several techniques have been
 165 proposed for representing the radial boundary condition in semi-infinite domains. For example,
 166 in [15], the coupled boundary element and finite element method (referred as 2.5D FEM-BEM)
 167 has been used. However, this method leads to a nonlinear eigenvalue problem that requires an
 168 expensive solution strategy. In [33], an iterative procedure was proposed to solve the nonlinear
 169 eigenvalue problem derived from exact radiation condition. For the case of an immersed plate,
 170 the nonlinear eigenvalue problem with exact radiation condition was transformed into a cubic
 171 polynomial eigenvalue problem using a change of variables [34]. However, its extension to 3D cases
 172 is not trivial. Although using a PML within SAFE formulation requires a supplement layer with

173 a thickness about two wavelengths, it has some attractive advantages because (i) it allows us to
 174 avoid the nonlinear term in the exact radiation condition and thus the final eigenvalue problem
 175 established for the coupled fluid-solid system still has the quadratic polynomial form, which may
 176 efficiently be solved (as it will be shown in Sec. 3); (ii) the implementation of PMLs in the SAFE
 177 formulation is straightforward and requires very little modifications.

178 2.3. Weak formulation in the frequency-wavenumber domain

We look for the solution of harmonic waves propagating along the axial direction (\mathbf{e}_3) which may be expressed by the following form

$$\mathbf{u}^s(x_1, x_2, x_3, t) = \tilde{\mathbf{u}}^s(x_1, x_2)e^{i(k_3x_3 - \omega t)}, \quad (11a)$$

$$p_\alpha(x_1, x_2, x_3, t) = \tilde{p}_\alpha(x_1, x_2)e^{i(k_3x_3 - \omega t)}, \quad (11b)$$

179 where $i^2 = -1$; $\omega \in \mathbb{R}$ is the angular frequency; k_3 is the wavenumber in the \mathbf{e}_3 -direction; the
 180 vector $\tilde{\mathbf{u}}^s(x_1, x_2) = (\tilde{u}_1, \tilde{u}_2, \tilde{u}_3)^T$ and $\tilde{p}_\alpha(x_1, x_2) = \tilde{p}_\alpha$ which represents are the amplitudes of the
 181 displacement vector in the $\bar{\Omega}^s$ and of the pressures in $\bar{\Omega}_\alpha^f$, respectively. By applying harmonic forms
 182 (Eqs. 11a,11b), the problem presented in Sec. 2.1 can be transformed to a 2D system of equations
 183 with respect only to x_1 and x_2 (see Appendix A for the detailed development)

Upon integrating Eqs. (A.3)-(A.4) against test function $\delta\tilde{\mathbf{u}}^s$ and $\delta\tilde{p}_\alpha$, respectively, then applying the Gauss theorem and taking into account the interface conditions (A.5), the weak formulation of the boundary value problem in the solid layer $\bar{\Omega}^s$ and in the fluid domain $\bar{\Omega}_\alpha^f$ ($\alpha = \{1, 2\}$) may be derived as in [35]:

$$\begin{aligned} & -\omega^2 \int_{\bar{\Omega}^s} \delta\tilde{\mathbf{u}}^s \cdot \rho\tilde{\mathbf{u}}^s d\bar{\Omega}^s + \int_{\bar{\Omega}} (\mathbf{L}_1\partial_1 + \mathbf{L}_2\partial_2 - ik_3\mathbf{L}_3)\delta\tilde{\mathbf{u}}^s \cdot (\mathbf{C}(\mathbf{L}_1 + \mathbf{L}_2\partial_2 + ik_3\mathbf{L}_3)\tilde{\mathbf{u}}^s) d\bar{\Omega}^s \\ & + \int_{\bar{\Gamma}_\alpha^{sf}} \delta\tilde{\mathbf{u}}^s \cdot (\tilde{p}_\alpha\mathbf{n}^s) d\bar{\Gamma}_\alpha^{sf} = 0, \quad \forall \delta\tilde{\mathbf{u}}^s \in \mathcal{C}^{ad}. \end{aligned} \quad (12a)$$

$$\begin{aligned} & -\omega^2 \int_{\bar{\Omega}_\alpha^f} \delta\tilde{p}_\alpha^* \rho_\alpha^f \tilde{p}_\alpha d\bar{\Omega}_\alpha^f - \omega^2 \int_{\bar{\Gamma}_\alpha^{sf}} \delta\tilde{p}_\alpha^* \rho_\alpha^f K_\alpha^f \tilde{\mathbf{u}}^s \cdot \mathbf{n}^s d\bar{\Gamma}_\alpha^{sf} + k_3^2 \int_{\bar{\Omega}_\alpha^f} \delta\tilde{p}_\alpha K_\alpha^f \tilde{p}_\alpha d\bar{\Omega}_\alpha^f \\ & + \int_{\bar{\Omega}_\alpha^f} \left((\partial_1\delta\tilde{p}_\alpha)^* K_\alpha^f \partial_1\tilde{p}_\alpha + (\partial_2\delta\tilde{p}_\alpha)^* K_\alpha^f \partial_2\tilde{p}_\alpha \right) d\bar{\Omega}_\alpha^f = 0, \quad \forall \delta\tilde{p}_\alpha \in \mathcal{C}^{ad}. \end{aligned} \quad (12b)$$

184 3. NURBS-based isogeometric approximation

185 The formulation presented in Eqs. (12a)-(12b) is valid irrespective of the numerical discretiza-
 186 tion employed in the cross-section. In the framework of conventional SAFE, the Lagrange poly-
 187 nomials are used to discretized these equations. The main drawback of this approach, which is
 188 also called a SAFE method, is the fact that discretization should be fine enough to achieve the
 189 required accuracy. The consequence of fine discretization is a significant increase in computational
 190 cost. Furthermore, the Lagrange PML functions give a non-smoothness profile across the PML
 191 interfaces, which leads to imperfect absorption of the leaky modes. We propose to use a differ-
 192 ent technique for computational efficiency. The idea is based on the NURBS-based isogeometric
 193 analysis allowing the use of globally C^k -continuous basis functions, with $k \leq p - 1$, p being the
 194 polynomial degree. In this section, we briefly recall the concept of isogeometric analysis with the

195 main focus on the B-spline and NURBS basis functions, their properties, their use for the geo-
 196 metrical representation as well as incorporating the interface with the C^0 -continuity. For a more
 197 detailed review of these topics, we refer the interested reader to [36, 37].

198 3.1. Geometrical representation

NURBS are piecewise rational functions of degree p that are connected in so-called knots. Let $\Xi = \{\xi_1, \xi_2, \xi_3, \dots, \xi_{n+p+1}\}$ and $H = \{\eta_1, \eta_2, \eta_3, \dots, \eta_{m+p+1}\}$ be the knot vectors in the bi-dimensional parametric domain $\hat{\Omega}$, each consisting of nondecreasing real numbers ξ_i and η_i , respectively. The bivariate NURBS basis functions are defined by:

$$R_{ij}^{p,q}(\xi, \eta) = \frac{N_{i,p}(\xi)N_{j,q}(\eta)w_{ij}}{\sum_{k=1}^n \sum_{\ell=1}^m N_{k,p}(\xi)N_{\ell,q}(\eta)w_{k\ell}}, \quad (13)$$

where $N_{i,p}$ denotes the i th B-spline basis function of p -degree, $w_{k\ell} \in \mathbb{R}$ is the weight values and n, m are the numbers of basis functions used to construct the B-spline curve in each dimension. By using the well-known Cox-de Boor formula, the B-spline basis functions are defined recursively as:

$$N_{i,0}(\xi) = \begin{cases} 1 & \text{if } \xi_i < \xi < \xi_{i+1}, \\ 0 & \text{otherwise,} \end{cases} \quad (14a)$$

$$N_{i,p}(\xi) = \frac{\xi - \xi_i}{\xi_{i+p} - \xi_i} N_{i,p-1}(\xi) + \frac{\xi_{i+p+1} - \xi}{\xi_{i+p+1} - \xi_{i+1}} N_{i+1,p-1}(\xi). \quad (14b)$$

199 Note that the quotient $0/0$ is assumed to be zero. By introducing a set of control points $\mathbf{P}_{ij} \in \mathbb{R}^2$,
 200 the NURBS surfaces are constructed by:

$$\mathbf{S}(\xi, \eta) = \sum_{i=1}^n \sum_{j=1}^m R_{ij}^{p,q}(\xi, \eta) \mathbf{P}_{ij}. \quad (15)$$

201 3.2. Solution approximation

202 Let v^h denotes the approximation of a function $v(x_1, x_2)$ defined in the physical domain Ω .
 203 According to the isogeometric concept [37], the function v^h may be given by a composition between
 204 a function \hat{v}^h , defined in the parametric domain $\hat{\Omega}$, with the inverse of geometrical mapping:
 205 $v^h = \hat{v}^h \circ \mathbf{x}^{-1}$. The function \hat{v}^h is built over the parametric domain by:

$$\hat{v}^h(\xi, \eta) = \sum_{i=1}^n \sum_{j=1}^m R_{ij}^{p,q}(\xi, \eta) V_{ij}, \quad (16)$$

206 where the coefficients $V_{ij} \in \mathbb{C}$ are the corresponding control variables (values at the control points
 207 \mathbf{P}_{ij}). The properties of the function \hat{v}^h follow those of the NURBS basis functions.

In this study, by using Galerkin's method, the same approximations are applied for both functions $\tilde{\mathbf{u}}^h$ and $\delta \tilde{\mathbf{u}}^h$ (as well as for \tilde{p}_α^h and $\delta \tilde{p}_\alpha^h$) on each patch:

$$\tilde{\mathbf{u}}^h = \mathbf{R}^u \mathbf{U}, \quad \delta \tilde{\mathbf{u}}^h = \mathbf{R}^u \delta \mathbf{U}, \quad (17a)$$

$$\tilde{p}_\alpha^h = \mathbf{R}_\alpha^p \mathbf{P}_\alpha, \quad \delta \tilde{p}_\alpha^h = \mathbf{R}_\alpha^p \delta \mathbf{P}_\alpha, \quad (17b)$$

where \mathbf{R}^u , \mathbf{R}^p are the interpolation matrix containing the NURBS basis functions (Eq. 13); \mathbf{U} and $\delta\mathbf{U}$ are the vectors of control displacements; \mathbf{P}_α and $\delta\mathbf{P}_\alpha$ are the vectors of control pressures. By substituting the approximations (Eqs. 17a-17b) into the weak formulations (Eqs. 12a-12b), then assembling the elementary matrices, one obtains

$$(-\omega^2\mathbf{M} + \mathbf{K}_0 + ik_3\mathbf{K}_1 + k_3^2\mathbf{K}_2)\mathbf{V} = \mathbf{0}, \quad (18)$$

where $\mathbf{V} = (\mathbf{P}_1, \mathbf{U}, \mathbf{P}_2)^T$ containing the global eigenvectors of pressure (\mathbf{P}_1 , \mathbf{P}_2) and of displacement (\mathbf{U}); the global matrices \mathbf{M} , \mathbf{K}_0 , \mathbf{K}_1 , \mathbf{K}_2 are defined by:

$$\mathbf{M} = \begin{bmatrix} \mathbf{M}^{f_1} & \mathbf{M}^{f_1s} & \mathbf{0} \\ \mathbf{0} & \mathbf{M}^s & \mathbf{0} \\ \mathbf{0} & \mathbf{M}^{f_2s} & \mathbf{M}^{f_2} \end{bmatrix}, \quad \mathbf{K}_0 = \begin{bmatrix} \mathbf{K}_0^{f_1} & \mathbf{0} & \mathbf{0} \\ \mathbf{K}_0^{sf_1} & \mathbf{K}_0^s & \mathbf{K}_0^{sf_2} \\ \mathbf{0} & \mathbf{0} & \mathbf{K}_0^{f_2} \end{bmatrix}, \quad (19a)$$

$$\mathbf{K}_1 = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{K}_1^s & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix}, \quad \mathbf{K}_2 = \begin{bmatrix} \mathbf{K}_2^{f_1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{K}_2^s & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{K}_2^{f_2} \end{bmatrix}, \quad (19b)$$

in which the sub-matrices are determined from the assembling of corresponding elementary matrices in solid and fluid domains: the sub-matrices (with superscript s) representing the behavior of the solid domain are defined by:

$$\mathbf{M}^s = \bigcup_e \int_{\bar{\Omega}^s(e)} (\mathbf{R}^u)^T \rho \mathbf{R}^u d\bar{\Omega}^s, \quad (20a)$$

$$\mathbf{K}_0^s = \bigcup_e \int_{\bar{\Omega}^s(e)} \left(\partial_i (\mathbf{R}^u)^T \mathbf{A}_{ij} \partial_j \mathbf{R}^u \right) d\bar{\Omega}^s, \quad \text{for } i, j = \{1, 2\}, \quad (20b)$$

$$\mathbf{K}_1^s = \bigcup_e \int_{\bar{\Omega}^s(e)} \left(-(\mathbf{R}^u)^T \mathbf{A}_{3i} \partial_i \mathbf{R} + \partial_i (\mathbf{R}^u)^T \mathbf{A}_{i3} \mathbf{R}^u \right) d\bar{\Omega}^s, \quad \text{for } i = \{1, 2\}, \quad (20c)$$

$$\mathbf{K}_2^s = \bigcup_e \int_{\bar{\Omega}^s(e)} (\mathbf{R}^u)^T \mathbf{A}_{33} \mathbf{R}^u d\bar{\Omega}^s, \quad (20d)$$

the sub-matrices (with superscript f) representing the behavior of the fluid domains are defined by

$$\mathbf{M}^{f_\alpha} = \bigcup_e \int_{\bar{\Omega}_\alpha^f} (\mathbf{R}^p)^T \rho_\alpha^f \gamma_1 \gamma_2 \mathbf{R}^p d\bar{\Omega}_\alpha^f, \quad (21a)$$

$$\mathbf{K}_0^{f_\alpha} = \bigcup_e \int_{\bar{\Omega}_\alpha^f} \partial_2 (\mathbf{R}^p)^T K_\alpha^f \gamma_1 \gamma_2 \partial_2 \mathbf{R}^p d\bar{\Omega}_\alpha^f, \quad (21b)$$

$$\mathbf{K}_2^{f_\alpha} = \bigcup_e \int_{\bar{\Omega}_\alpha^f} (\mathbf{R}^p)^T K_\alpha^f \gamma_1 \gamma_2 \mathbf{R}^p d\bar{\Omega}_\alpha^f, \quad (21c)$$

and sub-matrices $\mathbf{M}^{f\alpha s}$ and $\mathbf{K}^{sf\alpha}$ representing the coupling operator at fluid-solid interfaces:

$$\mathbf{M}^{f\alpha s} = \int_{\bar{\Gamma}_\alpha^{sf}} (\mathbf{R}^p)^T \rho_\alpha^f K_\alpha^f \mathbf{R}^u d\bar{\Gamma}_\alpha^{sf}, \quad (22a)$$

$$\mathbf{K}^{sf\alpha} = \int_{\bar{\Gamma}_\alpha^{sf}} (\mathbf{R}^u)^T \mathbf{R}^p d\bar{\Gamma}_\alpha^{sf}. \quad (22b)$$

208 The matrices \mathbf{M} , \mathbf{K}_0 , \mathbf{K}_1 , \mathbf{K}_2 are computed by using two-dimensional Gauss–Legendre quadra-
 209 ture formula with $r = p + 1$ quadrature nodes per element along each parametric direction which
 210 has been shown to be efficient [38]. Due to the fact that $\mathbf{A}_{\alpha\beta} = \mathbf{A}_{\beta\alpha}^T$, it can be shown that that \mathbf{M} ,
 211 \mathbf{K}_0 , \mathbf{K}_2 are symmetric while the matrix \mathbf{K}_1 is anti-symmetric. In this paper, where the material
 212 is assumed to be elastic, these matrices are real and constant with respect to ω and k_3 . In the
 213 case where viscoelastic materials are considered, the elasticity tensor \mathbf{C} can be by replaced by a
 214 complex tensor $\mathbf{C}(\omega)$, which depends to the frequency. Then same formulations (Eqs. 20a-22b)
 215 can be used for taking into account viscosity effects. In that case, the matrices \mathbf{K}_0 , \mathbf{K}_1 , \mathbf{K}_2 will
 216 depend on ω , but still independent to k_3 .

217 It is worth to note that the quadratic eigenvalue Eq. (18) can be converted into a generalized
 218 linear eigenvalue problem for the media possessing orthorhombic symmetry (more detail can be
 219 found in [18]).

220 3.3. Dispersion analysis

The system of characteristic equations (18) is an eigenvalue problem which is used to determine
 the relationship between the pulsation ω and the wavenumber k_3 . By noting that all global matrices
 (\mathbf{M} , \mathbf{K}_0 , \mathbf{K}_1 , \mathbf{K}_2) do not depend on k_3 , Eq. (18) is a quadratic eigenvalue problem with respect
 to k_3 and could be solved by reformulating it under following linearized eigenvalue problem:

$$\left(\begin{bmatrix} \mathbf{0} & -\omega^2 \mathbf{M} + \mathbf{K}_0 \\ -\omega^2 \mathbf{M} + \mathbf{K}_0 & i\mathbf{K}_1 \end{bmatrix} - k_3 \begin{bmatrix} -\omega^2 \mathbf{M} + \mathbf{K}_0 & \mathbf{0} \\ \mathbf{0} & -\mathbf{K}_2 \end{bmatrix} \right) \begin{pmatrix} \mathbf{V} \\ k_3 \mathbf{V} \end{pmatrix} = \mathbf{0}. \quad (23)$$

For each value of the angular frequency ω , solving Eq. (23) allows us to determine the eigenvalues
 k_3 and their associated eigenvectors (also called by wave structures), $\mathbf{V}(\omega, k_3)$ of guided modes.
 The frequency-dependent phase velocity (C_{ph}) and the attenuation (att) of a guided mode are
 given by:

$$C_{\text{ph}} = \frac{\omega}{\text{Re}(k_3)} [\text{m.s}^{-1}], \quad \text{att} = \text{Im}(k_3) [\text{Np.m}^{-1}], \quad (24)$$

221 where $\text{Re}()$ and $\text{Im}()$ denote the real and imaginary parts of a complex function.

222 Three kinds of modes can be found for the immersed waveguides: the trapped mode, the
 223 leaky mode and and the radiation mode. As the name indicates, trapped modes propagate in
 224 the waveguide with energy concentrating in the waveguide. Leaky modes propagate along the
 225 waveguide with some energy leaking into the surrounding fluid. Radiation modes resonate mainly
 226 in the fluid domain, and they are of less interest in practical applications [39]. For the studies
 227 presented in this paper, the following filtering condition is applied in post-processing to identify
 228 and remove the radiation modes: $\frac{|E_k^f|}{|E_k|} > \eta$, where E_k^f and E_k are the kinetic energy of the fluid

229 domain and the total kinetic energy of all domains, respectively; η is a user-defined parameter,
 230 identifying the criterion of the model and depends on PML parameters (in this paper, a value
 231 $\eta=0.98$ was used for all examples).

232 4. Numerical results

233 This section presents some numerical examples in order to validate the accuracy of the proposed
 234 SAIGA formulations for the analysis of the dispersion of guided waves in 3D elastic structures.
 235 First, a hollow cylinder in vacuum will be considered (Fig. 2(a)). Second, the wave dispersion
 236 of the waveguides coupled with fluids (inner and outer) will be studied (Fig. 2(b) and Fig. 2(c)).
 237 The PML is applied to absorb the leaky wave in the infinite exterior fluid. Third, the wave
 238 dispersion within a 3D waveguide with a complex section representing an anisotropic cortical bone
 239 will be analyzed (Fig. 2(d)). The validations were done by comparing the solutions obtained using
 240 the proposed SAIGA approach, by the conventional SAFE method and by analytical analysis
 241 (which are only exist for the homogeneous or layered plates and cylinders). In this study, all
 242 analytical solutions were obtained by using the software DISPERSE [40] and the conventional
 243 SAFE solutions by implementing the weak formulations (Sec. 2.3) into the software COMSOL
 244 Multiphysics (COMSOL AB, Stockholm, Sweden), **in which the isoparametric elements were used**
 245 **for the discretization**. In all examples, the order of basis functions in the direction \mathbf{e}_1 and \mathbf{e}_2 are
 246 assumed to be the same and denoted by p .

The convergence analysis was performed for two cases: free hollow cylinder and fluid-filled cylinder. In order to carry out a convergence analysis of the proposed method at a given frequency, we introduced a function err which is estimated as the summation of the relative errors of the numerical solutions of the first m modes:

$$\text{err} = \sqrt{\sum_{i=1}^m \frac{(C_{\text{ph}}^{i,num} - C_{\text{ph}}^{i,ref})^2}{(C_{\text{ph}}^{i,ref})^2}}, \quad (25)$$

247 where $C_{\text{ph}}^{i,num}(f)$ is the phase velocity of i -th mode at a frequency f calculated by using SAFE
 248 or SAIGA, $C_{\text{ph}}^{i,ref}$ is the corresponding reference solution. For the case of a homogeneous cylinder,
 249 the reference solution can be obtained analytically by using Disperse software (homogeneous
 250 cylindrical case). When the section has irregular shapes, the analytical solution doesn't exist, then
 251 the reference solutions were numerically computed by using the conventional SAFE method with
 252 a very fine mesh.

253 4.1. Dispersion of guided-waves in a hollow cylinder

254 Let us first consider a free hollow cylinder, which is a well-known case and has been studied
 255 in many works using analytical or the conventional SAFE methods (see *e.g* [1]). Although the
 256 solutions of dispersed guided-waves in a free hollow cylinders were well-known, this benchmark
 257 example will allow us to validate and show the effectiveness of the proposed SAIGA method in
 258 compared with the conventional SAFE method, especially for the evaluation of the phase velocities
 259 and mode shapes at the high-frequency range.

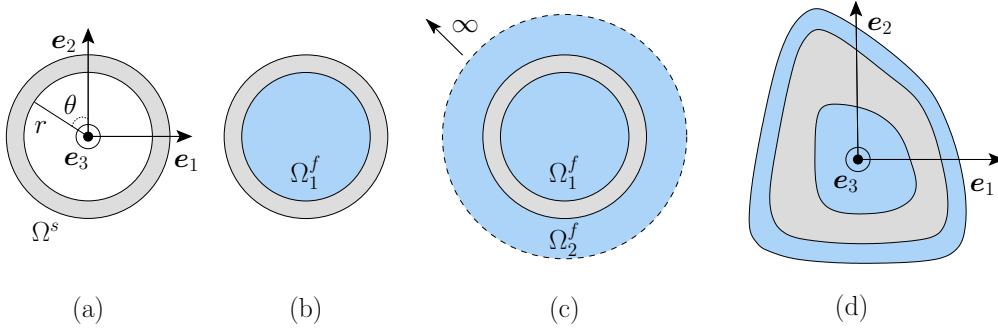


Figure 2: Overview of the studied waveguides: (a) hollow cylinder (without fluid); (b) fluid-filled cylinder; (c) immersed fluid-filled cylinder; and (d) cortical bone with arbitrary cross-section.

260 In this example, the hollow cylinder is made by 2 mm-thickness steel material with the inner
 261 radius of 5 mm Fig. 2(a). The isotropic elastic properties of the steel are characterized by the
 262 density $\rho = 7840 \text{ kg.m}^{-3}$, the longitudinal wave velocity $c_P = 5900 \text{ m/s}$ and the shear wave
 263 velocity $c_S = 3200 \text{ m/s}$. Cubic NURBS basis functions are used for SAIGA and cubic Lagrange
 264 polynomials are used for SAFE analysis, so that the number of degrees of freedom $N_{\text{dof}} = 1080$
 265 of both numerical approaches. When employing SAIGA, four patches are used to represent the
 266 geometry of the annular section. Phase velocities are computed within a frequency range from 0
 267 to 2 MHz.

268 The results of both numerical approaches are shown in Figs. 3 and are compared with the
 269 analytical solution using global matrix method. Fig. 3(a) depicts all propagating modes (of which
 270 k_3 are reals and positives) computed by SAIGA or IGA methods. Among them, three fundamental
 271 modes were interested : axisymmetric longitudinal modes $L(0, n)$, non-axisymmetric flexural modes
 272 $F(1, n)$ and torsional modes $T(0, n)$, where integer n represents the group order of a mode. The
 273 filtering criteria to select these modes are presented in Appendix B. After applying the filtering
 274 procedure to the solutions presented in Fig. 3(a), the mentioned modes may be separated
 275 as shown in Fig. 3(b). One may notice that in the dispersion curves obtained by using SAFE
 276 method, some modes are missed, namely in high frequency range. It is because when the mesh
 277 is not sufficiently fine, the fundamental modes defined by proposed criteria may not be identified
 278 due to numerical errors. On contrary, SAIGA solutions show an very well agreement with the
 279 analytical ones over entire frequency range and for all the considered modes. It means that by
 280 using SAIGA formulation allows to obtain better estimations not only of the eigenvalues but also
 281 of the eigenvectors of the considered system.

282 To quantify the accuracy of the phase velocity computed by SAIGA, we first study the p -
 283 convergence of SAIGA method at two different frequencies: $f = 0.2 \text{ MHz}$ and $f = 2 \text{ MHz}$. **To do**
 284 **so, the section is discretized into 4 patches (with C^0 continuity between the patches) and one single**
 285 **span (element) in each patch (e.g. the knot vectors for $p = 2$ and $p = 3$ are $\Xi = H = \{0, 0, 0, 1, 1, 1\}$**
 286 **and $\Xi = H = \{0, 0, 0, 0, 1, 1, 1, 1\}$, respectively). The p -refinement is then applied in which the order**
 287 **of NURBS basis functions is varied: $p = 2, 3, 4, 6$. The numerical errors of the phase velocities**
 288 **(Eq. (25)) associated to different modes (see Fig. 3(b)) are presented in Table 1. At the low**
 289 frequency $f = 0.2 \text{ MHz}$, the p -convergence could be quickly achieved. For example, using the

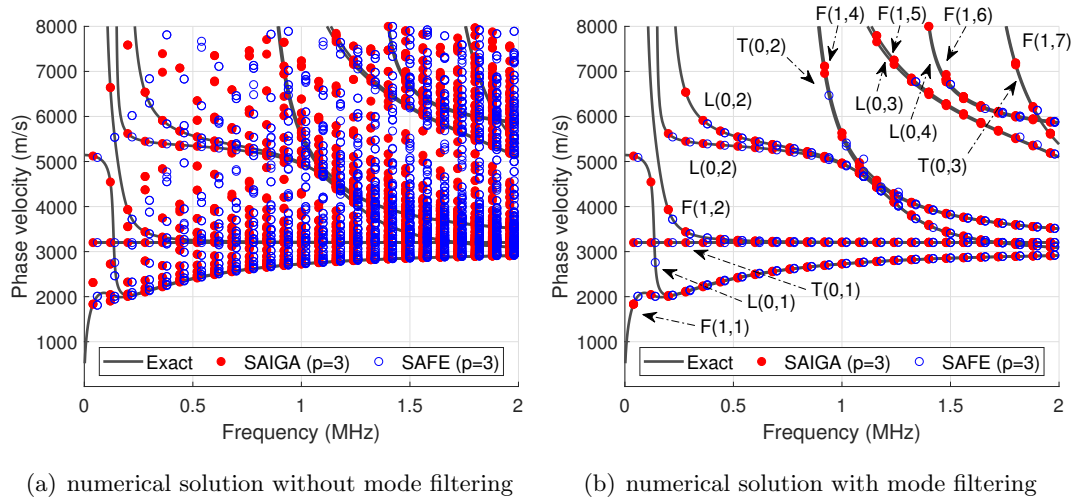


Figure 3: Dispersion curves of a steel cylinder: comparison between SAIGA solution (red marker), SAFE (blue marker) and analytical solutions (grey line); both SAIGA and SAFE used $p = 3$, $N_{\text{dof}}=1080$

Table 1: Case of a homogeneous hollow cylinder: error analysis of SAIGA solutions based on p -refinement (one single element per patch)

Mode	f (MHz)	$C_{\text{ph}}^{\text{Exact}}$ ($\text{m}\cdot\text{s}^{-1}$)	$\text{err}^{\text{SAIGA}}$	$\text{err}^{\text{SAIGA}}$	$\text{err}^{\text{SAIGA}}$	$\text{err}^{\text{SAIGA}}$
			($p=2$, $N_{\text{dof}}=72$)	($p=3$, $N_{\text{dof}}=144$)	($p=4$, $N_{\text{dof}}=240$)	($p=6$, $N_{\text{dof}}=504$)
L(0,1)	0.2	2027.64	0.0082	1.8020×10^{-4}	2.5573×10^{-6}	8.8473×10^{-7}
F(1,1)	0.2	2007.44	0.0156	5.4788×10^{-4}	2.3993×10^{-5}	8.5000×10^{-7}
L(0,1)	2	2910.29	0.0526	0.0241	0.0044	4.7768×10^{-5}
L(0,3)	2	5095.31	0.2002	0.0926	0.0135	1.1250×10^{-4}
L(0,4)	2	5845.90	0.1381	0.0250	0.0068	9.0413×10^{-5}
F(1,1)	2	2912.34	0.0528	0.0241	0.0044	4.7835×10^{-5}
F(1,5)	2	5103.56	0.2021	0.0938	0.0134	1.1138×10^{-4}
F(1,6)	2	5864.83	0.1392	0.0251	0.0069	9.1223×10^{-5}
F(1,7)	2	5403.87	-	-	0.0070	3.7099×10^{-5}
T(0,3)	2	5385.32	-	-	0.0068	3.5097×10^{-5}

290 cubic function ($p = 3$) is sufficient to obtain a good estimation (with errors of order 10^{-4}) of the
291 phase velocities of L(0,1) and F(1,1) modes. At a high frequency ($f = 2$ MHz), the errors are
292 significant and have orders of 10^{-1} and 10^{-2} when using the quadratic and cubic NURBS basis
293 functions, respectively. Eventually, the higher modes F(1,7) and T(0,3) cannot be identified by
294 filtering the numerical solutions of eigenvectors. Using higher-order NURBS basis functions ($p = 4$
295 and $p = 6$) clearly allows to obtain much smaller errors for all modes, showing the p -convergence
296 of NURBS-based formulations.

Table 2: Case of a homogeneous hollow cylinder: error analysis of SAIGA and SAFE solutions based on h -refinement with basis function's order $p=3$

Mode	f (MHz)	$C_{\text{ph}}^{\text{Exact}}$ (m.s ⁻¹)	err ^{SAIGA} ($N_{\text{dof}}=360$)	err ^{SAIGA} ($N_{\text{dof}}=504$)	err ^{SAIGA} ($N_{\text{dof}}=1080$)	err ^{SAFE} ($N_{\text{dof}}=504$)	err ^{SAFE} ($N_{\text{dof}}=1080$)
L(0,1)	0.2	2027.64	7.3118×10^{-7}	1.4290×10^{-7}	5.7945×10^{-9}	3.4599×10^{-5}	2.9142×10^{-6}
F(1,1)	0.2	2912.34	8.8493×10^{-6}	1.5434×10^{-6}	4.3057×10^{-8}	1.5836×10^{-4}	1.8420×10^{-5}
L(0,1)	2	2910.29	0.0016	3.5389×10^{-4}	1.5429×10^{-5}	0.0047	9.0892×10^{-4}
L(0,3)	2	5095.31	0.0046	7.6841×10^{-4}	1.5385×10^{-5}	0.0110	0.0025
L(0,4)	2	5845.90	0.0014	2.8549×10^{-4}	5.7720×10^{-6}	0.0043	3.3196×10^{-4}
F(1,1)	2	2912.34	0.0016	3.5434×10^{-4}	1.5451×10^{-5}	0.0047	9.1307×10^{-4}
F(1,5)	2	5103.56	0.0047	7.7035×10^{-4}	1.5441×10^{-5}	0.0100	0.0054
F(1,6)	2	5864.83	0.0014	2.8804×10^{-4}	5.8241×10^{-6}	0.0043	3.4168×10^{-4}
F(1,7)	2	5403.87	0.0078	8.1188×10^{-4}	1.8269×10^{-5}	0.0171	8.0496×10^{-5}
T(0,3)	2	5385.32	0.0078	8.0536×10^{-4}	1.8147×10^{-5}	0.0197	4.2228×10^{-4}

Table 2 presents the numerical errors of SAIGA and SAFE solutions obtained when applying the h -refinement. The basis function's order was fixed at $p = 3$ for all cases. **Note that when using SAFE with a mesh of $N_{\text{dof}} = 360$, the studied modes could not be identified as the annular geometry cannot correctly be described by few quadratic elements.** As it would be expected, the numerical errors of all modes, computed by SAFE or by SAIGA methods, decrease when N_{dof} increases. At low frequency ($f = 0.2$ MHz), the SAFE results of modes L(0,1) and F(1,1) have good precision when $N_{\text{dof}} = 504$, and the errors significantly decrease when $N_{\text{dof}} = 1080$. The precision of SAIGA solutions is even better: for example, the error of L(0,1) obtained by SAIGA with $N_{\text{dof}} = 360$ is smaller than the one obtained by SAFE with $N_{\text{dof}} = 1080$. At high frequency ($f = 2$ MHz), the h -convergence is achieved much faster when using SAIGA. The errors of SAIGA solution are typically about hundreds of times smaller than the SAFE's ones based on the same N_{dof} . Using SAIGA method with $N_{\text{dof}} = 504$ leads to similar precision than the one computed by using SAFE with $N_{\text{dof}} = 1080$.

In Fig. 4, we present the mode shapes of displacements computed for the mode L(0,1) at the frequency of 2 MHz. To be able to compare to the analytical solution which is derived in cylindrical coordinates system, the normalized displacement components U_r (radial), U_θ (circumferential), U_3 (longitudinal) along the thickness's direction are determined from the eigenvectors \mathbf{U} (see Appendix B). The graphs show that the mode shapes obtained by the proposed SAIGA approach are in very good agreement with the exact solutions. It can also be checked that the SAIGA's solutions of circumferential displacement U_θ have vanished over the cylinder's section, which is consistent with our expectation for a longitudinal mode L(0,1) (Eq. (B.3)). By using the SAFE method with the same numerical parameters ($p = 3$, $N_{\text{dof}} = 1080$), the differences between numerical and exact solutions of mode shapes are visibly much more important. In particular, one may see that the U_θ components estimated by SAFE method are not zeros along the thickness's direction, which does not give a good representation of the axisymmetric feature of the considered mode L(0,1). Though,

322 a good precision of the phase velocity computed by the SAFE method using the same numerical
 323 parameters has been achieved (error = 9.0892×10^{-4} , see Tab. 2). Note that in this example, in
 324 comparison to the SAIGA method, using SAFE method disposes of some disadvantages due not
 325 only to the interpolation function but also to the description of the geometry.

326 To further investigate the efficiency of the proposed SAIGA formulation, we focus on some
 327 specific points in the dispersion curves of the phase velocity (C_{ph}). Fig. 5(a) depicts the numerical
 328 errors of C_{ph} of the mode L(0,1) at $f = 2$ MHz with respect to the total number of DOFs, which
 329 are computed by using Lagrange or NURBS basis functions with different orders (p). Conformity
 330 to the results presented in Tab. 2, it may be checked that the error obtained by employing SAIGA
 331 method is much smaller than the one obtained by the conventional SAFE method which has the
 332 same p and N_{dof} . Using NURBS-based basis function, even with low order ($p = 2, 3$), is shown
 333 to significantly improve the precision of C_{ph} 's numerical results. Moreover, convergence rates of
 334 the SAIGA solutions are much faster than SAFE's ones, especially when using high-order basis
 335 functions, *i.e.* $p = 6$. Fig. 5(b) shows the k -refinement (increasing the polynomial order and
 336 then inserts knots (or elements)) of NURBS and Lagrange basis function at $N_{dof} = 1080$ and
 337 $N_{dof} = 1872$. It is shown that by increasing the order of basis function p , which increase the
 338 continuity of the NURBS basis function across the span, the global slope of the convergence curve
 339 of SAIGA's results is higher than SAFE's one.

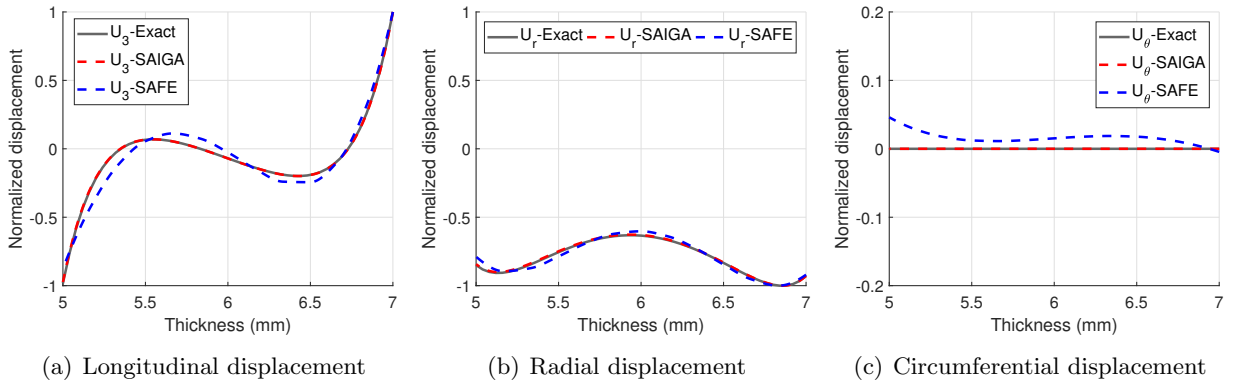


Figure 4: Mode shape of the mode L(0,1) at $f = 2$ MHz of a steel hollow cylinder: comparison between SAIGA solution (red dashed line), SAFE solution (blue dashed line) and analytical solution (grey line); both SAIGA and SAFE solutions are computed with $p = 3$, $N_{dof}=1080$.

340 4.2. Dispersion of guided-waves in fluid-filled cylinder

341 Guided-wave propagation in a steel cylinder filled by water, as shown in Fig. 2(b), is considered
 342 in this example. The steel cylinder has the same geometry and material properties as the one
 343 studied in the previous section. The acoustic properties of water are given by the density $\rho_1^f =$
 344 998 kg.m^{-3} and the sound speed $c_1^f = 1478 \text{ m.s}^{-1}$. The outer surface of the cylinder is assumed to
 345 be free.

346 Fig. 6 depicts illustrations of meshes used for simulations using SAIGA approach. For the
 347 geometrical description using IGA, four patches are used for the solid domain and five patches

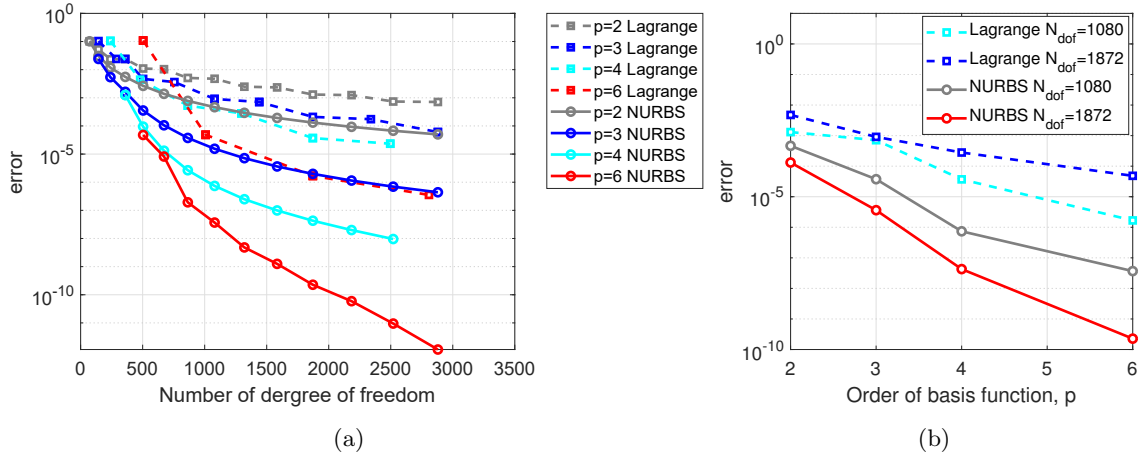


Figure 5: Case of a homogeneous hollow cylinder: (a) relative error of the phase velocity of the mode L(0,1) at $f = 2$ MHz (b) k -refinement at the $N_{\text{dof}}=1080$ and $N_{\text{dof}}=1872$

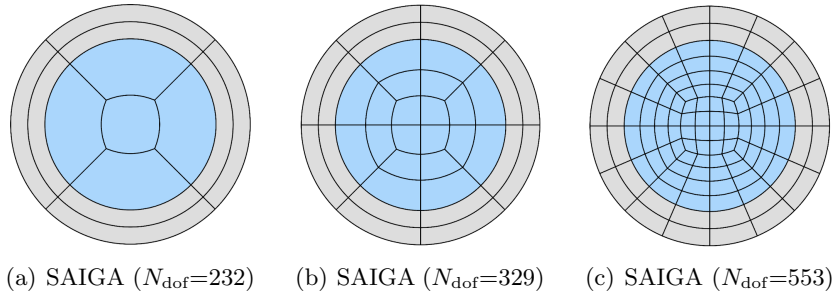


Figure 6: Mesh discretization of a fluid filled cylinder using NURBS and Lagrange basis functions of order $p = 3$

348 are used for the fluid's one (Figs. 6 (a),(b),(c)). It is worth noting that the fluid domain cannot
 349 be meshed by using one single patch because it leads to the singularity of the stress due to the
 350 collinearity of the control points. For the SAFE modeling, both fluid and solid domains are meshed
 351 by using quadrilateral elements.

352 We first validate the proposed FE formulations for calculating C_{ph} 's dispersion curves. Thanks
 353 to the simple cylindrical geometry, the considered fluid-filled hollow cylinder can also be solved by
 354 using the semi-analytical method [41] which was implemented in the Disperse software [40]. Note
 355 that when analyzing a coupled fluid-solid system, solving the dispersion equation may sometimes
 356 have numerical issues due to singularities when establishing transfer matrices. For this example,
 357 DISPERSE software starts to have difficulties to compute analytical solutions at frequencies higher
 358 than 1 MHz, and consequently, some modes are missed. On the contrary, using the finite element
 359 method, which requires higher computational costs, allows to avoid this kind of difficulty.

360 In Fig. 7, we compare the numerical solutions of C_{ph} obtained by SAIGA or SAFE methods
 361 to the analytical ones over the frequency range from 0 to 1 MHz. The discretization is performed

362 by using cubic NURBS or Lagrange basis functions, respectively. The total number of degrees of
363 freedom for both models equals to $N_{\text{dof}}=1321$. A similar filtering procedure as presented in the
364 previous section was applied to extract the modes of interest according to their symmetries. It is
365 shown that the dispersion curves computed by both SAIGA and SAFE methods agree well with
366 the analytical dispersion ones. However, at high frequencies, some of the modes obtained from
367 SAFE solutions couldn't be identified due to imprecise computed shape modes.

368 To quantify the accuracy SAIGA and SAFE methods in guided wave dispersion analysis, the

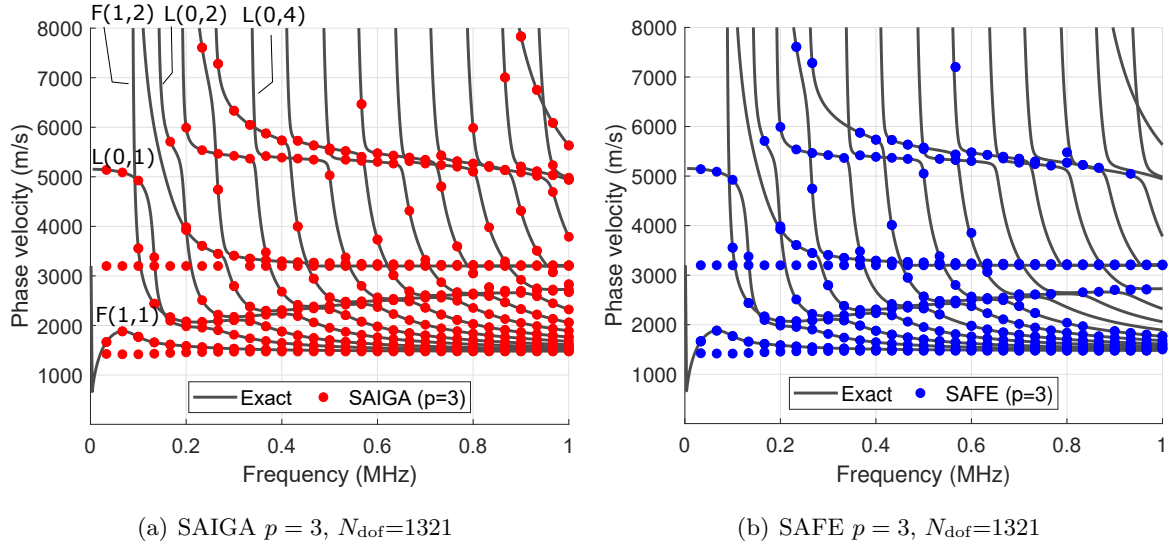


Figure 7: Dispersion curves of a fluid-filled steel cylinder: comparison between the analytical solution (grey line) and: (a) SAIGA solution (red marker), (b) SAFE solution (blue marker)

Table 3: Case of a fluid-filled cylinder: error analysis based on h -refinement of SAIGA and SAFE ($p = 3$)

Mode	f (MHz)	$C_{\text{ph}}^{\text{Exact}}$ (m.s ⁻¹)	err ^{SAIGA} $N_{\text{dof}}=329$	err ^{SAIGA} $N_{\text{dof}}=553$	err ^{SAFE} $N_{\text{dof}}=616$	err ^{SAFE} $N_{\text{dof}}=1321$
L(0,1)	0.2	1983.24	6.4898×10^{-6}	6.3073×10^{-6}	5.7993×10^{-5}	2.542×10^{-6}
L(0,1)	2	1584.89	2.7430×10^{-4}	4.484077×10^{-6}	1.6472×10^{-4}	5.2118×10^{-5}
L(0,2)	0.2	3986.96	1.6823×10^{-4}	3.5105×10^{-6}	0.0076	8.8835×10^{-5}
L(0,2)	2	1497.71	1.2490×10^{-4}	1.0421×10^{-4}	0.0042	7.2485×10^{-5}
L(0,4)	2	1567.14	0.00753	5.2335×10^{-4}	-	0.0016
F(1,1)	0.2	1584.89	1.0500×10^{-5}	4.4841×10^{-6}	1.3547×10^{-5}	3.4830×10^{-5}
F(1,1)	2	1478.05	1.9460×10^{-4}	1.9460×10^{-4}	9.1997×10^{-5}	1.2939×10^{-4}
F(1,2)	0.2	2078.20	5.0153×10^{-5}	7.0398×10^{-6}	3.0751×10^{-4}	3.2647×10^{-6}
F(1,2)	2	1488.92	1.0684×10^{-4}	1.0433×10^{-4}	3.1601×10^{-4}	5.7643×10^{-5}

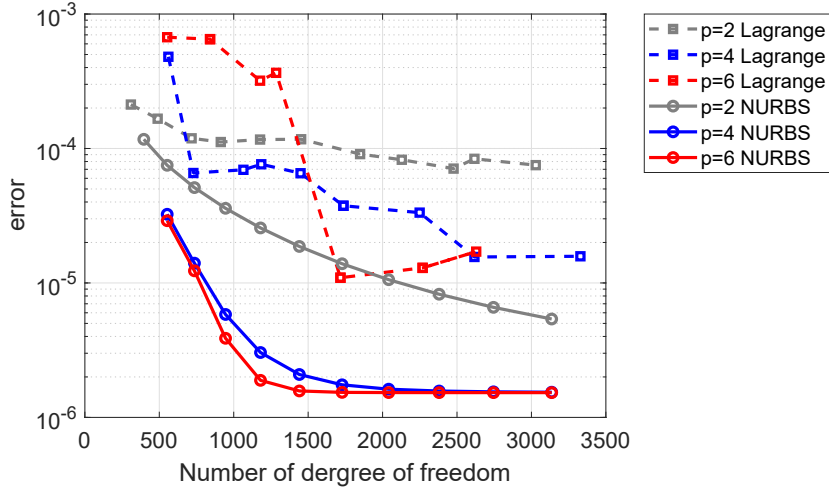


Figure 8: Case of a fluid-filled cylinder: relative error of the phase velocity of the mode L(0,1) at $f = 2$ MHz

369 relative errors of the phase velocity are investigated (Eq. (25)) at two frequencies of 0.2 MHz and
 370 2 MHz. Similar to the previous study on the free hollow cylinder, we perform a h -refinement when
 371 fixing the basis function to be of order 3. Table 3 shows that the SAIGA solutions using $N_{\text{dof}} = 329$
 372 or $N_{\text{dof}} = 553$ have negligible errors at both low and high frequencies. On the contrary, by
 373 employing the conventional SAFE approach with $N_{\text{dof}} = 616$ or $N_{\text{dof}} = 1321$, which are much bigger
 374 than the ones used for SAIGA, the errors have greater values at both low and high frequencies.
 375 The results of a convergence analysis of C_{ph} of the mode L(0,1) at $f = 2$ MHz, in which SAFE
 376 and SAIGA methods are employed by using different orders of basis functions ($p = 2, 4, 6$), are
 377 presented in Fig. 8. It is shown that by using the same basis function order p , the global slope of
 378 the convergence curve of SAFE's results is much lower than SAIGA's one. Moreover, using higher-
 379 order basis functions in SAIGA allows to manifestly reduce the numerical errors. For the SAFE
 380 analysis, while the results using $p = 4$ clearly have better convergence than the ones obtained with
 381 $p = 2$, the advantage of using $p = 6$ is not clearly found in this example. **Note that the asymptotic**
 382 **behavior of the errors when the becomes smaller than 10^{-6} is due to inaccurate reference solution,**
 383 **which was calculated using the Disperse software. In fact, although the Disperse's solutions were**
 384 **computed by using an analytical method, the results were exported in a format with limited digits**
 385 **after decimal point.**

386 Fig. 9 presents the displacement shapes of the mode (L(0,4)) at high frequency $f = 2$ MHz. As
 387 the eigenvectors of the fluid domain derived from (Eq. 23) is defined in terms of pressures, the fluid
 388 displacements shape modes may be determined using Euler's equation: $\mathbf{u}^f = -\frac{1}{\omega^2} \nabla p$. In this figure,
 389 the color isolvalue surface represents the radial displacement component U_r of particles in both fluid
 390 and solid domains which are expected to be axisymmetric. Moreover, the radial displacements at
 391 the solid-fluid interface should theoretically be continuous to verify the interface condition (Eq. 4).
 392 The SAIGA solution of U_r field, computed with $N_{\text{dof}} = 553$ and has a C_{ph} 's error of 0.052%
 393 (see Tab. 3), shown to be axisymmetric and continuous in agreeing with mentioned conditions.
 394 However, the solution of U_r in the fluid domain computed by SAFE method with $N_{\text{dof}} = 1321$ is

395 unlikely axisymmetric (Fig. 9(b)), despite the fact that a correct solution of C_{ph} has been found
 396 (Tab. 3). Moreover, the fluid’s radial displacements, computed by deriving the pressure solutions,
 397 do not conform to tho the solid’s ones at the solid-fluid interface. The expected axisymmetry and
 398 continuity can be found by refining the FE mesh as shown in Fig. 9(c), requiring $N_{\text{dof}} = 1696$ which
 399 is about three times more than the number of DOFs required by SAIGA method ($N_{\text{dof}} = 553$).
 400 Using SAIGA has a greater advantage in this coupled problem thanks to the higher continuity of
 401 NURBS basis functions across the elements.

402 To further investigate the robustness of using SAIGA for mode shape calculation, in Fig. 10,
 403 we present the results of a higher-order mode (F(2,2)) at $f = 0.5$ MHz, computed by using SAIGA
 404 and SAFE approaches with different discretizations as plotted in Figs. 6. The isovalue surface
 405 represents the radial displacement in the solid or fluid domains. Again, the displacement fields
 406 in the fluid domain, which are derived from the pressure eigenvector, can be found to be more
 407 smooth when using SAIGA, even with much fewer elements. The mode shape obtained by using
 408 SAIGA with $N_{\text{dof}} = 329$ seems to have better quality than the one obtained by using SAFE with
 409 $N_{\text{dof}} = 1321$. Figs. 11 depicts the variation of the radial displacement components U_r of both fluid
 410 and solid phases along the solid-fluid interface. It can be checked that while fluid and solid solutions
 411 using SAIGA are perfectly matched from each to other (Fig. 11(b)), significant discontinuities are
 412 obtained when the SAFE method is used (Fig. 11(b)). Note that the errors of the phase velocity
 413 evaluated with SAIGA and SAFE methods are 3.297×10^{-5} and 8.867×10^{-5} , respectively.

414 4.3. Dispersion of guided-waves in a hollow cylinder immersed in fluid

415 Let consider a water-filled steel cylinder immersed in an infinite water domain (Fig. 2(c)),
 416 which has been considered in numerous works [42] to investigate a numerical solution of leaky
 417 wave propagation in fluid immersed structures. The material properties of the steel cylinder and
 418 of the water are the same as the ones presented in the previous section. The infinite water domain
 419 is described by introducing a Perfectly Matched Layer (PML) as described in Section 2.2. The
 420 PML functions in the simulation are chosen as $\gamma_1 = \gamma_2 = 3 + 12i$ as given in [16]. The minimum
 421 thicknesses of the PMLs can be based on Eq. (10), which allows us to choose $h_1^{\text{pml}} = h_2^{\text{pml}} = 3.5$
 422 mm for both PML domains in \mathbf{e}_1 and \mathbf{e}_2 directions.

423 We first perform a validation of the proposed FE formulations for calculating the dispersion
 424 curves of the phase of the considered fluid-filled and immersed cylinder within the frequency range

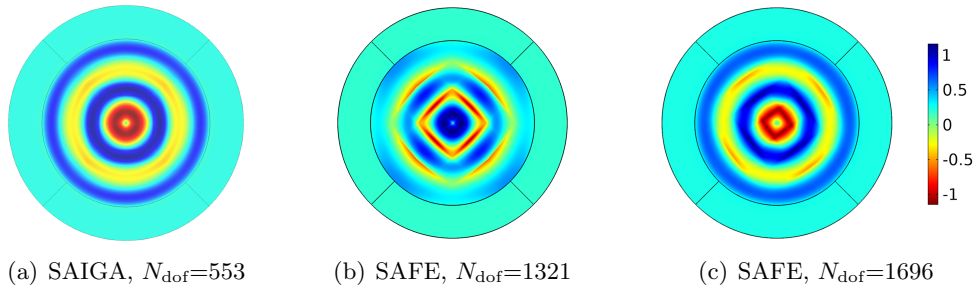


Figure 9: Displacement U_r mode shape for L(0,4) at the frequency $f = 2$ MHz (a) SAIGA with $p = 3$, $N_{\text{dof}} = 553$, (b) SAFE with $p = 3$, $N_{\text{dof}} = 1321$ and (c) SAFE with $p = 3$, $N_{\text{dof}} = 1696$

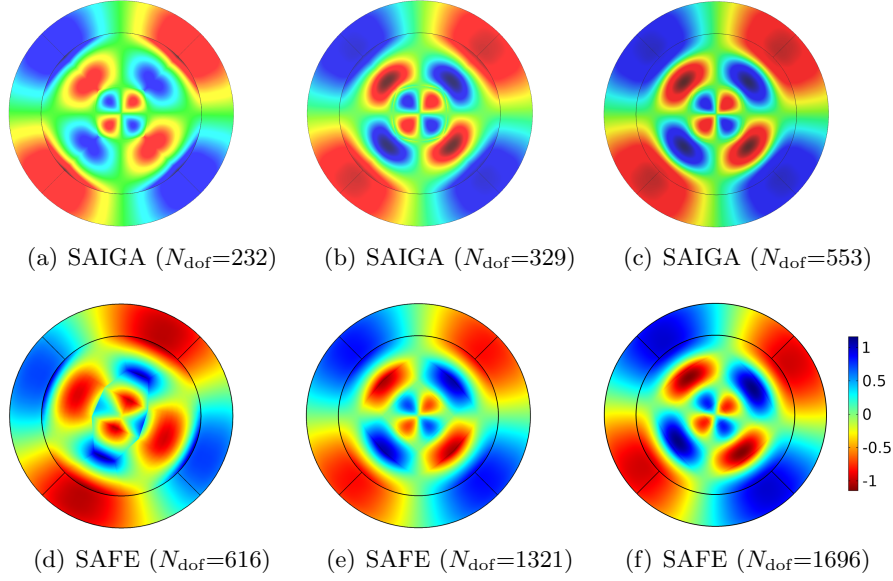


Figure 10: Displacement mode shape (U_r) of a flexural mode (F(2,2)) at low frequency ($f=0.5$ MHz) and $C_{ph}=2452.5$ m/s: (a,b,c) SAIGA with $p = 3$; (d,e,f) SAFE with $p = 3$

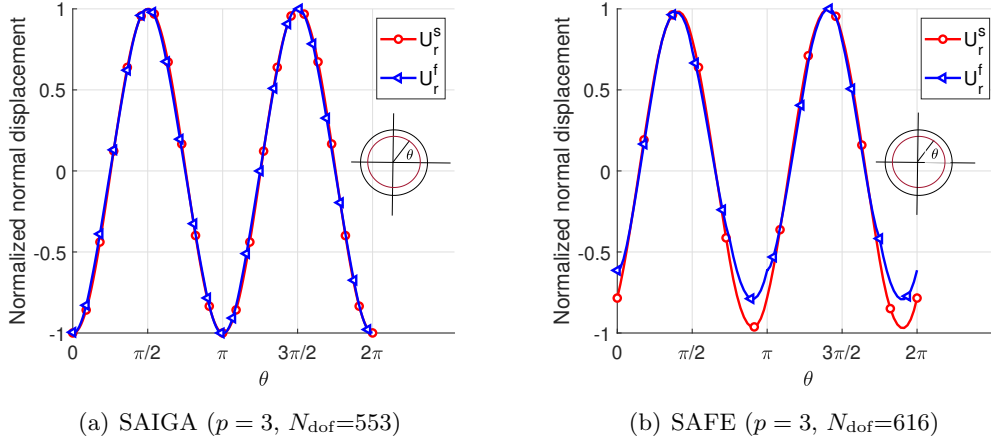


Figure 11: Continuity of displacement for the solid-fluid interfaces of flexural mode (F(2,2)) U_r^s (solid domain) and U_r^f (fluid domain) of fluid-filled cylinder (a) SAIGA solution (b) SAFE solution

425 $f=0-1$ MHz. In Fig. 12, we compare the analytical solutions of the phase velocities over the
 426 frequency range from 0 to 1 MHz with the numerical ones obtained by SAIGA and SAFE methods.
 427 The discretization is performed by using cubic NURBS basis functions. The total number of degrees
 428 of freedom required for the SAIGA and for SAFE equal to $N_{dof}=1508$ and $N_{dof}=1945$, respectively.
 429 A similar filtering procedure as presented in the previous section has been applied to extract the
 430 modes of interest according to their symmetries. It is shown that the dispersion curves obtained
 431 from the proposed approach agree well with the analytical dispersion curves.

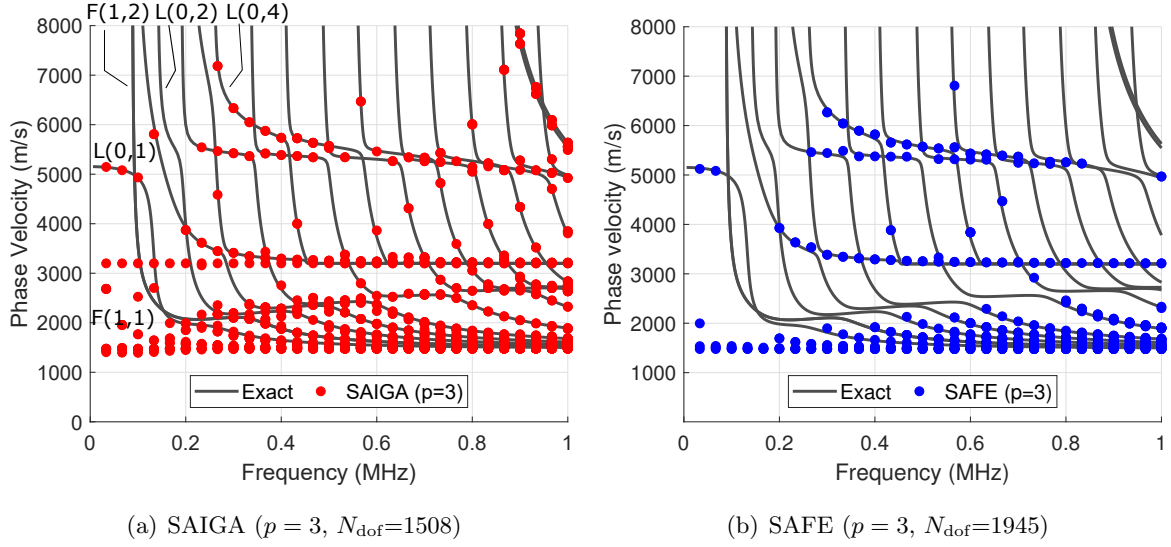


Figure 12: Dispersion curves of a fluid-filled and immersed steel cylinder: comparison between (a) SAIGA solution (red marker), (b) SAFE solution (blue marker) and analytical solution (grey line).

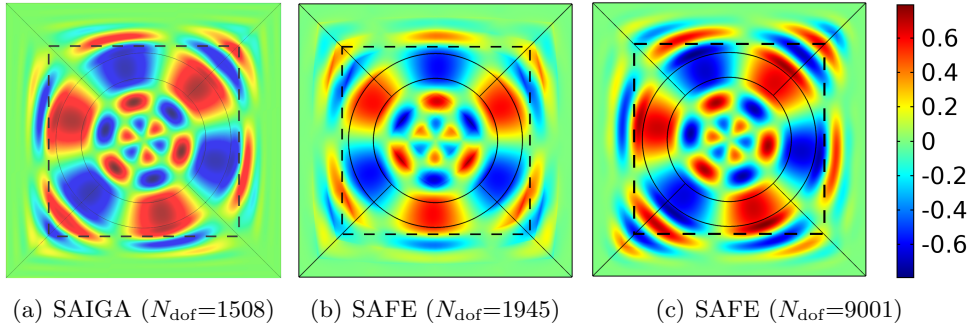


Figure 13: Displacement U_r shape for a flexural mode (F(3,3)) at the frequency of $f=0.5$ MHz and $C_{\text{ph}}=3791$ m/s (a) SAIGA with $p = 3, N_{\text{dof}}=1508$ (b) SAFE with $p = 3, N_{\text{dof}}=1945$ (c) SAFE with $p = 3, N_{\text{dof}}=9001$

432 Mode shapes in terms of radial displacements are presented in Fig. 13. One may observe
 433 that leaky waves are totally damped in a very short distance when they go into the PML. It
 434 has been numerically checked with several larger domains (data not shown) that the solutions of
 435 interests don't depend on the location of the PMLs, showing the PML could efficiently attenuate
 436 leaky waves. As discussed previously, the displacement field in the fluid domains simulated by
 437 using the conventional SAFE method may have some discontinuities, as shown *e.g* in Fig. 13b
 438 ($N_{\text{dof}} = 1945$), due to the smoothness drawback of the Lagrange polynomials. Consequently,
 439 the FE mesh should be extensively refined to obtain an accurate calculation of differentiation
 440 operators (Fig. 13c with $N_{\text{dof}} = 9001$). Using SAIGA approach allows significantly improves the
 441 smoothness of fluid's displacement with much fewer elements (Fig. 13a with $N_{\text{dof}} = 1508$) thanks
 442 to the higher continuity of NURBS basis functions across the elements. Moreover, the conventional

443 SAFE approach suffers from a non-smoothness profile across the PML interfaces, which leads to
 444 imperfect absorption of the leaky modes. Using NURBS leads to better continuity of solid and fluid
 445 normal displacements at the interface, and consequently, it would be preferable to use higher-order
 446 NURBS basis functions for modeling leaky wave in fluid-solid coupled systems.

447 4.4. Anisotropic waveguide with arbitrary cross-section coupling with fluids

448 As a final example, we demonstrate the merits of using SAIGA in the analysis of guided wave
 449 propagation in a more complex geometry structure. Let us consider a waveguide representing a
 450 long cortical bone with real cross-section, which has been reported by many authors using the
 451 conventional SAFE method in the literature [13, 43]. The domain of interests consists of one
 452 transversely isotropic elastic solid sandwiched between two homogeneous fluid domains, which
 453 represents the cortical bone, the marrow (inner fluid) and the soft tissue (outer fluid), respectively.
 454 The cortical has a mean thickness of 7 mm . The inner fluid (bone marrow) has approximately
 455 15 mm-diameter and the outer fluid (soft tissue) has a thickness of 3 mm-thickness (Fig. 2(d))
 456 [44]. The mass density of the cortical bone is given by $\rho = 1722 \text{ kg.m}^{-3}$. The non-zero entries of
 457 the elasticity tensor taken by: $C_{11} = C_{22} = 15.1 \text{ GPa}$, $C_{13} = C_{23} = 8.7 \text{ GPa}$, $C_{33} = 23.5 \text{ GPa}$,
 458 $C_{44} = C_{55} = 4.7 \text{ GPa}$ and $C_{66} = 3.3 \text{ GPa}$ [14]. The bone marrow and soft tissue were modeled
 459 as homogeneous idealized acoustic fluids. The bone marrow is characterized with the density
 460 $\rho_1^f = 930 \text{ kg.m}^{-3}$ and sound speed $c_1^f = 1480 \text{ m.s}^{-1}$, while the characteristics of soft tissue are
 461 given by the density $\rho_2^f = 1043 \text{ kg.m}^{-3}$ and sound speed $c_2^f = 1561 \text{ m.s}^{-1}$ [13].

462 The geometry is built from 13 patches including four patches for the outer fluid domain, four
 463 patches for the bone's section and five patches for the inner fluid domain. Thus, we have several
 464 interfaces with C^0 -continuity between the patches and C^{p-k} -continuity between the elements within
 465 the patches, which is a useful feature in the majority of practical applications. The NURBS
 geometries and positions of control points for these three domain are depicted in Figs. 14(a,b,c).

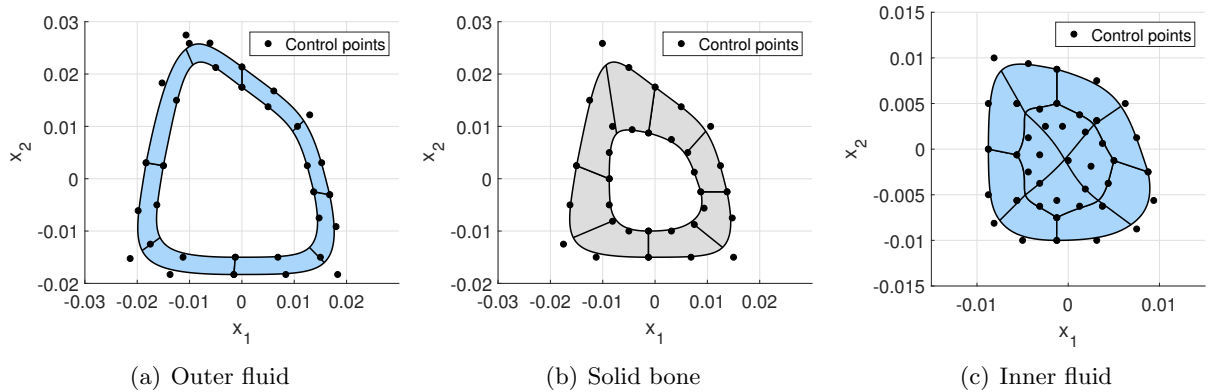


Figure 14: Geometry illustration and position of control points created from NURBS of $p=3$ for (a) outer soft tissue (b) core of cortical bone and (c) inner bone marrow.

466 As the analytical solution doesn't exist for this coupled system, the validation of SAIGA method
 467 was performed by comparing to SAFE solution. Figs. 15 depicts the fine FE mesh of cubic La-
 468 grangian elements ($N_{\text{dof}} = 12929$) used to obtain a "reference" solution. We also present Figs. 15a
 469

470 and 15b two NURBS-based meshes which are built with cubic and eight-order NURBS basis func-
 471 tions, respectively. Note that the numbers of DoFs associated to these meshes are nearly equal,
 472 which are $N_{\text{dof}} = 2683$ and $N_{\text{dof}} = 2689$, respectively. As NURBS allows represent exact curvature
 473 of this geometry, the SAIGA meshes shown in Fig. 15b,c require less number of elements and
 474 are much more regular than the FE mesh with triangular elements (Fig. 15a) which requires fine
 475 refinement at high curvature zones. When using $p = 8$, only one element is needed for each patch.

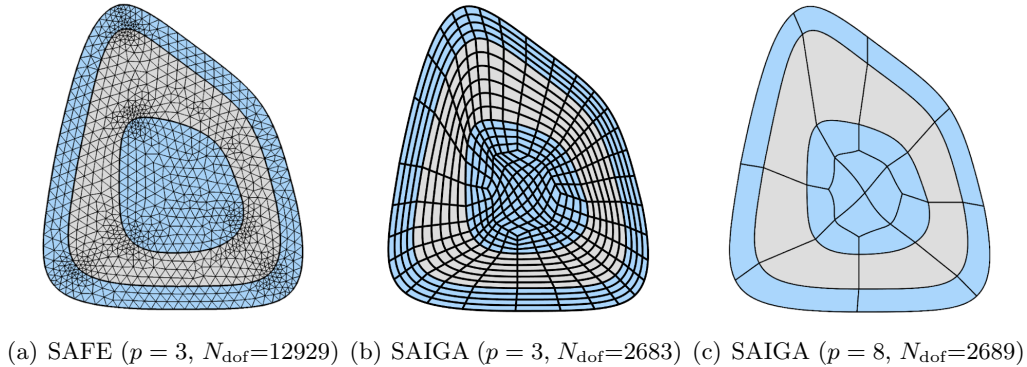


Figure 15: Meshes of a bone's section coupled with marrow and soft tissue

476 Fig. 16 presents the dispersion curves of phase velocity in the frequency range 0-100 kHz,
 477 computed by using SAFE and SAIGA methods. The SAIGA solutions were computed with $p = 3$,
 478 $p = 6$, $p = 8$ and $p = 10$. Similar to the examples previously presented, using SAIGA with all orders
 479 $p = 3, 6, 8, 10$ allows to capture very well the reference solution at low frequencies. Figs. 17(a,b,c)
 480 depict the mode shape in terms of the radial displacement computed by using SAIGA method
 481 with ($p = 3$ or $p = 8$) or using SAFE method ($p = 3$) with very fine discretization. The presented
 482 mode is found at $f = 50$ KHz and $C_{\text{ph}}=1160$ m/s. It may be observed that SAIGA results of the
 483 considered mode shape, which are computed by using $p = 3$ and $p = 8$ and have almost the same
 484 N_{dof} , have both good agreements with the reference solution.

485 The calculation of higher-order modes at high frequency requires more refinement. Hence, we
 486 focus to examine the solutions of the phase velocity in a zoomed window as shown in Fig. 16. It
 487 is clearly seen that the difference between SAIGA solution with $p = 3$ and the reference ones are
 488 significant. The solutions with $p = 6$ are better, yet they still do not match well with the reference
 489 values at some locations. The solutions obtained with $p = 8$ and $p = 10$ are perfectly in agreement
 490 with reference ones, showing that the convergence is archived. Note that while the N_{dof} of the
 491 cases $p = 3, 6, 8$ are quite similar (about 2600 dofs), using $p = 10$ requires a greater value of N_{dof}
 492 ($N_{\text{dof}} = 4249$). Therefore, employing NURBS with $p = 8$, for which only one element per patch is
 493 needed, would be the best choice for this example for calculating the phase velocities.

494 In Fig. 18, we investigated the continuity of solid and fluid displacements at the interfaces
 495 of a mode at $f = 100$ kHz. To do so, the solid and fluid displacement components, which are
 496 normal to the fluid-solid interfaces, are compared. It is worth noting that the SAFE's mesh used
 497 for the simulation has a similar N_{dof} to the SAIGA's one, in which each patch is modeled by only
 498 one eight-order NURBS element. It was shown that while fluid-solid displacement's continuity

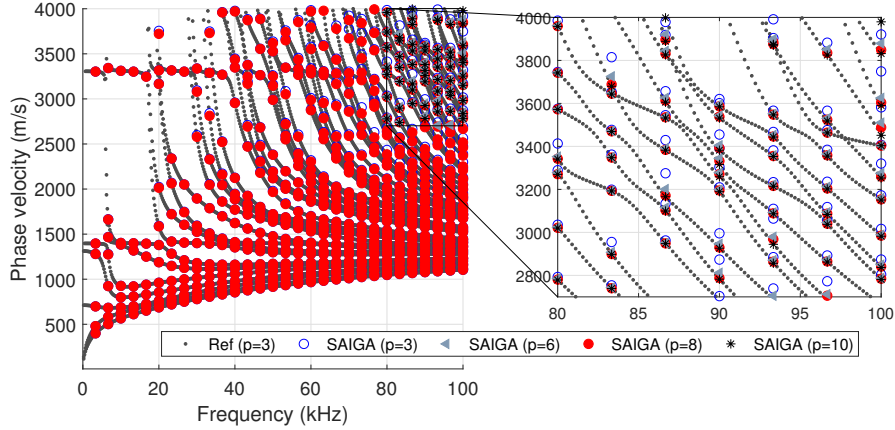
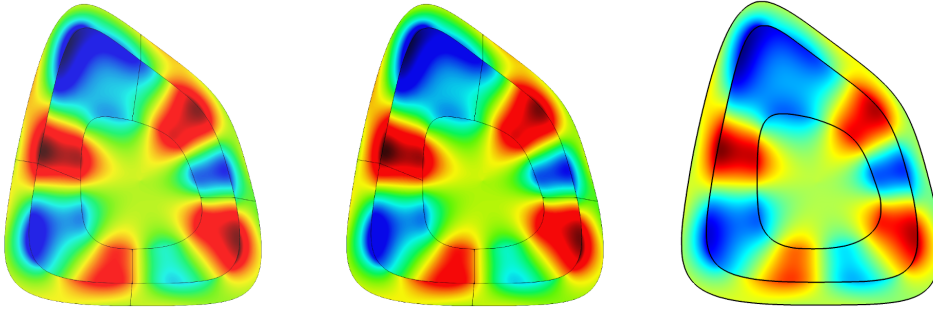


Figure 16: Phase velocity *versus* frequency in a coupled soft tissue-cortical bone system: comparison between the reference SAFE solution (grey marker, $p = 3$, $N_{\text{dof}} = 12929$) and SAIGA solutions (blue marker: $p = 3$, $N_{\text{dof}}=2689$; grey marker: $p=6$, $N_{\text{dof}}=2683$; grey marker: $p = 6$, $N_{\text{dof}}=2689$; black marker: $p = 10$, $N_{\text{dof}}=4249$)



(a) SAIGA ($p = 3$, $N_{\text{dof}}=2683$) (b) SAIGA ($p = 8$, $N_{\text{dof}}=2689$) (c) SAFE ($p = 3$, $N_{\text{dof}}=12929$)

Figure 17: Displacement U_r of flexural mode at the $f = 50$ kHz and $C_{\text{ph}}=1160$ m/s for cortical bone coupled with bone marrow and soft tissue (a) SAIGA with $p = 3$, $N_{\text{dof}}=2683$, (b) SAIGA with $p = 8$, $N_{\text{dof}}=2689$ and (c) SAFE with $p = 3$, $N_{\text{dof}}=12929$.

499 condition may be precisely satisfied by using SAIGA's results, SAFE's solution suffers significant
 500 errors at both interior and exterior interfaces. Furthermore, the errors of the fluid's displacement
 501 seemed to be more significant than the solid's ones. This would be due to the numerical errors of
 502 the fluid pressure's gradient estimation. This comparison confirms again the advantage of using
 503 NURBS for studying the mode shapes in coupled fluid-solid system.

504 5. Conclusion

505 In this paper, a semi-analytical isogeometric analysis (SAIGA) for the anisotropic elastic wave-
 506 guides coupled with fluid was proposed to improve the efficiency of guided wave modeling in com-
 507 parison with the conventional SAFE method. The convergence analysis for isotropic homogeneous

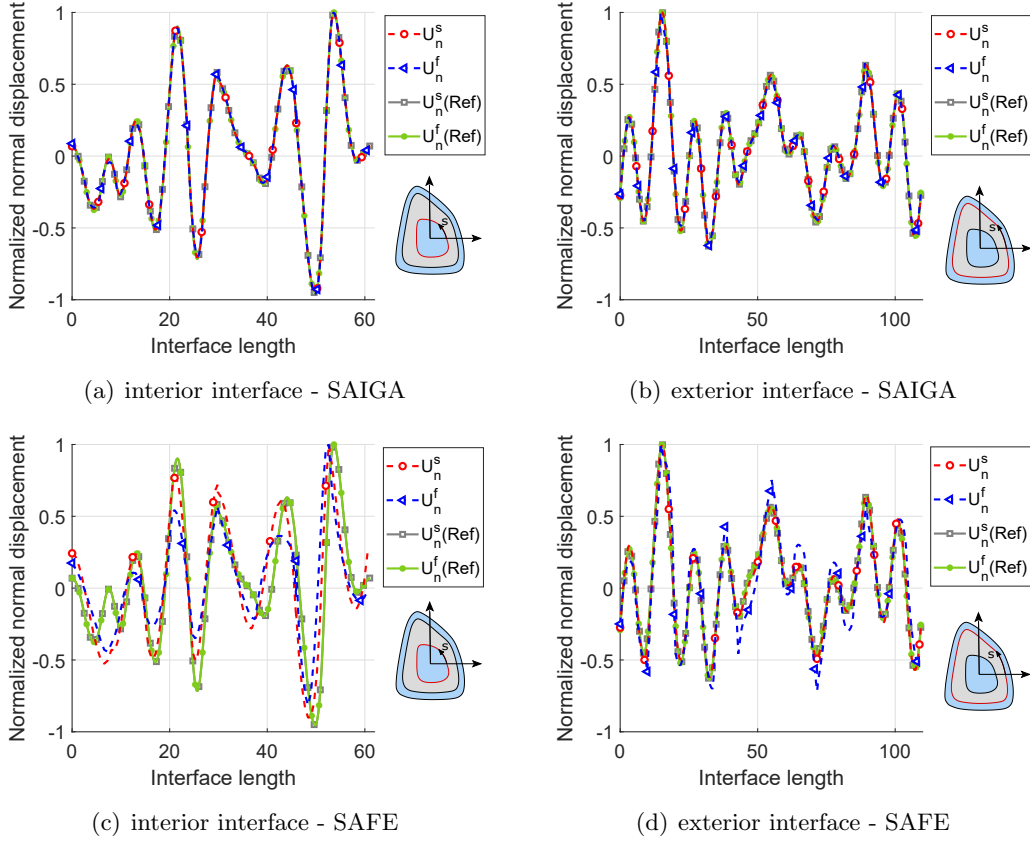


Figure 18: Continuity of displacement for the solid-fluid interfaces of a flexural mode at the $f = 100$ kHz and $C_{ph}=3980$ m/s : U_n^s (solid domain) and U_n^f (fluid domain) of cortical bone (a,b) SAIGA solution ($p = 8$, $N_{dof}=2689$); (c,d) SAFE solution ($p = 3$, $N_{dof}=2675$)

508 hollow cylinder and fluid-filled cylinder showed that increasing the order of NURBS basis function
509 yields a much faster convergence rate in comparison with a similar process using Lagrange poly-
510 nomials. When considering elastic waveguides coupled with fluids, using NURBS basis functions
511 can significantly improve the evaluation of not only dispersed wavenumbers but also mode shapes.
512 In particular, using SAIGA allows to obtain excellent continuity at the solid-fluid interface, which
513 is much more difficult to achieve when using conventional SAFE. Moreover, it has been shown
514 that waveguides immersed in an infinite fluid may efficiently be modeled by introducing perfectly
515 matched layers (PML) and then employing SAIGA procedure.

516 In terms of computational time, our numerical experiences showed that using SAGA allows to
517 reduce significantly the computational time to archive a similar precision. It was also observed
518 that using the same number of DOFs, SAFE and SAIGA required similar computational times.
519 However, a rigorous comparison of computational times requires that both methods should be
520 implemented in a same environment. Note that in this work, while the SAFE simulation was
521 performed in Comsol Multiphysics, the SAIGA method was implemented in an in-house Matlab
522 code.

523 For wave propagation simulation using FEM, it is well-known that the discretization needs to
 524 ensure a number of elements which is sufficient not only for simulating interested wavelengths, but
 525 also for correctly describing the curved boundaries. The advantage of SAIGA in this context, be-
 526 sides its better continuity feature as mentioned before, is that using NURBS allows can representing
 527 exact curvature, hence it does not require as much number of elements as the one required by the
 528 conventional SAFE method for describing the curved interface between fluid and solid domains.
 529 The proposed SAIGA procedure was shown to be particularly interesting for studying arbitrary
 530 cross-section waveguides in terms of computational cost as well as of accuracy.

531 Extensions of the proposed SAIGA formulation may be developed for studying guided-wave
 532 propagation in poroelastic [45] or second-gradient media [46], for which the high-order continuity
 533 of NURBS would be interesting to improve the simulation efficiency.

534 Appendix A. Equations in the frequency-wavenumber domain

Noting that in the frequency-wavenumber $(\omega - k_3)$ domain, the time derivative and the spatial
 derivative with respect to x_1 can be replaced by: $(\dot{\star}) \rightarrow -i\omega(\star)$ and $\partial_1(\star) \rightarrow ik_3(\star)$, respectively,
 then the amplitudes of the strain and stress vectors ($\tilde{\mathbf{e}}$ and $\tilde{\mathbf{s}}$) are expressed by:

$$\tilde{\mathbf{e}} = (\mathbf{L}_1\partial_1 + \mathbf{L}_2\partial_2 + ik_3\mathbf{L}_3)\tilde{\mathbf{u}}^s, \quad (\text{A.1})$$

$$\tilde{\mathbf{s}} = \mathbf{C}\tilde{\mathbf{e}}, \quad (\text{A.2})$$

and the balance equation Eq. (1) can be reformulated as a 2D system of equations with respect to
 (x_2, x_3) :

$$-\rho\omega^2\tilde{\mathbf{u}}^s - (\mathbf{L}_1^T\partial_1 + \mathbf{L}_2^T\partial_2 + ik_3\mathbf{L}_3^T)\mathbf{C}(\mathbf{L}_1\partial_1 + \mathbf{L}_2\partial_2 + ik_3\mathbf{L}_3)\tilde{\mathbf{u}}^s = 0, \quad \forall \tilde{\mathbf{x}} \in \bar{\Omega}^s. \quad (\text{A.3})$$

Similarly, by substituting Eq. (11b) into Eq. (3), the wave equations in the fluid domains are
 simplified into a two-dimensional problem:

$$(-\rho_\alpha^f\omega^2 + k_3^2K_\alpha^f)\tilde{p}_\alpha - K_\alpha^f(\partial_1^2 + \partial_2^2)\tilde{p}_\alpha = 0, \quad \forall \tilde{\mathbf{x}} \in \bar{\Omega}_\alpha^f. \quad (\text{A.4})$$

The boundary conditions in Eqs. (4)-(5) reads:

$$\left. \begin{aligned} \tilde{\mathbf{u}}^s \cdot \mathbf{n}^s &= \frac{1}{\rho_\alpha^f\omega^2}(n_1\partial_1 + n_2\partial_2)\tilde{p}_\alpha \\ \tilde{\mathbf{t}} &= -\tilde{p}_\alpha\mathbf{n}^s \end{aligned} \right\} \quad \forall \tilde{\mathbf{x}} \in \bar{\Gamma}_\alpha^{sf} (\alpha = \{1, 2\}), \quad (\text{A.5})$$

$$\tilde{p}_\alpha \rightarrow 0 \quad \text{when } |\tilde{\mathbf{x}}| \rightarrow \infty, \quad (\text{A.6})$$

535 where $\tilde{\mathbf{t}} = (n_1\mathbf{L}_1^T + n_2\mathbf{L}_2^T)\tilde{\mathbf{s}}$.

536 Appendix B. Mode filtering for cylindrical waveguides

In hollow cylinders, there exist several guided waves denoted by longitudinal modes $L(m, n)$,
 torsional modes $T(m, n)$ and flexural modes $F(m, n)$ where the integer m denotes the circumferen-
 tial order of a mode and the integer n represents the group order of a mode [1]. The guided waves

contain axisymmetric modes ($m = 0$) and non-axisymmetric modes (also known as flexural modes). A modal filtering must be processed to identify and separate two fundamental modes: axisymmetric longitudinal modes $L(0, n)$ and non-axisymmetric flexural modes $F(1, n)$. In order to apply a filtering criteria, the Cartesian displacement solution $\mathbf{U} = \{U_1, U_2, U_3\}^T$ must be converted into the cylindrical coordinates solution $\mathbf{U} = \{U_r, U_\theta, U_3\}^T$ as follow:

$$U_r = U_1 \cos(\theta) + U_2 \sin(\theta), \quad U_\theta = -U_1 \sin(\theta) + U_2 \cos(\theta), \quad (\text{B.1})$$

where $\theta = \arctan(\frac{x_2}{x_1})$. For modes with $m = 0$, the derivatives of the displacement components with respect to θ vanish [47]

$$\partial_\theta U_r = \partial_\theta U_\theta = \partial_\theta U_3 = 0, \quad (\text{B.2})$$

additionally, the longitudinal waves have dominant particle motions in either the r and/or \mathbf{e}_3 direction which means :

$$U_\theta = 0. \quad (\text{B.3})$$

For modes with $m = 1$, the filtering criteria can be written as:

$$\partial_\theta U_r = \partial_\theta U_\theta \text{ and } \partial_\theta U_3 = 0 \quad (\text{B.4})$$

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